Potential-model predictions for the Υ particles

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We discuss predictions for the energy spectrum, leptonic width, and some representative $E1$ radiative transitions specific to $Q\bar{Q}$ bound states of quark mass $M_Q = 5.4$ GeV within the context of a plausible universal potential model.

In a recent paper,[1] hereafter referred to as I, we presented a model of meson masses which incorporated both the high-momentum asymptotically free behavior of quantum chromodynamics and the known long-distance Regge behavior of mesons into a consistent, universal potential model for meson binding. In I the potential parameters were fitted to the low-lying and charmed mesons, and the qualitative spectrum for heavy-quark states was discussed. The recent discovery[2] in proton-nucleus collisions at the Fermi National Accelerator Laboratory of narrow resonances at 9.44 and 10.17 GeV (the Υ particles) has renewed interest in specific potential-model predictions. In this paper we would like to report the detailed predictions of our model for the Υ system.

After the potential parameters are fitted to the low-lying and charmed-meson states (see I), only the heavy quark mass and charge are free to vary to fit the Υ system. In I the ground-state energy of 9.44 GeV determines the quark mass to be 5.4 GeV. Then the spectrum of states is completely determined. This detailed spectrum is given in Table I. The first, second, and possibly third s-wave radial excitations are predicted to lie at 600, 930, and 1210 MeV above the ground state. The $D^0\bar{D}^0$ threshold lies at ~10.76 GeV. The hyperfine and fine structure are calculated by adopting the modified DGG philosophy of paper I and assuming a generalized Fermi-Breit Hamiltonian for the form of the interaction. This gives hyperfine splittings between the $^1S_0$ and $^3S_1$ s-wave states of 90, 40, 30, and 20 MeV for the ground state and three radial excitations. The center of gravity of the p-, d-, and f-wave states in Table I should only be taken as estimates in that we neglect throughout the orbital-angular-momentum-dependent dyadic term in the generalized Fermi-Breit Hamiltonian.

The leptonic decay widths and radiative decays are sensitive to the new quark charge. We present values for quark charge $+\frac{2}{3}$ for definiteness. If the quark charge e_Q/e is $-\frac{1}{3}$, the values given here are reduced by a factor of 4. The leptonic decay widths are calculated using

$$\Gamma = 16\pi\alpha^2\left(\frac{e_Q}{e}\right)^2 \frac{[\psi(0)]^2}{M_V^2}\,, \tag{1}$$

where $\alpha = \frac{1}{137}$ is the fine-structure constant, M_V is the mass of the vector meson, and $[\psi(0)]^2$ is the nonrelativistic wave function at the origin squared. For the ground state and three radial excitations of the Υ system, we find values of 0.76, 0.34, 0.26, and 0.23 for the wave function at the origin squared, yielding leptonic widths of 10, 4, 3, and 2 keV, respectively. It has been demonstrated,[3] however, that the wave function at the origin is sensitive to higher-order relativistic corrections. For ψ and ψ' with $m_Q = 1.98$ GeV, hard-core effects induced by direct integration of the Fermi-Breit Hamiltonian reduce the naive value by a factor of 2.[1,3] For heavier quarks we expect that the relativistic corrections are less important and thus the above lowest-order calculations should overestimate the experimental value for low radial excitations by no more than that in charmonium, i.e., ~a factor of 2. For even heavier systems, $m_Q = 10$ GeV for instance, the relativistic corrections should be negligible making the lowest-order result, in principle, a good test of the model.

We calculate some representative $E1$ radiative decays for the heavy-quark system by using

TABLE I. Mass spectrum for heavy-quark states with $M_Q = 5.4$ GeV. (All masses are in GeV units; N is the principal quantum number; l and J are the orbital and total angular momentum, respectively.)

N/l	0	1 (J)	2 (J)	3 (J)
1	9.42			
2	10.02	9.81 (0)		
		9.86 (1)		
		9.90 (2)		
3	10.35	10.21 (0)	10.12 (1)	
		10.24 (1)	10.14 (2)	
		10.27 (2)	10.17 (3)	
4	10.64	10.51 (0)	10.42 (1)	10.33 (2)
		10.54 (1)	10.44 (2)	10.35 (3)
		10.57 (2)	10.47 (3)	10.37 (4)

TABLE II. Representative $E1$ radiative decays.

Process	E_1 (GeV)	E_2 (GeV)	ω (GeV)	$(R_{nl}^{n'l'})^2$	Γ (keV)
$\Upsilon' \to {}^3P_0 + \gamma$	10.02	9.81	0.21	2.4	11
$\Upsilon' \to {}^3P_1 + \gamma$	10.02	9.86	0.16	2.4	14
$\Upsilon' \to {}^3P_2 + \gamma$	10.02	9.90	0.12	2.4	10
${}^3P_0 \to \Upsilon + \gamma$	9.81	9.42	0.39	1.0	86
${}^3P_1 \to \Upsilon + \gamma$	9.86	9.42	0.44	1.0	123
${}^3P_2 \to \Upsilon + \gamma$	9.90	9.42	0.48	1.0	160
${}^3D_1 \to {}^3P_0 + \gamma$	10.12	9.81	0.31	3.4	97
${}^3D_1 \to {}^3P_1 + \gamma$	10.12	9.86	0.26	3.4	43
${}^3D_1 \to {}^3P_2 + \gamma$	10.12	9.90	0.22	3.4	2

$$\Gamma(\Upsilon' \to {}^3P_J + \gamma) = \tfrac{4}{3}\,\alpha\omega^2\left(\frac{e_Q}{e}\right)^2 \frac{2J+1}{9}\,|R_{nl}^{n'l'}|^2\,,$$
$$\Gamma({}^3P_J \to \Upsilon + \gamma) = \tfrac{4}{9}\,\alpha\omega^3\left(\frac{e_Q}{e}\right)^2 |R_{nl}^{n'l'}|^2\,, \qquad (2)$$
$$\Gamma({}^3D_1 \to {}^3P_J + \gamma) = \tfrac{4}{3}\,\alpha\omega^3\left(\frac{e_Q}{e}\right)^2 C_J\,|R_{nl}^{n'l'}|^2\,,$$

where ω is the emitted-photon energy, $\alpha = \frac{1}{137}$, e_Q/e is the quark charge,

$$R_{nl}^{n'l'} = \int_0^\infty r^3 R_{nl} R_{n'l'}\,dr$$

is the radial overlap integral, and

$$C_J = \begin{pmatrix} \frac{2}{9} \\ \frac{1}{6} \\ \frac{1}{90} \end{pmatrix} \text{ for } J = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

are the Clebsch-Gordan coefficients appropriate for the specific ${}^3D_1$ decay mode. In Table II we state the widths for some of these decays (again using for definiteness quark charge $+\frac{2}{3}$). As is well known from experience with charmonium, the precise widths calculated from Eq. (2) are extremely sensitive to the emitted-photon energy and thus the precise location of the higher-orbital-angular-momentum states. We can, however (as discussed above in I), only estimate the positions of these states due to the contribution of the dyadic term in the Hamiltonian and the theoretical ambiguities concerning the nature of the p-, d-, and f-state splittings. Thus the numbers in Table II should be considered only rough estimates which can be refined, as in the case of charmonium, once the higher-orbital-angular-momentum states are observed. There is still, however, a difficulty with the naive calculation. The higher-orbital-angular-momentum states may be sensitive to higher-order corrections which could introduce a J dependence into the radial overlap integrals. This is due to the fact that different angular momentum states see different effective potentials.[1,3] Again we invoke the belief that the nonrelativistic approximation should be becoming better and better as the quark mass increases. A nonrelativistic calculation of the charmonium $E1$ decays for $\psi' \to {}^3P_J + \gamma$ with the present model[3] is within a factor of 2 of that experimentally observed. We thus expect the nonrelativistic calculation of the square of the radial overlap integrals for the heavy quark system to be no worse than this.

Nature has again provided us with a laboratory in which to test the ideas of quark chemistry. Encouraged by the good agreement of our model with experiment for the Υ-Υ' energy difference, we have reported other predictions of our model for the Υ system. It will be interesting to watch the experimental evolution of this system both as a test of old ideas of meson structure and as a guide towards the understanding of the until now elusive nature of such things as hyperfine and fine-structure interactions in hadronic systems.

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[1] W. Celmaster, H. Georgi, and M. Machacek, Phys. Rev. D 17, 879 (1978).

[2] S. Herb et al., Phys. Rev. Lett. 39, 252 (1977).

[3] W. Celmaster, Harvard University thesis, 1977 (unpublished).