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# Potential model of meson masses

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We use ideas about quantum chromodynamics at large and small momenta to find a plausible explicit form for a $Q\bar{Q}$ potential. We fit the observed $s$-wave meson masses and predict the masses of bound states of yet heavier quarks.

## I. INTRODUCTION

Quark chemistry is as old as the concept of quarks.[1] Long before the development of any theoretical understanding of interquark forces, the quark model had been used with great success to systematize hadron spectroscopy.[2] The advent of quantum chromodynamics (QCD) revolutionized the field by suggesting the nature of quark-quark forces at long and short distances. At long distances, lattice gauge theories[3] suggested that quarks are confined by a constant spin-independent[4] force carried by strings of color flux. At short distances, asymptotic freedom[5] suggested that the quark-quark interaction can be described by color-gluon exchange.

Modern quark chemistry began with the charmonium interpretation/prediction of the $J/\psi$.[6] Shortly afterwards, the picture of interquark forces suggested by QCD, long-range spin-independent forces, and short-range color-gluon exchange was used to qualitatively explain the mass spectrum of low-lying normal and strange hadrons and to predict the masses of charmed mesons and baryons.[4] The subsequent discovery of charmed particles silenced most of the doubters and established the QCD quark model as the theory of choice for hadron spectroscopy.

There are three obvious ways to improve on the qualitative successes of the QCD quark model:

(1) At present, the connection between hadron masses and the underlying field theory is indirect at best, based on a number of plausible but unproven speculations. More theoretical work is needed to verify these guesses.[7]

(2) There is room for more quantitative work with realistic potentials, to see how far the potential idea can be pushed to describe hadrons accurately.[8]

(3) Finally, we may want to apply the same ideas to hadrons containing quarks even heavier than the charmed quark. If nature supplies such heavy hadrons, they will be an excellent laboratory for testing and refining the QCD model of quark chemistry.

The third direction has been vigorously pursued by Eichten and Gottfried.[9] They study heavy-quark-antiquark bound states in the potential

$$-\frac{4}{3}\frac{\alpha_s(M_Q)}{r}+kr\,, \tag{1.1}$$

where $\alpha_s(M_Q)$ is the effective quark-gluon coupling constant evaluated at the heavy-quark mass [at the charmed-quark mass they take $\alpha_s(M_c)=0.19$], and $k$ is the slope of the linear potential, which they take to be

$$k=0.20\ \text{GeV}^2\,. \tag{1.2}$$

Eichten and Gottfried solve the Schrödinger equation for the potential of Eq. (1.1). They address the following questions: How many narrow charmonium-type states will appear below the threshold for the production of hadrons carrying the quantum number of the heavy quark? How do the leptonic widths of $Q\bar{Q}$ states depend on the heavy-quark mass $M_Q$? At what $M_Q$ is the spectrum of $Q\bar{Q}$ states dominated by the Coulomb part of the potential, etc.?

In this paper, we approach some of the same questions from a different direction, trying to implement the first and second approaches described above. We will first try to make the best possible guess, with present theoretical ideas, for the $Q\bar{Q}$ potential. Then we will apply the potential to normal and heavy hadrons. At the very least, we will learn how sensitive the Eichten-Gottfried results are to the particular potential they have chosen.

One might argue that it is foolish to dwell on the details of the $Q\bar{Q}$ potential, because the potential description itself is bound to be wrong. Certainly, we cannot hope for more than qualitative success in the description of normal hadrons without a serious attempt to include relativity. But for heavy quarks, as the potential description (hopefully) becomes more accurate, it is more and more important to find the right one. And if such heavy quarks exist, quark chemistry will be a valuable laboratory in which to test the ideas of QCD.

In Sec. II, we will describe our guess for the $Q\bar{Q}$

potential and fit the parameters in the potential by comparing the predictions for light-quark and charmed-quark bound states with the observed spectrum. We will then compare our predictions for heavy-quark bound states with those of Eichten and Gottfried.

In Sec. III, we will discuss some technical questions which are particularly important for the detailed comparison of potential-model results for the charmonium system with experiment.

## II. A GUESS FOR THE $Q\bar{Q}$ POTENTIAL—AND ITS IMPLICATIONS FOR HEAVY-QUARK STATES

The asymptotic freedom (AF) of QCD makes it plausible to assume that the interaction between a quark-antiquark pair at large momentum transfer ($q^2$) is given by the one-gluon-exchange formula

$$T_a^Q T_a^{\bar{Q}} g^2(q^2)\frac{1}{q^2}\,, \tag{2.1}$$

where $g(q^2)$ is the effective quark-gluon coupling constant and $T_a^Q(T_a^{\bar{Q}})$ are the color generators for the quark (antiquark) representation. Since quarks (antiquarks) transform as 3's ($\bar{3}$'s) under color SU(3), $T_a^Q=\frac{1}{2}\lambda_a$ and $T_a^{\bar{Q}}=-\frac{1}{2}\lambda_a^*$, where $\lambda_a$ are the standard SU(3) matrices. They satisfy $T_a^Q T_a^Q = T_a^{\bar{Q}} T_a^{\bar{Q}}=\frac{4}{3}$. In mesons, we are presumably interested in $Q\bar{Q}$ in a color-singlet state in which $T_a^Q+T_a^{\bar{Q}}=0$, so that $T_a^Q T_a^{\bar{Q}}=-\frac{4}{3}$ and the force is attractive. At large $q^2$ and neglecting quark masses, $g(q^2)$ is given by

$$\frac{g^2(q^2)}{4\pi^2}=\frac{12}{33-2n}\frac{1}{\ln(q^2/\Lambda^2)}\,, \tag{2.2}$$

where $n$ is the number of quark flavors and $\Lambda^2$ is the one parameter which characterizes the strength of the interaction. From analyses of scaling violations in electroproduction and $e^+e^-$ annihilation we know that (for $n=3$)

$$\Lambda \simeq 500 \text{ MeV}.^{10} \tag{2.3}$$

In fact, we are interested in $q^2$ which are not large compared to heavy-quark masses. At moderate $q^2$, the effect of heavy quarks is small (by the Appelquist-Carrazone theorem[11]), so we include only the light quarks $u$, $d$, and $s$ and take $n=3$ in Eq. (2.2). Then Eq. (2.1) becomes

$$-\frac{64\pi^2}{27}\frac{1}{\ln(q^2/\Lambda^2)}\frac{1}{q^2}\,. \tag{2.4}$$

In position space, the Fourier transform of Eq. (2.4) becomes a Coulomb potential modified by logarithmic terms. If we replace the logarithm in Eq. (2.4) by a function which is nonzero everywhere, for example, $\ln(\xi+q^2/\Lambda^2)$ for $\xi>1$, we can calculate the Fourier transform at short distances in an asymptotic expansion in $1/\ln(1/r^2\Lambda^2)$. The result is the potential

$$V_{\rm AF}(r)=\left[-\frac{16\pi}{27}\frac{1}{\ln(1/r^2\Lambda^2e^{2\gamma})}+O\left(\frac{1}{\ln^3(1/r^2\Lambda^2e^{2\gamma})}\right)\right]\frac{1}{r}\,. \tag{2.5}$$

The surprising feature is the appearance of $\Lambda e^{\gamma}$ (where $\gamma$ is the Euler-Mascheroni constant) as the scale in the effective coupling constant. The potential is stronger than $-\frac{4}{3}\alpha_s(1/r^2)\times 1/r$ at short distances. We expect $V_{\rm AF}(r)$ to dominate the $Q\bar{Q}$ potential for $r\ll 1/\Lambda$.

The hypothesis that QCD confines quarks in color-singlet states is unproven but very attractive. The only detailed clues to the possible nature of confinement come from lattice gauge theories.[3] On the lattice, quarks are confined because the color must be transmitted from quark to antiquark (in $Q\bar{Q}$) by a string of color flux with constant energy per unit length. Thus the energy of the $Q\bar{Q}$ system grows linearly with the separation for large distances. This suggests that the $Q\bar{Q}$ potential should be linear at large distances:

$$V_{\rm S}(r)=kr \tag{2.6}$$

(subscript S for string) where $k$ is the energy per unit length of the string.

This picture of quark confinement is suggestive because it may provide a field-theoretical interpretation of the string picture of hadrons. It is well known that a rotating string gives rise to a linearly rising Regge trajectory with inverse slope

$$\alpha^{-1}=2\pi k\,. \tag{2.7}$$

Experimentally one finds $\alpha^{-1}\simeq 1$ GeV$^2$; so we may expect that

$$k=0.16 \text{ GeV}^2\,. \tag{2.8}$$

The reader may object: The value of $k$ in Eq. (2.8) is obtained by neglecting the mass of the quarks on the end of the string, tacitly assuming that the light-quark mass is small. But in a sensible potential-model picture, the light-quark mass must be several hundred MeV. Our reply is that a mass renormalization is taking place. The effective quark mass in a system with high orbital angular momentum is smaller than that in an $l=0$ hadron. The $s$-wave quark, moving more slowly, carries a larger load of gluons.[12]

We expect the $Q\bar{Q}$ potential at large distances to be dominated by the linear term Eq. (2.6) with $k=0.16$ GeV$^2$. This value of the slope of the linear potential is smaller than that used by Eichten and Gottfried.

We have made a plausible guess for the $Q\bar{Q}$ potential at very small and very large separations.

Alas, our theoretical prejudices do not apply to intermediate (INT) distances. The most general possibility is

$$V(r) = V_{AF}(r) + V_{INT}(r) + V_S(r)\,, \tag{2.9}$$

where $V_{INT}(r)$ is some function which is negligible compared to the modified Coulomb interaction, Eq. (2.5), at short distances and negligible compared to the linear potential, Eq. (2.6)–(2.8), at large distances. We take

$$V_{INT}(r) = V_0 - kr\,e^{-ar}\,, \tag{2.10}$$

but hope that the results for heavy-quark states will be insensitive to the precise form of $V_{INT}(r)$.

We will also include "hyperfine-splitting" corrections, some suitable generalization of the Fermi-Breit interaction:

$$\begin{aligned} H_t = {} & \frac{1}{8}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}\right)\nabla^2 V(r) + \frac{2}{3m_1m_2}\vec{S}_1\cdot\vec{S}_2\nabla^2 V(r) \\ & + \frac{1}{2}\left[\left(\frac{1}{m_1^2}+\frac{2}{m_1m_2}\right)\vec{S}_1\cdot\vec{L} + \left(\frac{1}{m_2^2}+\frac{2}{m_1m_2}\right)\vec{S}_2\cdot\vec{L}\right]\frac{1}{r}\frac{dV(r)}{dr} \\ & - \frac{1}{3m_1m_2}(3\vec{S}_1\cdot\hat{r}\,\vec{S}_2\cdot\hat{r} - \vec{S}_1\cdot\vec{S}_2)\left[\frac{d^2V(r)}{dr^2} - \frac{1}{r}\frac{dV(r)}{dr}\right]. \end{aligned} \tag{2.11}$$

It is not entirely obvious that this is the right form to use for the hyperfine corrections. In particular, it may be that the linear part of the potential should not be included in Eq. (2.11). We wish to postpone any discussion of such delicacies until after we have fitted our potential to experiment. For the present, we will include hyperfine corrections for $s$-wave states by treating Eq. (2.11) as a perturbation, but we will leave $\langle\nabla^2 V\rangle$ as a free parameter to be fit to the experimental hyperfine splitting. This gives an effective "center-of-mass" (which depends on the quark masses) for each hyperfine pair.

We can now attempt to fit the mass spectrum of observed $s$-wave mesons. The parameters at our disposal are $a$ and $V_0$ (in $V_{INT}$) and the quark masses. To construct such a fit, choose a reasonable value for the charmed-quark mass $M_c$, by which we mean a mass slightly larger than half the $\psi$ mass (this is larger than the mass appropriate to a field-theoretic treatment of charm production in $e^+e^-$ annihilation because the potential description involves quarks at low momentum dressed with extra gluons). Fit parameter $a$ in Eq. (2.10) by requiring that the energy difference between the ground state and the first excited $s$-wave state is equal to the $\psi'$-$\psi$ mass difference. Fit $V_0$ in Eq. (2.10) to the center-of-mass of the $\psi'$-$\eta_c$ system. This completely determines $V(r)$. To determine the light-quark ($u$ and $d$) mass, fit to the center-of-mass of the $D^*$-$D$ system. If the predicted and observed $D$ masses are comparable for a reasonable value of the light-quark mass, the fit is acceptable. If not, choose a different $M_c$ and start again. One can then go on to determine the strange-quark mass by fitting to the $K^*$-$K$ system.

TABLE I. Potential fit (all energies in GeV).

| State | $M$ (exp) | $\Delta E_{HF}$ (exp) | $M$ (fit) |
|---|---|---|---|
| $\psi$ | 3.10 | 0.25 | 3.1 |
| $D$ | 2.01 | 0.14 | 2.01 |
| $\rho, \omega$ | 0.77 | 0.63 | 0.77 |
| $K^*$ | 0.89 | 0.40 | 0.89 |
| $\phi$ | 1.02 | 0.40 (?) | 1.02 |
| $F^*$ | ? | 0.14 (?) | 2.03 |

Table I summarizes one such fit, with

$$\begin{aligned} & a = 0.16\ \text{GeV}^2, \quad V_0 = 0.39\ \text{GeV} \\ & M_c = 1.98\ \text{GeV}, \quad M_{u,d} = 0.315\ \text{GeV} \\ & M_s = 0.675\ \text{GeV}. \end{aligned} \tag{2.12}$$

The resulting potential is displayed in Fig. 1. This gives a satisfactory description of the masses of all the observed $s$-wave mesons.

We will return to the hyperfine splittings in some detail in Sec. III. For now, we simply state that we will calculate the hyperfine splittings (and as-

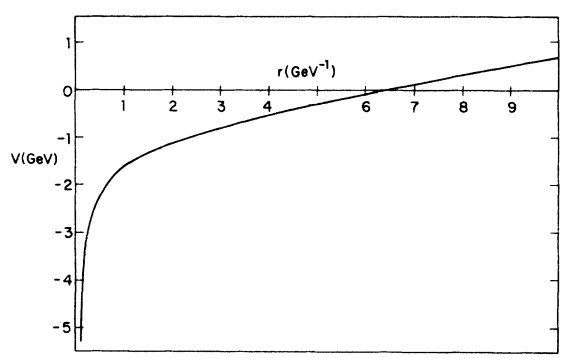


FIG. 1. Nonrelativistic potential for meson binding.

sociated center-of-gravity shifts) for heavy-quark states by inserting $V_{AF}$ in Eq. (2.11) and using first-order perturbation theory. This procedure works reasonably well for the lighter-quark bound states.

We are finally ready to discuss our results for heavy quarks. We give predictions for the masses of the spin-1 $s$-wave bound states which should be produced as narrow resonances in $e^+e^-$ annihilation. In Fig. 2 we plot the mass of the lightest such state on the horizontal axis. On the vertical axis we plot the mass difference between the excited states and the lowest state, and also the position of the threshold for production of states with the new quantum number of the heavy quark, $(\bar{u}Q)+(Q\bar{u})$ or $(\bar{d}Q)+(d\bar{Q})$ states. Our results are qualitatively similar to those of Eichten and Gottfried, but quite different in detail. Precise comparison is difficult because their results are stated in terms of the quark mass rather than the observable mass of the lightest spin-1 $Q\bar{Q}$ states. In Fig. 3, we have used their formula for the mass of the lightest $\bar{u}Q$ state to construct the analog of Fig. 2 for their potential. Since in the present model the short-range part of the force is stronger than that of Eichten-Gottfried, the binding energy between quarks of a given mass is greater. The lowest-energy state of the system is pushed down in energy relative to that expected by Eichten and Gottfried. Thus we predict a much larger energy difference between threshold and the lightest $Q\bar{Q}$ state. Further, this allows the qualitative features of the spectrum, such as the number of states below threshold, to be similar to that of the Eichten-Gottfried model, although the detailed splittings between excited states are quite different. In the present model these splittings do not decrease significantly as the mass of the quark increases. In fact, in the potential of Eq. (2.9) the energy difference between the lowest state and the first excited state remains roughly constant for lower quark masses as well. We predict $\phi'$ and $\rho'$ states 600–700 MeV above the $\phi$ and $\rho$.

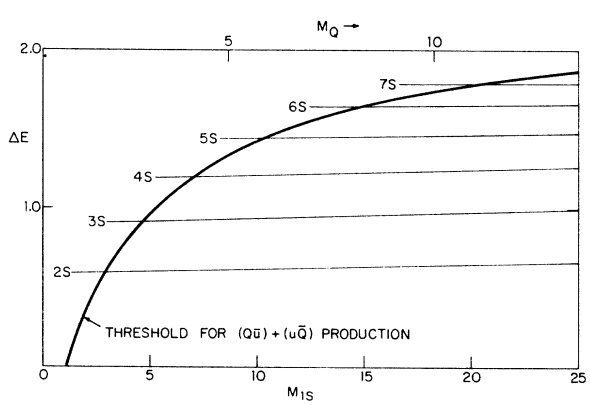


FIG. 2. Heavy-quark states below threshold: present model (all energies in GeV units).

### III. HYPERFINE STRUCTURE

Little is known theoretically about the precise nature of the hyperfine interaction in quantum chromodynamics. The simplest phenomenological prescription is to borrow from positronium the Fermi-Breit form for the spin-dependent interactions and lump all non-Abelian complications of the theory into the definitions of the appropriate effective potential and the quark masses. Although this approach is naive, it may be a valuable first approximation and phenomenological guide. Even when we adopt this simple procedure, the next step is ambiguous. We learn from lattice gauge theories that the longest-range component of quark-antiquark forces is spin independent. We do not learn where or how spin dependence begins to evolve. Two extreme philosophies have been applied to the problem. The first (DGGS) assumes that the spin-dependent forces are extremely short-range and can be characterized by a Coulomb interaction with an effective coupling constant evaluated at the bound-state mass of the system.[4,13] Thus the hyperfine splitting between the spin-0 and spin-1 $s$-wave states is proportional to the wave function at the origin. The tensor and spin-orbit terms are proportional to $\langle 1/r^3\rangle$ [see Eq. (2.11)]. The second philosophy (total $V$) (Ref. 14) is that the approach to spin independence is sufficiently slow so that over the region where the hadron wave function resides the whole potential $V$ contributes to the spin-dependent terms in Eq. (2.11). We suggest that a third, intermediate philosophy (modified DGG), in which the Fourier transform $V_{AF}$ of a single-gluon exchange governs the spin dependence, is also reasonable. This differs from DGGS in that the effective coupling constant is allowed to vary over the momentum distribution within the hadron. Thus the $r$-space

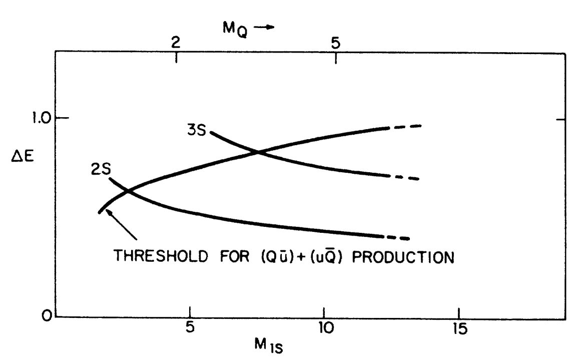


FIG. 3. Heavy-quark states below threshold: Eichten-Gottfried model (all energies in GeV units).

form of the potential is no longer strictly Coulombic and the interaction Hamiltonian is the same as Eq. (2.11) with $V$ replaced by $V_{AF}$. Using the fit of Sec. II, we calculate the hyperfine splitting predicted by first-order perturbation theory for each of the above philosophies. The results are given in Table II. The predicted hyperfine splittings of the low-mass mesons are consistently too large. This discrepancy is not serious because in this low-mass region higher-order effects are important. For instance, the effect of directly integrating the Fermi-Breit Hamiltonian (rather than treating it as a first-order perturbation) reduces the $D$-$D^*$ splitting by ~30%. It also dramatically reduces the wave function at the origin for the spin-1 state, making the first-order DGGS calculation unreliable. In the charmonium region the total $V$ and modified DGG approaches are becoming indistinguishable in that 90% of the hyperfine splittings comes from $\langle\nabla^2 V_{AF}\rangle$. Higher-order effects, although still present, are less important in this region as demonstrated by the fact that direct integration now reduces the splitting by only 15%. Thus the discrepancy with experiment may be signaling the onset of a new physics.[15] By $M_Q \gtrsim 5$–10 GeV the predictions become unambiguous. Lowest-order perturbation theory is good and the predictions are roughly independent of the philosophy chosen to treat hyperfine splittings within the Fermi-Breit Hamiltonian. If a discrepancy between theory and experiment persists for these higher quark-mass systems it would indicate the need for theoretical modifications to the form of the hyperfine interaction.

Any treatment of the fine structure for higher-orbital angular momentum states is beset by the same difficulties that were encountered above for hyperfine splittings. In Table III the location of the $p$ states for charmomium and heavy-quark states (with quark masses of 5 and 10 GeV) are tabulated.

TABLE II. Hyperfine structure for $s$-wave mesons (all energies in GeV).

| System | $\Delta E_{exp}$ | $\Delta E_V$ | $\Delta E_{V_{AF}}$ | $\Delta E_{DGG}$ |
|---|---|---|---|---|
| $\pi$-$\rho$ | 0.631 | 1.41 | 0.70 | 1.52 |
| $K$-$K^*$ | 0.398 | 0.84 | 0.48 | 0.75 |
| $D$-$D^*$ | 0.139 | 0.31 | 0.20 | 0.14 |
| $\phi$ | | 0.52 | 0.35 | 0.55 |
| $F$-$F^*$ | 0.14 (?) | 0.25 | 0.19 | 0.15 |
| $\eta_c$-$\psi$ | 0.24–0.26 | 0.17 | 0.15 | 0.13 |
| $\eta_c'$-$\psi'$ | 0.23 (?) | 0.10 | 0.08 | 0.07 |
| $H_5$-$H_5^*$ | | 0.09 | 0.09 | 0.07 |
| $D_5$-$D_5^*$ | | 0.13 | 0.09 | 0.04 |
| $F_5$-$F_5^*$ | | 0.11 | 0.09 | 0.05 |
| $G_5$-$G_5^*$ | | 0.10 | 0.10 | 0.07 |
| $H_{10}$-$H_{10}^*$ | | 0.07 | 0.07 | 0.06 |

TABLE III. Lowest-lying $p$-wave states for charmonium and heavy-quark states (all energies in GeV).

| $M_Q$ | State | $E_V$ | $E_{AF}$ | $E_{DGGS}$ | $E_{exp}$ |
|---|---|---|---|---|---|
| 1.98 | $3P_2$ | 3.50 | 3.47 | 3.44 | 3.55 |
| 1.98 | $3P_1$ | 3.38 | 3.38 | 3.39 | 3.51 |
| 1.98 | $3P_0$ | 3.26 | 3.28 | 3.33 | 3.42 |
| 5 | $3P_2$ | 9.13 | 9.13 | 9.11 | |
| 5 | $3P_1$ | 9.08 | 9.08 | 9.09 | |
| 5 | $3P_0$ | 9.03 | 9.04 | 9.06 | |
| 10 | $3P_2$ | 18.85 | 18.85 | 18.84 | |
| 10 | $3P_1$ | 18.82 | 18.82 | 18.83 | |
| 10 | $3P_0$ | 18.79 | 18.79 | 18.81 | |

The center-of-gravity of the charmonium $p$-state system is lower than the observed value. However, we have neglected throughout the effect of the dyadic term in the Fermi-Breit Hamiltonian which is orbital-angular-momentum dependent. A more instructive quantity to consider is the magnitude and ratio of the splittings between these states. These are given in Table IV, again for charmonium and heavy-quark states ($M_Q = 5$ and 10 GeV). The most striking difference between the predictions of the three phenomenological approaches is that the splitting in the total $V$ and modified DGG approaches is a factor of 2 larger than that in DGGS. The ratios $(3P_2 - 3P_1)/(3P_1 - 3P_0)$ calculated using first-order perturbation theory are 1.1, 0.9, and 0.8 for the total $V$, modified DGG, and DGGS approaches, respectively. Each of these ratios differs significantly from the experimentally observed ratio of 0.5. But direct integration of the Fermi-Breit form for $p$ states shows that higher-order effects are significant, particularly for the $J=0$ state where the angular momentum coefficients are large and negative. In heavy-quark states, where higher-order effects are less important, the factor of 2 difference in magnitude persists between modified DGG and DGGS thus making it possible in principle to choose between these two philosophies, provided that the ratio of splittings is close to 0.9. (DGGS unambiguously predicts this ratio to be 0.8.) If

TABLE IV. $P$-wave splittings for charmonium and heavy-quark states (all splittings in MeV).

| $M_Q$ (GeV) | Splitting | $\Delta E_V$ | $\Delta E_{AF}$ | $\Delta E_{DGGS}$ | $\Delta E_{exp}$ |
|---|---|---|---|---|---|
| 1.98 | $3P_2$-$2P_1$ | 122 | 92 | 50 | 40 |
| 1.98 | $3P_1$-$3P_0$ | 115 | 100 | 62 | 90 |
| 5 | $3P_2$-$3P_1$ | 48 | 44 | 21 | |
| 5 | $3P_1$-$3P_0$ | 49 | 46 | 24 | |
| 10 | $3P_2$-$3P_1$ | 28 | 26 | 12 | |
| 10 | $3P_1$-$3P_0$ | 31 | 31 | 16 | |

the ratio continues to be much smaller, as now observed in charmonium, it will require a new understanding of the physics involved in the fine-structure interaction.

The $p$-state situation might be still more complicated. There could be significant mixing between the $p$-wave $Q\bar{Q}$ states and $s$-wave bound states of $Q\bar{Q}$ plus gluon ($G$). In the $Q\bar{Q}G$ system, there is a small *repulsive* force between the quark and antiquark because

$$T_a^Q T_a^{\bar{Q}} = \tfrac{1}{2}(T_a^G T_a^G - T_a^Q T_a^Q - T_a^{\bar{Q}} T_a^{\bar{Q}}) = \tfrac{1}{6}\,, \qquad (3.1)$$

($T_a^G T_a^G = 3$). There is a large attractive force between quark or antiquark and gluon:

$$T_a^Q T_a^G = \tfrac{1}{2}(T_a^{\bar{Q}} T_a^{\bar{Q}} - T_a^G T_a^G - T_a^Q T_a^Q) = -\tfrac{3}{2} = T_a^{\bar{Q}} T_a^G\,. \qquad (3.2)$$

The quark-spin dependence of this system is similar to that of the $p$ wave with the gluon spin playing the role of the orbital angular momentum. The $QG$ and $\bar{Q}G$ forces give rise to a "spin-orbit" interaction, while the $Q\bar{Q}$ interaction is similar to the tensor term. It seems unlikely that mixing between $p$-wave $Q\bar{Q}$ states and $s$-wave $Q\bar{Q}G$ states can resolve the $p$-wave charmonium puzzle. We mention it as a reminder that more interesting physical possibilities may exist due to the rich non-Abelian structure of QCD.

So far we have not discussed predictions for leptonic and hadronic decay widths in our model. We can summarize these predictions in one word: *wrong*, if we naively relate the widths to the wave function at the origin taken from lowest-order perturbation theory ignoring spin-dependent corrections. Our width predictions are consistently larger than the experimental values. The disagreement is a factor of 3 for leptonic widths of charmonium and more for hadronic widths. Including the effect of the short-range spin-dependent interaction (which in the spin-1 states acts like a repulsive "hard core")[8] which improves the predictions for the vector states (by a factor of 2 for charmonium), but probably does not completely resolve all of the difficulties.

The source of these problems may be in the standard width calculations, which treat the quarks as point particles. It is possible that a constituent quark, particularly in an $s$-wave bound-state hadron, is a rather soft object. The quarks exhibit pointlike properties at large $Q^2$ in electroproduction, but inside hadrons they are bound to a gluon cloud which may shield the quarks and suppress the annihilation amplitude.

The question for phenomenological quark chemistry is whether this kind of effect can be included in the potential description in some way. One obvious possibility is to allow for the momentum dependence of the effective quark mass. In QCD, we expect the effective quark mass to decrease at large momentum or short distance.[12] If the quarks "undress" and shed their mass at short distances, the kinetic-energy term in the Hamiltonian can become the "hard core" interaction required to reduce the width predictions.

We conclude with a brief summary. We have tried to use QCD as a guide in constructing a nonrelativistic potential model description of $s$-wave meson masses. Impressed by the qualitative successes of QCD in explaining the light-hadron mass spectrum, we required that our description apply to light- as well as heavy-quark bound states. We applied our model to quarks heavier than the charmed quark and found predictions rather different from the earlier results of Eichten and Gottfried.

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[1]M. Gell-Mann, Phys. Lett. 8, 214 (1964); G. Zweig (unpublished).
[2]See for example, J. J. J. Kokkedee, *The Quark Model* (Benjamin, New York, 1969).
[3]K. Wilson, Phys. Rev. D 10, 2445 (1974); J. Kogut and L. Susskind, *ibid.* 9, 3501 (1974).
[4]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975).
[5]H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973); D. J. Gross and F. Wilczek, Phys. Rev. D 8, 3633 (1973).
[6]T. Appelquist and H. D. Politzer, Phys. Rev. Lett. 34, 43 (1975).
[7]An interesting attempt in this direction is A. Duncan, Phys. Rev. D 13, 2866 (1976).
[8]H. J. Schnitzer, Brandeis University report, 1974 (unpublished); J. S. Kang and H. J. Schnitzer, Phys. Rev. D 12, 841 (1975); 12, 2791 (1975); E. Eichten *et al.*, Phys. Rev. Lett. 34, 369 (1975); B. J. Harrington *et al.*, *ibid.* 34, 706 (1975); J. Kogut and L. Susskind, *ibid.* 34, 767 (1975); Phys. Rev. D 12, 2821 (1975); R. Barbieri *et al.*, Nucl. Phys. B105, 125 (1976); M. Machacek and Y. Tomozawa, Ann. Phys. (N.Y.) (to be published); W. Celmaster, Harvard University thesis, 1977 (unpublished).
[9]E. Eichten and K. Gottfried, Phys. Lett. 66B, 286 (1977).

[10]A. De Rújula, H. Georgi, and H. D. Politzer, Ann. Phys. (N.Y.) 103, 315 (1977); E. C. Poggio, H. R. Quinn, and S. Weinberg, Phys. Rev. D 13, 1958 (1976); R. Shankar, *ibid.* 15, 755 (1977); R. G. Moorhouse, M. R. Pennington, G. G. Ross, Nucl. Phys. B124, 285 (1977).
[11]T. Appelquist and J. Carazzone, Phys. Rev. D 11, 2856 (1975).
[12]H. Georgi and H. D. Politzer, Phys. Rev. D 14, 1829 (1976); H. D. Politzer, Nucl. Phys. B117, 397 (1976).
[13]H. J. Schnitzer, Phys. Lett. 69B, 477 (1977).
[14]H. J. Schnitzer, Phys. Rev. D 13, 74 (1976).
[15]H. J. Schnitzer, Phys. Lett. 65B, 239 (1976).