Hadronic fragmentation as a probe of the underlying dynamics of hadron collisions

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We demonstrate that measurements of the power-law behavior of single and double fragmentation cross sections ($A + B \to C + X$ and $A + B \to C + D + X$) at high energies are capable of distinguishing between different models of hadronic interactions. Present evidence supports the idea that quark exchange or annihilation is the dominant strong interaction at present energies; further tests are proposed.

I. INTRODUCTION

The striking success of the quark-parton model in describing the dynamics of lepton-induced reactions has led to increased emphasis on developing an equally fundamental explanation of the regularities of purely hadronic collisions. Within the general framework of quantum chromodynamics there are two distinguishable perturbation-theory mechanisms which could characterize the *initial* interaction between colliding hadrons:

(a) vector-gluon exchange[1-3] [Fig. 1(a)], or

(b) quark exchange or annihilation[4,5] [Figs. 1(b)–1(e)].

In each case one assumes that the separating color systems are neutralized by final-state interactions in analogy to the case of $e^+e^- \to q\bar{q} \to$ hadrons.

The gluon-exchange mechanism, which is the starting point for the Pomeron model of Low[1] and Nussinov,[2] has the advantage of automatically producing a roughly constant high-energy cross section (because of the vector coupling), and is often identified with the dual (cylinder) model of the Pomeron. It is, however, remote from the standard hadronic models based on short-ranged interactions in rapidity, and because of the pointlike gluon-quark interaction, does not lead in any obvious way to a transverse-momentum cutoff of the forward jets.

The quark-exchange mechanism[6] is the quantum-chromodynamic realization of Feynman's wee-parton-exchange ansatz.[5] This mechanism can be identified with either a simple quark-exchange amplitude [Fig. 1(c)] or the annihilation of a wee quark and wee antiquark of the target and projectile into a color-singlet system [Fig. 1(e)]. This quark-exchange (or annihilation) mechanism will yield a logarithmically increasing cross section at high energies if one assumes the distribution functions $G_{q/H}(x) = dN_{q/H}(x)/dx$ for wee quarks in a hadron have the Feynman form $G_{q/H}(x) \sim x^{-1}$ for small x, and it can be made consistent with the short-range-correlation, low-transverse-momentum properties of the multiperipheral model, as well as the large-transverse-momentum phenomenology of the constituent-interchange model.[6,7] [Here x is the light-cone variable, $x = (k_0 + k_3)/(p_0 + p_3)$, where $k^\mu (p^\mu)$ is the momentum of the quark (hadron).]

The form of the cross section (after integrating over transverse momenta) is

$$\sigma_{AB}(s) = \int_0^1 dx_a \int_0^1 dx_b\, G_{q/A}(x_a) G_{\bar{q}/B}(x_b) \hat{\sigma}(\hat{s}_{ab})$$

with

$$\hat{s} = x_a x_b s + \frac{m_a^2 m_b^2}{4 x_a x_b s} ;$$

$\hat{\sigma}(\hat{s})$ is a decreasing function of s. In the case of valence-sea $q\bar{q}$ interactions [Figs. 1(b), 1(d)], only one $G(x)$ has x^{-1} behavior and the cross section $\sigma_{AB}(s) \to$ const.

It is interesting to note that if the quark-exchange mechanism is correct, then the same dynamics which produces hadrons in association with the Drell-Yan mechanism[8] ($q\bar{q} \to l\bar{l}$) for lepton pairs (at small $\mathfrak{M}_{l^+l^-}^2/s$) is also responsible for the

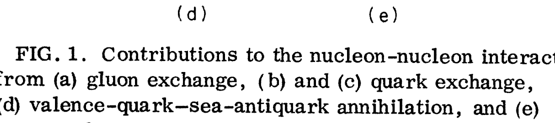

FIG. 1. Contributions to the nucleon-nucleon interaction from (a) gluon exchange, (b) and (c) quark exchange, (d) valence-quark–sea-antiquark annihilation, and (e) sea-quark–sea-antiquark annihilation. Only the minimum quark lines are shown.

production of hadrons in ordinary hadron collision; if gluon exchange is correct, then there are two quite distinct hadron-production mechanisms.

As we have discussed in an earlier paper,[5] the observed universality of inelastic multiplicities in lepton- and hadron-incuded reactions at the same available energy, i.e., the apparent equality of the hadronic and current plateaus, can be regarded as evidence of a quark-exchange or annihilation mechanism as the primary hadronic interaction. This is because the final state of a lepton-induced reaction always begins as a separating 3 and $\overline{3}$ of color, leading to gluon radiation and subsequent hadron production, dependent on separating color "charges", proportional to $\frac{4}{3}\alpha_s$ ($\alpha_s = g_s^2/4\pi$). Hadron-hadron interactions which start with quark exchange also begin with separating 3 and $\overline{3}$ systems, whereas for gluon exchange the final states begin with separation of color octets and, accordingly, gluon radiation, dependent on separating charges proportional to $3\alpha_s$. Thus at similar available energies, hadronic collisions should evidently have the same central hadron multiplicities as leptonic-induced reactions ($e^+e^- \to$ hadrons, $lp \to l'X$) only if the wee-quark exchange or annihilation mechanism dominates. This color separation argument can also be supported by the following physical considerations. If two quarks in pp collisions are scattered to large transverse momentum by the exchange of a gluon with four-momentum q^μ, then the resulting total hadron multiplicity will be just double that of deep-inelastic lepton-proton scattering at the same q^2. As q^2 becomes small and the two quarks scatter forward, the multiplicity of their jets will combine with the multiplicity of the spectator quarks to give at least double the usual jet multiplicity—in apparent conflict with experiment. (We rule out the possibility of destructive interference of the multiplicity chains since the initial gluon interactions leaves color separated.) It is clear, though, that all these arguments are based on the isolation of perturbation-theory terms, and thus are only suggestive and nonrigorous.

The debate between gluon exchange and quark exchange also enters into the analysis of large-transverse-momentum reactions. Phenomenologically, both the Landshoff contribution[9] to large-angle elastic pp scattering and the scale-invariant $p_T^{-4}f(x_T,\theta)$ contribution to large-transverse-momentum cross sections, such as $E d\sigma/d^3p(pp \to \pi X)$ appear to be absent. This, together, with the success of the constituent-interchange model[6] and dimensional-counting[10] parametrizations of these reactions again points to the dominance of a quark-exchange mechanism. A fundamental reason why low-momentum-transfer gluon exchange should be suppressed in models based on quantum chromodynamics has not been given, although in strong-coupling lattice models[11] and in Tye's covariant string model[12] the quark-exchange mechanism is either the dominant or the sole hadronic interaction. It should be noted that standard estimates[13] of gluon-exchange contributions to quark-quark scattering predict that the order α_s^2 contribution will not be dominant in $pp \to \pi^+ X$ unless $p_T \gtrsim 8$ GeV/c, if $\alpha_s \lesssim 0.4$.

It is clear from the above discussion that it is important to propose further empirical tests which distinguish the gluon- and quark-exchange mechanisms, and also to compare both approaches to standard Regge phenomenology. As we shall discuss here, the power-law behavior of hadronic fragmentation associated with underlying jet structures appears to provide a critical tool for this purpose.[14-19]

II. JET FRAGMENTATION

It is already well known that jets with properties associated with the fragmentation of quarks into hadrons have been observed in both e^+e^- collisions and in the photon (or W) fragmentation region of deep-inelastic lepton scattering, i.e., $\gamma_v + p \to X$ or $W + p \to X$ (see Ref. 18 for a summary). It is interesting to note that qq jets are expected in the target-fragmentation region in these current-induced reactions when $x_{Bj} \gtrsim 0.2$. Similarly, in the Drell-Yan scaling region (assuming valence q + sea $\bar{q}$ annihilation) for $pp \to \mu^+\mu^- X$, one expects to see the fragments of both qq and $qqqq$ underlying jets in the fragmentation region of the target and projectile.

It is immediately clear that the quark- and gluon-exchange mechanisms lead to strikingly different jet behavior in pp collisions. In the case of gluon exchange, the jets in the target and projectile region both correspond to an underlying (qqq) in a color octet. In the case of quark exchange (or $q\bar{q}$ annihilation into singlets) the underlying jet structure is the same [for example, (qq) and ($qqqq$)] as in the Drell-Yan process (see Fig. 1).

A critical method for distinguishing underlying jet structure is the $x \to 1$ power-law behavior of its hadronic fragments. We will utilize the dimensional-counting rule,[15,16,20]

$$G_{a/A}(x) \propto (1-x)^{2n_{\bar{a}A}-1}, \quad (x \to 1) \qquad (1)$$

for the production of the hadronic fragment a with light-cone (or infinite momentum) fraction $x = (p_0^a + p_z^a)/(p_0^A + p_z^A)$, where $\vec{p}^A$ is taken along the z axis. Here $n_{\bar{a}A}$ is the minimum number of quark spectators which are left behind with total light-cone fraction $(1-x)$. (Equivalently, the $\bar{a}A$ state is the minimal quark state with the quantum numbers of $\bar{a}A$.) It is intuitively clear that the probability for finding a fragment with a large fraction of the momentum must decrease as the number of particles in the Fock space increases. Equation (1) follows from evaluating the lowest-order terms in a renormalizable perturbation theory assuming the Bethe-Salpeter wavefunction of A is finite at the origin and contains the quarks necessary to form a. It is also consistent with a smooth exclusive-inclusive connection (correspondence principle[21]) and the dimensional-counting rules for exclusive large-p_T processes and elastic form factors. In certain cases [when $n(\bar{a}A)$ is odd] spin effects can lead to an extra single power of $(1-x)$ suppression, but in practice this can probably be ignored[22] because of the effect of nonscaling terms. Another complication is that resonance production and decay can lead to the production of certain hadrons which might otherwise be suppressed.

Some simple applications of (1) are[15,16,20] $\nu W_{2p} = G_{q/p} \sim (1-x)^3$ (two spectators), $G_{\bar{q}/p} \sim (1-x)^7$, $\nu W_{2\pi} \sim (1-x)$, and $G_{\pi/p} \sim (1-x)^5$. Further, for quark-jet fragmentation $D_{\pi/u} \equiv G_{\pi/u} \sim (1-x)$, $D_{p/u} \equiv G_{p/u} \sim (1-x)^3$. All of these results seem to be consistent with experiment. These rules have also been successfully applied to nuclear processes (Fermi motion, beam fragmentation), e.g.,[23-25] $G_{p/D} \sim (1-x)^5$ and $G_{N/A} \sim (1-x)^{6(A-1)-1}$. In general we expect the simple $(1-x)$ behavior to be a good approximation for x beyond the "quasielastic" peak, e.g., $x>0.3$ for $G_{q/p}$. This is discussed further in Ref. 6.

In this paper we will consider the application of the fragmentation counting rules to high-energy low-transverse-momentum inclusive reactions:

$$\text{(I) } A+B \to C+X\ , \qquad \text{(II) } A+B \to C+D+X\ , \tag{2}$$

and emphasize the features of the cross sections which can discriminate between various mechanisms. We will primarily be interested in the behavior of the center-of-mass cross sections in the beam- and target-fragmentation regions. These will be considered as functions of x_C and x_D where, ideally, x_C and x_D are the light-cone fractions of the observed particles C and D relative to the underlying source jets. Since in the gluon-exchange and quark-annihilation models a certain amount of transverse momentum is imparted to the jets before fragmentation, the jets do not necessarily lie along the beam axis. In the absence of an event-by-event jet-direction analysis the best approximation to the light-cone fraction is the radial scaling variable $x_R = E^*/E^*_{\max}$ employed extensively in the analysis of Taylor *et al.*[26] At $s\to\infty$, x_R becomes equal to the usual Feynman scaling variable $x_L = |p^*_z|/|p^*_{\max}|$. However, at finite energy the variable x_R, which maintains a constant distance from the phase-space boundary, is to be preferred for analysis in the context of the fragmentation models.

Conventionally, the fragmentation region is identified with the (energy-independent) triple-Regge contribution; e.g., for $A+B\to C+X$, we have

$$\frac{d\sigma}{d^3p_C/E_C} \propto (1-x_C)^{1-2\alpha_{A\bar{C}}(t_{AC})} \quad (x_C \to 1) \tag{3}$$

(x_C is usually identified with x_L for this theory), where $\alpha_{A\bar{C}}$ is the leading Regge trajectory in the $A+\bar{C}$ channel identified in exclusive processes $A+H\to C+H'$. This form successfully describes the triple-Pomeron region in $pp\to pX$ for $x_L>0.8$ and, apparently, the quantum-number-exchange reactions, e.g., $\pi^+p\to K^+X$, $pp\to\pi^+X$, for $x_L>0.9$, recently measured by Anderson *et al.* at Fermilab.[27] In the case of the double inclusive reactions (II), the dominance of the Pomeron trajectory in the scattering of the Reggeon implies that the beam and target fragmentations are independent (absence of long-range correlations). The triple-Regge prediction is then

$$\frac{d\sigma(A+B\to C+D+X)}{(d^3p_C/E_C)(d^3p_D/E_D)} \propto (1-x_C)^{1-2\alpha_{A\bar{C}}(t_{AC})}(1-x_D)^{1-2\alpha_{B\bar{D}}(t_{BD})} \quad (x_C\to 1, x_D\to 1)\ .$$

It is, however, clear from experiment that at high energies the triple-Regge description can only be applied to $|x_C|$ or $|x_D|$ very close to 1. For example, the forward fragmentation data[28] for $pp\to\pi^+X$ measured at the CERN ISR, 23 GeV $\sqrt{s}$ < 62 GeV, where $x_R \simeq x_L$ are consistent with the form

$$\frac{d\sigma}{d^3p/E}(pp\to\pi^+X) \sim (1-x_\pi)^{3.34\pm0.06} \tag{5}$$

for $x_\pi>0.6$, $0.55 \le p_T \le 0.95$ GeV. The interpretation in terms of the triple-Regge model presents difficulties and it seems plausible to associate this contribution with the power-law tail from a fragmentation process. The x_R analysis of Ref. 26 for all data $10<\sqrt{s}<62$ is also consistent with (5).

We thus investigate the possibility that the fragmentation cross sections $d\sigma/dx$ in the $|x|\le 0.9$ region can be identified with the fragmentation distribution appropriate to the underlying jets J_A and J_B produced in the beam and target regions and carrying virtually all the momentum of p_A

and p_B. Thus in the simplest model where the jet J_A is defined as the system produced by the dissociation of beam A via a Pomeron one predicts[29] that $[x_C=(p_0^C+p_z^C)/(p_0^A+p_z^A)]$

$$x_C\frac{dN}{dx_C}(A+B\to C+X)\equiv\frac{1}{\sigma_{\rm inel}}\int d^2k_T^C\frac{d\sigma}{d^3p_C/E_C}$$
$$\cong x_C G_{C/A}(x_C)$$
$$\propto(1-x_C)^{2n_{A\bar{C}}-1}\quad(x_C\to 1)\,,\tag{6}$$

e.g., $dN/dx(pp\to\pi^+X)\propto(1-x)^5$ (corresponding to three spectators in $p\to\pi^+$), $dN/dx(pp\to K^-X)\propto(1-x)^9$, and $dN/dx(pp\to\bar{p}X)\propto(1-x)^{11}$. The dissociation model also predicts that the fragmentation regions in the double inclusive reaction are uncorrelated:

$$\frac{dN}{dx_C dx_D}(A+B\to C+D+X)\cong\frac{dN}{dx_D}(A+B\to D+X)\times\frac{dN}{dx_C}(A+B\to C+X)\,.\tag{7}$$

It is also clear that the predictions for a quantum-chromodynamic model based on color-gluon exchange leads to the same fragmentation distributributions (6) and (7) as the dissociation model. After color-gluon exchange the underlying jets J_A and J_B are each a color-octet state, but they still contain the same numbers of quarks as the initial A and B states, respectively. Although this model introduces a long-range correlation in color, there are no long-range correlations in momentum or flavor.

Predictions for the fragmentation powers in the model based on quark exchange (or annihilation), on the other hand, differ substantially from those of diffractive dissociation or gluon-exchange mechanisms. For example, a π^+ observed in the fragmentation region in pp collisions can arise in either of two ways:

(1) The four-quark jet remaining after the annihilation or exchange of a wee quark fragments into the observed π^+, giving the Feynman-scaling contribution $dN/dx(pp\to\pi X)\sim G_{\pi^+/q\bar{q}qq}\sim(1-x)^3$. For example, if the five-quark Fock state of the proton $|uudd\bar{d}\rangle$ interacts via exchange or annihilation of the wee d quark, then the remaining $uud\bar{d}$ system may fragment into the $\pi^+(u\bar{d})$ leaving behind just two spectator quarks (ud). The crucial point is that the number of spectators which must be slowed down is reduced by one, and hence the $(1-x)$ power by 2 relative to the dissociation or gluon-exchange predictions. This result requires that the π^+ arises from a component of the incoming proton Fock space with at least one additional $q\bar{q}$ pair from the sea so that the interacting quark (the wee d quark in the above example) will have a dx/x spectrum.

(2) The π^+ arises directly from the collision subprocess, i.e., from the annihilation of a valence u quark from the beam with a $\bar{d}$ in the sea of the target.[30] In this case, we have $dN/dx\sim G_{u/p}(x)\sim(1-x)^3$. This last mechanism can be used as an explicit justification of Ochs[17] conjecture that the quark distributions directly determine the meson distributions.

Thus we predict that

$$\frac{dN}{dx}(pp\to\pi^+X)\sim\begin{cases}(1-x)^3 & \text{quark exchange or annihilation}\\ (1-x)^5 & \text{gluon exchange or dissociation}\end{cases}\,.\tag{8}$$

In both mechanisms (1) and (2), the ratio of π^+ and π^- production is given by

$$\frac{dN/dx(pp\to\pi^+X)}{dN/dx(pp\to\pi^-X)}=\frac{G_{u/p}(x)}{G_{d/p}(x)}\,.\tag{9}$$

Ochs[17] has shown that this result is in reasonably good agreement with experiment. (Note, however, that we do not predict $G_{\pi^+/p}\propto G_{u/p}$.) The ISR results,[26,28] Eq. (5), appear to favor the quark-exchange (annihilation) prediction. We should emphasize that even in this model, those events with a large rapidity gap characteristic of diffractive dissociation are still predicted to have the $(1-x)^5$ behavior. We do not attempt to predict the relative normalization of these contributions.

III. CORRELATIONS IN DOUBLE-FRAGMENTATION CROSS SECTIONS

The quark-exchange (or annihilation) model can lead to a dramatic long-range correlation between the fragmentation behavior of the beam and target in the double inclusive cross section $A+B\to C+D+X$. The correlation occurs whenever there is a mismatch between the remaining quarks in the Fock space of A and B when C and D are produced. In this section we shall make the additional strong assumption that the exchanged wee quark can be found in Fock-space states of the hadron with the minimum number of $q\bar{q}$ pairs.

A typical prediction which we discuss in detail below is

$$\frac{dN}{dx_1dx_2}(pp\to\pi^+_{(1)}\pi^+_{(2)}X)\sim(1-x_1)^3(1-x_2)^7+(1\to 2)\tag{10}$$

which represents a strong violation of factorization (7). The scaling variables x_1 and x_2 are evaluated in the fragmentation region of the beam and target, respectively. In fact, the predicted behavior in cases where there is a quark mismatch is usually

$$\frac{dN/dx_C dx_D}{(dN/dx_C)(dN/dx_D)} \sim (1-x_D)^4 \qquad (11)$$

for small $1-x_C$.

We begin the detailed discussion with the special example $\pi^+ p \to K^+ + \pi^- + X$. Consider first the dissociation or gluon-exchange model [Fig. 2(a)]. For $\pi^+ \to K^+$, the minimum number of quark spectators is two, for $p \to \pi^-$; the minimum number is three. Thus for this model we have

$$\frac{dN}{dx_K dx_\pi}(\pi^+ p \to K^+ \pi^- X) \sim (1-x_K)^3(1-x_\pi)^5 ,$$

(gluon exchange, dissociation) . (12)

In the case of the quark-exchange mechanism, Fig. 2(b), let us first consider $1-x_C$ small. The minimal π^+-fragmenting state is $u\bar{d}s\bar{s}$. The $\bar{d}$ quark in the π^+ can interact leaving the $u\bar{s}s$ system as the jet J_A which provides the K^+ fragment plus one quark spectator:

$$\frac{dN}{dx_K}(\pi^+ p \to K^+ X) \sim (1-x_K) . \qquad (13)$$

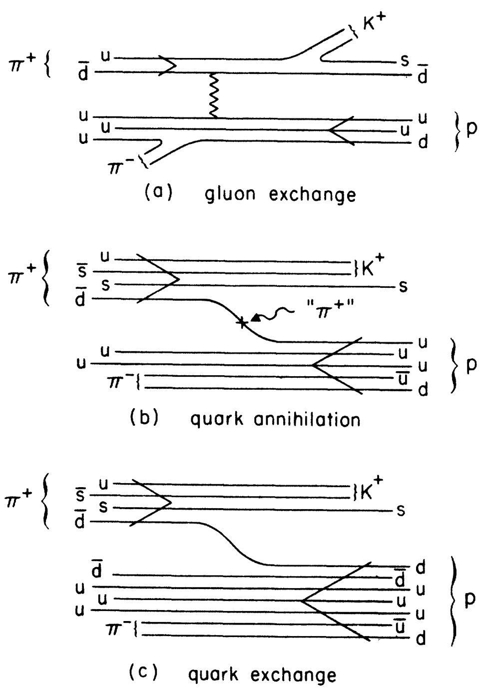

FIG. 2. Contributions to $\pi^+ p \to K^+\pi^- X$ from (a) gluon exchange, (b) $q\bar{q}$ annihilation, and (c) quark exchange. Only the minimum quark lines are shown.

For the double-fragmentation process (12), the proton begins in a Fock state $|uud\bar{u}u\rangle$ from which the π^- can be fragmented. One of the u quarks in the $\bar{D}B$ state $|uuu\rangle$ which is not part of the π^- can interact with the $\bar{d}$ of the $\bar{C}A$ state $|\bar{d}s\rangle$ by annihilation (*not exchange*) into a π^+. Thus the jet J_B will be $|uud\bar{u}\rangle$, and there will be two spectators for the π^- emission. This yields the factorizing form

$$\frac{dN}{dx_K dx_\pi}(\pi^+ p \to K^+ + \pi^- + X) \sim (1-x_K)(1-x_\pi)^3 ,$$

($q\bar{q}$ annihilation) (14)

However, if we insist on quark exchange, Fig. 2(c) (or annihilation into a flavorless state), then the $\bar{D}B$ state must be augmented by a $\bar{d}d$ pair so that J_B consists of $|uuu\bar{d}d\bar{u}\rangle$, and four spectators will be left after π^- emission—severely increasing the $(1-x_\pi)$ power[31]: For $(1-x_K)$ small compared to $1-x_\pi$),

$$\frac{dN}{dx_K dx_\pi}(\pi^+ p \to K^+ + \pi^- + X) \sim (1-x_K)(1-x_\pi)^7 ,$$

(q exchange) . (15)

We now state the general rules for the quark-exchange and annihilation models:

(a) Consider the $\bar{C}A$ overlap state (the minimal quark state with the quantum numbers of $\bar{C}A$) and compute $n_{\bar{C}A}$ (the number of quarks in $\bar{C}A$); then for both single and double fragmentation the $(1-x_C)$ power is

$$(1-x_C)^{2n_{\bar{C}A}-3} ,$$

after accounting for the interacting quark.

(b) For double fragmentation $A+B\to C+D+X$, consider the $\bar{D}B$ state and find $n_{\bar{D}B}$:

(i) If $\bar{D}B$ can interact with $\bar{C}A$ either via quark exchange[3] or annihilation then we obtain $(1-x_D)^{2n_{\bar{D}B}-3}$.

(ii) If there is a mismatch and an extra $q\bar{q}$ pair is necessary in B then the power is $(1-x_D)^{2n_{\bar{D}B}+1}$. There is, of course, the additional contribution, obtained by the symmetrical process which dominates when $(1-x_D)$ is small compared to $(1-x_C)$.

We also summarize the earlier results for gluon exchange or dissociation. The $A\to C$ and $B\to D$ fragmentations are independent of one another and we have

$$(1-x_C)^{2n_{\bar{C}A}-1}$$

and

$$(1-x_C)^{2n_{\bar{C}A}-1}(1-x_D)^{2n_{\bar{D}B}-1}$$

for single and double fragmentation distributions, respectively.

One could also consider the possibility of central baryon production, analogous to $q\bar{q}$ annihilation, obtained from the fusion of a slow (qq) system from A or B with a quark from the other. Although this can lead in some cases to favorable fragmentation powers, the fractional percentage of central baryon production is known to be very small.

Finally we note that all the predicted fragementation powers should be essentially independent of the momentum transfer in the reaction, at low t or p_T. This is because the square of the momentum transfer t_{AC} or t_{BD} tends to be absorbed by the exchanged quark or gluon prior to the beginning of fragmentation. We emphasize that the triple-Regge contributions which usually produce $(1-x)$ power laws dependent on t are distinguishable from the t-independent jet-fragmentation contributions.[32] In general both contributions can be important when x is very near one, although the jet-fragmentation terms appear to dominate at high energies at moderate x.

IV. COMPARISON WITH EXPERIMENT

In order to summarize the model predictions, we present the power laws for various fragmentation processes in tabular form. Three reactions are considered in Tables I, II, and III[33]: pp interactions; π^+p interactions with $\pi^+ \to C$; and $p\pi^+$ with $p \to C$. Tables IV and V give the fragmentation powers for the gluon-exchange and dissociation models, a summary of the less restrictive annihilation predictions of Tables I, II, and III, as well as the triple-Regge $t=0$ predictions, and predictions for central baryon production. It should be noted that one subtlety has been omitted from the Tables: Deep-inelastic scattering indicates that effectively, $G_{d/p} \sim (1-x)G_{u/p}$ for $x \to 1$ (although this could be due to resonance or scale-breaking effects). Thus the $p \to \pi^-$ fragmentations power predictions should be ~1 unit higher than the results of Table IV.

TABLE I. Quark annihilation (exchange) predictions. Fragmentation powers for $dN/dx_C\,dx_D\,(pp \to C+D+X) \sim (1-x_C)^{N_C}(1-x_D)^{N_D}$. The number for N_D in parentheses applies to the more restrictive quark-exchange model.

D \ C	π^+	π^-	K^+	K^-	n	Λ	$\bar{p}$	$\bar{\Lambda}$
	$N_C=3$	3	3	7	1	1	9	9
π^+	$N_D=7$	7	7	3 (7)	3	3 (7)	7	7
π^-	$N_D=7$	7	7	3 (7)	3 (7)	3 (7)	7	7
K^+	$N_D=7$	7	7	3	3	3	7	7
K^-	$N_D=7$ (11)	7 (11)	7	7 (11)	7	7 (11)	7 (11)	7
p	$N_D=1$	1	1	1	1	1	1	1
n	$N_D=1$	1 (5)	1	1	1 (5)	1 (5)	1	1
Λ	$N_D=1$ (5)	1 (5)	1	1 (5)	1 (5)	1 (5)	1 (5)	1
$\bar{p}$	$N_D=13$	13	13	9	9	9 (13)	13	13
$\bar{\Lambda}$	$N_D=13$	13	13	9 (13)	9	9	13	13

TABLE II. Quark annihilation (exchange) predictions. Fragmentation powers for $dN/dx_C\,dx_D\,(\pi^+p \to C+D+X) \sim (1-x_C)^{N_C}(1-x_D)^{N_D}$.

D \ C	π^0	K^+	$\bar{K}^0$	p	Λ	$\bar{p}$	$\bar{\Lambda}$
	$N_C=1$	1	1	3	3	3	3
π^+	$N_D=3$	3	3(7)	3	3	7	7
π^-	$N_D=3$ (7)	3 (7)	3(7)	3	3 (7)	7	7
K^+	$N_D=3$	3	3	3	3	7	7
K^-	$N_D=7$	7	7 (11)	7	7	7 (11)	7
p	$N_D=1$	1	1	1	1	1	1
n	$N_D=1$ (5)	1 (5)	1 (5)	1	1 (5)	1 (5)	1 (5)
Λ	$N_D=1$ (5)	1	1 (5)	1	1 (5)	1 (5)	1
$\bar{p}$	$N_D=9$	9	9 (13)	9	9	13	13
$\bar{\Lambda}$	$N_D=9$	9	9	9	9	13	13

TABLE III. Quark annihilation (exchange) predictions. Fragmentation powers for $dN/dx_C dx_D\,(p\pi^+ \to C+D+X) \sim (1-x_C)^{N_C}(1-x_D)^{N_D}$.

$D \backslash C$	π^+	π^-	K^+	K^-	n	Λ	$\bar{p}$	$\bar{\Lambda}$
	$N_C=3$	3	3	7	1	1	9	9
π^+	$N_D=1$	1	1	1	1	1	1	1
π^0	$N_D=1$	1	1	1	1 (5)	1 (5)	1	1
K^+	$N_D=1$	1 (5)	1	1	1 (5)	1	1	1
$\bar{K}^0$	$N_D=1$ (5)	1 (5)	1	1 (5)	1 (5)	1 (5)	1 (5)	1
p	$N_D=3$	3	3	3	3 (7)	3	3	3
Λ	$N_D=3$	3 (7)	3	3	3 (7)	3	3	3
$\bar{p}$	$N_D=7$	7	7	3	3 (7)	3 (7)	7	7
$\bar{\Lambda}$	$N_D=7$	7	7	3 (7)	3 (7)	3	7	7

From Tables IV and V a number of interesting facts emerge. First, an accurate determination of the $(1-x_C)$ power in $dN/dx(A+B\to C+X)$ and its p_T dependence should eliminate most of the models. The results of the Table correspond to minimum powers so that some mixture of higher powers could conceivably occur. Thus we can argue that the t-independent ISR results[28] $dN/dx \sim (1-x_C)^{3.5}$ for $p\to\pi^+$ and K^+, first of all, do not favor Regge theory and central baryon production. It is also below the minimal gluon-exchange or dissociation prediction by a significant amount, given the high statistics of the experiment. The results are, however, consistent with the quark-exchange or annihilation mechanisms. The ISR results $dN/dx(p\to\pi^-)\sim(1-x_C)^4$ and $dN/dx(p\to K^-)\sim(1-x_C)^{6\pm1}$ (again, apparently independent of t) also support the quark-exchange or annihilation models.

The results from the Fermilab experiments require more discussion. Anderson *et al.*[27] have analyzed $dN/dx(pp\to\pi^+X)$ in terms of powers of $1-x_L$. Their results, which are averaged over t, cannot rule out the triple Regge model predictions. However, as we stated earlier, the variable which best approximates the light-cone fraction (relevant to our approach) at lower energies is the radial scaling variable x_R. Taylor *et al.*[26] have analyzed all available $p\to\pi^+,\pi^-,K^+,K^-,\bar{p}$ data using x_R (including very low Brookhaven energies) and have found complete consistency between ISR and Fermilab energies with p_T-independent powers which agree with those obtained by Sens *et al.*[28] [consistent with the quark-exchange or annihilation predictions]. Analyzed in terms of x_L there is a manifest inconsistency between ISR (where $x_L\approx x_R$) and Fermilab $(1-x_L)$ powers for $d\sigma/dx(pp\to\pi^+X)$.

The experimental results of the x_R analysis and of the individual experiment of Sens *et al.*[28] are given in Table IV. Results are also available from Fermilab for $dN/dx(pp\to\Lambda,\bar{\Lambda}X)$,[34] the latter of which falls at least as fast as $(1-x_{\bar{\Lambda}})^{10}$, slightly above the quark-annihilation prediction, but not inconsistent with either quantum chromodynamics model. The Regge prediction is not known since $A\bar{C}$ is exotic for the $p\to\bar{\Lambda}$ transition. The quark-exchange (or annihilation) prediction is consistent with the $pp\to\bar{p}X$ data, as summarized by Taylor *et al.* in Table IV. Note that the ratios of all the measured fragementation cross sections are within the range of the predictions of the fragmentation rules, although ratios alone do not discriminate between the gluon-exchange (or dissociation) and quark-exchange models: The data from ISR (analyzed by Taylor *et al.*) give

$$\frac{dN/dx(pp\to K^-X)}{dN/dx(pp\to K^+X)} \sim (1-x)^{3.5\pm1},\quad 0<x<0.8$$
$$\frac{dN/dx(pp\to \bar{p}X)}{dN/dx(pp\to pX)} \sim (1-x)^{7.5\pm1},\quad 0<x<0.3\ . \tag{16}$$

The corresponding predictions for both gluon and quark exchange are $(1-x)^4$ and $(1-x)^8$, respectively, taking $dN/dx(pp\to pX)\sim(1-x)$ for $x\lesssim 0.3$. However, we note again that the absolute powers of $(1-x)$ for $dN/dx(pp\to\pi^+$ or $K^+,\pi^-)$ are observed to be in the range 2.8 to 3.5 and thus favor the quark-exchange (annihilation) model.

Although the results for single-fragmentation distributions in $pp\to hX$ seem to be reasonably consistent with the predictions of the quark-exchange model, other tests involving the use of other beams (mesons, photons, etc.) are necessary. It should be noted that at low energies, complications are expected from finite-mass and phase-space effects; and at high $x\gtrsim 0.9$, triple-Regge contributions must become important.[32] In some cases resonance production and decay can also provide a background to the direct fragmentation distributions. In view of the possible complications and backgrounds to the single fragmentation distributions, the importance of the double-

TABLE IV. Fragmentation powers for the proton. I. (dN/dx_C) $(p=A\rightarrow C+X)\sim(1-x_C)^{N_C}$; II. (dN/dx_D) $(p=B\rightarrow D+X)\sim(1-x_D)^{N_D}$ for given $A\rightarrow C+X$. $(1-x_C)\ll(1-x_D)$.

Model	Fragment (C or D)	π^+	π^-	K^+	K^-	n	Λ	$\bar{p}$	$\bar{\Lambda}$
Gluon or dissociation	$N_C=N_D=$	5	5	5	9	3	3	11	11
	N_D values are independent of $A\rightarrow C$								
Quark annihilation	$N_C=$	3	3	3	7	1	1	9	9
	$N_D=$	3	3	3	7	1	1	9	9
	for $(A\rightarrow C)=[(\pi^+\rightarrow\pi^0,K^+,\bar{K}^0,\Lambda,p,n);\ (K^+\rightarrow \text{all});\ (\pi^-\rightarrow\pi^0,K^-,K^0,K^+,p,\Lambda);\ (p\rightarrow K^-,n,\Lambda)]$								
	$N_D=$	7	7	7	7	1	1	13	13
	for $(A\rightarrow C)=[(\pi^+\rightarrow\bar{p},\bar{\Lambda});\ (\pi^-\rightarrow\bar{p},\bar{\Lambda});\ (p\rightarrow\pi^+,\pi^-,K^+,\bar{p},\bar{\Lambda})]$								
Triple Regge $(t=0)$	$N_C=N_D=$	~ 2	~ 2.3	~ 2.5	Exotic trajectory	~ 0	~ 0.3	Exotic trajectories	
Experiment (average values)									
Sens ISR (Ref. 28)	$N_C=$	3.5 ± 0.3	3.9 ± 0.4	3.1 ± 0.3	6.0 ± 1.0	...	...	...	...
Taylor x_R analysis (Ref. 26)	$N_C=$	3.1 ± 0.3	4.3 ± 0.3	2.8 ± 0.6	6.3 ± 1.0	...	...	8.5 ± 1	...
Fermilab x_L analyses	$N_C=$	2.3[a]	...	2.3[a]	...	1[b]	...	...	≥ 10[b]

[a] From Ref. 27.
[b] From Ref. 34.

fragmentation measurements $A+B\rightarrow C+D+X$ becomes apparent because of the possibility of a long-range correlation in flavor induced by the quark-exchange (annihilation) mechanisms. The predictions for several cases are given in Tables IV and V, but because of the possible high statistics, the single experiment $pp\rightarrow\pi^+\pi^+X$ is probably the best candidate for deciding whether the quark-exchange mechanism is correct. In this case $dN/dx(pp\rightarrow\pi^+X)\sim(1-x)^3$ for single fragmentation, but for $1-x_1$ small the double-fragmentation prediction is

$$\frac{dN/dx_1dx_2(pp\rightarrow\pi^+_{(1)}\pi^+_{(2)}X)}{dN/dx_1(pp\rightarrow\pi^+_{(1)}X)}\propto(1-x_2)^7\ ,$$

since one requires at minimum a seven-quark Fock-space state of the proton in order to provide the annihilating $q\bar{q}$ system or wee-quark exchange. We note that there is also a background factorizing contribution $\sim(1-x_1)^5(1-x_2)^5$ from dissociation contributions, but this should be negligible at small $1-x_1$. We also see from Table I, that the process $pp\rightarrow\pi^+K^-X$ can discriminate between the quark-exchange versus the less restrictive annihilation mechanism: at small $1-x_1$,

$$\frac{dN/dx_1dx_2(pp\rightarrow\pi^+_{(1)}K^-_{(2)}X)}{dN/dx_1(pp\rightarrow\pi^+_{(1)}X)}\propto\begin{cases}(1-x_2)^{11} & q \text{ exchange}\\(1-x_2)^7 & q\bar{q}\rightarrow\text{mesons}\end{cases}.$$

Many other possibilities for distinguishing these two mechanisms using double fragmentation are apparent from Tables I–V.

Correlations involving two fast particles in the same fragmentation direction can also be an important discriminant of models.[35] In the case of proton fragmentation it is easy to find Fock states containing $\pi^+\pi^+$, or $\pi^+\pi^-$ but not $\pi^-\pi^-$. Defining $x=x_{\pi_1}+x_{\pi_2}$, the natural predictions of the quark-exchange or annihilation model approach are $(1-x)^3$ for $\pi^+\pi^\pm$ and $(1-x)^7$ for $\pi^-\pi^-$ excluding resonance decay contributions. These predictions would not be expected to hold in the $q\bar{q}$ fusion model (II) discussed in Sec. II. The gluon-exchange model predicts $(1-x)^5$ and $(1-x)^9$, respectively.

We also wish to emphasize the importance of measurements of the fragmentation distributions for particles produced in association with massive lepton pairs. Assuming the dominance of the Drell-Yan $q\bar{q}\rightarrow l\bar{l}$ mechanism, the ambiguity of the interaction mechanism is eliminated and the fragmentation powers can be precisely tested. The physics of the fragmentation region is continuous for $\mathfrak{M}^2_{l^+l^-}/s\rightarrow 0$ if the quark annihilation or exchange mechanism correctly describes the total hadronic cross section. It is also very interesting to compare the fragmentation distributions on and

TABLE V. Fragmentation powers for the π^+. I. (dN/dx_C) $(\pi^+ = A \to C+X) \sim (1-x_C)^{N_C}$; II. $(dN/dx_D)(\pi^+ = B \to D+X) \sim (1-x_D)^{N_D}$ for given $A \to C+X$. $(1-x_C) << (1-x_D)$.

Model	Fragment (C or D)	π^0	K^+	$\bar{K}^0$	p	Λ	$\bar{p}$	$\bar{\Lambda}$
Gluon or dissociation	$N_C = N_D =$	3	3	3	5	5	5	5
	N_D values are independent of $A \to C$							
Quark annihilation	$N_C =$	1	1	1	3	3	3	3
	$N_D =$	1	1	1	3	3	3	3
	for $(A \to C) = (p \to K^-, n, \Lambda)$							
	$N_D =$	1	1	1	3	3	7	7
	for $(A \to C) = (p \to \pi^+, \pi^-, K^+, \bar{p}, \bar{\Lambda})$							
Triple Regge ($t=0$)	$N_C = N_D =$	0	0	0	~ 2.0	~ 2.5	~ 2.3	~ 2.5
Experiment Fermilab x_L analysis (Ref. 27)	$N_C =$	...	0.6 ± 0.12	...	2.0 ± 0.3	...	...	...

off resonance $(\rho, \phi, \psi, \Upsilon)$ production in order to discriminate the various proposed mechanisms.

In summary, we have demonstrated that observations as basic as single- and double-fragmentation power laws may well unravel the nature of the underlying dynamical mechanism responsible for hadronic collisions. The success of the quark-exchange (annihilation) model in yielding a natural explanation of multiplicity universality and of the observed single-fragmentation power laws at the ISR argues strongly in its favor. The difference between the Fermilab and ISR results is eliminated by using the radial-scaling variables. An open theoretical question is whether quark confinement will affect the fragmentation predictions of the type we have given. Furthermore, is one justified in abstracting dynamical predictions from the simplest gluon (quark) diagrams? We have assumed that, as appears to be the case in the one-space one-time quantum-chromodynamic calculations,[36] the full hadron dynamics will mimic the naive quark level diagrams in their dynamical dependence if not in normalization.

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[30]Alternately, one can consider the π^+ to arise from the $u\pi^+ \to \pi^+ u$, $u\bar{u} \to \pi^+ M^-$, $u\bar{d} \to \pi^+ M^0$ subprocesses of the constituent-interchange and quark-fusion models (see Refs. 6 and 7). Again, $dN/dx(pp \to \pi^+ X) \propto G_{u/p}(x)$. The $q\bar{q}$ fusion model for vector-meson production has also been discussed by K. Böckmann, Bonn Report No. BONN-HE-77-25, 1977 (unpublished), and references therein.

[31]The predictions for quark exchange also apply to mechanism (2) where one of the mesons is produced directly by $q\bar{q}$ annihilation.

[32]At large negative t, Regge trajectories are predicted in the constituent-interchange model (Ref. 6) to fall to negative integers: $\alpha(t) \to -n$. This ensures continuity with fixed-angle scaling laws.

[33]The tables can be easily extended to the case of π^- (or K^-) production in $\pi^+ p$ reactions using the dominant resonance decay with $C = \rho_0 \to \pi^- + \pi^+$ (or $C = \bar{K}^{0*} \to K^- + \pi^+$). This adds roughly 1 unit to the $(1 - x_C)$ power since the decays are two body. The predictions for ϕ, D, $\bar{D}$, ψ, etc. and also other beam types may be easily worked out from the rules given in Sec. II. We neglect here indirect production of particles from the decay of baryon resonances.

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