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Scale-violating quark model for large-p_T processes. Single-hadron inclusive reactions*

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A model of quark-parton distributions with a number of features dictated by asymptotically free field theories (AFFT) is presented. It leads to a logarithmic pattern of scale violations which, although not exactly the same, is close to that of AFFT and accounts for recent data on lepton-nucleon inelastic structure functions. The model, together with the basic Berman-Bjorken-Kogut mechanism, accounts fairly well for large-transverse-momentum (p_T) data on single-hadron production in proton-proton collisions. At least in order of magnitude, the essential large-p_T production parameters can be determined via AFFT considerations and lepton-nucleon deep-inelastic data; there is some difficulty, however, in understanding the magnitude of baryon production. Simple explanations of the p_t dependence of the data on inclusive-cross-section ratios for charged-hadron production are also given.

I. INTRODUCTION

A theory of elementary processes based on a fundamental interaction between spin-$\frac{1}{2}$ particles (quarks) and vector bosons (gluons) has become recently very appealing. In this context the parton model of Berman, Bjorken, and Kogut (BBK),[1] that unifies large-transverse-momentum (p_T) hadron reactions and deep-inelastic lepton-hadron processes, is of particular importance. Furthermore this model predicts a two-jet structure for the large-p_T events that now seems to be well supported by experiment.[2,3,4]

Unfortunately the original BBK model, constructed on the basis of exact Bjorken scaling for the inelastic structure functions, naturally leads to an inclusive cross section for, say, $pp \to \pi + X$ decreasing as p_T^{-4}, in contrast with the known experimental behavior $\propto p_T^{-8}$. There are other important quark models, notably the constituent-interchange model (CIM),[5] leading to $\propto p_T^{-8}$; however, these models arbitrarily neglect the quark-gluon interaction and also lead to essentially one-jet structure, thus facing difficulties *vis-à-vis* the large-p_T correlation data.[6]

An important theoretical development has been the realization that asymptotically free field theories (AFFT) as well as conventional ones lead to breaking of Bjorken scaling, i.e., to inelastic structure functions which in the Bjorken limit depend not only on the scaling variable x but also on the 4-momentum Q of the probe. This scale breaking is now confirmed by experiment[7,8] and in fact with a patterm quite consistent with the logarithmic Q^2 dependence of AFFT.

It is then of particular interest to examine whether the original BBK mechanism (i.e., quark-quark scattering via single-gluon exchange) properly supplemented by scale breaking (i.e., quark probability distributions depending on both x and Q^2) can account for the existing wealth of experimental data on large-p_T hadron production. This problem was first attacked in Ref. 9 by means of a particular AFFT solution, but unfortunately the conclusion was negative. More recently Hwa *et al.*[10] have shown that the introduction of scale breaking in a phenomenological way leads to inclusive $pp \to \pi + X$ cross sections with a p_T and s dependence in accord with available data in the CERN ISR energy range. However, the scaling violation of Hwa *et al.* is powerlike rather than logarithmic in Q^2; and this, although it does not contradict present data, is in disagreement with AFFT.

The purpose of this work is, first of all, to introduce a simple scale-breaking pattern which, although not exactly the same, is very close to that of AFFT and certainly logarithmic in Q^2. Thus we present a specific model for quark-parton distributions depending on both x and Q^2 (Sec. II). The model is in accord with data on lepton-nucleon deep-inelastic structure functions; in particular we present detailed comparisons of our predicted $\nu W_2^p(x, Q^2)$ with recent data for several Q^2 (Sec. II). Then using the same quark-parton distributions and the basic BBK mechanism we calculate single-hadron inclusive cross sections at large p_T (Sec. III). We present detailed comparison with ISR and Fermilab data, and we show that in our approach all the essential parameters determining large-p_T production of hadrons are determined, at least in order of magnitude, from either theoretical (AFFT) considerations or lepton-nucleon deep-inelastic data (Sec. IV). Also we show that the forms of our quark distributions and fragmentation functions offer very simple explanations of the p_T dependence of the data on inclusive-cross-section ratios for charged-particle production (Sec. IV). Finally, in the Appendix we determine the $Q^2 \to \infty$ behavior of the moments of our structure functions.

While this work was in progress we became aware of certain independent similar analyses of large-p_T processes.[11] Those analyses use quark-parton distributions and functions $F_{a/A}$, $G_{C/c}$ with no scale breaking (only x dependent or y dependent), but modify the basic BBK quark-quark scattering so that it contributes $\propto p_T^{-8}$. However, so far, these modifications are with no theoretical basis; moreover, exact scaling is now contradicted by inelastic lepton-nucleon data, in particular on νW_2^p.

This paper focuses on single-hadron production in proton-proton collisions. We intend to present elsewhere detailed analysis of two-hadron production (correlations) and of single-hadron production in pion-proton collisions. We only state that our first results on correlations are very satisfactory.[12]

II. QUARK DISTRIBUTIONS AND SCALE BREAKING

To describe hadron production by protons we need the distribution fuctions for the u, d, s, $\bar{u}$, $\bar{d}$, $\bar{s}$ quarks. We assume an SU(3)-symmetric sea contribution so that

$$u=2v_u(x,Q^2)+t(x,Q^2), \quad d=v_d(x,Q^2)+t(x,Q^2)\,, \tag{2.1}$$

and

$$\bar{u}=\bar{d}=s=\bar{s}=t(x,Q^2). \tag{2.2}$$

Then conservation of charge and third component of isospin requires[13]

$$\int_0^1 v_i(x,Q^2)dx=1\,, \tag{2.3}$$

where $i=u,d$; with (2.2) strangeness is also conserved.

For the valence distributions v_i we adopt the following scale-breaking form:

$$v_i(x,Q^2)=\beta_i(Q^2)\left(\frac{Q^2}{Q_0^{\,2}}\right)^{-bx}x^{-1/2}p_i(x), \tag{2.4}$$

and for the sea distribution

$$t(x,Q^2)=\beta_t(Q^2)\left(\frac{Q^2}{Q_0^{\,2}}\right)^{-bx}x^{-1}(1-x)^{11/2}. \tag{2.5}$$

The constants b and Q_0 (the same for all u, d, and t) enter as free parameters; nevertheless, as we discuss in Sec. IV, the values we adopt (Table I) are in accord with independent determinations. The functions $p_i(x)$ are given in Table I; as can be seen at $Q=Q_0$, the x dependence and magnitude of our quark distributions is taken from a modified Kuti-Weisskopf model[14] with only the sea contribution changed according to Ref. 15 to account for the results of certain neutrino experiments.[16] In view of (2.3), the functions $\beta_i(Q^2)$ are given by

$$\beta_i^{-1}(Q^2)=\int_0^1\left(\frac{Q^2}{Q_0^{\,2}}\right)^{-bx}x^{-1/2}p_i(x)dx. \tag{2.6}$$

The function $\beta_t(Q^2)$ is determined below.

We note that, in view of the conditions (2.3), the distributions have the following basic property: As Q^2 increases, at small x ($\lesssim 0.2$), v_i increase, but at large x, v_i decrease, so that the area under $v_i(x,Q^2)$ is kept constant. This is an essential feature of certain scale-breaking parton models which also satisfy basic requirements of AFFT or of conventional field theories[17]; and it is one of the main motivations for our choice of the scale breaking in the form (2.4). Our sea distribution (2.5) (as specified just below) has a similar property.

In terms of the quark distributions v_i and t the structure function $\nu W_2^p(x,Q^2)$ for $ep\to eX$ or $\mu p \to \mu X$ takes the form

$$\nu W_2^p(x,Q^2)=\frac{x}{9}\left[4v_u(x,Q^2)+v_d(x,Q^2)+12t(x,Q^2)\right]. \tag{2.7}$$

Its nth moment is

$$M_n(Q^2)=\int_0^1 x^n \nu W_2^p(x,Q^2)dx. \tag{2.8}$$

Subsequently we set

$$b\ln(Q^2/Q_0^{\,2})=\xi. \tag{2.9}$$

Consider first the contribution $M_n^v(\xi)$ to M_n from either of the valence parts v_i; this has the form

$$M_n^v(\xi)\sim\beta_i(\xi)\int_0^1 e^{-\xi x}x^{n+1/2}p_i(x)dx \tag{2.10}$$

with $\beta_i(\xi)$ given by (2.6). It is shown in the Appendix that, in the limit $\xi\to\infty$,

$$M_n^v(\xi)\sim\xi^{-n-1}. \tag{2.11}$$

Thus the leading contributions to the nth moment M_n from the valence parts are of order $(\ln Q^2)^{-n-1}$.

TABLE I. Functions and constants determining quark distributions.

Type of quark	Function $p_i(x)$
valence $i=u$	$0.895(1-x)^3(1+2.3x)$
valence $i=d$	$1.107(1-x)^{3.1}$
sea: $c_0=0.2$, $c_1=5/56$	
In all valence and sea: $b=1.2$, $Q_0^{\,2}=1.5\ \text{GeV}^2$	

The contribution $M_n^t(\xi)$ from the sea is

$$M_n^t(\xi) = \tfrac{12}{9}\beta_t(\xi)\int_0^1 e^{-\xi x}x^n(1-x)^{11/2}dx \qquad (2.12)$$

and, for $\xi \to \infty$,

$$M_n^t(\xi) \sim \tfrac{12}{9}\beta_t(\xi)\xi^{-n-1}. \qquad (2.13)$$

To determine the form of $\beta_t(\xi)$ we consider the predictions of AFFT for $M_0(Q^2)$. For a Lagrangian containing n_ψ fermion fields (quarks) of charge Q_j and n_v vector fields (gluons) these theories predict, for $Q^2 \to \infty$,[18,19]

$$M_0(Q^2) \sim \frac{\sum_j Q_j^2}{2n_v + n_\psi} + \frac{A}{(\ln Q^2)^\delta} \qquad (2.14)$$

with A and δ constants, $A>0$ and $0<\delta<1$. Then the simplest choice of β_t leading to essentially this form is

$$\beta_t(\xi) = c_0 + c_1\xi \qquad (2.15)$$

with c_0 and c_1 constants. We take $n_\psi = 4\times 3$(= quark flavors × color) and $n_v = 8$ so that $M_0(Q^2) \to \frac{5}{42}$; this completely fixes c_1 to the value $c_1 = \frac{5}{56}$. The constant c_0 is determined by fitting the data on νW_2^p; its value (Table I) is close to the values of Refs. 14 and 15. With these choices our $M_0(Q^2)$ is of the form (2.14) with $A>0$ and $\delta = 1$.

In view of (2.13) and (2.15), for $\xi \to \infty$,

$$M_n^t(\xi) \propto \xi^{-n}.$$

Comparing with (2.11) we see that the leading term in $M_0(Q^2)$ arises from the sea distribution $t(x,Q^2)$ and not from the valence $v_i(x,Q^2)$. This is in agreement with the results of recent detailed analyses of the contributions to the structure functions from the nonsinglet Wilson operators (associated only with valence) and from the singlet operators (where the sea enters).[19] Then, as Q^2 varies, this property implies a change in the composition of the proton with respect to valence and sea quarks. Notice that this can be directly tested by comparing, e.g., data on the ratio of $e^-p \to e^-K^- + X$ to $e^-p \to e^-\pi^- + X$ and taking into account that K^- originates mostly from the proton sea, whereas π^- mostly from the valence (see also Secs. III and IV).

The resulting expression of νW_2^p is compared with experimental data for various values of Q^2 (Fig. 1)[8]; on the whole agreement is fair. Notice in particular that at small x ($\lesssim 0.2$) νW_2^p increases with Q^2 but at large x it decreases, in agreement with the behavior of $\nu W_2^p(x,Q^2)$ in scale-breaking parton models of Ref. 17. Also, the ratio $\nu W_2^n/\nu W_2^p$ of neutron to proton structure function is reasonably well accounted, as can be seen from the fact that for $Q^2 = Q_0^2$ our valence- and sea-quark distributions are essentially the same as those of Refs. 14 and 15. Better fits are certainly possible by using more complicated $\beta_t(Q^2)$ and x dependences, but this is outside the scope of the present work.

With respect to higher moments AFFT predict, that, for $Q^2 \to \infty$, $M_n(Q^2) \propto (\ln Q^2)^{-d_n}$ with $d_n \propto \ln n$ for $n \gg 1$. In our case Eqs. (2.11), (2.13), and (2.15) imply

$$M_n(Q^2) \propto (\ln Q^2)^{-n}. \qquad (2.16)$$

Thus, apart from $n=0$, our scale breaking somewhat differs but it is still of logarithmic character.

III. SINGLE-HADRON INCLUSIVE CROSS SECTIONS

The differential probability dP that a hadron A is seen by a probe of 4-momentum Q to contain a quark a with a fraction x of its longitudinal momentum is written

$$dP = F_{a/A}(x,Q^2)\frac{dx}{x}. \qquad (3.1)$$

Likewise the differential probability that a quark c is seen by a probe of 4-momentum Q to produce a hadron C carrying a fraction y of the quark's

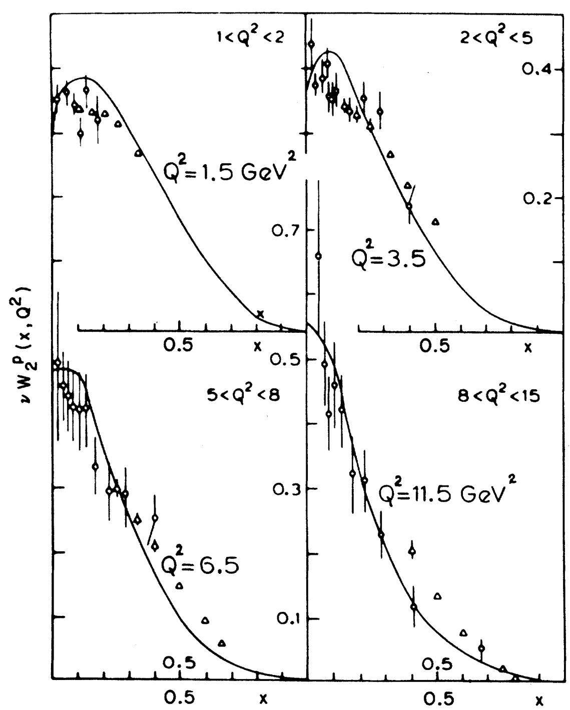

FIG. 1. The structure function νW_2^p as a function of x and Q^2. Data: ○, Ref. 8; Δ, E. M. Riordan *et al.*, SLAC Report No. SLAC-PUB-1634, 1975 (unpublished).

momentum is written

$$dP = G_{C/c}(y, Q^2)\,\frac{dy}{y}\,. \tag{3.2}$$

We assume that the inclusive reaction $AB \to C+X$ takes place via the quark scattering subprocesses $ab \to cd$ of which the differential cross section is denoted by $d\sigma/d\hat{t}$. Then the invariant inclusive cross section for $AB \to C+X$ with C produced at an angle θ in the c.m. of A and B is[1,20]

$$E\frac{d\sigma}{d^3p}(p_T,\theta,s) = \frac{4}{\pi x_T^{\,2}} \sum_{a,b,c} \int_{x_1}^{1} dx_a \int_{x_2}^{1} dx_b\, F_{a/A}(x_a,Q^2) F_{b/B}(x_b,Q^2)\,\frac{d\sigma}{d\hat{t}}\, G_{C/c}(y,Q^2)\,\frac{\eta}{(1+\eta)^2} + (\theta \to \pi - \theta)\,. \tag{3.3}$$

With p_T and m_C the transverse momentum and mass of hadron C we use

$$x_T = \frac{2(p_T^{\,2}+m_C^{\,2})^{1/2}}{\sqrt{s}} \tag{3.4}$$

so that

$$\begin{aligned}
x_1 &= \cot\frac{\theta}{2}\,x_T\left(2-\tan\frac{\theta}{2}\,x_T\right)^{-1},\\
x_2 &= \tan\frac{\theta}{2}\,x_a x_T\left(2x_a-\cot\frac{\theta}{2}\,x_T\right)^{-1},\\
y &= \frac{x_T(1+\eta)}{2x_a\tan(\theta/2)},\\
\hat{t} &= -Q^2 = -\frac{x_a s\tan(\theta/2)}{\tan(\theta/2)/x_a+\cot(\theta/2)/x_b}\,,\\
\eta &= \frac{x_a}{x_b}\tan^2\frac{\theta}{2}\,.
\end{aligned} \tag{3.5}$$

Throughout our work $d\sigma/d\hat{t}$ is taken to be

$$\frac{d\sigma}{d\hat{t}} = \frac{2\pi\alpha_{\text{eff}}^{\,2}}{\hat{s}^2}\left[\frac{1}{\eta^2}+\left(1+\frac{1}{\eta}\right)^2\right] \tag{3.6}$$

and here $\hat{s}^{1/2}$ is the total c.m. energy of the subprocess $ab \to cd$. The expression (3.6) corresponds to scattering of two spin-$\frac{1}{2}$ quarks by exchange of a massless vector gluon. In (3.3) the contributions of the t and u channel have been added incoherently.[20] α_{eff} is the fine-structure constant for the quark-gluon interaction.

For hadron production by protons the probability functions $x^{-1}F_{a/A}(x,Q^2)$ and $x^{-1}F_{b/B}(x,Q^2)$ are given by the quark distributions (2.1) and (2.2). For the quark fragmentation functions $G_{C/c}(y,Q^2)$ we assume a scale breaking similar to that of Eq. (2.5). Thus we take

$$G_{C/c}(y,Q^2) = A(C,c)\beta_{C/c}(Q^2)\left(\frac{Q^2}{Q_0^{\,2}}\right)^{-by}(1-y)^{m(C,c)} \tag{3.7}$$

with b and Q_0 the *same* as in Sec. II (Table I). The function $\beta_{C/c}(Q^2)$ is determined from the condition

$$\beta_{C/c}^{\;-1}(Q^2) = \int_0^1 \left(\frac{Q^2}{Q_0^{\,2}}\right)^{-by}(1-y)^{m(C,c)}dy\,, \tag{3.8}$$

which is much the same as (2.6); $m(C,c)$ and $A(C,c)$ are constants.

Our exponents $m(C,c)$ and the ratios of coefficients $A(C,c)/A(\pi^+,u)$ are given in Table II. To explain and justify our choice of many of these constants we distinguish between the case in which c is a valence quark of hadron C and the case in which c is a nonvalence quark of C.

In the first case $m(C,c)$ is determined from the counting rules of Ref. 20. Thus, when $C=\pi^\pm, K^\pm$, $m(C,c)=1$; when $C=p,\bar{p}$, $m(C,c)=3$. Notice that if we assume that, near $y=1$, $G_{C/c}(y) \sim F_{c/C}(y)$ ("crossing" relation); for $C=\pi^\pm$ and $C=p$ our values of $m(C,c)$ are in accord with the Drell-Yan relation connecting the behavior of $\nu W_2^p(y)$ with the asymptotic behavior of the electromagnetic form factor $F^C(q^2)$ of the hadron C [assuming that $F^\pi(q^2) \propto q^{-2}$ and $F^p(q^2) \propto q^{-4}$].

In the case where c is a nonvalence quark of C, in general the corresponding $G_{C/c}$ are relatively small. Nevertheless analysis of deep-inelastic neutrino-nucleon data[22] shows that, for $C=\pi^\pm$, $G_{C/c}$ are non-negligible. Thus we use a simple form consistent with the results of Ref. 22. For $C=K^\pm$ there is no similar analysis, but rough SU(3) arguments suggest forms essentially similar to those for $C=\pi^\pm$. Finally, for $C=p$ and $\bar{p}$ we anticipate very small contributions and simply take $A(C,c)=0$.

TABLE II. Values of the constants $m(C,c)$ (above) and $A(C,c)/A(\pi^+,u)$ (below).

$c \backslash C$	π^+	π^-	K^+	K^-	p	$\bar{p}$
u	1	1.5	1	2	3	...
	1	0.5	0.5	0.25	10	0
d	1.5	1	2	2	3	...
	0.5	1	0.25	0.25	10	0
s	1.5	1.5	2	1	...	...
	0.5	0.5	0.25	0.5	0	0
$\bar{u}$	1.5	1	2	1	...	3
	0.5	1	0.25	0.5	0	10
$\bar{d}$	1	1.5	2	2	...	3
	1	0.5	0.25	0.25	0	10
$\bar{s}$	1.5	1.5	1	2	...	...
	0.5	0.5	0.5	0.25	0	0

In the case $C=\pi^0$, for every kind of quark c we take

$$G_{\pi^0/c}(y,Q^2)=\tfrac{1}{2}[G_{\pi^+/c}(y,Q^2)+G_{\pi^-/c}(y,Q^2)]. \qquad (3.9)$$

For a given quark c, the functions $G_{C/c}(y,Q^2)$ are subject to the sum rule

$$\sum_{\text{all } C}\int_0^1 G_{C/c}(y,Q^2)dy=1. \qquad (3.10)$$

In view of (3.8) this implies

$$\sum_{\text{all } C} A(C,c)=1. \qquad (3.11)$$

This sum rule together with the values of $A(C,c)/A(\pi^+,v)$ (Table II) fixes the magnitude of the coefficients $A(C,c)$.

IV. RESULTS AND DISCUSSION

With the parameters specified in Tables I and II we have calculated the inclusive cross sections for $pp\to C+X$, where $C=\pi^\pm,\pi^0,K^\pm,p,\bar{p}$.

First, at fixed s, the p_T dependence of these cross sections is sensitive to the value of the parameter b. The choice $b=1.2$, although here on phenomenological grounds, is in reasonable agreement with values resulting from experimental determinations of the quantity ($\omega=1/x$) $\partial^2\ln[\nu W_2^p(\omega,Q^2)]/\partial(\ln\omega)\partial(\ln Q^2)$.[7]

The p_T dependence of $pp\to CX$ is rather insensitive to the exact value of Q_0. Our choice $Q_0^2=1.5$ (Table I) has been dictated by the fact that at $Q=Q_0$ our valence and sea distributions (2.4) and (2.5) become practically the same as those of Refs. 14 and 15, which have been determined mainly from low-Q^2 data.

AFFT predict that α_{eff} decreases logarithmically with the momenta p_i of the particles involved.[18] However, this holds for momenta in the deep Euclidean region (i.e., all p_i^2 large and negative). In our case the quarks are nearly on their mass shells and the behavior of α_{eff} is not known. In our calculations we have fixed α_{eff} to a constant value. In certain similar calculations[9] α_{eff} is taken to decrease $\propto 1/\log Q^2$. Following Ref. 10, we prefer to fix α_{eff} to a constant value. Anyway, a slow variation of α_{eff} with Q^2 does not significantly change our results (see also Ref. 9).

Depending on what we include in the sum over hadrons C of (3.11), the fits we present in Figs. 2–5 correspond to $\alpha_{\text{eff}}=1$ or $\alpha_{\text{eff}}=1.8$ (see below). Although the value of α_{eff} is somewhat higher than that obtained in AFFT applications to deep-inelastic scattering, it is of the same order of magnitude. It can be said that in our approach, at least in order of magnitude, all essential parameters are either theoretically motivated or independently determined.

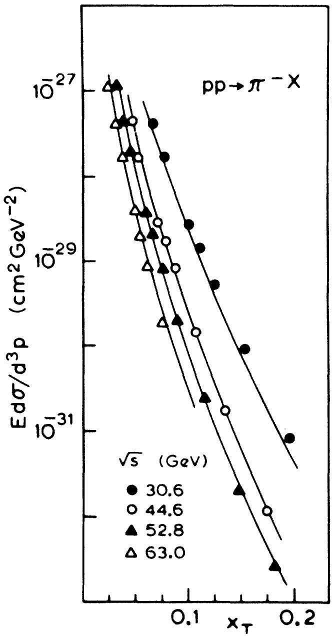

FIG. 2. Invariant inclusive cross sections for $pp\to\pi^-X$ at $\theta=89°$. Data are from B. Alper *et al.* (British-Scandinavian Collaboration); Nucl. Phys. B87, 19 (1975). We do not include comparison with data at $\sqrt{s}=23.4$ GeV, for they correspond to rather small p_T (< 2.34 GeV/c).

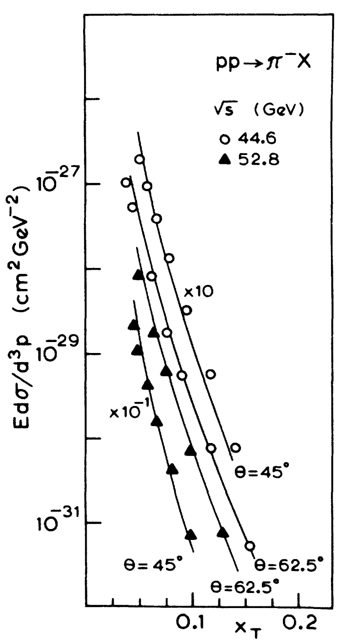

FIG. 3. Invariant inclusive cross sections for $pp\to\pi^-X$ at $\theta=62.5°$ and 45°. Data are from the same source as in Fig. 2.

Figures 2 and 3 present comparison with data on $pp \to \pi^- X$ at several ISR energies and c.m. angles θ. Our model accounts fairly well for the x_T as well as θ dependence, in particular at the highest ISR energies. Figure 4 presents comparison with data on $pp \to \pi^0 X$ and $pp \to (\pi^+ + \pi^-)/2X$ at 52.7 GeV [ISR and 19.4 (Fermilab)]. While the 52.7 data are well accounted for, our predictions at 19.4 fall somewhat too low, showing that our s dependence (at fixed p_T) is somewhat too strong. Nevertheless, we believe that, on the whole, the results are acceptable.

Figure 5 presents comparison with data on ratios of inclusive cross sections for charged-hadron production. The model accounts very well for the cross-section ratios of K^-/π^- and K^+/π^+, but at intermediate x_T predicts a ratio π^+/π^- somewhat too large; nevertheless, the basic trends of the data are well reproduced. Finally, with the values of $A(p,c)$ and $A(\bar{p},c)$ of Table II, the model accounts very well for p/π^+ and reasonably well for $\bar{p}/\pi^-$.

In our model the increase with x_T (at fixed s) of the cross-section ratio of π^+/π^- is due to the fact that the initial protons contain two quarks u (valence quark of π^+), but only one quark d (valence of π^-); as x_T increases the contributions of those valence quarks of the initial protons which are also valence of the final pion dominate over the rest of the contributions.

The increase of K^+/π^+ in the range $p_T \gtrsim 1$ is accounted for in our fit by including the mass m_c in the expression (3.4), i.e., it is understood as a mass effect ($m_K > m_\pi$). As x_T increases beyond $x_T \simeq 0.4$, K^+/π^+ tends to flatten.

The fact that K^-/π^- is predicted to be much smaller than K^+/π^+ is due to the fact that none of the valence quarks of K^- is a valence quark of the colliding hadrons (protons). The same holds for the ratio of $\bar{p}/\pi^-$ compared with p/π^+. The decrease of K^-/π^- with p_T (for $p_T \gtrsim 2.5$) is mainly due to the higher exponent ($=\frac{11}{2}$) of $1-x$ in the sea contribution [Eq. (2.5)].

For $p_T \gtrsim 2.5$, the decrease with p_T of p/π^+ results from the higher exponent $m(p,c)=3$ of $1-y$ in $G_{p/c}(y,Q^2)$ (Table II); and the fast decrease of $\bar{p}/\pi^-$ is due to higher exponents of both $1-x$ in the sea and $1-y$ in $G_{\bar{p}/c}(y,Q^2)$. Finally, for $1 \lesssim p_T \lesssim 2.5$ the increase with p_T of all K^-/π^-, p/π^+, and $\bar{p}/\pi^-$ is understood as a mass effect [due to m_C in (3.4)].

To account for the data on the cross-section ratios of p/π^+ and $\bar{p}/\pi^-$ we had to choose large values for the coefficient ratios $A(p,c)/A(\pi^+,u)$ and $A(\bar{p},c)/A(\pi^+,u)$ [in Table II: $A(p,u)=A(p,d)$ $=A(\bar{p},\bar{u})=A(\bar{p},\bar{d})=10A(\pi^+,u)$]. We have no justification for such a choice. In fact, considering the sum rule (3.11), e.g., for the quark $c=u$ it is

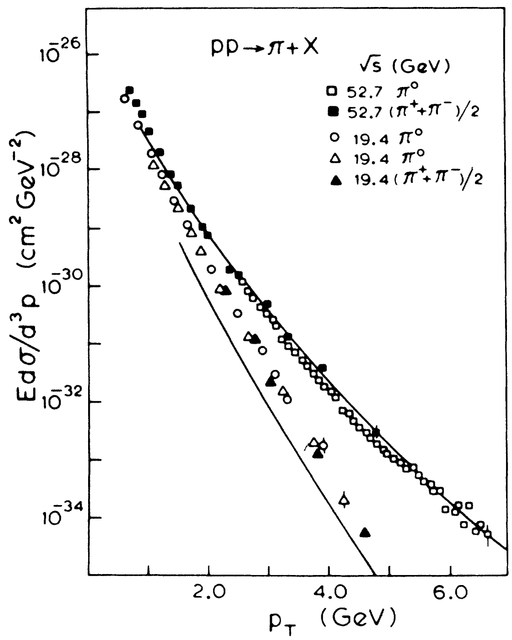

FIG. 4. Invariant inclusive cross sections at $\theta = 90°$ for $pp \to \pi^0 X$ and $pp \to \frac{1}{2}(\pi^+ + \pi^-)X$. Data: □, K. Eggert *et al*. (Aachen-CERN-Heidelberg-Munich Collaboration), Nucl. Phys. B98, 49 (1975); ■, B. Alper *et al*. (British-Scandinavian Collaboration), Nucl. Phys. B100, 237 (1975); ○, D. C. Carey *et al*., Fermilab Report No. Fermilab-Pub-75/20 (unpublished); Phys. Rev. D 14, 1196 (1976); △, G. Donaldson *et al*., Phys. Rev. Lett. 36, 1110 (1976); ▲, J. W. Cronin *et al*. (Chicago-Princeton Collaboration), Phys. Rev. D 11, 3105 (1975).

clear that $A(p,u)$ will give the dominant contribution. This is inconsistent with SPEAR data which indicate that mesons dominate in the quark fragmentation.

There are further problems concerning the energy dependence at fixed p_T of $p+p \to p+X$. For K^-/π^-, $\bar{p}/\pi^-$, as well as p/π^+ the model predicts an *increase* of the cross-section ratio with energy. The predicted energy dependence is not very strong and for K^-/π^- and $\bar{p}/\pi^-$ is in agreement with the data (Fig. 5). However, for p/π^+ the data indicate a *decrease* with energy. Thus although the model appears to correctly predict the p_T dependence at fixed s, it fails with respect to the s dependence.

The same difficulties in connection with $p+p \to p+X$ and $p+p \to \bar{p}+X$ appear in other models based on quark-quark interaction (which completely neglect scale-violating effects.[11] It is possible that for baryon production the quark-quark scattering subprocess does not provide the dominant dynamical mechanism; e.g., bremsstrahlung-type mechanisms[23] may play an important role. Still $p+p \to \bar{p}+X$ remains a puzzle since it is not simply explained by bremsstrahlung.

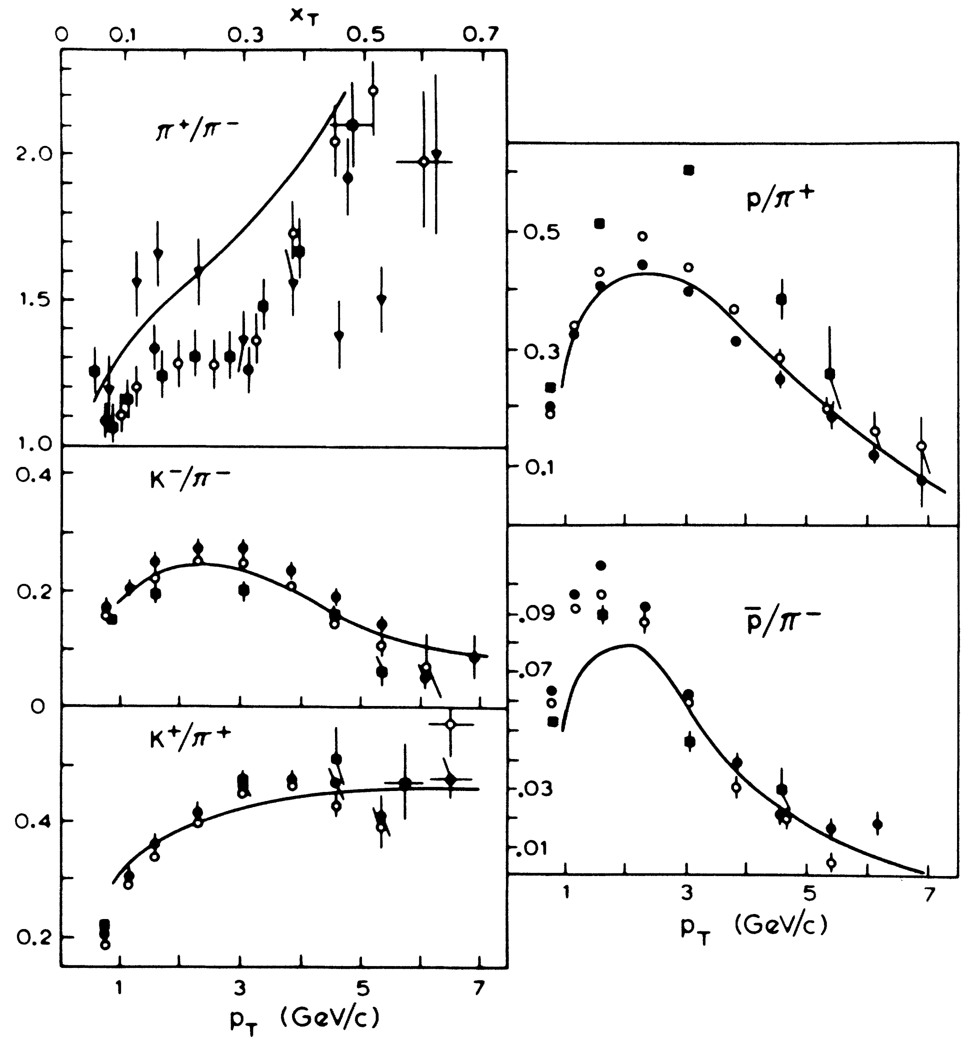

FIG. 5. Ratios of invariant inclusive cross sections at $\theta = 90°$ for $pp \to$ charged hadron $+X$. Data: ■ at incident proton laboratory momentum $p_{inc} = 200$ GeV/c; ○, $p_{inc} = 300$; ●, $p_{inc} = 400$ all from A. Andreasyan *et al*. (Chicago-Princeton Collaboration), submitted to the Tbilisi Conference (see Ref. 3) (unpublished). Data: ▼ correspond to p +nucleon $\to \pi^+/\pi^- + X$ with $p_{inc} = 200$ GeV/c and are taken J. W. Cronin *et al*. (Chicago-Princeton Collaboration), Phys. Rev. D 11, 3105 (1975). All our calculations correspond to $p_{inc} = 300$ GeV/c; the model predicts a weak energy dependence.

Returning to the sum rule (3.11) for $c = u$, the fact that $A(p,u)$ dominates much affects the determination of the absolute magnitude of the other coefficients such as $A(\pi^+,u)$, $A(\pi^0,u)$, etc. In turn, the absolute magnitude of these coefficients is important in the determination of the value of α_{eff} required to fit the data. Thus the value $\alpha_{eff} = 1$ quoted above corresponds to completely neglecting p and $\bar{p}$ in the sum rules (3.11), and $\alpha_{eff} = 1.8$ corresponds to including p and $\bar{p}$.

In closing, we would like to stress that we do not aim at producing perfect fits to either the lepton-nucleon structure functions or to the large-hadron data; in fact, we have made no systematic search for the overall best values of our parameters (within the allowed ranges). However, we believe that our fits are satisfactory enough in quality and detail to support our point, that the BBK mechanism together with scale breaking of logarithms pattern provides a good account of large-p_T hadron production.

APPENDIX

The contributions from the valence parts v_i to the n momentum are given by (2.10). With ξ defined in (2.9), these may be written

$$M_n^v(\xi) = \beta_i(\xi)(-1)^{n+1}\frac{d^{n+1}}{d\xi^{n+1}}\int_0^1 dx\, e^{-\xi x}x^{-1/2}p_i(x). \quad \text{(A1)}$$

In view of Eq. (2.6) the integral in (A1) is just β_i^{-1} so that

$$M_u^\nu(\xi)=(-1)^{n+1}\beta_i(\xi)\frac{d^{n+1}}{d\xi^{n+1}}\beta_i^{-1}(\xi). \tag{A2}$$

The asymptotic expansion for $\beta_i^{-1}(\xi)$ as $\xi\to\infty$ is obtained by changing the integration variable to $y=\xi x$. We find

$$\beta_i^{-1}(\xi)=\xi^{-1/2}\int_0^\xi dy\, e^{-y}y^{-1/2}p_i\left(\frac{y}{\xi}\right) \underset{\xi\to\infty}{\sim} \xi^{-1/2}\int_0^\infty dy\, e^{-y}y^{-1/2}p_i(0)=\sqrt{\pi}\,p_i(0)\xi^{-1/2}. \tag{A3}$$

Substituting (A3) into (A2) we find the leading behavior

$$M_n^\nu(\xi)\to\frac{(2n+1)!!}{2^{n+1}}\xi^{-n-1}. \tag{A4}$$

The contribution from the sea to the n moment is given by (2.12). With the change of variable $y=\xi x$ this becomes

$$M_n^t(\xi)=\tfrac{12}{9}\beta_t(\xi)\xi^{-n-1}\int_0^\xi dy\, e^{-y}y^n\left(1-\frac{y}{\xi}\right)^{11/2}. \tag{A5}$$

The integral may be written as

$$\int_0^\xi dy\, e^{-y}y^n\left(1-\frac{y}{\xi}\right)^{11/2} =\int_0^\infty dy\, e^{-y}y^n\left(1-\frac{11}{2}\frac{y}{\xi}+\cdots\right) -(-1)^{11/2}\int_\xi^\infty dy\, e^{-y}y^n\left(\frac{y}{\xi}-1\right)^{11/2}. \tag{A6}$$

The second term in (A6) vanishes faster than any inverse power of ξ as $\xi\to\infty$. The first term gives the asymptotic expansion for the integral in (A5):

$$\int_0^\infty dy\, e^{-y}y^n\left(1-\frac{11}{2}\frac{y}{\xi}+\cdots\right) =\Gamma(n+1)-\tfrac{11}{2}\xi^{-1}\Gamma(n+2)+O(\xi^{-2}). \tag{A7}$$

If we substitute (A7) into (A5) and use the definition (2.15) for $\beta_t(\xi)$ we obtain

$$M_n^t(\xi)\sim\tfrac{12}{9}\Gamma(n+1)\xi^{-n}\{c_1+(1/\xi)[c_0-\tfrac{11}{2}c_1(n+1)] +O(\xi^{-2})\}. \tag{A8}$$

The expression for the asymptotic behavior of the n moment is obtained from (A4) and (A8) with

$$M_n(\xi)=M_n^t(\xi)+(2)(\tfrac{4}{9})M_n^{\nu_u}(\xi)+\tfrac{1}{9}M_n^{\nu_d}(\xi). \tag{A9}$$

Since $M_n^{\nu_u}$ and $M_n^{\nu_d}$ have the same leading term we find

$$M_n(\xi)\sim\tfrac{12}{9}\Gamma(n+1)c_1\xi^{-n} +\left\{\tfrac{12}{9}\Gamma(n+1)[c_0-\tfrac{11}{2}c_1(n+1)]+\frac{(2n+1)!!}{2^{n+1}}\right\}\xi^{-n-1}. \tag{A10}$$

The moment $n=0$ is

$$M_0(\xi)\sim\tfrac{12}{9}c_1+[\tfrac{12}{9}(c_0-\tfrac{11}{2}c_1)+\tfrac{1}{2}]\xi^{-1}=\tfrac{5}{42}+0.112\xi^{-1}, \tag{A11}$$

where c_0 and c_1 are chosen as in Table I.

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[1] S. Berman, J. Bjorken, and J. Kogut, Phys. Rev. D 4, 3388 (1971).

[2] P. Darriulat, in *High Energy Physics*, proceedings of the European Physical Society International Conference, Palermo, 1975, edited by A. Zichichi (Editrice Compositori, Bologna, 1976), p. 840; lecture at the Summer Institute on Particle Physics, McGill University, Montreal, 1976 (unpublished).

[3] P. Darriulat, in *Proceedings of the XVIII International Conference on High Energy Physics, Tbilisi, 1976*, edited by N. N. Bogolubov *et al.* (JINR, Dubna, 1977), Vol. I, p. A4-23.

[4] L. Di Lella, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California*, edited by W. T. Kirk (SLAC, Stanford, 1976), p. 939; M. Della Negra, review Talk at the VII International Colloqium on Multiparticle Reactions, Tutzing/Munich, 1976 [CERN Report No. CERN/EP/PHYS 76-52 (unpublished)].

[5] R. Blankenbecler and S. Brodsky, Phys. Rev. D 10, 2973 (1974); D. Sivers, S. Brodsky, and R. Blankenbecler, Phys. Rep. 23, 1 (1976).

[6] S. Brodsky, in proceedings of Stanford Summer Institute, 1975 [SLAC Report No. 191 (unpublished)]. However, inclusion of resonance production as required by theory and experiment, removes several difficulties of the simple formulations of CIM.

[7] C. Chang *et al.*, Phys. Rev. Lett. 35, 901 (1975).

[8] Chicago-Harvard-Illinois-Oxford Collaboration, H. Anderson *et al.*, contribution to the 18th International Con Conference on High Energy Physics, Tbilisi, USSR, 1976 (unpublished).

[9] R. Cahalan, K. Geer, J. Kogut, and L. Susskind, Phys. Rev. D 11, 1199 (1975).

[10] R. C. Hwa, A. Spiessbach, and M. Teper, Phys. Rev. Lett. 36, 1418 (1976).

[11] R. Field and R. P. Feynman, Phys. Rev. D 15, 2590 (1977); G. C. Fox, Caltech Report No. CALT-68-573 (unpublished); R. Baier *et al.*, Nucl. Phys. B118, 139 (1977).

[12] A. P. Contogouris and R. Gaskell, Nucl. Phys. B (to be published).

[13]R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, New York, 1972).
[14]R. McElhaney and S. F. Tuan, Phys. Rev. D 8, 2267 (1973).
[15]F. M. Renard, Nuovo Cimento 29A, 64 (1975).
[16]D. C. Cundy, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974*, edited by J. R. Smith (Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p. IV-131.
[17]J. Kogut and L. Susskind, Phys. Rev. D 9, 697 (1974); and 9, 3391 (1974).
[18]H. D. Politzer, Phys. Rep. 14, 129 (1974); C. Llewellyn Smith, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies* (Ref. 4), p. 709.
[19]M. Glück and E. Reya, Phys. Rev. D 14, 3034 (1976); Phys. Lett. 69B, 77 (1977); Phys. Rev. D 16, 3242 (1977).
[20]S. Ellis and M. Kisslinger, Phys. Rev. D 9, 2027 (1974).
[21]S. Brodsky and G. Farrar, Phys. Rev. Lett. 31, 1153 (1973); and Phys. Rev. D 11, 1309 (1975); V. Matveev, R. Muradyan, and A. Taukhelidze, Lett. Nuovo Cimento 5, 907 (1972).
[22]L. Sehgall, Nucl. Phys. B90, 471 (1975). In particular, the values $m(\pi^-,u)=m(\pi^+,d)=m(\pi^+,s)=m(\pi^-,s)\simeq 1.5$ and $A(\pi^-,u)=A(\pi^+,d)=A(\pi^+,s)=A(\pi^-,s)\simeq 0.5A(\pi^+,u)$ come from the analysis of Sehgall. The values of $m(K^-,u)=m(K^+,d)=m(K^+,s)=2$ and the corresponding coefficient ratios $A(K^\pm,c)/A(\pi^+,u)$ of Table II were chosen partly by fitting the data and partly on the basis of rough SU(3) considerations [with some SU(3) breaking]. We note that dimensional counting rules would predict, e.g., $m(\pi^-,u)=m(K^-,u)=5$.
[23]H. M. Fried and T. K. Gaisser, Phys. Rev. D 7, 741 (1973); A. P. Contogouris, J. P. Holden, and E. N. Argyres, Phys. Lett. 51B, 251 (1974); R. Gaskell, Phys. Rev. D 16, 3320 (1977).