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Implication of the Chou-Yang mdoel in πp elastic scattering

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With some knowledge of the pion form factor, we study πp elastic scattering within the Chou-Yang model. We find it agrees very well with recent Fermilab measurements and we predict the existence of a dip at $-t=4$–5 GeV^2. The sensitivity of the dip structure due to the variation of the shape of the pion form factor is studied.

I. INTRODUCTION

The success of the Chou-Yang model[1] in explaining the features of pp elastic scattering at CERN ISR in the range $P_{1ab}\sim 1000$ GeV prompts us to restudy the problem of other hadronic scattering.[2] In particular, Fermilab has recently produced measurements of $\pi p, kp$ elastic scattering in the region of small $-t$.[3] It is interesting to ask what the Chou-Yang model can explain and predict in view of the forthcoming measurement of the large-t elastic cross section.

Let us first briefly review the Chou-Yang model. The elastic differential cross section is given by the following formulas:

$$\frac{d\sigma}{dt}=\pi|a|^2, \tag{1.1}$$

$$a=\int\frac{d^2\vec{b}}{(2\pi)}[1-S(b)]e^{i\vec{k}\cdot\vec{b}}, \tag{1.2}$$

where $\vec{k}$ is the two-dimensional transverse momentum and $\vec{k}^2=-t$. The $S(b)$ exponential is

$$S(b)=e^{-\Omega(b)}, \tag{1.3}$$

where $\Omega(b)$ is the opaqueness function. For πp elastic scattering the Fourier transform of the opaqueness function is given by

$$(\Omega)=\mu_{\pi p}F_\pi(k^2)F_p(k^2), \tag{1.4}$$

where $\mu_{\pi p}$ is the interacting strength and $F_\pi(k^2)$, $F_p(k^2)$ are the form factors of the pion and proton. Then the scattering amplitude can be given explicitly in terms of the form factors as

$$a_{\pi p}(k^2)=\mu_{\pi p}F_\pi(k^2)F_p(k^2)-\frac{\mu_{\pi p}^2}{2!}F_\pi(k^2)F_p*F_\pi(k^2)F_p(k^2)+\frac{\mu_{\pi p}^3}{3!}F_\pi F_p*F_\pi F_p*F_\pi F_p-\cdots. \tag{1.5}$$

The interaction strength is given by normalizing to the total-cross-section measurements:

$$a_{\pi p}(0)=\frac{\sigma_T(\pi p)}{4\pi}. \tag{1.6}$$

Then the only unknown for predicting the behavior of the differential cross section lies in the pion form factor.

II. PION FORM FACTOR

The pion form factor has been measured[4] in the last several years in the region of $0<|t|\lesssim 1.2$ GeV^2. However, there is no measurement above $|t|\gtrsim 1.2$ GeV^2 as compared with our knowledge of the proton form factor up to $-t\sim 25$ GeV^2. It is then necessary to use some analytic formula for the pion to extrapolate above $|-t|\sim 1.2$ GeV^2. There exist at present two possible forms for the pion form factor that agree with the experimentally measured one in the lower-t region:

(a) Vector-meson dominance gives

$$F_\pi^V(q^2)=\frac{1}{1+q^2/m_\rho^2}. \tag{2.1}$$

(b) Some versions of the quark model[5] give

$$F_\pi^Q(q^2)=F_p^{2/3}(q^2), \tag{2.2}$$

the pion form factor being a $\frac{2}{3}$ power of the proton form factor.

The two forms, although similar for low q^2 values, differ from one another at large q^2 values by

q^2 (GeV^2)	1	5	10
difference	13%	35%	70%

If a smooth behavior is expected from the pion factor as a function of q^2, it is not too unrealistic to expect that the two forms of (1.1) and (1.2) will be more than a factor 2 away from the real pion form factor, say at $q^2\sim 5$ GeV^2.

To facilitate numerical computation, we use a sum of Gaussians to represent the product of the pion and proton form factors:

$$F_p(q^2)F_\pi^V(q^2) = 0.8503e^{-4.42q^2} + 0.1470e^{-1.10q^2} + 0.0026e^{-2.25q^2} + 0.0001e^{-6.5q^2}, \tag{2.3}$$

$$F_p(q^2)F_\pi^Q(q^2) = 0.7963e^{-5.00q^2} + 0.200e^{-1.5q^2} + 0.0036e^{-0.30q^2} + 0.0001e^{-0.48q^2}. \tag{2.4}$$

They are numerically equivalent to Eqs. (2.1) and (2.2) for the range of $q^2 = 0.0$ to 20 GeV^2.

To obtain the above two equations, we use the Sachs magnetic form factor for the proton:

$$F_p(q^2) = \frac{G_M(q^2)}{\mu_p}. \tag{2.5}$$

In the following, we shall use the more familiar pion form factor given by vector-meson dominance Eq. (2.1). This form (2.1) will be used only as an illustration.

III. THE DIP IN THE ELASTIC πp SCATTERING

Using the pion form factor given in the previous section, the scattering amplitude for $\pi^{1/2}p \to \pi^{1/2}p$ can be expressed simply as

$$a_{\pi p} = \mu_{\pi p} F_\pi F_p - \frac{\mu_{\pi p}^2}{2} \sum_{ij} \frac{a_i a_j}{2Y_{ij}} \exp\left(-\frac{b_i b_j}{Y_{ij}} k^2\right) + \frac{\mu_{\pi p}^3}{3!} \sum_{ijl} \frac{a_i a_j a_l}{2^2 Y_{ijl}} \exp\left(-\frac{b_i b_j b_l}{Y_{ijl}} k^2\right) - \frac{\mu_{\pi p}^4}{4!} \sum_{ijlm} \frac{a_i a_j a_l a_m}{2^3 Y_{ijlm}} \exp\left(-\frac{b_i b_j b_l b_m}{Y_{ijlm}} k^2\right) + \cdots, \tag{3.1}$$

where the a_i are defined by

$$F_\pi(k^2)F_p(k^2) = \sum_{i=1}^{4} a_i e^{-b_i k^2} \tag{3.2}$$

and

$$Y_{ij} = b_i + b_j, \quad Y_{ijl} = Y_{ij} b_l + b_i b_j, \quad Y_{ijlm} = Y_{ijl} b_m + b_i b_j b_l. \tag{3.3}$$

In Eq. (3.1) we have retained up to five terms for our calculation.

In Table I we list the values of interaction strength obtained from total cross section measurements for incident momentum $P_{lab} = 50$ to 175 GeV/c. We display in Figs. 1 and 2, for π^+p and π^-p, the elastic scattering differential cross sections, and comparison between theory and experiment is good. The experiments are for $P_{lab} = 50$ to 170 GeV/c and the momentum-transfer range is $-t \cong 0$–1.0 GeV^2. The curves are no-parameter fits.

At Fermilab, it is expected that measurements at large $-t$ for $\pi^\pm p$ are possible. It is very interesting to investigate the behavior of the differential cross section in a momentum-transfer range larger than the present $-t \sim 1$ GeV^2 limit. Especially, one finds the first dip in pp elastic scattering at about $-t = 1.5$ GeV^2, as predicted by the Chou-Yang model. The natural question to ask is where is the first dip in πp scattering in the

TABLE I. Values of the interaction strength $\mu_{\pi p}$.

	P_{lab} (GeV)	σ_T (mb)	$\mu_{\pi p}$ (GeV^{-2})
π^+p	50	23.07 ± 0.12	5.61
	70	23.16 ± 0.12	5.64
	100	23.29 ± 0.12	5.68
	140	23.43 ± 0.12	5.72
	175	23.60 ± 0.12	5.78
π^-p	50	24.01 ± 0.12	5.89
	70	24.00 ± 0.12	5.89
	100	23.96 ± 0.12	5.875
	140	24.00 ± 0.12	5.89
	175	24.17 ± 0.12	5.945

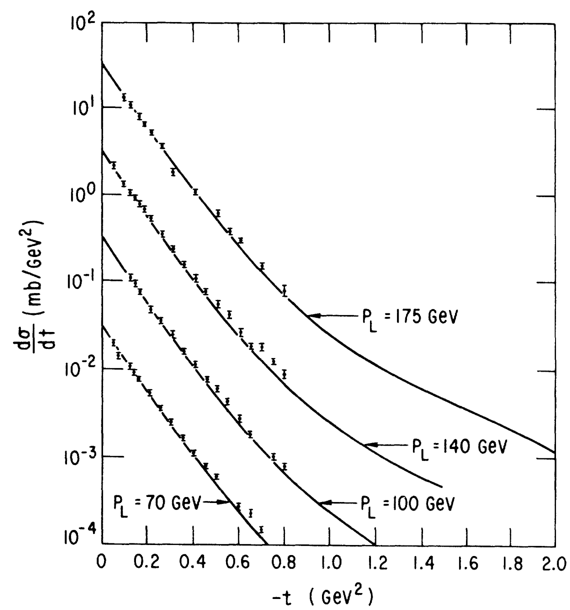

FIG. 1. The comparison of the π^+p elastic differential cross section with the Chou-Yang model at $P_{lab} = 175$, 140, 100, 70 GeV. The solid lines are theoretical curves. To facilitate presentation, each line and data point is shifted by a factor of 10 from the line above it. The pion form factor is given by vector-meson dominance.

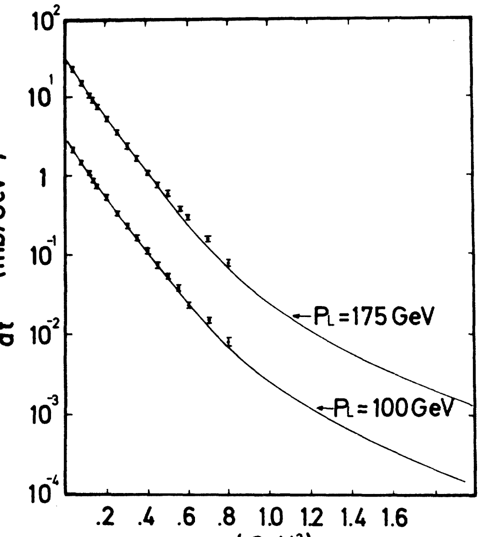

FIG. 2. The comparison of the $\pi^- p$ elastic differential cross section with the Chou-Yang model at P_{1ab} = 175, 100 GeV. The bottom theoretical line and the experimental data points are shifted by a factor 10 from the top line to facilitate easy viewing. Other data points at 140, 70 GeV are not plotted, because of the close resemblance and accuracy of agreement. The pion form factor is given by vector-meson dominance.

framework of the Chou-Yang model. Does it also predict dip, and if a dip exists, where is it? In Fig. 3, we plot the theoretical prediction for larger $-t$ values for $\pi^+ p$ scattering at P_{1ab} = 175 GeV/c. It is interesting to note that the dip is a shallow one even if there is no real part, spin-flip part, or energy-dependent term added to it to fill up the zero in the scattering amplitude. This is partly due to the nature of the pion form factor (we assume vector-meson dominance) and partly due to the smallness of the interaction strength $\mu_{\pi p}$ = 5.78 GeV^{-2} as compared with that ($\mu_{pp} \sim$ 9.5 GeV^{-2}) of pp elastic scattering. With a smaller $\mu_{\pi p}$, the second term in Eqs. (1.5) or (3.1) of power $\mu_{\pi p}{}^2$, although negative, is big enough to compensate the positive terms contributed by the first term and the other odd terms. However, as $\mu_{\pi p}$ becomes larger, as required by a increasing total cross section $\sigma_{\pi p}$, one expects the dip to sharpen and eventually drop to zero. This is also displayed in Fig. 3 for a value of $\mu_{\pi p}$ = 7.7 GeV^2, which corresponds to the normalization point of the total cross section $\sigma_{\pi p}$ = 30 mb, given by Eq. (1.6). Then the zero of the scattering amplitude produces a much deeper dip and one notices that the position of the dip also shifts from $-t \cong 5.2$ GeV^2 to a smaller value $-t \simeq 3.8$ GeV^2.

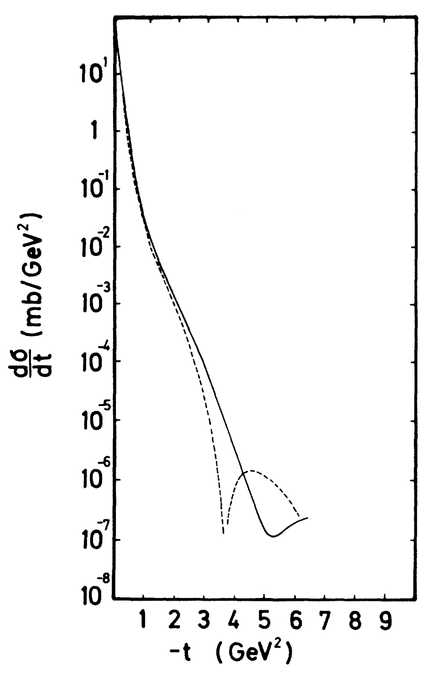

FIG. 3. The dip of $\pi^+ p$ elastic scattering at high energy. The solid line is for P_{1ab} = 175 GeV, with σ_T = 23.6 mb, and the broken line is for $\mu_{\pi p}$ = 7.7 GeV^{-2}, when $\sigma_T(\pi p)$ reaches 30 mb at even higher energy.

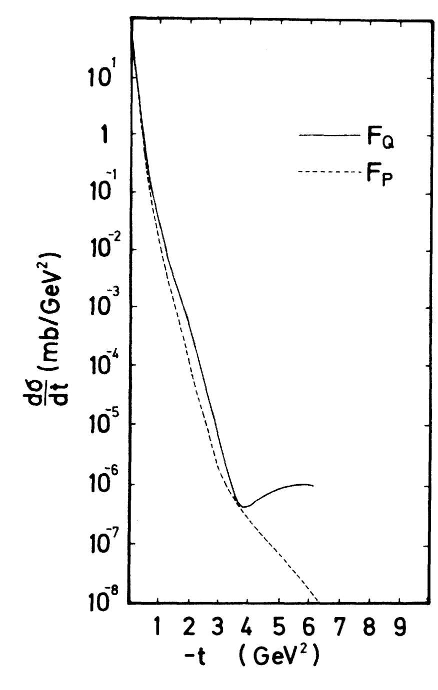

FIG. 4. The study of dip structure from two different shapes of pion form factor: (a) The quark model $F_\pi^Q(q^2) = F_p{}^{2/3}(q^2)$, solid line; (b) the assumption that $F_\pi(q^2) = F_p(q^2)$, dashed line.

In order to understand the sensitivity of the dip of the differential cross section due to the variation of the pion form factor, we use that form given in Eqs. (2.2) and (2.4) obtained by some arguments in the quark model. Then one can calculate its values for πp scattering at larger $-t$ values. This is shown in Fig. 4. The dip is still a shallow one, but it has been shifted to smaller values $-t \simeq 4$ GeV^2. This is due to the fact that the quark model version F_π^Q for the pion form factor has a shallower dependence on $t \sim t^{-2/3}$, as against that of t^{-1}, given by vector-meson dominance F_π^V. However, both of them give similar values of the differential cross section at $-t < 1$ GeV^2. The fits from F_π^Q are not as good as those from F_π^V, however.

In addition, we also consider the hypothetical case of $F_\pi(q^2) = F_p(q^2)$. Hence it has a dependence $\sim t^{-2}$ and an even steeper behavior in t than that of the quark-model F_π^Q form factor. We show this result also in Fig. 4. There is *no* dip in the differential cross section, and only a change of slope in the region $-t = 3-4$ GeV^2. We call this case hypothetical because $F_\pi = F_p$ does not fit the measurement of the pion form factor and the differential cross section for $t \lesssim 1$ GeV^2 does not agree with the experimental measurement. Nevertheless, it teaches us that *a priori* there may or may not be a dip for πp elastic scattering in the Chou-Yang model if some knowledge of the pion form factor is not known. Conversely, one can argue that the existence of the dip and its position place a stringent condition on the shape of the pion form factor.

IV. DISCUSSION AND CONCLUSION

Based on the advancement in the knowledge of the pion form factor from electromagnetic interaction in the region of small $-t$ it is possible now to make some reasonable extrapolation to the region of larger $-t$ for the pion form factor. Using the Chou-Yang model, we look for the dip structure in πp elastic scattering and find that the dip should occur at around $-t = 4-5$ GeV^2. The precise position depends on the detailed structure of the pion form factor. Nevertheless, the dip position occurs at a much larger $-t$ value than that in pp elastic scattering, where the dip occurs at $-t = 1.5$ GeV^2. Of course, the energy-dependent term (nonvacuum-exchange term) may mask such a dip, and may make it look like a change of slope only. But at higher energy the energy-dependent term decreases and the dip should emerge. Lack of a reliable estimate of the energy-dependent term at such a large $-t$ value makes the precise determination of the energy when the dip emerges difficult. In the pp elastic scattering case, the dip at $-t = 1.5$ GeV^2 starts to show up at $P_{\rm lab} = 200$ GeV/c and above. It may not be unreasonable then to expect that the dip structure in πp elastic scattering should also be detected at Fermilab.

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