PHYSICAL REVIEW D VOLUME 17, NUMBER 3 1 FEBRUARY 1978

Electroproduction of pions on pions and the possibility of experimental investigation of the $\rho\pi\gamma$ vertex

W. Gajewski and A. Jurewicz*
Institute for Nuclear Research, Warsaw, Poland
(Received 26 January 1977)

Possibilities of experimental investigation of the reaction of electroproduction of pions on pions are examined. For this purpose the absolute cross section of the reaction $\pi e \rightarrow \pi\pi e$ is calculated with the use of analytic properties of an inelastic pion form factor. It is shown that the discussed reaction can be a good source of information on the $\rho\pi\gamma$ vertex. Pion beams from the doubler at Fermilab will have sufficient energy to allow for experimental studies of this reaction.

INTRODUCTION

The progress achieved in the last few years in accelerator technique provided facilities for producing meson beams of high energy and high density. This in turn created favorable conditions to study the structure of hadrons other than nucleons, by means of scattering these hadrons on atomic electrons. First measurements of the electromagnetic radius of the pion have already been performed by this technique.[1] Under way are further measurements of this radius in a larger range of four-momentum transfers and also measurements of the kaon radius.

In the present note we examine some possibilities of extending these experimental studies to other exlusive form factors of the π meson, in particular to that connected with electroproduction of an additional pion.

The majority of hitherto collected data on hadronic structure pertain to nucleons. Most information has been drawn by using leptons as probes of this structure. Although the experimental results can be surprisingly well understood in terms of various theoretical models, the latter should be nevertheless generalized and also experimentally tested for other hadrons. In the nearest future such experimental tests will be limited to π mesons since only pions, the lightest hadrons, when scattered off electrons, can get four-momentum transfers so large that the effects of the form factor become measurable.

Pairs of pions produced with small invariant masses in the process $\pi e \rightarrow \pi\pi e$ can be well described by a single amplitude with $I=J=1$, dominated by the ρ resonance. It should be emphasized that this conclusion is model-independent since it follows only from basic conservation laws (to lowest order in e). Due to such simplicity of the final state the process in question can be an excellent source of information on the $\rho\pi\gamma$ vertex. The reaction offers a possibility of measuring the dependence of the $\rho^{\pm}\pi^{\pm}\gamma$ structure function on the mass of the pion pair (mass of the ρ) and on the squared four-momentum of the photon. Owing to kinematic constraints, however, the latter quantity is limited to small absolute values.

In this note we use some well established theoretical concepts, and also the latest experimental data, to examine the possibilities of measuring the electroproduction of pions on pions. For this purpose we use, first of all, a dispersion approach to calculate the dependence of the cross section on the invariant mass of the final pair of pions. Our approach is very traditional, but we believe that dispersion relations proved themselves so effective in the study of photoproduction processes that the use of this theoretical tool puts us on a very safe ground.

We have no equally reliable means to calculate the dependence of the $\rho\pi\gamma$ structure function on λ^2, the squared four-momentum of the photon. Vector-meson dominance (VMD) connects it to the structure function of the $\rho\pi\omega$ vertex[2] which has been studied mainly with the aid of current algebra.[3] Since the results do not seem to be conclusive enough we tentatively assume that the cross section depends on λ^2 through kinematical factors only. This allows us to give an estimate of absolute values of the cross section. The important question about its dependence on λ^2 has to be answered by experiment.

KINEMATICS AND THE INELASTIC FORM FACTOR

The kinematics of the five-body reaction $\pi e \rightarrow \pi\pi e$ in the one photon-exchange approximation are shown in Fig. 1. We introduce the following invariants:

$$S=(p_1^\nu+l_1^\nu)^2\,,\quad t=(p_1^\nu-p_2^\nu)^2\,,$$
$$s=(p_2^\nu+p_3^\nu)^2\,,\quad \lambda^2=-(l_1^\nu-l_2^\nu)^2=2ml^0_{2\,\mathrm{lab}} \tag{1}$$

and the following angle

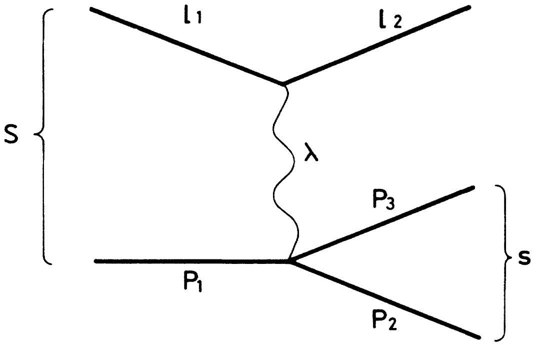

FIG. 1. The kinematics of electroproduction of pions on pions in the one-photon-exchange approximation.

$$\cos\psi = \vec{l}_2\cdot\vec{p}_1/(l_2\cdot p_1) \tag{2}$$

in the c.m. system of the final pions. The masses of electrons and pions are denoted m and μ, respectively.

The hadronic vertex of Fig. 1 is described by the matrix element with the shape[4]

$$\langle p_2 p_3 | j_\mu | p_1\rangle = \epsilon_{\mu\nu\rho\sigma} p_1^\nu p_2^\rho p_3^\sigma F(s,t;\lambda^2)\,. \tag{3}$$

This defines a function $F(s,t;\lambda^2)$, which in the following is called the inelastic form factor. It is well known that it has to be symmetric under crossing $s \leftrightarrow t$. For the mass of the two-pion system $\sqrt{s} \leqslant 1$ GeV the form factor is completely dominated by the p state and can be fairly approximated by the lowest term of the partial-wave expansion, giving

$$F(s,t;\lambda^2) \cong f_1(s;\lambda^2)\,. \tag{4}$$

Summation and averaging over spins of the electrons and simple integration over angles yields the following expression for the cross section:

$$d\sigma = \frac{e^2}{2\pi m p_{1\,\mathrm{lab}}}\,\frac{2l_2{}^2\sin^2\psi+\lambda^2}{\lambda^4}\left(\frac{p_1\cdot p_2{}^3}{8}\left|\frac{F}{4\pi}\right|^2\right) \times \tfrac{4}{3}\sqrt{s}\,p_1 l_2\, dl_2^0\, d\cos\psi\,, \tag{5}$$

where the factor in the parentheses corresponds to the cross section for production of pions on pions by real photons. The momentum $p_{1\,\mathrm{lab}}$ of the primary pion refers to the system where $\vec{l}_1 = 0$; all other quantities refer to the c.m. system of the final pion pair.

The dependence of the form factor on s can be found from fixed-t dispersion relations. A slight generalization of earlier results[4,5] yields

$$f_1(s;\lambda^2) = \int_{4\mu^2}^{\infty}\frac{\mathrm{Im} f_1(s';\lambda^2)}{s'-s}ds' + \int_{4\mu^2}^{\infty}\frac{\mathrm{Im} f_1(u';\lambda^2)}{u'-u}du'\,, \tag{6}$$

where $u = (7\mu^2-\lambda^2)/2 - s$.

With the aid of the elastic unitarity condition the dispersion relation (6) can be transformed into a singular integral equation whose solution is well known.[6] By assuming that the phase shift of pion-pion scattering in the state $I = J = 1$ satisfies the conditions $\delta_1(s) \to 0$ for $s \to 4\mu^2$ and $\delta_1(s)\to\pi$ for $s\to\infty$, as indicated by experimental data,[7] we get

$$f_1(s;\lambda^2) = A_{2k}(s;\lambda^2)\exp[\rho(s;\lambda^2) + i\delta_1(s)]\,, \tag{7}$$

where

$$\rho(s;\lambda^2) = \frac{1}{\pi}[s-\tfrac{1}{4}(7\mu^2-\lambda^2)]\,\mathrm{P}\int_{4\mu^2}^{\infty}\frac{\delta_1(s')}{s'-\frac{1}{4}(7\mu^2-\lambda^2)}\left[\frac{1}{s'-s}-\frac{1}{s'+s-\frac{1}{2}(7\mu^2-\lambda^2)}\right]ds' \tag{8}$$

and $A_{2k}(s;\lambda^2)$ is an arbitrary polynomial of order $2k$ in the variable s. We easily find for $s\to\infty$ that $f_1(s;\lambda^2) \sim s^{2k-1}$ and that the cross section $\sigma(s;\lambda^2=0) \sim s^{-4+2+4k}$; hence, in order to have an asymptotically decreasing cross section we should take $k=0$. The polynomial $A_{2k}(s;\lambda^2)$ thus reduces to a function $A(\lambda^2)$, independent of s and still undetermined. Its estimate is given below.

The function $\rho(s;\lambda^2)$ has been evaluated numerically by using the phase shifts $\delta_1(s)$ as determined by Grayer and by Hyams.[7] For $s \gtrsim 4$ GeV^2 an extrapolating analytic expression for $\delta_1(s)$ has been used to reproduce its presumed asymptotic behavior. In this connection it is worth mentioning that the existing results concerning the p-wave phase shifts are susceptible to certain ambiguities:

(1) It has been pointed out[8] that at small values of s the phase shifts are too large, inconsistent with the bounds of Roy[9] and that the measured scattering length is in disagreement with Weinberg's estimate.[10]

(2) In the region of $\sqrt{s} > 1$ GeV the coupling of $\rho'(1600)$ to the dipion system still seems to be an open question.[7]

In our calculation of $\rho(s:\lambda^2)$ the uncertainties in the low-energy region were relevant only for values of $\rho(s;\lambda^2)$ close to threshold, whereas the influence of the presumed $\rho'(1600)$ did not contribute to the value of $\rho(s;\lambda^2)$ by more than a few percent. The dependence of the function $\rho(s;\lambda^2)$ on λ^2 was found to be very weak. Its values at $s = m_\rho{}^2$ change by about 2% over the range $0.01 < \lambda^2 < 0.1$ GeV^2.

ABSOLUTE ESTIMATE OF THE CROSS SECTION

In the sharp-resonance approximation the hadronic vertex of Fig. 1 is dominated by ρ exchange

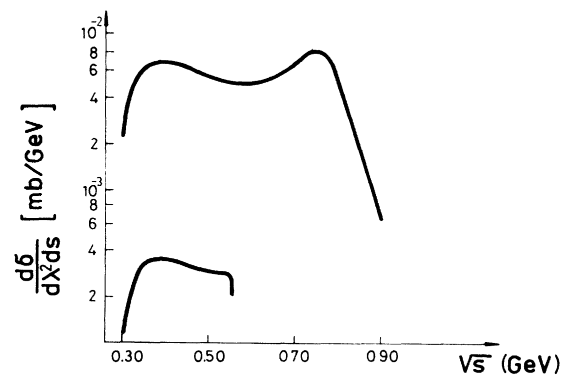

FIG. 2. Absolute cross section for electroproduction of pions on pions vs the mass of the dipion system. The upper curve refers to $p_{1\mathrm{lab}}=800$ GeV, the lower one to $p_{1\mathrm{lab}}=200$ GeV (in both cases $\lambda^2=0.02$ GeV^2).

whose contribution to the matrix element for $\lambda^2 = 0$ is the following:

$$\langle p_2 p_3 | j_\mu | p_1 \rangle = \frac{2e}{\mu} f_{\rho\pi\pi} f_{\rho\pi\gamma} \frac{1}{s - m_\rho^2 + i m_\rho \Gamma_\rho} \times \epsilon_{\mu\nu\rho\sigma} p_1^\nu p_2^\rho p_3^\sigma , \qquad (9)$$

where $f_{\rho\pi\pi}$ and $f_{\rho\pi\gamma}$ denote the respective coupling constants, m_ρ is the ρ mass, and Γ_ρ its width.

By direct comparison of (3), (4), and (9) we get

$$|f_1(s=m_\rho^2;\lambda^2=0)| = \left| \frac{2e f_{\rho\pi\pi} f_{\rho\pi\gamma}}{\mu m_\rho \Gamma_\rho} \right| . \qquad (10)$$

which can also be expressed through partial widths of the ρ decay $\Gamma_{\rho\pi\pi}$ and $\Gamma_{\rho\pi\gamma}$, yielding

$$|f_1(s=m_\rho^2;\lambda^2=0)| = \frac{96\pi (2\Gamma_{\rho\pi\pi}/\Gamma_{\rho\pi\gamma})^{1/2}}{m_\rho^2 (1-4\mu^2/m_\rho^2)^{3/4} (1-\mu^2/m_\rho^2)^{3/2}} . \qquad (11)$$

The decay $\rho \to \pi\gamma$ has not been experimentally observed so far, and only upper bounds on its probability are available. There exists an estimate of this probability[11] based on the half width of the decay $\pi^0 \to 2\gamma$. This estimate, updated with the aid of the 1976 tables of the Particle Data Group,[12] reads

$$\Gamma_{\rho\pi\gamma}/\Gamma_\rho = (5.7 \pm 0.7) \times 10^{-4} , \qquad (12)$$

giving the value

$$|f_1(s=m_\rho^2;\lambda^2=0)| \cong 26 \text{ GeV}^{-3} . \qquad (13)$$

The validity of the estimate (13) for $\lambda^2 \neq 0$ and, more generally, the shape of the dependence of $f_1(s;\lambda^2)$ on λ^2 remain open questions. There seems to be no reliable way as to how they should be answered, and we assume tentatively that the dependence of the cross section (5) on λ^2 is mostly kinematic. Since $\rho(s;\lambda^2)$ as calculated before turned out to be almost independent of λ^2, our assumption amounts to saying that $f_1(s;\lambda^2)$ is a slowly varying function of λ^2. When the primary pion momentum in the lab frame is around 10^3 GeV the estimate (13) can therefore make sense over the whole kinematically allowed range

$$0 \lesssim \lambda^2 \lesssim 2mp_{1\mathrm{lab}}(2mp_{1\mathrm{lab}} + \mu^2 - s)/(2mp_{1\mathrm{lab}} + \mu^2)$$

Note, that for $p_{1\mathrm{lab}} = 800$ GeV and $s = m_\rho^2$ we have $\lambda^2 \lesssim 0.2$ GeV^2.

Our view can be supported by some calculations based on current algebra[3] which suggest that the $\rho\omega\pi$ structure function should depend linearly on the squared masses of the external particles. When combined with the known VMD relation between "coupling constants"[13] this result leads to the conclusion that the dependence of $f_1(s;\lambda^2)$ on λ^2 may indeed be weak in the relevant range of λ^2. It would be important to have these assumptions checked by direct measurements of the reaction $e\pi \to e\pi\pi$.

From (13) we get immediately the estimate $A(\lambda^2) = 9.2$ GeV^{-3} which enables us to calculate absolute values of the cross section as given by formula (9). The dependence of the cross section on $\sqrt{s}$ for $\lambda^2 = 0.02$ GeV^2 and for two different values of the initial pion momentum in the lab frame is shown in Fig. 2. We feel that the given values of the cross section are credible to about 20%.

A few final remarks are due in this connection. For the presently available pion momenta the cross section is strongly suppressed by phase space. For this reason the inelastic process does not contaminate appreciably the elastic $e\pi \to e\pi$ measurements. In order to study electroproduction of pions over a larger range of s including the ρ peak, it is necessary to have pion beams of energy at least twice as large as those available right now. When the energy doubler at Fermilab provides us with such beams, it will be interesting to study the process $e\pi \to e\pi\pi$ in order to get, in a clean way, such information on the $\rho\pi\gamma$ vertex, which is now available only from rare vector-meson decays.

We express our thanks to M. Swiecki for fruitful discussions.

*Work supported in part by the U. S. National Science Foundation under Grant No. GF-41958.

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