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Photoproduction of charged intermediate vector bosons

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We consider the photoproduction of charged intermediate vector bosons W^+ or W^- with an arbitrary anomalous magnetic moment $\kappa e/2M_W$. The cross sections at high energies are shown to increase logarithmically with s/M_W^2 except for the case $\kappa = 1$, where a constant cross section equal to $\sqrt{2}\alpha\ G_F$ is reached. The deep-inelastic photoproduction of $W^\pm$ is calculated in a quark-parton model and we present numerical results for the total and the differential cross sections.

I. INTRODUCTION

The prime target for the next generation of accelerators will be the production of weak intermediate vector bosons. Since gauge theories predict rather large masses for these particles, the energy required for their production is beyond existing accelerators but well within the energy range of machines planned for the near future (e.g. ISABELLE[1]). This expectation has led to an increase in theoretical calculations of $W^\pm$ and Z^0 production processes. Proton-proton or proton-antiproton colliding beams are particularly promising because theoretical calculations[2,3] predict $\sigma \approx 10^{-34}$ cm^2. Electron-positron colliding beams also are very suitable because the cross section for pair production of charged bosons is calculated[4] to be around 10^{-35} cm^2.

In this paper we consider the photoproduction of a single W^+ or W^- off a nucleon target. Experimentally it will be difficult to obtain the high-energy photon beams required for the production of a massive W, and a colliding γ-p facility may be the only way to achieve the necessary center-of-mass energy. Should such a facility become possible in the future, the numerical results presented here will be useful to the experimentalists looking for the reaction

$$\gamma + p \to W^\pm + X\,. \tag{i}$$

Much of the present work, however, is theoretically oriented, and motivated by the following simple observation: It is well known that certain production cross sections behave badly at high energies, reflecting the nonrenormalizability of the theory, unless new particles, fermions or bosons, are introduced. For example, the weak neutral current in the Weinberg-Salam theory[5,6] regularizes the high-energy behavior of the amplitude for $e^+ + e^- \to W^+ + W^-$. However, in reaction (i) there are only three basic diagrams (see Fig. 1) no matter what the underlying theory is. Our first aim is therefore to calculate the cross section for this process in the very-high-energy limit.

The second aim of this paper is to explore the possibility of measuring the magnetic moment of the charged boson. This turns out to be an important property of the W bosons for the following reason: Though in principle the W can have a magnetic moment $(1+\kappa)e/2M_W$ with any value of κ, the Weinberg-Salam model fixes[7] the value of κ uniquely to be $\kappa = 1$. By studying the differential and the total cross sections one may extract the value of κ ($\kappa e/2M_W$ is usually referred to as the anomalous magnetic moment of the W). Since the weak charged currents have been extensively studied the couplings of the W to the quarks and the leptons are very well known. For the Feynman diagrams shown in Fig. 1 the only unknown coupling is κ.

We shall briefly review earlier work on the photoproduction of W. The inelastic photoproduction of single W^+ bosons was considered[8] using crude parton distribution functions and for $\kappa = 0$ only. The elastic reactions $\gamma + p \to W^+ + n$ and $\gamma + n \to W^- + p$ were considered in detail by Fearing *et al.*,[9] who discussed the question of gauge invariance including form factors. Earlier work on these elastic reactions by Reiss and Cha[10] and by Williamson and Deck[11] concentrated on relatively low center-of-mass energies. Fearing *et al.* also derive the high-energy limit of the total cross section. However, when simplified for the case of an elementary target, their expressions do not agree with our result.

In Sec. II we consider the elastic photoproduction of $W^\pm$. We have included no form factors, but we have let the charge of the target be a variable number Q, in units of e, because our plan is first to get the high-energy behavior of the total cross section off a point target, and second to use our formulas to calculate the inelastic photoproduction of W^+ or W^- in a quark-parton model. This is done in Sec. III, where numerical results are presented for different values of κ. Finally, we discuss our formulas and make a number of comments and conclusions in Sec. IV.

II. THE ELASTIC PHOTOPRODUCTION OF SINGLE W BOSONS

A. The amplitude and its square

In this section we consider the reaction

$$\gamma(k)+q(p)\to W^{\pm}(k')+q'(p')\,, \tag{ii}$$

where q, later to be identified with a quark, stands for a structureless target of charge Qe. Wherever possible we shall drop the masses m and m' of q and q' since we are interested in reaction (ii) only at very high energies.

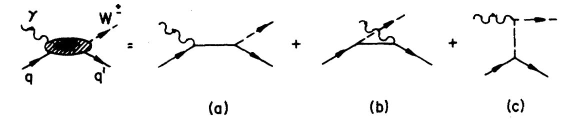

FIG. 1. Feynman diagrams for the reaction $\gamma+q \to W^{\pm}+q'$.

The basic amplitude corresponding to the three Feynman diagrams of Fig. 1 is given by[12]

$$A_{W^+}(Q)=ieg\epsilon^{\mu}(k)\epsilon^{*\alpha}(k')\bar{u}(p')\left\{-Q\Gamma_{\alpha\mu}(-p)+(Q-1)\Gamma_{\mu\alpha}(p')+\frac{\gamma^{\delta}}{2k\cdot k'}[-2k'_{\mu}g_{\delta\alpha}+(1+\kappa)(g_{\mu\alpha}k_{\delta}-g_{\mu\delta}k_{\alpha})]\right\}(1-\gamma_5)u(p)\,, \tag{1}$$

where g is the semiweak coupling constant equal to $(M_W\sqrt{G_F}/2^{1/4})\cos\theta_C$ and to $(M_W\sqrt{G_F}/2^{1/4})\sin\theta_c$ for strangeness-conserving and strangeness-changing transitions, respectively. The ϵ^{μ} are polarization four-vectors satisfying $k_{\mu}\epsilon^{\mu}(k)=k'_{\mu}\epsilon^{\mu}(k')=0$. $\Gamma_{\alpha\mu}$ is defined as

$$\Gamma_{\alpha\mu}(p)=\gamma_{\alpha}\frac{1}{\not p-\not k}\gamma_{\mu}=\frac{\gamma_{\alpha}(\not p-\not k)\gamma_{\mu}}{-2p\cdot k}\,. \tag{2}$$

$A_{W^{\pm}}$ is the amplitude for the photoproduction of $W^{\pm}$. To obtain A_{W^-} from A_{W^+}, the following should be done: The charge factor $Q-1$ multiplying the second term in Eq. (1) must be replaced by $Q+1$, and the sign of the third term in Eq. (1) should be reversed. The net result can be expressed by

$$A_{W^-}(Q)=-A_{W^+}(-Q)\,. \tag{3}$$

We now square the amplitudes $A_{W^{\pm}}$, average over the initial spin s_i and photon helicity λ_{γ}, and sum over the final spin s_f and boson helicity λ_W, to obtain

$$\overline{\sum_{s_i}}\,\overline{\sum_{\lambda_{\gamma}}}\sum_{s_f}\sum_{\lambda_W}|A_{W^{\pm}}|^2=\frac{1}{4}\left(\frac{1}{4mm'}\right)e^2g^2T(\kappa,\pm Q,s,t)\,. \tag{4}$$

The calculation of T requires taking traces of γ matrices and is rather lengthy but straightforward. The result is

$$\begin{aligned}T(\kappa,Q,s,t)=&-16(Q-1)^2\frac{s}{u}-16Q^2\frac{u}{s}-32Q(Q-1)\frac{tM_W^{\,2}}{su}+16[(Q-1)/u-Q/s][2tM_W^{\,2}-(1+\kappa)su]/(M_W^{\,2}-t)\\&-8\frac{t}{M_W^{\,2}}+8[2s(s+t)/M_W^{\,2}+(1+\kappa)(t-(s+t)^2/M_W^{\,2})]/(M_W^{\,2}-t)\\&-2[8s^2-16tM_W^{\,2}-4(1+\kappa)s^2(1+t/M_W^{\,2})+(1+\kappa)^2(4su+(s^2+u^2)\,t/M_W^{\,2})]/(M_W^{\,2}-t)^2\,.\end{aligned} \tag{5}$$

The notation in Eqs. (4) and (5) is standard: $s=(p+k)^2$, $t=(k-k')^2$, and $u=M_W^{\,2}-s-t$.

As discussed in the remainder of this paper, $T(\kappa,Q,s,t)$ is the basic function which enters all our calculations. It represents the square of the sum of the three Feynman diagrams Figs. 1(a)–1(c), for the case when the initial massless quark has charge Q and the final massless quark has charge $Q-1$ or $Q+1$ depending on the charge of the emitted W.

We have tried to further simplify Eq. (5) by considering a few special cases. In all cases the result was almost equally complicated. Only for the case $\kappa=1$ was an exceptionally simple expression obtained:

$$T(1,Q,s,t)=-16\left(Q-\frac{1}{1+u/s}\right)^2(s^2+u^2+2tM_W^{\,2})/su\,. \tag{6}$$

B. The elastic cross section

The differential cross section for the elastic photoproduction of $W^{\pm}$ is

$$\frac{d\sigma^{\pm}}{dt}=\frac{\alpha g^2}{16s^2}T(\kappa,\pm Q,s,t)\,. \tag{7}$$

We shall calculate the total cross section $\sigma(\kappa,Q,s)$ analytically to find its large-s behavior for arbitrary values of κ. Integrating Eq. (7) over t, we find

$$\sigma(\kappa, Q, s) = \frac{\alpha G_F}{8\sqrt{2}}\left[2(\kappa-1)^2 \ln\frac{s}{M_W{}^2} + \frac{\kappa^2}{2} + 15\kappa + \tfrac{1}{2}\right], \qquad (8)$$

in the limit $s \gg M_W{}^2$. We have set $g^2 = G_F M_W{}^2/\sqrt{2}$. Nonleading terms are given in the Appendix.

As given by Eq. (8) to leading order in $M_W{}^2/s$, $\sigma(\kappa, Q, s)$ is independent of Q for all values of κ. It increases logarithmically with s except for the case $\kappa = 1$, where it reaches the constant value

$$\sigma(1, Q, s) = \sqrt{2}\,\alpha G_F, \qquad (9)$$

which is about 4.6×10^{-35} cm^2.

III. THE INELASTIC PHOTOPRODUCTION OF SINGLE W BOSONS

High-energy experiments on $W^\pm$ production are likely to be highly inelastic. The physical picture is the complete breakup of the target by high-energy photons and the creation of a vector boson during that process. We shall calculate the deep-inelastic cross section for the photoproduction of $W^\pm$ using a quark-parton model. As indicated in Fig. 2, the basic production process is off a quark or antiquark of charge Qe inside the target. The final quark or antiquark of charge $(Q \pm 1)e$ recombines to form a hadronic state of variable mass m_f. Let $P_i(x)$ be the probability of finding a quark of charge $Q_i e$ inside a nucleon of mass m_T, and denote its coupling to W by g_i. Then

$$\frac{d\sigma^\pm}{dt\,dm_f{}^2} = \frac{\alpha}{16 s^2 t}\sum_i g_i{}^2 P_i(x) T(\kappa, \pm Q_i, xs, t), \qquad (10)$$

where $x = t/(t + m_T{}^2 - m_f{}^2)$ is the usual scaling variable and the function T is given in Eq. (5). Assuming that nucleons are made up of the ordinary quarks u, d, and s, we obtain

$$\frac{d\sigma^+}{dt\,dm_f{}^2} = \frac{\alpha G_F M_W{}^2}{16\sqrt{2}\, s^2 t}\{P_u(x) T(\kappa, \tfrac{2}{3}, xs, t) + [P_{\bar d}(x)\cos^2\theta_c + P_{\bar s}(x)\sin^2\theta_c] \times T(\kappa, \tfrac{1}{3}, xs, t)\} \qquad (11)$$

and

$$\frac{d\sigma^-}{dt\,dm_f{}^2} = \frac{\alpha G_F M_W{}^2}{16\sqrt{2}\, s^2 t}\{P_{\bar u}(x) T(\kappa, \tfrac{2}{3}, xs, t) + [P_d(x)\cos^2\theta_c + P_s(x)\sin^2\theta_c] \times T(\kappa, \tfrac{1}{3}, xs, t)\}. \qquad (12)$$

In the above two equations we have used the fact that for photoproduction off an antiparticle we simply let $Q \to -Q$ in the trace. Another sign change comes when we go from W^+ to W^- production as dictated by Eq (3). These two changes

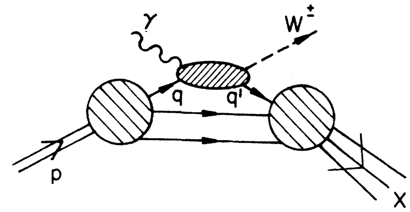

FIG. 2. The quark-parton model for the reaction $\gamma + p \to W^\pm + X$.

explain why $T(\kappa, \tfrac{2}{3}, xs, t)$ occurs in the photoproduction of a W^- off an up antiquark $\bar u$.

To proceed we need explicit forms for the probability functions $P_i(x)$. In our numerical calculations we have used the parametrization given by Okada, Pakvasa, and Tuan,[13] which is reproduced here for completeness:

$$\begin{aligned} &P_s = P_{\bar s} = P_{\bar u} = P_{\bar d} = 0.1(1-x)^{7/2}/x, \\ &P_u = 1.74(1-x)^3(1+2.3x)/\sqrt{x} + P_s, \qquad (13)\\ &P_d = 1.11(1-x)^{3.1}/\sqrt{x} + P_s. \end{aligned}$$

To obtain the differential cross section $d\sigma^\pm/dt$ we need to integrate Eq. (11) or Eq. (12) over $m_f{}^2$. The limits are given by

$$\begin{aligned} &(m_f{}^2)_{\min} = (m_T + m_\pi)^2, \\ &(m_f{}^2)_{\max} = s + M_W{}^2 - 2\sqrt{s}\left[\frac{(M_W{}^2 - t)}{4K_0} + \frac{M_W{}^2 K_0}{(M_W{}^2 - t)}\right], \end{aligned} \qquad (14)$$

where $K_0 = (s - m_T{}^2)/2\sqrt{s}$.

In Fig. 3 we plot $d\sigma^+/d|t|$ for $\kappa = -1$, 0, and 1. We have set $M_W = 70$ GeV/c^2, which is close to the value obtained in the Weinberg-Salam model with $\sin^2\theta_W = 0.3$. The square of the center-of-mass

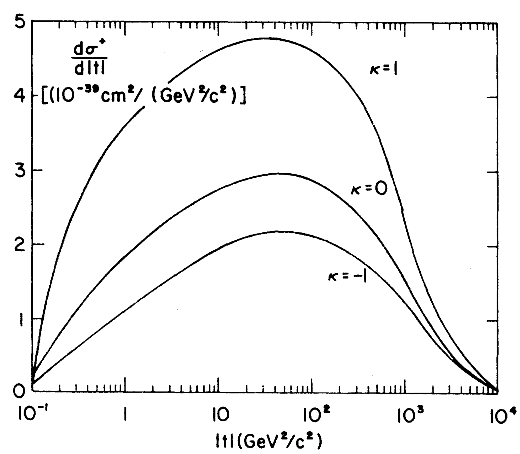

FIG. 3. The differential cross sections $d\sigma^+/d|t|$ for the reaction $\gamma + p \to W^+ + X$, in units of 10^{-39} cm^2/(GeV^2/c^2), as a function of $|t|$ in units of GeV^2/c^2. $M_W = 70$ GeV/c^2 and $s = 25000$ GeV^2. The three curves correspond to $\kappa = 1$, $\kappa = 0$, and $\kappa = -1$ as indicated.

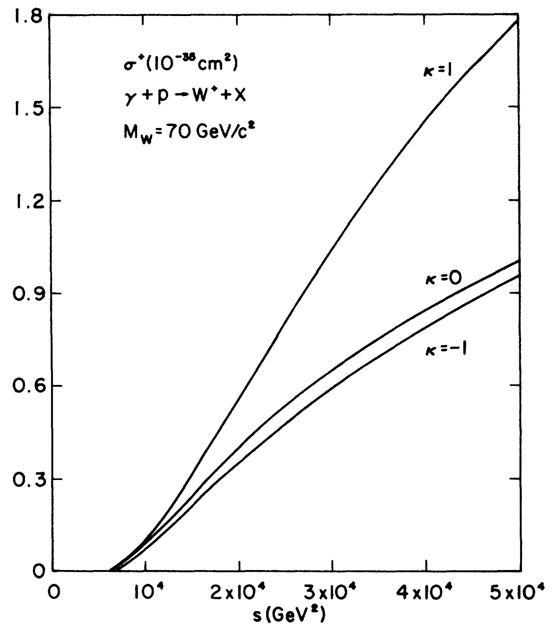

FIG. 4. For $M_W = 70$ GeV/c^2, the total cross section for $\gamma + p \to W^+ + X$, in units of 10^{-35} cm^2, as a function of s in units of GeV^2. The three curves correspond to $\kappa = 1$, $\kappa = 0$, and $\kappa = -1$ as indicated.

energy s is 25 000 GeV^2 in Fig. 3.

The total cross sections as functions of s are plotted in Fig. 4, again for the production of a positively charged W boson of mass 70 GeV/c^2. These curves were obtained by integrating $d\sigma^+/dt$ over t with the following limits:

$$(t)_{\min} = M_W^2 - 2K_0(L_0 + L),$$
$$(t)_{\max} = M_W^2 - 2K_0(L_0 - L), \tag{15}$$

where

$$L_0 = [s + M_W^2 - (m_f^2)_{\min}]/2\sqrt{s} \tag{16}$$

and

$$L = (L_0^2 - M_W^2)^{1/2}. \tag{17}$$

IV. COMMENTS AND CONCLUSIONS

We shall first comment on the inelastic photoproduction of $W^\pm$ which was considered in the preceding section:

(a) We have presented numerical results only for the case of the W^+ because we find that at high energies the photoproduction cross sections of W^+ and W^- are the same to within 10%. Only near threshold do the two cross sections differ appreciably, σ_{W^-} being somewhat smaller than σ_{W^+}.

(b) Since the cross sections are quadratic in κ, the reader interested in values of κ different from -1, 0, or 1 can use Figs. 3 and 4 to get

$$\sigma(\kappa) = (1-\kappa^2)\sigma(0) + \kappa[\sigma(1) - \sigma(-1)]/2 + \kappa^2[\sigma(1) + \sigma(-1)]/2, \tag{18}$$

and similarly for $d\sigma/dt$.

(c) We have tried other values of M_W and found that to a good (~10%) approximation the cross sections scale in s/M_W^2. Using this property one can easily find from Fig. 4 the photoproduction cross section for a W with a mass different from 70 GeV/c^2.

(d) Though we have used a left-handed $1-\gamma_5$ coupling for the W, the same cross sections are obtained for the photoproduction of a boson which couples to the quark (s) in a right-handed manner. More generally, replacing $1-\gamma_5$ by $g_V + g_A\gamma_5$ simply multiplies our cross sections by $(g_V^2 + g_A^2)/2$.

(e) From Figs. 3 and 4 we see that the rates for $\kappa = 1$ are larger than the rates for $\kappa = 0$ or $\kappa = -1$. This is of course encouraging since we are biased towards $\kappa = 1$. Eventually all cross sections for $\kappa \neq 1$ will exceed $\sigma(\kappa = 1)$ since they will grow logarithmically with s. This happens at far too large energies to be of any practical interest. For a crude estimate one may use Eq. (8) to calculate the *ratios* of the cross sections for different values of κ. We find, for example, that $\sigma(-1)$ and $\sigma(0)$ exceed $\sigma(1)$ only for $s \geq M_W^2 e^{3.75}$ and for $s \geq M_W^2 e^{7.75}$, respectively.

(f) The photoproduction of a W^+ or W^- seems to be the best means of measuring κ, provided that a precision better than a factor of 2 can be achieved. Of course there are other reactions where the $W^\pm$ may be produced electromagnetically and therefore the rates will again depend on κ, but these reactions involve a virtual photon and hence one must add at least one more Feynman diagram replacing the virtual γ by a virtual Z_0. The interpretation of the experimental results will then not be as clear as in this case.

(g) To estimate the cross section for $ep \to eW^\pm X$, we may use the Weizsäcker-Williams approximation. Very crudely, we multiply our photoproduction cross sections by $\alpha/\pi \ln S/m_e^2$, where S is the square of the center-of-mass energy for the ep system. More precisely,

$$\sigma^\pm_{ep}(\kappa, S) \simeq \frac{\alpha}{\pi} \ln\frac{S}{m_e^2} \int_{M_W^2}^{S} \frac{ds}{s}\left(1 - \frac{s}{S} + \frac{s^2}{2S^2}\right)\sigma^\pm_{\gamma p}(\kappa, s). \tag{19}$$

This reaction has obvious experimental advantages over both $\gamma p \to W^\pm X$ and $ep \to \nu W^- X$.

This brings us to the elastic process treated in Sec. II. We emphasize that since form factors have not been included, the results of Sec. II do not represent the rates for $\gamma + p \to W^+ + n$ or $\gamma + n \to W^- + p$. As discussed in Ref. 9, these rates are substantially smaller because of the form factors entering at the weak and the electromagnetic vertices.

(a) For any value of κ we find that the total cross section for (ii) increases at most logarithmically. However, $\kappa=1$ stands out as a rather special case: $T(1,Q,s,t)$ is an impressive simplification of Eq. (5), and the $\ln s/M_W{}^2$ term in Eq. (8) drops out leaving a constant cross section for $\kappa=1$. This behavior is the opposite of the usual role which the anomalous magnetic moment of the W plays in other reactions, where it contributes divergent terms to be canceled by other diagrams, as in $e^+ + e^- \to W^+ + W^-$.

(b) It is remarkable that the leading terms are independent of Q. One may obtain the same result for σ by neglecting diagram (a) or diagram (b), but not both, in Fig. 1: Simply choose $Q=0$ or $Q=1$.

(c) Finally, we mention a few checks of our calculations. Equation (5) agrees with the trace given in Ref. 8 for $\kappa=0$. For $\kappa=1$ the cross section was obtained by integrating over t first and then setting $\kappa=1$. The same answer was obtained by reversing this procedure, that is, by using $T(1,Q,s,t)$ as integrand. We find, however, that our expression for general κ given in Eq. (8) is different[14] from the result of Ref. 9.

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APPENDIX

There are nonleading terms in the elastic cross section which involve both κ and Q, and which break the scaling behavior through $\ln s/m'^2$. A more complete expression replacing Eq. (8) is the following:

$$\sigma(\kappa,Q,s)=\frac{\alpha G_F}{8\sqrt{2}}\left\{2(\kappa-1)^2\ln\frac{s}{M_W{}^2}+\frac{\kappa^2}{2}+15\kappa+\tfrac{1}{2}\right.$$
$$+2\left(\frac{M_W{}^2}{s}\right)\left[4(Q-1)^2\ln\frac{s}{m'^2}+[(1-\kappa)(3+\kappa)-8Q(1+\kappa)]\ln\frac{s}{M_W{}^2}+2Q^2+4Q(1+\kappa)-\frac{\kappa^2}{2}-7\kappa-\tfrac{1}{2}\right]$$
$$\left.-16\left(\frac{M_W{}^2}{s}\right)^2(Q-1)^2\ln\frac{s}{m'^2}+O\left(\frac{M_W{}^4}{s^2}\ln\frac{s}{M_W{}^2}\right)\right\}. \quad (A.1)$$

It is interesting to note that the scale-breaking terms come from Fig. 1(b) and vanish for $Q=1$.

[1] BNL Report No. 50648, 1977 (unpublished).

[2] C. Quigg, Rev. Mod. Phys. 49, 297 (1977).

[3] L. B. Okun and M. B. Voloshin, Nucl. Phys. B120, 459 (1977).

[4] W. Alles, Ch. Boyer, and A. J. Buras, Nucl. Phys. B119, 125 (1977); F. Blezacker and H. T. Nieh, *ibid.* 124, 511 (1977).

[5] S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[6] S. Weinberg, Phys. Rev. Lett. 27, 1688 (1971).

[7] This is true in other models also. Elsewhere we will pursue the question whether all gauge theories require $\kappa=1$. The calculation reported in this paper suggests that other values of κ may also be accommodated in renormalizable theories.

[8] K. O. Mikaelian, Phys. Rev. D 5, 70 (1972).

[9] H. W. Fearing, M. Pratap, and J. Smith, Phys. Rev. D 5, 177 (1972).

[10] H. R. Reiss and M. H. Cha, Phys. Rev. Lett. 14, 399 (1965).

[11] W. Williamson, Jr. and R. T. Deck, Phys. Rev. 174, 1986 (1968).

[12] We follow the conventions of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965).

[13] J. Okada, S. Pakvasa, and S. F. Tuan, Lett. Nuovo Cimento 16, 555 (1976).

[14] Setting $\kappa^{n}=\kappa^{p}=F_1^n=0$, $F_1^p=F_1^W=F_2^W=F_V=-F_A=1$, Eqs. (4.3)–(4.6) of Ref. 9 give

$$\sigma=\frac{\alpha G_F}{8\sqrt{2}}\left[2(1+\kappa)(\kappa-3)\ln\frac{s}{M_W{}^2}+\frac{\kappa^2}{2}+15\kappa+\tfrac{1}{2}+14\right],$$

which can become negative because the coefficient of the $\ln s/M_W{}^2$ term is not positive definite. The suppression by many orders of magnitude for $\kappa=-1$ reported in Ref. 9 may be explained by noting that the above expression vanishes for $\kappa=-1$. When no form factors are introduced but κ^n and κ^p are kept finite a large cross section, $\sim 10^{-32}$ cm^2, is obtained because κ^n and κ^p appear in σ with the extra factor s/m^2.