Radiative corrections in electron-positron pair photoproduction

Guido Corbò
Istituto di fisica Guglielmo Marconi, Università di Roma, Roma, Italy and Istituto Nazionale di Fisica Nucleare, Sezione di Roma, Roma, Italy
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We discuss the shape of the cross section for the process of electron-positron pair production with radiative emission. We conclude that the so-called peaking approximation is valid only when the radiated photon is much less energetic than the emitted electrons.

The aim of this paper is to briefly discuss some general features of radiative corrections to the basic Bethe-Heitler mechanism which describes electron-positron pair production by γ rays. These features are only sketched in a previous short paper and deserve a more accurate analysis which we try to do here in view of the exact results obtained (Ref. 1).

The experiments on pair production are usually performed in a symmetric configuration, namely a γ ray hits a nucleus and the outcoming electron and positron are observed when they have the same energy and are produced symmetrically with respect to the direction of the incoming photon. It can be shown that, in this situation, nuclear effects are negligible and therefore the cross section is dominated by electromagnetic effects. Lowest-order diagrams for pair production are drawn in Fig. 1, where the cross denotes the nuclues, l_0 is the momentum of the incoming photon, and $p_-\equiv(\vec{p}_-, W_-)$, $p_+\equiv(\vec{p}_+, W_+)$ are the momenta of the electron and positron, respectively. q is the momentum of the virtual photon. If q is small, one can consider the nucleus to be always at rest during the process. The cross section $d\sigma_{\rm BH}$ one writes down from Fig. 1, in a general configuration, is given by Eq. (1.1) of Ref. 2. Knowing the momenta of e_-e_+, one can calculate the minimum energy $k_{0_{\rm min}}$ for the photon to produce such an event. In the symmetric configuration $d\sigma_{\rm BH}$ shows a minimum which is the more marked the higher the energies of e_-e_+ and this minimum is not resolved experimentally so that it is necessary, in comparing with experiments, to integrate $d\sigma_{\rm BH}$ over the acceptance of the apparatus both in energy and solid angle. If the energy of the pair is less than $k_{0_{\rm max}}$ (maximum value in the energy spectrum of incident photons) one has also processes in which a photon of energy k_{0i} (such that $k_{0_{\rm min}}<k_{0i}<k_{0_{\rm max}}$) gives a pair e_-e_+ with the emission of an unobserved photon with energy $k=k_{0i}-W_--W_+$. To study the "elastic" processes described by the Bethe-Heitler formula one could try to reduce the "inelastic" ones making $k_{0i}\approx k_{0_{\rm max}}$. But, in this way, one would reduce enormously the number of recorded events so that one looks for a compromise between the reduction of inelastic processes and a relevant number of countings. Typically, in the experiments, k_{0i} is such that $k_{0i}/k_{0_{\rm max}}=\frac{4}{5}$. One sees, for instance, that for a photon beam for which $k_{0_{\rm max}}=5$ GeV, the emission of a ~1-GeV photon which escapes observation is possible. It is then clear one needs a thorough treatment of radiative corrections due to the emission of real hard photons. One also needs the same treatment for soft and virtual photons because it can be shown that the cross section of radiative emission is proportional to $1/k$ for low k and it becomes infinite in the limit $k\to 0$ (infrared catastrophe). But one has to add to the contribution due to real photons the contribution due to the exchange of virtual photons; it can be seen that the latter has an infrared divergence too. These contributions are infinite of the same order and their sum goes to a finite limit so that the correction due to virtual photons eliminates the infrared catastrophe.

The problem of radiative corrections is a very complicated matter and it has been handled by introducing various approximations and simplifications. Bjorken, Drell, and Frautschi (Ref. 3) have calculated approximately the radiative corrections to symmetric pair production due to soft-photon emission. They give the following expression which includes also virtual-photon effects

$$d\sigma_{\rm BH}^{\rm rad}=-\frac{2\alpha}{\pi}\ln\frac{2p_-\cdot p_+}{m^2}\times\left(\ln\frac{W_\pm}{k_{0_{\rm max}}-2W_\pm}-\frac{13}{12}\right)d\sigma_{\rm BH}\ . \tag{1}$$

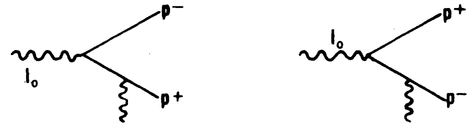

FIG. 1. Lowest-order diagrams for pair production.

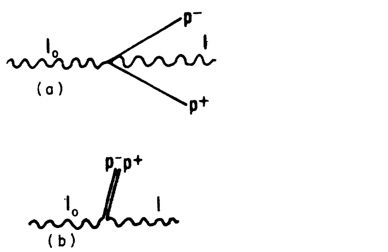

FIG. 2. (a) The first configuration studied by Ferrari and Thurnauer [Eq. (2)]; (b) the second configuration [Eq. (3)].

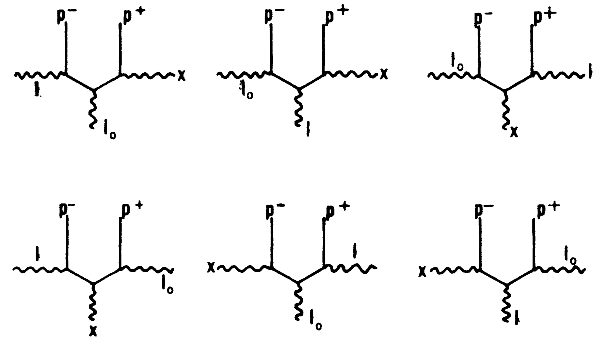

FIG. 3. The six diagrams contributing to pair production with radiative emission.

But the extension of Eq. (1), valid when $(k_{0_{max}} - 2W_\pm)/W_\pm \ll 1$ to the case $K_{0_{max}} - 2W_\pm \sim W_\pm$, has not been adequately justified. In the hypothesis that Eq. (1) is not adequate to describe hard-photon effects, attempts have been made to calculate approximately the dominant contribution due to these same photons.

Brodsky has made a calculation[4] in connection with an experiment by Asbury *et al.*[5] adopting the approximation that the dominant contributions are those due to the emission of the hard photon around the direction of the electron and the positron. In fact, as we shall see, the differential cross section of the radiative process shows a peak around the direction of the electron and positron which we hereafter call the Brodsky peak. This peaking approximation is justified by Huld's works (Refs. 2,6) in which there is also a treatment of virtual photons and of the infrared problem. Huld's works deserve special consideration because his results are very satisfactory, in particular, his formula which furnishes the differential cross section. The comparison between the results obtained by Huld's formula and those obtained by exact computation is discussed in Ref. 1. Before Huld's work, attempts to do an exact calculation have brought such complicated results that they are not useful in practice for comparison with experiments.[7] The only exact and handy calculation has been done by Ferrari and Thurnauer[8] who give two formulas, valid, however, only for two particular configurations, which furnish the differential cross section for pair production with radiative emission. The first formula Eq. (2) is valid when one has incident and radiated photons in the same direction, coplanar with the pair emitted symmetrically Fig. 2(a). The cross section is

$$d\sigma^{\text{rad}} = Z^2 G_E{}^2 (4\lambda^2)\,\frac{\alpha^4}{4\pi^4}\,\frac{p^2 k}{k_0}\, dW_+ dW_- d\Omega_+ d\Omega_- d\Omega_k\, \frac{1}{8W_\pm{}^2\lambda^4}\left(1+\frac{4Q^2W_\pm}{\lambda k k_0}+\frac{8Q^4W_\pm{}^2}{\lambda^2k^2k_0{}^2}\right). \tag{2}$$

The second formula is valid for incident and emitted photons in the same direction but with the $e_- e_+$ pair emitted in the same direction Fig. 2(b). This second configuration has been studied by Ferrari and Thurnauer after similar calculations by de Tollis, Jona-Lasinio, and Liotta.[9] The cross section is

$$d\sigma^{\text{rad}} = Z^2 G_E{}^2 [4(\lambda^2+Q^2)]\,\frac{\alpha^4}{4\pi^4}\,\frac{p^2 k}{k_0}\, dW_+ dW_- d\Omega_+ d\Omega_- d\Omega_k\, \frac{1}{8W_\pm{}^2(\lambda^2+Q^2)}\left(1+\frac{Q^2(2W_\pm+\lambda)}{\lambda^3}+\frac{Q^2m^2W_\pm{}^2}{4\lambda^4}\,\frac{k_0{}^2+k^2}{k_0{}^2k^2}\right). \tag{3}$$

Here, as in the preceding formula, $p = |\vec{p}_+| = |\vec{p}_-|$, $Q = p\sin\theta$, $\lambda = W_\pm - p\cos\theta$, and θ is the angle between the electron and the forward photon. Incidentally, Eqs. (2) and (3) have been used as a test for our exact calculation (Ref. 1).

Pair production in a Coulomb field with emission of radiation can be described by third-order Feynman diagrams. More precisely, we have six contributions as illustrated in Fig. 3 where, as before, $l_0 \equiv (\vec{k}_0, k_0)$ is the four-momentum of the incident photon, p_-, p_+ the four-momentum of the electron and positron, and $l \equiv (\vec{k}, k)$ is the four-momentum of the emitted photon. These diagrams differ for permutation of the vertices. In principle there are two other diagrams in which the photon is emitted directly by the nucleus. However, it has been shown (Ref. 2) that the contributions coming from these diagrams are very small for all configurations in which one can neglect nuclear effects. In particular, in the symmetric configuration, these contributions are rigorously zero. From Feynman rules one can write down the differential cross section for the situation in Fig. 3; namely,

$$d\sigma^{\mathrm{rad}}=\frac{(2\pi)^{-8}}{4kk_0}\frac{Z^2G_E(q^2)e^8}{q^4}\tfrac{1}{2}\sum_{\mathrm{spins}}|\bar{u}_-(p_-)Ou_+(p_+)|^2$$

$$\times\,\delta(k_0-k-W_--W_k)d^3k\,d^3p_+\,d^3p_-\ , \qquad (4)$$

where O is defined in Eq. (1) of Ref. 9. In Eq. (4), $G_E(q^2)$ is the form factor which contains the nuclear effects. For small q^2, $G_E(q^2)$ can be put equal to 1. The exploitation of Eq. (4) is obviously very hard and that is the reason why one has introduced various approximations. With an exact calculation at our disposal we can now discuss the general features of Eq. (4). At this point, the reader should refer to Ref. 1 and the figures therein to get an idea of the situation. The shape of the cross section depends essentially on the parameter $k/W_\pm$. More precisely, for $k/W_\pm \ll 1$ we are in the peaking approximation: The cross section rises sharply around the direction of the electron and positron. It should be noted that tridimensionally the Brodsky peak looks like a crater around the electron and positron. The dip of the crater corresponds to the emission in the exact direction of the electron and positron. This structure is understandable on physical grounds. In the relativistic case, one can say that when the photon is emitted it comes from the bremsstrahlung of the electron and positron. The well-known formula[10] which gives the bremsstrahlung cross section shows that for low k and relativistic electrons the emission is preferably around the direction that the electron has in the initial and final states with a structure which depends on a factor

$$\sim\frac{p_i{}^2\sin^2\theta_i}{(E_i-p_i\cos\theta_i)^2}$$

and

$$\frac{p_f{}^2\sin^2\theta_f}{(E_f-p_f\cos\theta_f)^2}\ ,$$

respectively, where θ_i,θ_f are the angles between the direction of k and those of the electrons with momentum p and energy E in the initial and final states, respectively. In our case of pair production, initial and final states mean positron and electron states. This justifies the peaking approximation for low $k/W_\pm$ with bremsstrahlung shape. This physical interpretation is also obtained by an examination of Huld's formula Eq. (5) for the differential cross section:

$$d\sigma_{\mathrm{Huld}}=\frac{\alpha^4Z^2}{16\pi^4}\frac{W_-W_+k}{k_0}M\,dW_-\,dW_+\,dk\,d\Omega_-\,d\Omega_+\,d\Omega_k\ , \qquad (5)$$

$$M=\left[\frac{2p_-\cdot p_+}{(l\cdot p_-)(l\cdot p_+)}\left(1+\frac{1}{2}\,\frac{4(l\cdot p_-)^2+4(l\cdot p_+)^2}{Q^2(Q^2+2l\cdot p_-+2l\cdot p_+)}\right)-\frac{m^2}{(l\cdot p_-)^2}-\frac{m^2}{(l\cdot p_+)^2}\right]\frac{1}{q^4}M_{\mathrm{BH}}(p'_+,p'_-)\ ,$$

where

$$p'_++p'_-=p_++p_-+l\ ,$$

$$p'_+-p'_-=p_+\left(1+\frac{l\cdot p_-}{p_+\cdot p_-}\right)-p_-\left(1+\frac{l\cdot p_+}{p_+\cdot p_-}\right)\ ,$$

$$Q=p_-+p_+\ ,$$

m is the mass of the electron, and M_{BH} is linked to $d\sigma_{\mathrm{BH}}$ as pointed out in Ref. 2. When k is very small with respect to $W_\pm$, the momenta p'_-,p'_+ remain nearly the same on changing the direction of l and therefore M_{BH} can be considered a constant. The structure of $d\sigma$ is consequently independent of M_{BH}. Moreover, the term $[2(l\cdot p_-)^2+2(l\cdot p_+)^2]/[Q^2(Q^2+2l\cdot p_-+2l\cdot p_+)]$ is negligible compared with 1, so that the structure of $d\sigma$ is determined by the latter. From Eq. (5) one can see that it determines the crater structure with a very deep dip. In practice, $d\sigma$ is equal, apart from a constant of proportionality, to the bremsstrahlung one. When k increases, the above interpretation no longer applies: The term

$$\frac{2p_-\cdot p_+}{(l\cdot p_-)(l\cdot p_+)}-\frac{m^2}{(l\cdot p_-)^2}-\frac{m^2}{(l\cdot p_+)^2}$$

maintains the same structure but the dip is "filled up" by the term $[2(l\cdot p_-)^2+(l\cdot p_+)^2]/[Q^2(Q^2+2l\cdot p_-+2l\cdot p_+)]$, which is no more negligible and, moreover shows a peak when the direction of l coincides with that of e_- or e_+. Besides this, when l is not negligible with respect to p_-,p_+ the term $M_{\mathrm{BH}}(p'_+,p'_-)$ is no more a constant on variation of the direction of l, and, consequently, when l is close to the forward direction, where bremsstrahlung effects are negligible, $d\sigma$ behaves like the Bethe-Heitler cross section near the symmetric configuration with a dip when the photon is rigorously emitted forward and with an abrupt climb when one goes away from symmetry. One can conclude that when $k/W_\pm \ll 1$ one observes a bremsstrahlung around the direction of emitted electron and positron and the peaking approximation is fully justified; on the other hand, as $k/W_\pm$ increases, one sees the Bethe-Heitler structure around the forward direction and must be careful in that we have other important contributions besides the Brodsky peak.

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