Modified quantum chromodynamics: Exact global color symmetry and asymptotic freedom*

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A theory of modified quantum chromodynamics is presented in which color is realized as a global rather than local SU(3) symmetry. Quarks and gluons are then free to exist, but below a certain energy threshold they are still effectively confined by the same dynamics as in quantum chromodynamics. In addition to the massive vector-gluon octet, the theory also contains an octet and two singlets of Higgs particles, or scalar gluons. Asymptotic freedom is maintained by the imposition of eigenvalue conditions on all coupling constants such that only one of them, say the gauge coupling g, is an independent parameter. This is shown to imply the existence of extraordinary quarks which are not triplets under color SU(3).

I. INTRODUCTION

In order to understand the totality of particle physics, it has often been assumed that hadrons are composed of quarks and that there is an underlying field theory of quark interactions which is ultimately responsible for all strong-interaction phenomena. This fundamental theory is now believed to be a non-Abelian generalization of quantum electrodynamics, and is called *quantum chromodynamics* because it is a dynamical realization of the by-now familiar concept of "color" as an internal SU(3) symmetry for each quark species, of which there are at least four: u, d, s, and c. To be more precise, one assumes that each quark species (or "flavor") is a color SU(3) triplet and that it interacts with itself and others via an octet of massless Yang-Mills vector gauge bosons A_a^μ ($a=1,\ldots,8$) called gluons. In fact, the Lagrangian of this theory is very simply given by

$$\mathcal{L} = -\tfrac{1}{4} F_a^{\mu\nu} F_{a\mu\nu} + i\bar{q}\gamma_\mu D^\mu q - m\bar{q}q\,, \tag{1.1}$$

where

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - g f_{abc} A_b^\mu A_c^\nu \tag{1.2}$$

and

$$D^\mu q = \partial^\mu q + \tfrac{1}{2} i g A_a^\mu \lambda_a q\,. \tag{1.3}$$

In the above, g is the gauge coupling, f_{abc} the conventional set of SU(3) structure constants, and λ_a the corresponding set of 3×3 representation matrices. The summation over quark flavors is implied.

The Lagrangian (1.1) is constructed to be locally gauge-invariant with respect to color SU(3), therefore the gluons are required to be massless. However, since neither quarks nor gluons have been identified as such experimentally, they must somehow be rather effectively confined and not easily exist as isolated objects. The mechanism for achieving this confinement is unknown, except that it is believed to come from quantum chromodynamics. There are two possible ways for this to occur. (1) The infrared divergence of this theory might be so drastic that it is impossible to define asymptotic states for either quarks or gluons, and so they simply cannot exist as such. However, a careful examination of the infrared problem in non-Abelian gauge theory shows[1] that, with the proper presecription, it can be handled order by order in perturbation theory, and hence cannot be invoked by itself to show confinement. (2) Since the gauge fields are self-coupled, and color is an exact local symmetry, confinement might be a property of this particular type of dynamics. There is no rigorous proof of this, only hints that it might be true; but in the absence of any contradiction, it can simply be accepted as a premise.

If gluons are indeed massless as in quantum chromodynamics, then confinement has to be absolute. This means that isolated quarks of fractional charges cannot exist. Experimentally, such a statement is impossible to prove; it can only be contradicted. In fact, a recent search for fractional charge on matter has come up with some positive results.[2] Whether or not this should be taken as evidence for quarks, it is still an important theoretical question as to how quantum chromodynamics can best be modified to incorporate the existence of physical quarks and gluons. The subject matter of this paper is a detailed account of how it can be done.

In Sec. II, the model is described in length. All essential features are reported; only mathematical details have been left out, to be presented in the next two sections. In Sec. III, the Higgs mechanism which brings about the local to global color transmutation is discussed in detail. In Sec. IV, the problem of maintaining asymptotic freedom is solved by imposing eigenvalue conditions on all coupling constants. This turns out to imply the existence of extraordinary quarks. Finally in Sec. V, there are some concluding remarks.

II. DESCRIPTION OF MODEL

If gluons are massive, then there is no reason to believe that they have to be permanently confined. However, an explicit mass term for A_a^μ cannot be added to the quantum chromodynamics Lagrangian (1.1), because it destroys the local gauge invariance, and hence the renormalizability of the field theory. The only proven way of circumventing this difficulty is to make use of the Higgs mechanism.[3] Therefore, I propose to introduce three color SU(3) triplets of scalar particles, such that their interaction results in a complete spontaneous breakdown of the local SU(3) gauge symmetry, thereby endowing all gluons with mass, but at the same time allowing an exact *global* SU(3) symmetry to survive.[4] The resulting Lagrangian in the U gauge (where all unphysical degrees of freedom have been eliminated) will be globally SU(3)-invariant, and in addition to the vector-gluon octet, there will be one octet and two singlets of physical Higgs particles. Details of this local to global color transmutation will be given in Sec. III.

Since the vector gauge fields A_a^μ in the present theory are still self-coupled in the same way as in quantum chromodynamics, and color remains an exact non-Abelian symmetry, the dynamics of confinement is evidently the same for both at short distances. Thus, for example, the effective interaction between a quark and an antiquark separated by a distance of the order of fermis could very well be described in both theories by a linearly rising potential as is often assumed. The difference is that in the present theory, the potential will not be absolutely confining, and given enough energy, quarks will be able to get out; in other words, instead of forever rising, the potential will turn over at some point.

However, the theory has a serious drawback as it stands, because unlike a purely non-Abelian gauge theory, it is not asymptotically free.[5] The trouble comes about because of the quartic scalar couplings.[6] Fortunately, there is a way to get around this, and that is to impose eigenvalue conditions on all coupling constants.[7] This means that only one coupling, say g, is assumed independent, while all others are taken to be functions of g. Even so, there is no guarantee that for a particular model, asymptotically free solutions exist. They depend crucially on the ability of the Yukawa couplings to alter the behavior of the quartic scalar couplings in a significant way. In the present theory, it is in fact necessary to introduce additional quarks which are not triplets under color SU(3), in order to find asymptotically free solutions. Together with the requirement that the theory remains globally SU(3)-invariant after spontaneous symmetry breakdown, this results in only two possible solutions. In the quark sector, one solution has two color octets and one color singlet; the other has two $\underline{6}$'s and one $\underline{3}^*$. Details will be given in Sec. IV.

To summarize, the theory of modified quantum chromodynamics has the following notable features. (1) It is a renormalizable field theory. (2) It has normal behavior in the infrared limit, i.e., massive asymptotic states can be defined. (3) There is an exact global SU(3) symmetry identifiable with color. (4) For the purpose of confinement, its short-distance behavior is the same as quantum chromodynamics. (5) It is asymptotically free by virtue of the imposition of eigenvalue conditions on all coupling constants. (6) Because of (5), there is only one independent coupling g. All others are calculable in terms of it in the ultraviolet limit. (7) In addition to having a massive vector-gluon octet, and the usual quark triplets, the theory also contains one octet and two singlets of Higgs particles (or scalar gluons), as well as extraordinary quarks which are not color triplets. (8) The requirement of both (3) and (5) allows only two solutions: one has two octets and one singlet of quarks, the other two $\underline{6}$'s and one $\underline{3}^*$. Each of these groups must have common flavor quantum numbers, and they interact directly with the scalar gluons, whereas the usual quark triplets only do so with the vector gluons.

In the following two sections, it will be shown in detail how all of the preceding comes about.

III. THE LOCAL TO GLOBAL COLOR TRANSMUTATION

Let $\Phi = (\phi_1, \phi_2, \phi_3)$ be a global SU(3) triplet of scalar fields, each one of which is also a local color SU(3) triplet. Then the most general renormalizable Higgs potential invariant under $SU(3)_{\text{local}} \times SU(3)_{\text{global}}$ is given by[8]

$$V=\mu^2\,\mathrm{Tr}\Phi^\dagger\Phi+\tfrac{1}{2}\eta(\mathrm{Tr}\Phi^\dagger\Phi)^2+\tfrac{1}{2}\rho\,\mathrm{Tr}\Phi^\dagger\Phi\Phi^\dagger\Phi - \sqrt{6}\,(\delta\det\Phi+\mathrm{H.c.})\,, \tag{3.1}$$

where the quartic scalar couplings η and ρ must satisfy

$$\eta+\rho>0\,,\quad 3\eta+\rho>0\,, \tag{3.2}$$

in order that V be bounded from below. The coupling δ (which has the dimension of mass) can be chosen to be real and positive without loss of generality by a suitable redefinition of the phase of Φ. Notice also that if $\delta=0$, then V has an additional U(1) symmetry, and an unwanted Goldstone boson will appear.[4] To facilitate certain calculations, V can also be written as follows[4]:

$$V=\mu^2\sum_{a=1}^{3}\phi_a^\dagger\phi_a+\tfrac{1}{2}(\eta+\rho)\sum_{a=1}^{3}(\phi_a^\dagger\phi_a)^2 +\tfrac{1}{2}\eta\sum_{a\neq b}(\phi_a^\dagger\phi_a)(\phi_b^\dagger\phi_b)+\tfrac{1}{2}\rho\sum_{a\neq b}(\phi_a^\dagger\phi_b)(\phi_b^\dagger\phi_a) -\frac{1}{\sqrt{6}}(\delta\epsilon_{abc}\epsilon_{ijk}\phi_{ai}\phi_{bj}\phi_{ck}+\text{H.c.})\,. \tag{3.3}$$

Given the present Higgs structure, there are only two possible patterns of spontaneous symmetry breaking. (1) Only one component of one ϕ acquires a vacuum expectation value. This breaks the original SU(3) × SU(3) symmetry into SU(2) × SU(2) × U(1). (2) All three ϕ's acquire vacuum expectation values, but in different components. Then the residual symmetry is SU(3). The parameter ρ is crucial in determining which of these two is in fact chosen. In the limit $\delta=0$, the condition for spontaneous symmetry breakdown of V is simply $\mu^2<0$; and depending on whether ρ is negative or positive, either (1) or (2) will occur. If $\delta\neq 0$, the situation is somewhat more complicated. However, if we assume $\delta^2\ll|-\mu^2\eta|$, so that the tree approximation can be used in calculating the vacuum expectation values of Φ, then the condition for (1) simply becomes $\rho<-2\sqrt{6}\,\delta[\eta/(-\mu^2)]^{1/2}$, whereas for (2) it is changed to $\rho>-3\sqrt{2}\,\delta[\eta/(-\mu^2)]^{1/2}$.

The case of interest is obviously (2) since it has a residual global SU(3) symmetry which can be identified as color in place of the original local symmetry. After spontaneous breakdown, 8 out of 18 degrees of freedom contained in Φ are lost to the vector gluons, but the other 10 remain physical scalar particles, and they can be grouped together to form one octet $\vec{x}$ and two singlets x_0 and y_0. In the U gauge,

$$\Phi=\tfrac{1}{2}\vec{\lambda}\cdot\vec{x}+\frac{1}{\sqrt{6}}(x_0+iy_0)I\,, \tag{3.4}$$

where $\langle x_0\rangle=v\neq 0$. [The fact that $\langle\Phi\rangle$ is proportional to the identity matrix I could be easily shown.[4]] The Higgs potential V of Eq. (3.1) now becomes

$$V=\tfrac{1}{2}\mu^2(x_0^2+y_0^2+\vec{x}\cdot\vec{x})+\tfrac{1}{8}(\eta+\tfrac{1}{3}\rho)(x_0^2+y_0^2)^2 +\tfrac{1}{4}(\eta+\rho)x_0^2(\vec{x}\cdot\vec{x})+\tfrac{1}{4}(\eta+\tfrac{1}{3}\rho)y_0^2(\vec{x}\cdot\vec{x}) +\tfrac{1}{8}(\eta+\tfrac{1}{2}\rho)(\vec{x}\cdot\vec{x})^2-\tfrac{1}{3}\delta x_0(x_0^2-3y_0^2-\tfrac{3}{2}\vec{x}\cdot\vec{x}) +\frac{1}{\sqrt{6}}(\tfrac{1}{2}\rho x_0-\delta)d_{abc}\,x_ax_bx_c\,, \tag{3.5}$$

which is manifestly invariant under SU(3). Therefore, in breaking the original SU(3) × SU(3) symmetry down to just SU(3), the three scalar triplets of local color SU(3) have rearranged themselves to produce an octet and two singlets of global color SU(3). In the tree approximation, their masses are given by

$$m_{y_0}^2=3\delta v\,,\quad m_{x_0}^2=(\eta+\tfrac{1}{3}\rho)v^2-\tfrac{1}{3}m_{y_0}^2\,,\quad m_{\vec{x}}^2=\tfrac{1}{3}\rho v^2+\tfrac{2}{3}m_{y_0}^2\,, \tag{3.6}$$

where

$$v^2=\frac{-6\mu^2+2m_{y_0}^2}{3\eta+\rho}\,. \tag{3.7}$$

Notice of course that if $\delta=0$, y_0 becomes a truly massless Goldstone boson, which is clearly undesirable.

Now we turn to the part of the Lagrangian which describes the vector-scalar interactions,

$$\mathcal{L}_{V\text{-}S}=\mathrm{Tr}(D^\mu\Phi)^\dagger(D_\mu\Phi)\,, \tag{3.8}$$

where

$$D_\mu\Phi=(\partial_\mu+\tfrac{1}{2}ig\vec{A}_\mu\cdot\vec{\lambda})\Phi\,. \tag{3.9}$$

Using Eq. (3.4) for Φ, we find

$$\mathcal{L}_{V\text{-}S}=\tfrac{1}{2}(\partial^\mu x_0)(\partial_\mu x_0)+\tfrac{1}{2}(\partial^\mu y_0)(\partial_\mu y_0)+\tfrac{1}{2}(\partial^\mu\vec{x})\cdot(\partial_\mu\vec{x}) +\tfrac{1}{2}gf_{abc}A_a^\mu(\partial_\mu x_b)x_c +\frac{1}{\sqrt{6}}g\vec{A}_\mu\cdot[(\partial^\mu y_0)\vec{x}-(\partial^\mu\vec{x})y_0] +\tfrac{1}{12}g^2(x_0^2+y_0^2)(\vec{A}_\mu\cdot\vec{A}^\mu)+\tfrac{13}{168}g^2(\vec{x}\cdot\vec{x})(\vec{A}_\mu\cdot\vec{A}^\mu) +\tfrac{1}{21}g^2(\vec{x}\cdot\vec{A}_\mu)^2+\frac{1}{2\sqrt{6}}g^2x_0d_{abc}x_aA_b^\mu A_{\mu c}\,, \tag{3.10}$$

which is again manifestly invariant under global SU(3). The vector gluons are, of course, no longer massless, but they still transform as an octet with a common mass given by

$$m_{\vec{A}}^2=\tfrac{1}{6}g^2v^2\,. \tag{3.11}$$

In conclusion, we have managed to modify the quantum chromodynamics Lagrangian of Eq. (1.1) by the introduction of a scalar multiplet Φ, in such a way that the new Lagrangian which is the sum of Eqs. (1.1), (3.5), and (3.10) retains an exact (albeit global) color SU(3) symmetry. Since the vector gluons become massive via the Higgs mechanism, this theory is certainly renormalizable; it is at the same time free of infrared divergences, so there is no reason why quarks and gluons should not exist as bona fide physical states. Exactly how a free quark should behave is an open question, and models have been proposed[8,9] to explain the recent report[2] that fractional charge on matter has been found. However, the only difference between quantum chromodynamics and the present theory, disregarding the scalar interactions for the time being, is the appearance of a mass term for $\vec{A}_\mu$, so if the former leads to a confinement potential of infinite range, that due to the latter must be of finite range.[8] To make sure that this result holds even in the presence of scalar interactions, we must insist that they do not alter significantly the short-distance behavior of quantum chromodynamics. Now it is well known that a purely non-Abelian gauge theory is asymptotically free,[5] but in the present theory, there are quartic scalar couplings (η and ρ) as well, and the statement of asymptotic freedom becomes much more complicated.[6,7] The next section will deal with this problem in detail.

IV. ASYMPTOTIC FREEDOM AND EXTRAORDINARY QUARKS

Given a Lagrangian which is the sum of Eqs. (1.1), (3.1), and (3.8), and using the renormalization-group technique, the following differential equations for g, η, and ρ (considered as effective, or running, couplings) can be obtained[5,6]:

$$16\pi^2 \frac{dg}{dt} = -[\tfrac{11}{3}(3) - \tfrac{4}{3} S_3(F) - \tfrac{1}{3}(\tfrac{1}{2})(3)]g^3$$

$$= -\tfrac{4}{3}[\tfrac{63}{8} - S_3(F)]g^3 \equiv -\tfrac{1}{2} b_0 g^3 , \qquad (4.1)$$

$$8\pi^2 \frac{d\eta}{dt} = 13\eta^2 + 12\eta\rho + 3\rho^2 - 8\eta g^2 + \tfrac{11}{12} g^4 , \qquad (4.2)$$

and

$$8\pi^2 \frac{d\rho}{dt} = 6\rho^2 + 6\eta\rho - 3\rho g^2 + \tfrac{5}{4} g^4 . \qquad (4.3)$$

In the above, the parameter t is minus the logarithm of the scale by which the renormalization point is changed, and $S_3(F)$ is defined in terms of the representation matrices t_i of the total fermion (quark) multiplet as follows:

$$\mathrm{Tr}(t_i t_j) = S_3(F)\delta_{ij} . \qquad (4.4)$$

In order that the theory be asymptotically free, b_0 in Eq. (4.1) must be positive[5]; therefore S_3 must be less then $\frac{63}{8}$. For a given irreducible SU(3) representation of dimension N, $S_3 = \frac{1}{8} N S_2$, where S_2 is the value of the quadratic Casimir operator for that representation. Specifically, for the representation $D(p,q)$, $S_2 = \frac{1}{3}[p^2 + q^2 + pq + 3(p+q)]$, therefore the value of S_3 increases rapidly with increasing N, and so we need only consider fermion representations of the lowest dimensions, such as $\underline{3}$, $\underline{6}$, and $\underline{8}$. If quarks only come in color triplets and there are n of them, then $S_3 = \frac{1}{2} n$, implying that $n \leq 15$.

In the absence of contributions from Yukawa couplings (which would be the case if all quarks are color triplets), Eqs. (4.2) and (4.3) do not admit any asymptotically free solution, even if η and ρ are assumed to be functions of g^2 alone.[7] This means that it is necessary to have quark multiplets which couple to Φ. However, we must do it in such a way as to retain the global color SU(3) symmetry of the Lagrangian in the U gauge. It turns out that there are only two possible solutions to be discussed below.

Let $\Psi = (\psi_1, \psi_2, \psi_3)$ be a global SU(3) triplet of quark fields, each one of which is also a local color SU(3) triplet, in exact analogy to Φ. Let ξ be another quark multiplet which is a global singlet but a local octet. Then

$$\mathcal{L}_Y = -\tfrac{1}{2}\alpha(\mathrm{Tr}\bar{\Psi}\lambda_a \Phi)\xi_a + \mathrm{H.c.} , \qquad (4.5)$$

provided that both Ψ and ξ have the same flavor quantum numbers, such as electric charge, etc. Again using the technique of the renormalization group, we find

$$16\pi^2 \frac{d\alpha}{dt} = \alpha(\tfrac{49}{12}\alpha^2 - 13 g^2) . \qquad (4.6)$$

If α is independent of g, then the only asymptotically free solution is $\alpha = 0$, which means that it is forced to vanish in the ultraviolet limit faster than g. However, assuming that an eigenvalue condition exists for α as a function of g, we can look for a solution of the form

$$\alpha^2 = K g^2 + O(g^4) \qquad (4.7)$$

for small g, where K is a nonzero constant independent of t. Under this assumption, Eqs. (4.1) and (4.6) must be identical, and so

$$K = \tfrac{12}{49}(13 - \tfrac{1}{2} b_0) . \qquad (4.8)$$

In the presence of $\mathcal{L}_Y$, Eqs. (4.2) and (4.3) must now include additional contributions, and they become

$$8\pi^2 \frac{d\eta}{dt} = 13\eta^2 + 12\eta\rho + 3\rho^2 - 8\eta g^2 + \tfrac{16}{3}\alpha^2 g^2 + \tfrac{11}{12} g^4 - \tfrac{1}{36}\alpha^4 \qquad (4.9)$$

and

$$8\pi^2 \frac{d\rho}{dt} = 6\rho^2 + 6\eta\rho - 8\rho g^2 + \tfrac{16}{3}\alpha^2 g^2 + \tfrac{5}{4} g^4 - \tfrac{7}{12}\alpha^4 . \qquad (4.10)$$

Now using Eq. (4.8), and making the corresponding assumption that η/g^2 and ρ/g^2 are constants for small g, we find that in order for Eqs. (4.9) and (4.10) to be identical to Eqs. (4.1) and (4.6), we must have the following numerical solutions:

$\frac{1}{2}b_0$	K	ρ/g^2	η/g^2
$\frac{1}{2}$	$\frac{150}{49}$	0.396	−0.091
$\frac{7}{6}$	$\frac{142}{49}$	0.357	−0.091
$\frac{11}{6}$	$\frac{134}{49}$	0.318	−0.092

(4.11)

The value of $\frac{1}{2}b_0$ is determined by how many additional quark multiplets are put in besides Ψ and ξ. The minimum value of $\frac{1}{2}$ corresponds to six triplets, but since we know that there are at least four triplets (u, d, s, and c), the maximum value is $\frac{11}{6}$.

To show that exact global color SU(3) symmetry is retained by $\mathcal{L}_Y$ in the U gauge, we decompose Ψ into an octet $\vec{\psi}$ and a singlet ψ_0 as follows:

$$\Psi = \frac{1}{\sqrt{2}}\vec{\lambda}\cdot\vec{\psi} + \frac{1}{\sqrt{3}}\psi_0 I ; \tag{4.12}$$

then

$$\mathcal{L}_Y = -\frac{\alpha}{2\sqrt{3}}\left[\bar{\psi}_a\xi_a(x_0+iy_0)+\bar{\psi}_0\xi_a x_a + (\tfrac{3}{2})^{1/2}(d_{abc}+if_{abc})\bar{\psi}_a\xi_b x_c\right] + \text{H.c.} , \tag{4.13}$$

which is manifestly invariant under SU(3) as expected. The term $i\,\mathrm{Tr}\bar{\Psi}\gamma^\mu D_\mu\Psi$ which also appears in the total Lagrangian can be written in a similar way. Notice the similarity between Eqs. (3.4) and (4.12) for Φ and Ψ, respectively. In both cases, three local color triplets have rearranged themselves to form global color octets and singlets. The physical quark sector of this model has therefore two octets (linear combinations of $\vec{\psi}$ and $\vec{\xi}$) and one singlet ψ_0, plus up to six triplets. Present strong-interaction phenomenology establishes only four triplets (u, d, s, and c), so there is room for two more. These "ordinary" quarks do not interact directly with the scalr gluons $\vec{x}$, x_0, or y_0, so their hadron spectrum is not expected to be much different from that of usual quantum chromodynamics. On the other hand, the "extraordinary" quarks $\vec{\psi}$, $\vec{\xi}$, and ψ_0 have much more complicated direct interactions, which can be the case of significant change in their hadron dynamics. This problem will be dealt with in a separate paper.

The only other possible solution with exact global color symmetry and asymptotic freedom is as follows. Let ξ' be a quark multiplet which is a global singlet but a local sextet. Then

$$\mathcal{L}_Y = -\beta\bar{\xi}'_k(\mathrm{Tr}\Psi S_k\Phi) , \tag{4.14}$$

where S_k ($k=1,\ldots,6$) is a set of 3×3 real symmetric matrices, normalized to $\mathrm{Tr}S_kS_{k'}=\delta_{kk'}$. Proceeding as before, we find

$$16\pi^2\frac{d\beta}{dt} = \beta(\tfrac{13}{2}\beta^2 - 14g^2) , \tag{4.15}$$

so that

$$K = \tfrac{2}{13}(14 - \tfrac{1}{2}b_0) . \tag{4.16}$$

The corresponding differential equations for η and ρ are now

$$8\pi^2\frac{d\eta}{dt} = 13\eta^2 + 12\eta\rho + 3\rho^2 - 8\eta g^2 + 8\beta^2 g^2 + \tfrac{11}{12}g^4 - \tfrac{1}{4}\beta^4 \tag{4.17}$$

and

$$8\pi^2\frac{d\rho}{dt} = 6\rho^2 + 6\eta\rho - 8\rho g^2 + 8\beta^2 g^2 + \tfrac{5}{4}g^4 - \tfrac{5}{4}\beta^4 . \tag{4.18}$$

From these, we obtain the following numerical solutions:

$\frac{1}{2}b_0$	K	ρ/g^2	η/g^2
$\frac{1}{2}$	$\frac{27}{13}$	0.369	−0.019
$\frac{7}{6}$	$\frac{77}{39}$	0.334	−0.022
$\frac{11}{6}$	$\frac{73}{39}$	0.300	−0.026
$\frac{5}{2}$	$\frac{23}{13}$	0.265	−0.030

(4.19)

This time, $\frac{1}{2}b_0 = \frac{1}{2}$ corresponds to seven "ordinary" triplets. In the U gauge, instead of turning into an octet and a singlet, Ψ now becomes $\underline{6}$ and a $\underline{3}^*$, so that $\mathcal{L}_Y$ of Eq. (4.14) remains invariant under global color SU(3). The physical quark sector of this model has therefore two sextets (linear combinations of ξ' and $\psi_{\underline{6}}$) and one $\underline{3}^*$, plus up to seven $\underline{3}$'s.

In conclusion, the theory of modified quantum chromodynamics has two asymptotically free solutions with exact global color SU(3) symmetry. Although there are four dimensionless coupling constants, only one (the gauge coupling g) is independent, so that at least conceptually, simplicity has been maintained. Moreover, it predicts the existence of "extraordinary" quarks not belonging to the triplet representation of color SU(3), and the form of their fundamental dynamics. The number of "ordinary" quark flavors is also more severely limited. In the next section, I will discuss some of this theory's many possible implications.

V. CONCLUDING REMARKS

In the present theory of modified quantum chromodynamics, color appears as an exact global SU(3) symmetry, but confinement is only effective below a certain energy threshold as discussed in Ref. 8. There it was found that within a "bag" formalism, the mass of an isolated quark or gluon is *inversely* proportional to the vacuum expectation value v of Φ. This means that the symmetry breaking may in fact be very soft, and the necessity to introduce nontriplet quarks to obtain eigenvalue solutions for the quartic scalar couplings may be eliminated, if only "temporary" asymptotic freedom is desired.[10]

The idea that quarks may come in color nontriplets is not new,[11] but only in the context of the present theory (and other related ones given in Ref. 7) is it seen to be necessary. Furthermore, definite predictions are made as to the number and type of these quarks, together with the form of their basic interactions. Let us consider the first case presented in Sec. IV, where there are two quark octets $\vec{\psi}$ and $\vec{\xi}$, and one quark singlet ψ_0. Assuming that they do combine with quark triplets to form hadrons, then a color-singlet hadron state such as $q\bar{q}\xi$, where q is a usual quark triplet, will have whatever electric charge ξ has. The existence of fractionally charged hadrons cannot therefore be ruled out. Notice also that although ψ_0 is a color singlet, it can interact with $\vec{\xi}$ through the scalar-gluon octet $\vec{x}$. The statement that color singlets can only interact through some kind of residual color Van der Waals force applies only to bound states in this theory.

Finally, let me speculate on a possible interpretation, within the present context, of the recent observation[12] of a dimuon resonance at 9.5 GeV. This state could very well be a quark-antiquark bound state, similar to the J/ψ. The question is whether this fifth quark is "ordinary" (i.e., a color triplet) or something else. If the former, then we expect it to mix with the other four known quarks (u, d, s, and c) in their weak interactions. But since the standard four-quark model works so well in this area, the mixing must be very small or even zero. This could be an accident, but then on the other hand, if we assume that there are only *four* "ordinary" quarks, and the 9.5-GeV object is, say, a $\bar{\xi}\xi$ bound state, the zero mixing is easily understood. In fact, a state such as $q\bar{q}\xi$ will be stable (unless there are quark-lepton transitions as in superunification gauge models) if it is the lightest particle of its kind. Therefore, instead of finding more and more quark flavors of the usual type, something really new and different may be awaiting the next generation of accelerators.

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[1]Y.-P. Yao, Phys. Rev. Lett. 36, 653 (1976); T. Appelquist, J. Carazzone, H. Kluberg-Stern, and M. Roth, *ibid.* 36, 768 (1976); 36, 1161(E) (1976).

[2]G. S. LaRue, W. M. Fairbank, and A. F. Hebard, Phys. Rev. Lett. 38, 1011 (1977).

[3]For a review, see for example, J. Bernstein, Rev. Mod. Phys. 46, 7 (1974).

[4]E. Ma, University of Hawaii Report No. UH-511-239-77 (unpublished); R. N. Mohapatra, J. C. Pati, and A. Salam, Phys. Rev. D 13, 1733 (1976); R. N. Mohapatra and J. C. Pati, Phys. Lett. 63B, 204 (1976).

[5]H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973); D. J. Gross and F. Wilczek, *ibid.* 30, 1343 (1973); Phys. Rev. D 8, 3633 (1973); 9, 980 (1974).

[6]T. P. Cheng, E. Eichten, and L.-F. Li, Phys. Rev. D 9, 2259 (1974).

[7]N.-P. Chang, Phys. Rev. D 10, 2706 (1974); E. Ma, *ibid.* 11, 322 (1975); Prog. Theor. Phys. 54, 1828 (1975); Phys. Lett. 62B, 347 (1976); Nucl. Phys. B116, 195 (1976); E. S. Fradkin and O. K. Kalashnikov, Phys. Lett. 59B, 159 (1975); 64B, 177 (1976).

[8]A. De Rújula, R. C. Giles, and R. L. Jaffe, Phys. Rev. D xx, xxxx (197x).

[9]M. J. Longo, University of Michigan Reports No. UM-HE-77-3 and No. UM-HE-77-37 (unpublished).

[10]H. D. Politzer and G. G. Ross, Nucl. Phys. B75, 269 (1974).

[11]E. Ma, Phys. Lett. 58B, 442 (1975); G. Karl, Phys. Rev. D 14, 2374 (1976); F. Wilczek and A. Zee, *ibid.* 16, 860 (1977).

[12]S. W. Herb *et al.*, Phys. Rev. Lett. 39, 252 (1977).