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Nonrenormalizable interactions and an eigenvalue condition

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It is pointed out that a certain class of nonrenormalizable theories can be made renormalizable if a theory possesses an ultraviolet-stable fixed point. As an example, four-fermion theories of the Nambu–Jona-Lasinio-type are considered.

It is well known that in a class of nonrenormalizable field theories, interesting collective phenomena occur. In particular, in Nambu–Jona-Lasinio–type[1] four-fermion theories, one can show the existence of collective bosonic bound states by solving Bethe-Salpeter equations within a certain approximation. Thus these models are actually interacting theories of fermions and bosons despite the fact that the original Lagrangian contains only spinor fields.

Usually these four-fermion models are, of course, regarded as nonrenormalizable. In a case of current-current-type self-interaction, however, it was known for some time that one can formally develop a renormalizable perturbation series.[2,3] Here the basic idea is to expand the theory in terms of induced coupling constants between collective bosons and fermions instead of the original four-fermion coupling constant. In the model of Refs. 2 and 3, a collective bound state, a photon, appears in the vector channel. When one expands the theory in terms of photon-fermion coupling constant, one finds that the perturbation series is renormalizable and obtains the same S matrix as in quantum electrodynamics to all orders in perturbation theory.

Recently this type of renormalizability and correspondence of four-fermion models to certain renormalizable theories has been extended to other types of four-fermion interactions.[4] The original Nambu–Jona-Lasinio model, for instance, is shown to correspond to the linear σ model.

In these works, however, the renormalizability of perturbation series is yet formal. Although the series has a finite number of superficially divergent vertices and all the ultraviolet infinities of the theory are amalgamated into field, mass, and charge renormalizations, renormalized coupling constants are not automatically guaranteed to take finite and cutoff-independent values. This is because in four-fermion theories induced coupling constants become independent of the original Fermi coupling constant G after renormalization and cannot be made finite and arbitrary by making G cutoff dependent in a prescribed way. It is well known that in the lowest-order, Hartree-Fock, approximation the induced Yukawa coupling constant behaves as $g_R{}^2 \sim G/G \ln\Lambda = 1/\ln\Lambda$ (Λ is the ultraviolet cutoff). It is possible to prove that this feature persists to all orders.

In this paper we shall show that there is a Gell-Mann–Low eigenvalue condition on these radiatively created charges. If a theory possesses an ultraviolet-stable fixed point, they can take well-defined and cutoff-independent values. In this case Nambu–Jona-Lasinio theories become completely independent of ultraviolet cutoff and equivalent to corresponding renormalizable models. On the other hand, if a theory does not possess a fixed point, cutoff dependence persists in created charges and they vanish in the limit of infinite cutoff. In this case, four-fermion models become a free-field theory of collective bosons and fermions.

Let us first consider the Nambu–Jona-Lasinio model. The Lagrangian is given by

$$\mathcal{L} = \bar{\psi} i\gamma\cdot\partial\psi - \tfrac{1}{2}G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2]\,, \qquad (1)$$

where ψ is an isodoublet spinor field. The description of collective bound states can be facilitated if we introduce collective variables σ and π and a new Lagrangian,

$$\mathcal{L}' = \bar{\psi} i\gamma\cdot\partial\psi - g\bar{\psi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi - \tfrac{1}{2}\,\delta\mu^2(\sigma^2 + \vec{\pi}^2)\,, \qquad (2)$$

where

$$G = g^2/\delta\mu^2\,.$$

g is a bare induced Yukawa coupling constant, and a term proportional to $\delta\mu^2$ is interpreted as a boson self-energy counterterm. The new Lagrangian $\mathcal{L}'$ has the same dynamical content as $\mathcal{L}$ since if we perform a path integration over σ and $\vec{\pi}$ in

$$\int d\psi d\bar{\psi} d\sigma d\vec{\pi}\, \exp\{i[\mathcal{L}'(\psi,\bar{\psi},\sigma,\vec{\pi}) + \bar{\eta}\psi + \bar{\psi}\eta]\}\,, \qquad (3)$$

we obtain the original generating functional,[5]

$$W[\eta,\bar{\eta}] = \int d\psi d\bar{\psi}\, \exp\{i[\mathcal{L}(\psi,\bar{\psi}) + \bar{\eta}\psi + \bar{\psi}\eta]\}\,. \qquad (4)$$

A renormalized perturbation expansion for $\mathcal{L}'$ can be formally set up if we add and subtract vertices for bound states,

$$\mathcal{L}' = [\bar\psi_R i\gamma\cdot\partial\psi_R - g_R\bar\psi_R(\sigma_R + i\gamma_5\vec\tau\cdot\vec\pi_R)\psi_R] + [(Z_2-1)\bar\psi_R i\gamma\cdot\partial\psi_R - g_R(Z_1-1)\bar\psi_R(\sigma_R+i\gamma_5\vec\tau\cdot\vec\pi_R)\psi_R - \tfrac{1}{2}\delta\mu^2 Z_3({\sigma_R}^2+{\vec\pi_R}^2)] \tag{5}$$

$$= \{\bar\psi_R i\gamma\cdot\partial\psi_R - g_R\bar\psi_R(\sigma_R+i\gamma_5\vec\tau\cdot\vec\pi_R)\psi_R + \tfrac{1}{2}[(\partial_\mu\sigma_R)^2+(\partial_\mu\vec\pi_R)^2] - \tfrac{1}{2}{\mu_R}^2({\sigma_R}^2+{\vec\pi_R}^2) - \tfrac{1}{4}\lambda_R({\sigma_R}^2+{\vec\pi_R}^2)^2\} + \{(Z_2-1)\bar\psi_R i\gamma\cdot\partial\psi_R - g_R(Z_1-1)\bar\psi_R(\sigma_R+i\gamma_5\vec\tau\cdot\vec\pi_R)\psi_R - \tfrac{1}{2}[(\partial_\mu\sigma_R)^2+(\partial_\mu\vec\pi_R)^2] + \tfrac{1}{2}({\mu_R}^2-\delta\mu^2 Z_3)({\sigma_R}^2+{\vec\pi_R}^2) + \tfrac{1}{4}\lambda_R({\sigma_R}^2+{\vec\pi_R}^2)^2\}. \tag{6}$$

Here the notations are standard. Five terms in the first set of curly brackets in Eq. (6) are regarded as parts of the renormalized Lagrangian and those in the second set of curly brackets as counterterms. g_R and λ_R are renormalized induced coupling constants. Next let us compare the above expression with that of the linear σ model,

$$\mathcal{L}_\sigma = \bar\psi i\gamma\cdot\partial\psi - g\bar\psi(\sigma+i\gamma_5\vec\tau\cdot\vec\pi)\psi + \tfrac{1}{2}[(\partial_\mu\sigma)^2+(\partial_\mu\vec\pi)^2] - \tfrac{1}{2}\mu^2(\sigma^2+\vec\pi^2) - \tfrac{1}{4}\lambda(\sigma^2+\vec\pi^2)^2 \tag{7}$$

$$= \{\bar\psi_R i\gamma\cdot\partial\psi_R - g_R\bar\psi_R(\sigma_R+i\gamma_5\vec\tau\cdot\vec\pi_R)\psi_R + \tfrac{1}{2}[(\partial_\mu\sigma_R)^2+(\partial_\mu\vec\pi_R)^2] - \tfrac{1}{2}{\mu_R}^2({\sigma_R}^2+{\vec\pi_R}^2) - \tfrac{1}{4}\lambda_R({\sigma_R}^2+{\vec\pi_R}^2)^2\} + \{(Z_2-1)\bar\psi_R i\gamma\cdot\partial\psi_R - g_R(Z_1-1)\bar\psi_R(\sigma_R+i\gamma_5\vec\tau\cdot\vec\pi_R)\psi_R + \tfrac{1}{2}(Z_3-1)[(\partial_\mu\sigma_R)^2+(\partial_\mu\vec\pi_R)^2] - \tfrac{1}{2}[(Z_3-1){\mu_R}^2+Z_3\delta\mu^2]({\sigma_R}^2+{\vec\pi_R}^2) - \tfrac{1}{4}\lambda_R(Z_4-1)({\sigma_R}^2+{\vec\pi_R}^2)^2\}. \tag{8}$$

We notice that equations (6) and (8) differ in the coefficient of counterterms.

The elimination of ultraviolet infinities in the linear σ model is well understood.[6] Since divergent parts of radiative corrections have a strictly chiral-symmetric structure, they can be eliminated by a common wave-function renormalization factor Z_2 for σ and π and a common mass counterterm $\delta\mu$ if we choose appropriate subtraction points. Then renormalization factors are determined as a power series in g_R, λ_R, and $\ln\Lambda$,

$$Z_i - 1 + (\text{divergent part of radiative corrections})_i = 0, \quad i=1,2,3,4, \tag{9}$$

$$(Z_3-1){\mu_R}^2 + Z_3\delta\mu^2 + (\text{divergent part of self-energy}) = 0. \tag{10}$$

On the other hand, in the Nambu–Jona-Lasinio model the elimination of infinities has an unconventional feature. It is performed by

$$Z_i - 1 + (\text{divergent part of radiative corrections})_i = 0, \quad i=1,2, \tag{11}$$

$$-1 + (\text{divergent part of radiative corrections})_i = 0, \quad i=3,4, \tag{12}$$

$$-{\mu_R}^2 + Z_3\delta\mu^2 + (\text{divergent part of self-energy}) = 0. \tag{13}$$

Here the radiative corrections in (9), (10), and (11), (12), (13) have an identical structure since they are calculated using the same renormalized Lagrangian. In the Nambu–Jona-Lasinio model there are only three parameters Z_1, Z_2, and $\delta\mu$ to absorb infinities, and hence the above equations (11), (12), (13) impose nontrivial restrictions on g_R and λ_R. Comparing (9) and (12) we notice that the vanishing of Z factors,

$$\lim_{\Lambda\to\infty} Z_3(g_R, \lambda_R, \Lambda) = 0, \tag{14}$$

$$\lim_{\Lambda\to\infty} Z_4(g_R, \lambda_R, \Lambda) = 0, \tag{15}$$

is needed to eliminate infinities in Nambu–Jona-Lasinio theory. When these conditions are satisfied with cutoff-independent values of g_R and λ_R, the model becomes free of ultraviolet cutoff. Furthermore, since the same renormalized Lagrangian is used to compute Green's functions and the S matrix in both the Nambu–Jona-Lasinio model and the σ model, these quantities will have an identical structure when expanded in power series in g_R and λ_R. From this the equivalence of two theories follows.

The above equations (14) and (15) are nothing but familiar compositeness conditions. This is only reasonable since σ and π are fermion-antifermion composites in our theory. Although much literature[7] exists on the compositeness or bootstrap condition $Z=0$, its physical significance has been somewhat obscured owing to ultraviolet divergences in the case of a relativistic field theory. Next we analyze these conditions using renormalization-group equations and the idea of a Gell-Mann–Low fixed point. In the following we choose subtraction points at certain nonexceptional Euclidean momenta in order to avoid possible infrared problems.

It is well known that renormalization factors obey Callan-Symanzik equations,[8]

$$\left(\Lambda \frac{\partial}{\partial\Lambda} - \beta_g \frac{\partial}{\partial g_R} - \beta_\lambda \frac{\partial}{\partial\lambda_R} + 2\gamma_M\right) Z_3(g_R, \lambda_R, \Lambda) = 0\,, \tag{16}$$

$$\left[\Lambda \frac{\partial}{\partial\Lambda} - \beta_g \frac{\partial}{\partial g_R} - \beta_\lambda\left(\frac{\partial}{\partial\lambda_R} + \frac{1}{\lambda_R}\right) + 4\gamma_M\right] \times Z_4(g_R, \lambda_R, \Lambda) = 0\,. \tag{17}$$

Here $\gamma_M(g_R, \lambda_R)$ is the anomalous dimension of the meson (σ and $\vec{\pi}$) field. β_g and β_λ are the β functions related to the scale transformation of g_R and λ_R, respectively.

First let us consider a possibility that β_g and β_λ have an ultraviolet-stable fixed point at $(g_R^\infty, \lambda_R^\infty)$ in the two-dimensional coupling-constant plane,

$$\beta_g(g_R^\infty, \lambda_R^\infty) = \beta_\lambda(g_R^\infty, \lambda_R^\infty) = 0\,, \tag{18}$$

and renormalized coupling constants are equal to these values,

$$g_R = g_R^\infty, \quad \lambda_R = \lambda_R^\infty\,. \tag{19}$$

Then using Callan-Symanzik equations we obtain an exponentiation of $\ln\Lambda$,

$$Z_3(g_R, \lambda_R, \Lambda) \sim \Lambda^{-2\gamma_M(g_R^\infty, \lambda_R^\infty)}\,, \tag{20}$$

$$Z_4(g_R, \lambda_R, \Lambda) \sim \Lambda^{-4\gamma_M(g_R^\infty, \lambda_R^\infty)}\,. \tag{21}$$

Since $\gamma_M(g_R^\infty, \lambda_R^\infty)$ is non-negative due to unitarity,[9] it follows that[10]

$$\lim_{\Lambda\to\infty} Z_3 = 0\,, \tag{22}$$

$$\lim_{\Lambda\to\infty} Z_4 = 0\,. \tag{23}$$

Thus $g_R = g_R^\infty$, $\lambda_R = \lambda_R^\infty$ is a solution to our bootstrap condition.

Actually the above assumption (19) can be relaxed significantly. As is well known in the renormalization-group analysis, as long as g_R and λ_R lie within a domain of attraction of the fixed point there are so-called running coupling constants which interpolate between (g_R, λ_R) and $(g_R^\infty, \lambda_R^\infty)$ in the coupling-constant plane. When Callan-Symanzik equations are solved in terms of them, we obtain a minor correction to (20) and (21) which leaves our result (22) and (23) unchanged. Thus the bootstrap condition can be solved for a certain range of values for g_R and λ_R if the theory possesses an ultraviolet-stable fixed point.[11]

In this case we obtain an interesting phenomenon; although we start with a theory of a spinor field and dimensional constant G, we arrive at a theory of bosons and fermion with two dimensionless numbers g_R and λ_R as well as a dimensional constant μ_R. It appears that the dimensional constant G has somehow been traded for μ_R. On the other hand parameters g_R and λ_R are independent of G and are interpreted as being radiatively created. They are determined to be a solution to bootstrap conditions; however, these equations do not completely fix them and allow them to take values within a certain range. The theory exhibits a scaling behavior at high energy with anomalous dimensions.

Such a mechanism seems to be common to most of the models for dynamical bound states. For instance in the models of Refs. 12 and 13, the analysis is based on an assumed existence of a power-behaved, nonperturbative solution to Schwinger-Dyson integral equations. The residue of a bound state becomes cutoff independent when a Bethe-Salpeter kernel possesses a certain power behavior. All of this will happen if a theory has a fixed point.

On the other hand if a fixed point is absent in the theory, cutoff dependence persists in g_R and λ_R. In this case, the renormalization-group equation is useless and we have to inspect directly the expression of Z factors in terms of g_R, λ_R, and $\ln\Lambda$. In the nth-order perturbation theory Z factors have a structure,

$$Z = 1 + (C_{11}f + \cdots + C_{1n}f^n)\ln\Lambda + \cdots + C_{n1}f^n(\ln\Lambda)^n\,, \tag{24}$$

where f is either $g_R{}^2$ or λ_R. We notice that bootstrap conditions imply a cutoff dependence of g_R and λ_R as

$$g_R{}^2 \sim \frac{1}{\ln\Lambda}\,, \quad \lambda_R \sim \frac{1}{\ln\Lambda}\,, \tag{25}$$

in each order of perturbation theory. Hence both vanish as $\Lambda\to\infty$. Thus we obtain a trivial free-field result. Therefore the eigenvalue is a condition to support nontrivial values for radiatively generated charges.

Apparently the above result applies to other types of four-fermion theories as well. Unfortunately in most of the nongauge Yukawa theories the origin of the coupling-constant plane is ultraviolet unstable.[14] In these cases we may not be able to discuss fixed points within the realm of a weak-coupling perturbation expansion. On the other hand if we let a non-Abelian gauge field couple to our theories, it is possible to stabilize the origin. The analysis seems feasible in this case though it becomes complicated owing to gauge dependence of renormalization factors.

The above results become somewhat modified when one considers a four-fermion theory where a gauge field appears as collective bound states. Here the simplest example is the Abelian model of Refs. 2 and 3,

$$\mathcal{L} = \bar\psi i\gamma\cdot\partial\psi - m\bar\psi\psi - \tfrac{1}{2}G(\bar\psi\gamma_\mu\psi)^2\,. \tag{26}$$

Using the same technique as before we can introduce a collective variable A_μ and a new Lagrangian

$$\mathcal{L}' = \psi i \gamma \cdot \partial \psi - m\bar{\psi}\psi - e\bar{\psi}\gamma_\mu \psi A^\mu + \tfrac{1}{2}\delta\mu^2 A_\mu A^\mu . \quad (27)$$

Here the term proportional to $\delta\mu^2$ is again interpreted as a mass counterterm. It should be adjusted to cancel the photon self-energy when we use a noncovariant momentum-space cutoff. If a gauge-invariant regulator or dimensional regularization is used, we should put $\delta\mu = 0$. Then the above model becomes equivalent to quantum electrodynamics if a bootstrap condition,

$$\lim_{\Lambda\to\infty} Z_3(e_R, \Lambda) = 0 , \quad (28)$$

is satisfied.

Now we argue that the above equation (28) may not be satisfied even by imposing an eigenvalue condition,

$$\beta(e_R^\infty) = 0 . \quad (29)$$

Here the basic reason is that the anomalous dimension of a photon field vanishes at the fixed point owing to gauge invariance, and hence Z_3 remains nonzero in the limit $\Lambda \to \infty$.

Though this may appear obvious to those familiar with the finite theory of quantum electrodynamics,[15,16] we shall give a simple argument using dimensional regularization. In 't Hooft's scheme of renormalization[17] Z_3 is expanded in a Laurent series in $n-4$,

$$Z_3 = 1 + \sum_{\nu=1}^{\infty} \frac{a_\nu(e_D^2)}{(n-4)^\nu} , \quad (30)$$

where a_ν has a structure,

$$a_\nu(e_D^2) = a_{\nu 1}(e_D^2)^\nu + a_{\nu 2}(e_D^2)^{\nu+1} + \cdots . \quad (31)$$

a_1 is related to the β function,

$$\tfrac{1}{2} e_D^3 a_1'(e_D^2) = \beta(e_D^2) . \quad (32)$$

Then using renormalization-group constraints,[17]

$$(-e_D^2 a_{\nu-1}' + a_{\nu-1}) a_1' - a_\nu' = 0 , \quad (33)$$

and the fact that the zero of the β function is an infinite-order zero,[16] one obtains

$$a_\nu = 0, \quad \nu = 1, 2, 3, \ldots . \quad (34)$$

Hence[18]

$$Z_3(e_D^\infty) = 1 . \quad (35)$$

This is just the other extreme of the Lehmann bound opposite to the bootstrap $Z_3 = 0$. Thus it appears unlikely that a photon (Abelian gauge field) can be interpreted as a fermion-antifermion bound state.

It is also possible to consider a non-Abelian analog of the above example. For instance in the case of SU(2) it is given by

$$\mathcal{L} = \bar{\psi} i \gamma \cdot \partial\psi - m\bar{\psi}\psi - \frac{G}{2}\left(\bar{\psi}\gamma_\mu \frac{\vec{\tau}}{2}\psi\right)^2 . \quad (36)$$

Introducing a field $\vec{A}_\mu$ and putting $\delta\mu = 0$ we obtain

$$\mathcal{L}' = \bar{\psi} i \gamma \cdot \partial\psi - m\bar{\psi}\psi - g\bar{\psi}\gamma_\mu \tfrac{1}{2}\vec{\tau} \cdot \psi \vec{A}^\mu . \quad (37)$$

By adding a ghost term we write the Lagrangian as

$$\begin{aligned}
\mathcal{L}'' &= \bar{\psi} i\gamma\cdot\partial\psi - m\psi\bar{\psi} - g\bar{\psi}\gamma_\mu\tfrac{1}{2}\vec{\tau}\cdot\psi\vec{A}^\mu - \partial^\mu\vec{\phi}^* \cdot (\partial_\mu + g\vec{A}_\mu \times)\vec{\phi} \\
&= [\bar{\psi}_R i\gamma\cdot\partial\psi_R - m_R\bar{\psi}_R\psi_R - g_R\bar{\psi}_R\gamma_\mu\tfrac{1}{2}\vec{\tau}\cdot\psi_R\vec{A}_R^\mu - \tfrac{1}{2}\partial^\mu\vec{\phi}_R^* \cdot (\partial_\mu + g_R\vec{A}_\mu^R \times)\vec{\phi}_R - \tfrac{1}{4}(\partial_\mu\vec{A}_\nu^R - \partial_\nu\vec{A}_\mu^R + g_R\vec{A}_\mu^R\times\vec{A}_\nu^R)^2] \\
&\quad + [(Z_F - 1)\bar{\psi}_R i\gamma\cdot\partial\psi_R - m_R(Z_F - 1)\bar{\psi}_R\psi_R - \delta m Z_F\bar{\psi}_R\psi_R - g_R(Z_g - 1)\bar{\psi}_R\gamma_\mu\tfrac{1}{2}\vec{\tau}\cdot\psi_R\vec{A}^\mu \\
&\quad - (\tilde{Z}_3 - 1)\partial_\mu\vec{\phi}_R^*\cdot\partial^\mu\vec{\phi}_R - g_R(\tilde{Z}_1 - 1)\partial_\mu\vec{\phi}_R^*\cdot\vec{A}_R^\mu\times\vec{\phi}_R \\
&\quad - \tfrac{1}{4}(\partial_\mu\vec{A}_\nu^R - \partial_\nu\vec{A}_\mu^R)^2 - \tfrac{1}{2}g_R(\partial_\mu\vec{A}_\nu^R - \partial_\nu\vec{A}_\mu^R)\cdot(\vec{A}_R^\mu\times\vec{A}_R^\nu) - \tfrac{1}{4}g_R^2(\vec{A}_\mu^R\times\vec{A}_\nu^R)\cdot(\vec{A}_R^\mu\times\vec{A}_R^\nu)] .
\end{aligned} \quad (38)$$

Then, comparing Eq. (38) with the Lagrangian of an SU(2) Yang-Mills theory coupled with fermions, we notice that the two theories become equivalent if a bootstrap condition $Z_1 = 0$, $Z_3 = 0$ is satisfied. Renormalization factors Z_1, Z_3 are those associated with three-vector and two-vector vertices, respectively.

Although these parameters are gauge dependent, we can solve Callan-Symanzik equations and evaluate them using asymptotic freedom. The result depends on the choice of group and representations. It is possible to show that if the quantities

$$\bar{\beta} = \frac{1}{16\pi^2}\left(\tfrac{11}{3} C_2 - \tfrac{4}{3} T_2\right) , \quad (39)$$

$$\bar{\gamma} = \frac{1}{8\pi^2}\left(\tfrac{8}{3} T_2 - \tfrac{13}{3} C_2\right) \quad (40)$$

are both positive, the running coupling constant and gauge parameter go to zero, $g_R^\infty = \alpha_R^\infty = 0$, in the deep Euclidean limit.[19] Here C_2 is the value of a quadratic Casimir operator in the adjoint representation and T_2 is that of the representation of fermions. Then by using renormalization-group equations we obtain

$$Z_1 \sim \lim_{\Lambda\to\infty} \Lambda^{-3\bar{\gamma}/4\bar{\beta}} = 0 , \quad (41)$$

$$Z_3 \sim \lim_{\Lambda\to\infty} \Lambda^{-\bar{\gamma}/2\bar{\beta}} = 0 . \quad (42)$$

This result is gauge independent in the sense that it holds in any gauge. Thus bootstrap conditions are satisfied. On the other hand if $\bar{\beta}>0$ and $\bar{\gamma}<0$, the Z's remain nonzero in the limit of infinite cutoff. In the case of SU(2), for example, $\bar{\beta},\bar{\gamma}>0$ if there are F fermion doublets with $\frac{13}{2}<F<11$.

Our method described above has an interesting comparison with that of Wilson[20] who introduced an unconventional renormalization procedure for superrenormalizable theories in less than four dimensions. In this procedure dimensional coupling constants of a theory are let go to infinity while properly defined dimensionless coupling constants are held fixed as $\Lambda\to\infty$. Then a superrenormalizable theory is converted into a nontrivial renormalizable theory. There is an eigenvalue condition for the dimensionless coupling constant, and by satisfying it the theory exhibits a scaling behavior with anomalous dimensions. Here relevant eigenvalues are infrared-stable fixed points.

A great virtue of this method is the guaranteed existence of a fixed point close to the origin so long as $\epsilon=4-d$ is small. However, in the limit $\epsilon\to 0$ Wilson's prescription gives a free-field theory. Therefore, here the existence of an eigenvalue is achieved only by having a trivial theory at four dimensions.

Our work suggests an intimate connection between the idea of dynamical symmetry breakdown and the Gell-Mann–Low fixed point. If a theory possesses an ultraviolet-stable fixed point, some of its fields (presumably spin-zero mesons) may be interpreted as composites, and their vertices can be eliminated from the Lagrangian without spoiling renormalizability. Then we have a smaller number of fields and vertices and a more constrained theory than the original one. Allowed phases in such a theory may well be quite restricted. Hopefully such a procedure eliminates a high degree of arbitrariness in the conventional Higgs mechanism and gives us a new approach to dynamical symmetry breakdown.

After completion of this manuscript we received a report by C. Bender, F. Cooper, and G. Guralnik [Los Alamos Report No. 77-1093 (unpublished)], where some of the materials of this paper are discussed using mean-field theory.

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