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Regge behavior of spontaneously broken non-Abelian gauge theory*

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The high-energy, fixed-momentum-transfer behavior of spontaneously broken non-Abelain gauge theory, with SU(2) gauge group, has previously been calculated through tenth order in perturbation theory in the leading-logarithm approximation. We interpret the complicated results as due to the exchange of the gauge-meson Regge trajectory plus associated cuts generated by Reggeon field theory. There are only two Reggeon couplings consistent with the leading-logarithm perturbation theory calculation. The coupling strengths are highly overdetermined by the perturbation calculation, demonstrating the consistency of the assumption of the applicability of Reggeon field theory. For large values of the momentum transfer our results are in agreement with the calculations of Carruthers, Fishbane, and Zachariasen and Cornwall and Tiktopoulos to all orders in perturbation theory. When all of the contributions of the moving cuts are summed in the weak-coupling approximation, the leading J-plane singularity in the $I=0$ channel is a fixed cut. We show that this singularity remains fixed, at least in the weak-coupling approximation, even when asymptotic freedom is taken into account.

I. INTRODUCTION

At the present time non-Abelian gauge theories (NAGT) provide the leading candidates for a theory of the strong interactions. As a result, it is important for the study of high-energy diffraction scattering to understand the behavior of these theories in the Regge limit. In this paper we present evidence that the high-energy, fixed-momentum-transfer behavior of NAGT with spontaneously broken gauge symmetry is concisely described by Reggeon field theory (RFT). For the case of an SU(2) gauge group we calculate the Regge trajectories and coupling functions in the weak-coupling limit.

Our analysis rests on leading-logarithm calculations of Feynman diagrams,[1-3] which have been carried out to tenth order for a theory with SU(2) gauge symmetry and gauge-meson mass, λ^2, generated by giving a vacuum expectation value to a doublet of Higgs scalars.[1] The same calculation has been carried out through eighth order, when there are three triplets of Higgs scalars,[3] with very similar results. For fermion-fermion scattering (F_I) and meson-meson scattering (G_I) with a Higgs doublet and isospin-I exchange, the results are[1]

$$\begin{aligned}
F_0 \sim G_0 \sim isg^4\{&J_1(\Delta)+[2J_2(\Delta)-2(\Delta^2+\tfrac{5}{4}\lambda^2)J_1(\Delta)^2]g^2\ln s+[(J_3(\Delta)+\tilde{J}(\Delta))-4(\Delta^2+\tfrac{5}{4}\lambda^2)J_1(\Delta)J_2(\Delta)\\
&+2(\Delta^2+\tfrac{5}{4}\lambda^2)^2J_1(\Delta)^3](g^2\ln s)^2,\\
&+[\tfrac{2}{3}J_4(\Delta)-\tfrac{1}{3}J_A(\Delta)-J_B(\Delta)+\tfrac{4}{3}J_C(\Delta)+\tfrac{2}{3}J_D(\Delta)-\tfrac{4}{3}(\Delta^2+\tfrac{5}{4}\lambda^2)(J_2(\Delta)^2+J_1(\Delta)J_3(\Delta)+J_1(\Delta)\tilde{J}(\Delta))\\
&+4(\Delta^2+\tfrac{5}{4}\lambda^2)^2J_1(\Delta)^2J_2(\Delta)-\tfrac{4}{3}(\Delta^2+\tfrac{5}{4}\lambda^2)^3J_1(\Delta)^4](g^2\ln s)^3+\cdots\},\\
F_1 \sim G_1 \sim sg^2[&1/(\Delta^2+\lambda^2)-J_1(\Delta)g^2\ln s+\tfrac{1}{2}(\Delta^2+\lambda^2)J_1(\Delta)^2(g^2\ln s)^2-\tfrac{1}{6}(\Delta^2+\lambda^2)^2J_1{}^3(\Delta)(g^2\ln s)^3+\cdots],\qquad(1)\\
G_2 \sim isg^4\{&J_1(\Delta)+[-4J_2(\Delta)+(\Delta^2+2\lambda^2)J_1(\Delta)^2]g^2\ln s+[4J_3(\Delta)+4\tilde{J}(\Delta)-4(\Delta^2+2\lambda^2)J_1(\Delta)J_2(\Delta)\\
&+\tfrac{1}{2}(\Delta^2+2\lambda^2)^2J_1(\Delta)^3](g^2\ln s)^2\\
&+[-\tfrac{4}{3}J_4(\Delta)-\tfrac{4}{3}J_A(\Delta)-4J_B(\Delta)-\tfrac{8}{3}J_C(\Delta)-\tfrac{4}{3}J_D(\Delta)+\tfrac{8}{3}(\Delta^2+2\lambda^2)(J_2(\Delta)^2+J_1(\Delta)J_3(\Delta)+J_1(\Delta)\tilde{J}(\Delta))\\
&-2(\Delta^2+2\lambda^2)^2J_1(\Delta)^2J_2(\Delta)+\tfrac{1}{6}(\Delta^2+2\lambda^2)^3J_1(\Delta)^4](g^2\ln s)^3+\cdots\},
\end{aligned}$$

where $\Delta^2=-t$ is the momentum transfer squared, and

$$J_1(\Delta)=\int\frac{d^2k}{(2\pi)^3}\frac{1}{(k^2+\lambda^2)[(\Delta-k)^2+\lambda^2]},$$

$$J_n(\Delta)=\int\frac{d^2k}{(2\pi)^3}\frac{J_{n-1}(k)}{(\Delta-k)^2+\lambda^2},\quad n\geq 2$$

$$\tilde{J}(\Delta)=\int\frac{d^2k}{(2\pi)^3}\frac{(k^2+\lambda^2)J_1{}^2(k)}{(\Delta-k)^2+\lambda^2},$$

$$J_A(\Delta)=\int\frac{d^2k}{(2\pi)^3}\frac{(k^2+\lambda^2)^2J_1(k)^3}{(\Delta-k)^2+\lambda^2},\tag{2}$$

$$J_B(\Delta)=\int\frac{d^2k}{(2\pi)^3}(k^2+\lambda^2)J_1(k)^2J_1(\Delta-k),$$

$$J_C(\Delta)=\int\frac{d^2k}{(2\pi)^3}\frac{(k^2+\lambda^2)J_1(k)J_2(k)}{(\Delta-k)^2+\lambda^2},$$

$$J_D(\Delta)=\int\frac{d^2k_1}{(2\pi)^3}\frac{d^2k_2}{(2\pi)^3}\frac{(k_1{}^2+\lambda^2)J_1(k_1)}{(\Delta-k_1)^2+\lambda^2}\frac{(k_2{}^2+\lambda^2)J_1(k_2)}{(\Delta-k_2)^2+\lambda^2}\times[(\Delta-k_1-k_2)^2+\lambda^2]^{-1}.$$

Feynman diagrams and rules giving rise to these transverse integrals are presented in Ref. 1. When there are three triplets of Higgs scalars, the only changes in Eq. (1) are $(\Delta^2+\frac{5}{4}\lambda^2)\to\Delta^2$ in F_0 and G_0, and $(\Delta^2+2\lambda^2)\to(\Delta^2+3\lambda^2)$ in G_2.[3]

We immediately note that the $I=1$ amplitude can be interpreted as the first four terms in the $g^2\ln s$ expansion of the Regge-pole amplitude

$$F_1\sim G_1\sim\tfrac{1}{2}\pi g^4J_1(\Delta)s^{\alpha_1}(1-e^{-i\pi\alpha_1})/\sin\pi\alpha_1,\tag{3}$$

with

$$\alpha_1(\Delta^2)=1-g^2(\Delta^2+\lambda^2)J_1(\Delta).\tag{4}$$

It has been shown that for $\Delta^2\gg\lambda^2$ and $g^2\ll 1$, Eq. (3) holds to all orders in $g^2\ln s$.[4,5] The trajectory passes through $J=1$ at $t=-\Delta^2=\lambda^2$, and is obviously the Reggeized vector meson.

The $I=0,2$ amplitudes are more complicated. In this paper we show that they can be interpreted as the first four terms in the $g^2\ln s$ expansion of a Regge-cut amplitude. The cuts are generated by the exchange of two of the $I=1$ Regge poles exhibited in Eqs. (3) and (4). Our procedure is to write down the most general form of the cut amplitude which is consistent with the contraints of RFT. We then expand in powers of $g^2\ln s$ and compare with Eq. (1). The requirement that only the integrals of Eq. (2) emerge in each order in perturbation theory determines the dependence of the Reggeon coupling functions on the transverse momentum. There are three independent couplings in each isospin channel. Their strengths are completely determined by the information obtained from fourth- and sixth-order perturbation theory. We are able to use these coupling functions to calculate the eighth- and tenth-order contributions to the $I=0$ and $I=2$ amplitudes, and we find agreement with Eq. (1). We take this agreement to be a nontrivial confirmation of our assumption that RFT correctly gives the Regge limit of NAGT.

Bartels has also used the techniques of RFT to analyze the high-energy behavior of NAGT.[6] Note that there are terms in the $I=0$ and $I=2$ amplitudes which might appear to be the contributions of Regge poles in those channels on the trajectories

$$\alpha_0(\Delta^2)=1-2g^2(\Delta^2+\tfrac{5}{4}\lambda^2)J_1(\Delta),$$
$$\alpha_2(\Delta^2)=1+g^2(\Delta^2+2\lambda^2)J_1(\Delta).\tag{5}$$

As a first approximation Bartels retains only these Regge poles and writes down an RFT based on them. On the other hand, we find we can reproduce the perturbation-theory results only with Regge cuts in the $I=0$ and $I=2$ channels. To clarify this point, we repeat our analysis allowing for contributions from these poles. Now there are five independent couplings in each isospin channel. The five strengths must satisfy a set of six equations in order that each coefficient of $(g^2\ln s)^nf_i(\Delta)$ take the value given in Eq. (1), when $n\leq 4$. It is unnecessary to include the tenth-order contributions in this analysis because the contributions through the eighth order already overconstrain the coupling strengths. There is only one solution to these equations, the old one in which the poles of Eq. (5) decouple from the theory. Actually, there is no obvious reason from non-Abelian gauge theory why "elementary" Regge poles should be present in any channel except $I=1$, so their decoupling is reasonable.

The RFT couplings we have deduced are, of course, only the leading weak-coupling expressions. (To our knowledge, NAGT are the only field theories for which one can unambiguously extract the Regge trajectory and coupling functions order-by-order in perturbation theory. This is the case because all the singularities are near $J=1$ in the weak-coupling limit.) Retaining only the lowest-order couplings, there are always just two Reggeons in the t channel for $I=0,2$. (For $I=1$ there is simple Regge-pole exchange in the weak-coupling limit.) Since the Reggeons only scatter off each other, we can write an integral equation which sums all the diagrams. This equation has been given previously by Fadin, Kuraev, and Lipatov,[7] and Cheng and Lo,[1] without note of its connection to RFT. The kernel is determined by the two-Reggeon–two-Reggeon coupling, and when the weak-coupling expression is used, the kernel is not sufficiently damped at large transverse momentum to be square integrable. If the kernel were L^2, then there would only be moving singularities in the angular momentum plane. In fact it is argued in Refs. 1 and 7 that when one sums all of the leading-logarithm contributions to the $I=0$ amplitude, the right-most singularity in the angular momentum plane is a fixed cut with a branch point at $J=1+g^2(2\ln 2)/\pi^2$. It is suggested in Ref. 7 that if the theory is asymptotically free,[8] the fixed cut may no longer be present once the momentum dependence of the effective gauge coupling constant is taken into account. We have studied this suggestion with negative results. We follow the basic approach of Cardy[9] and Lovelace[10] who used the renormalization group to study the behavior of the two-particle irreducible Bethe-Salpeter kernel in $\phi^3_{D=6}$ field theory. They found that the kernel was square integrable and hence there were only moving singularities in the angular momentum plane, provided the theory was asymptotically free. For NAGT the Bethe-Salpeter equation does not appear to be useful for this type of analysis because multiparticle intermediate states are not negligible even in the weak-coupling limit. However, multi-Reggeon intermediate states are negligible in the weak-coupling limit, so we write down an analogous equation with a two-Reggeon irreducible kernel. We use the renormalization group to study the behavior of this kernel when the (Euclidean) momenta on which it depends become large. Our formal argument indicates that the kernel is sufficiently damped at large momenta to be square integrable provided that the gauge coupling constant is driven to zero in the deep-Euclidean limit. However, when we make use of the weak-coupling approximation to the RFT couplings, we find that the formal argument breaks down because the approximate kernel does not have sufficiently tame infrared behavior. Whether the infrared behavior of the kernel is tamed by higher-order corrections is an open question.

In understanding the Regge limit of NAGT it is important to see how our results are related to those of Carruthers, Fishbane, and Zachariasen,[4] and Cornwall and Tiktopoulos.[5] These authors have calculated the coefficient of the leading power of $\ln(\Delta^2/\lambda^2)$ in each order of perturbation theory and summed up the contributions. They find for $I=0$,

$$F_0 \sim \frac{isg^2}{\Delta^2 \ln s}\left[1-\exp\left(-\frac{g^2}{4\pi^2}\ln s\,\ln(\Delta^2/\lambda^2)\right)\right]. \qquad (6)$$

We have used our RFT integral equation to evaluate the leading contribution in powers of $g^2 \ln s \ln(\Delta^2/\lambda^2)$ for $\Delta^2 \gg \lambda^2$. Our coefficients agree with Eq. (6), which further establishes the correctness of the extrapolation to all orders of perturbation theory. Equation (6) does not agree with the leading-logarithm calculation of Refs. 1 and 7, presumably because it is not sufficient to merely sum the leading powers of $\ln(\Delta^2/\lambda^2)$ even for $\Delta^2 \gg \lambda^2$.

The layout of the paper is as follows. In Sec. II we calculate the couplings in RFT by comparison with Eq. (1). In Sec. III we write down the integral equation which sums the leading-logarithm contributions and discuss its solution. We also establish Eq. (6). In Sec. IV we derive the renormalization group equation for the two-Reggeon irreducible kernel, and use its solution to discuss the large-momentum behavior of the kernel. Finally, in Sec. V we recapitulate and comment on our results.

II. REGGEON FIELD THEORY COUPLINGS

We now show how the results of Eq. (1) arise naturally from the assumption that the leading singularities in the angular momentum plane are the $I=1$ Regge pole and its associated Regge cuts.

It will be convenient to work with the Sommerfeld-Watson representation of the scattering amplitude

$$G_I(s,\Delta^2)=s\int\frac{dE}{2\pi i}\,s^{-E}f_I(E,\Delta^2)\,\frac{1-e^{-i\pi E}}{\sin\pi E} \qquad (7)$$

for $I=0,2$. Here f_I is the even-signatured t-channel partial-wave amplitude for isospin I, and $E=1-J$. The contour of integration runs from $+i\infty$ to $-i\infty$ and is to the left of all singularities of f_I. Note that we have only included even-signatured contributions to the G_I. It is clear from Eq. (4) that the singularities of interest will be within a distance of order g^2 from the point $E=0$ ($J=1$). As a result, in order to obtain the leading power of $\ln s$ at each order in g^2, it will be sufficient to approximate the signature factors by their values at $E=0$. The even-signatured amplitude will therefore be pure imaginary and the odd-signatured amplitude will be pure real, but we see from Eq. (1) that G_0 and G_2 are in fact imaginary.

We start by considering G_0. The simplest RFT diagrams with one large rapidity gap are shown in Fig. 1. The wavy lines represent the $I=1$ Reggeized vector meson. Diagrams corresponding to the exchange of an odd number of Reggeons contribute only to the odd-signatured amplitude, so they do not concern us. The two-Reggeon-cut diagram of Fig. 1(a) has the general form

$$G_0^1(s,\Delta^2)=is\int\frac{dE}{2\pi i}\,s^{-E}\times\int\frac{d^2k}{(2\pi)^3}\,\frac{\rho_2(k,\Delta)}{E-g^2K(k)-g^2K(\Delta-k)}, \qquad (8)$$

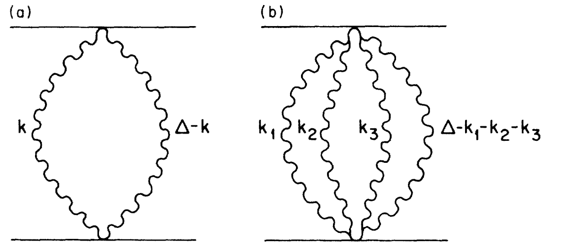

FIG. 1. Reggeon diagrams with one large rapidity gap. Diagram (a) contributes in the weak-coupling limit, but diagram (b) does not.

where

$$g^2K(k) = 1 - \alpha_1(k^2) = g^2(k^2+\lambda^2)J_1(k)\,. \tag{9}$$

We have approximated the signature factor by i, which, as we have just noted, is sufficient for a leading-logarithm or weak-coupling calculation.

In principle, the function $\rho_2(k,\Delta)$ is arbitrary. However, if we make use of the expansion

$$\int \frac{dE}{2\pi i}\,\frac{s^{-E}}{E-A} = 1 - A\ln s + \frac{1}{2!}\,(A\ln s)^2 - \frac{1}{3!}\,(A\ln s)^3 + \cdots\,, \tag{10}$$

we see that in order not to be in contradiction with the g^4 term in G_0 we must choose $\rho_2(k,\Delta)$ to have the form

$$\rho_2(k,\Delta) = \frac{a^2g^4}{(k^2+\lambda^2)[(\Delta-k)^2+\lambda^2]}\,, \tag{11}$$

where a is a constant which measures the strength of the coupling of two Reggeons to the external particles. The factors of $(k^2+\lambda^2)^{-1}$ and $[(\Delta-k)^2+\lambda^2]^{-1}$ are hardly surprising. We know from Gribov's general analysis[11] that ρ_2 must be proportional to $[\sin\pi\alpha_1(k)\sin\pi\alpha_1(\Delta-k)]^{-1}$, so it must have a pole when either of the Reggeons is on the spin shell. In writing the rules for evaluating RFT diagrams we shall assign a factor of $(k^2+\lambda^2)^{-1}$ to each Reggeon line carrying transverse momentum k. The two-Reggeon–two-particle vertex function is then simply ag^2.

Substituting Eqs. (10) and (11) into Eq. (8) and making use of Eq. (2), we expand the two-Reggeon-cut diagram of Fig. 1(a) in powers of $g^2\ln s$ and find

$$\begin{aligned} G_0^1(s,\Delta^2) = isa^2g^4\{&J_1(\Delta) - 2J_2(\Delta)g^2\ln s \\ &+ [J_3(\Delta)+\mathcal{J}(\Delta)](g^2\ln s)^2 \\ &- [\tfrac{1}{3}J_A(\Delta)+J_B(\Delta)](g^2\ln s)^3+\cdots\}. \end{aligned} \tag{12}$$

It is encouraging to note that all the terms found in Eq. (12) also appear in Eq. (1).

Since the coupling of two Reggeons to the external particles is of order g^2, we expect the coupling of four Reggeons to the external particles to be at least of order g^4. As a result, the four-Reggeon-cut diagram of Fig. 1(b) will not contribute in the weak-coupling limit. To be more specific, this diagram has the general form

$$G_0^{1\prime}(s,\Delta^2) = is\int\frac{dE}{2\pi i}\,s^{-E}\int\frac{d^2k_1}{(2\pi)^3}\,\frac{d^2k_2}{(2\pi)^3}\,\frac{d^2k_3}{(2\pi)^3}\,\frac{\rho_4(k_1,k_2,k_3,\Delta)}{E-g^2K(k_1)-g^2K(k_2)-g^2K(k_3)-g^2K(\Delta-k_1-k_2-k_3)}\,. \tag{13}$$

Since ρ_4 will be at least of order g^8 the nth-order term in the expansion of Eq. (13) will be proportional to $g^8(g^2\ln s)^n$, and therefore will be negligible compared to the corresponding terms in Eq. (12) in the weak-coupling limit. In addition we have been unable to find any nonzero form for ρ_4 which does not lead to a contradiction with the transverse-momentum dependence of G_0. The basic difficulty is that the coefficient of $(\ln s)^0$ arising from Eq. (13) involves a triple integral over the transverse momentum, whereas the corresponding coefficient in G_0 involves a single integral. The same arguments hold for the N-Reg-

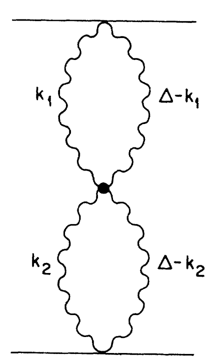

FIG. 2. Reggeon diagram with two large rapidity gaps.

geon-cut diagrams, so the diagram of Fig. 1(a) is the only one with one large rapidity gap which contributes in the weak-coupling limit.

We now turn to the RFT diagrams with two large rapidity gaps. Since only the two-Reggeon–two-particle vertex enters in the weak-coupling limit, the only relevant diagram is the one shown in Fig. 2. It has the general form

$$G_0^2(s,\Delta)=isa^2g^4\int\frac{dE}{2\pi i}s^{-E}\int\frac{d^2k_1}{(2\pi)^3}\frac{d^2k_2}{(2\pi)^3}C_0(k_1,k_2,\Delta)\prod_{i=1}^{2}(k_i^{\,2}+\lambda^2)^{-1}[(\Delta-k_i)^2+\lambda^2]^{-1}[E-g^2K(k_i)-g^2K(\Delta-k_i)]^{-1}. \tag{14}$$

$C_0(k_1,k_2,\Delta)$ is the two-Reggeon–two-Reggeon coupling function in the isospin-zero channel. The most general form for C_0, which reproduces the transverse-momentum dependence of the g^6 term in G_0, is

$$C_0(k_1,k_2,\Delta)=bg^2(\Delta^2+\tfrac{5}{4}\lambda^2)+cg^2\left\{\frac{(k_1^{\,2}+\lambda^2)[(\Delta-k_2)^2+\lambda^2]+(k_2^{\,2}+\lambda^2)[(\Delta-k_1)^2+\lambda^2]}{(k_1-k_2)^2+\lambda^2}\right.$$
$$\left.+\frac{(k_1^{\,2}+\lambda^2)(k_2^{\,2}+\lambda^2)+[(\Delta-k_1)^2+\lambda^2][(\Delta-k_2)^2+\lambda^2]}{(\Delta-k_1-k_2)^2+\lambda^2}\right\}. \tag{15}$$

b and c are constants whose numerical values will be determined shortly. Making use of the fact that

$$\int\frac{dE}{2\pi i}\frac{s^{-E}}{(E-A)(E-B)}=-\ln s+\frac{1}{2!}(A+B)(\ln s)^2-\frac{1}{3!}(A^2+AB+B^2)(\ln s)^3+\cdots, \tag{16}$$

we see that G_0^2 has the expansion

$$G_0^2=-isa^2g^4\{[b(\Delta^2+\tfrac{5}{4}\lambda^2)J_1(\Delta)^2+4cJ_2(\Delta)]g^2\ln s-[2b(\Delta^2+\tfrac{5}{4}\lambda^2)J_1(\Delta)J_2(\Delta)+4c(J_3(\Delta)+\mathcal{J}(\Delta))](g^2\ln s)^2$$
$$+[\tfrac{2}{3}b(\Delta^2+\tfrac{5}{4}\lambda^2)(J_1(\Delta)J_3(\Delta)+J_1(\Delta)\mathcal{J}(\Delta)+J_2(\Delta)^2)$$
$$+\tfrac{2}{3}c(J_4(\Delta)+2J_A(\Delta)+6J_B(\Delta)+2J_C(\Delta)+J_D(\Delta))](g^2\ln s)^3-\cdots\}. \tag{17}$$

Next, we must consider the RFT diagrams with three large rapidity gaps. The one shown in Fig. 3 has the form

$$G_0^3(s,\Delta)=isa^2g^4\int\frac{dE}{2\pi i}s^{-E}\int\prod_{i=1}^{3}\frac{d^2k_i}{(2\pi)^3}C_0(k_1,k_2,\Delta)C_0(k_2,k_3,\Delta)\prod_{i=1}^{3}(k_i^{\,2}+\lambda^2)^{-1}[(\Delta-k_i)^2+\lambda^2]^{-1}$$
$$\times[E-g^2K(k_i)-g^2K(\Delta-k_i)]^{-1}. \tag{18}$$

Making use of the expansion

$$\int\frac{dE}{2\pi i}\frac{s^{-E}}{(E-A)(E-B)(E-C)}=\frac{1}{2!}(\ln s)^2-\frac{1}{3!}(A+B+C)(\ln s)^3+\cdots, \tag{19}$$

we find that

$$G_0^3(s,\Delta)=isa^2g^4\{[\tfrac{1}{2}b^2(\Delta^2+\tfrac{5}{4}\lambda^2)^2J_1(\Delta)^3+4bc(\lambda^2+\tfrac{5}{4}\lambda^2)J_1(\Delta)J_2(\Delta)+4c^2J_3(\Delta)+4c^2\mathcal{J}(\Delta)](g^2\ln s)^2$$
$$-[b^2(\Delta^2+\tfrac{5}{4}\lambda^2)^2J_1(\Delta)^2J_2(\Delta)+\tfrac{8}{3}bc(\Delta^2+\tfrac{5}{4}\lambda^2)(J_2(\Delta)^2+J_1(\Delta)J_3(\Delta)+J_1(\Delta)\mathcal{J}(\Delta))$$
$$+4c^2(\tfrac{1}{3}J_A(\Delta)+J_B(\Delta))+\tfrac{8}{3}c^2(J_4(\Delta)+2J_C(\Delta)+J_D(\Delta))](g^2\ln s)^3+\cdots\}. \tag{20}$$

We now argue that the diagram of Fig. 3 is the only one with three large rapidity gaps that enters in the leading-logarithm approximation. First we note that the coupling function for an odd number of odd-signature Reggeons must vanish by signature conservation.[11,6] Diagrams involving the coupling of six or more Reggeons do not contribute because such couplings are at least of order g^4. Furthermore, such diagrams would not have the proper transverse-momentum dependence. The only other possibility would be for the three-Reggeon–one-Reggeon coupling function to be of order g^2. If this were the case then the diagram of Fig. 4 would contribute to the isospin-one exchange amplitude in the sixth order, and spoil the simple exponentiation. Since this does not happen, we conclude that the three-Reggeon–one-Reggeon coupling is at least of order g^3. As a result, the only RFT diagrams which contribute to the isospin-zero (and isospin-two) amplitude in the leading-logarithm approximation are those which correspond to strings of two-Reggeon bubbles as illustrated by Figs. 1(a), 2, and 3.

The last Reggeon diagram to consider is the one with four large rapidity gaps. Its contribution is

$$G_0^4 = -isa^2g^4(g^2\ln s)^3\{\tfrac{1}{6}b^2(\Delta^2+\tfrac{5}{4}\lambda^2)^3J_1(\Delta)^4 + 2b^2c(\Delta^2+\tfrac{5}{4}\lambda^2)^2J_1(\Delta)^2J_2(\Delta) + \tfrac{8}{3}bc^2[J_1(\Delta)J_3(\Delta)+J_1(\Delta)\tilde{J}(\Delta)+J_2(\Delta)^2] + \tfrac{8}{3}c^3[J_4(\Delta)+2J_C(\Delta)+J_D(\Delta)]\}+\cdots . \qquad (21)$$

The complete amplitude given by RFT is the sum of Eqs. (12), (17), (20), and (21). The three coupling strengths are determined by the fourth- and sixth-order perturbation terms of Eq. (1),

$$a=1,\quad b=2,\quad c=-1. \qquad (22)$$

Since all parameters are now determined, we have in effect "predicted" the results of the eighth- and tenth-order calculations. In order that the various functions of momentum transfer have the correct coefficients, b and c must satisfy the following eight equations:

eighth order

$$J_3+\tilde{J} \qquad (2c+1)^2=1,$$
$$J_1J_2 \qquad 2b(2c+1)=-4,$$
$$J_1^3 \qquad \tfrac{1}{2}b^2=2,$$

tenth order (23)

$$J_2^2+J_1(J_3+\tilde{J}) \qquad \tfrac{2}{3}b+\tfrac{8}{3}bc(1+c)=\tfrac{4}{3},$$
$$J_1^2J_2 \qquad b^2+2b^2c=-4,$$
$$J_1^4 \qquad \tfrac{1}{6}b^3=\tfrac{4}{3},$$
$$\tfrac{1}{3}J_A+J_B \qquad (2c+1)^2=1,$$
$$J_4+2J_C+J_D \qquad \tfrac{2}{3}c(1+2c)^2=-\tfrac{2}{3}.$$

These equations are satisfied by the values of b and c given in Eq. (22), which we believe lends strong support to our hypothesis that the high-energy behavior of the $I=0$ amplitudes is controlled by the two-Reggeon cut. As we mentioned in the Introduction, in Sec. III we shall extend the comparison of perturbation theory and RFT to all orders in the limit $\Delta^2 \gg \lambda^2$.

The isospin-two exchange amplitude can be treated in exactly the same way as the isospin-zero amplitude. The coupling of two Reggeons to two external particles in the $I=2$ channel will be denoted $a'g^2$. The two-Reggeon–two-Reggeon coupling function $C_2(k_1,k_2,\Delta)$ can be gotten from $C_0(k_1,k_2,\Delta)$ by the replacements

$$b\to b',\quad c\to c',\quad \Delta^2+\tfrac{5}{4}\lambda^2\to\Delta^2+2\lambda^2. \qquad (24)$$

It then follows that the contributions from Reggeon diagrams with one through four rapidity gaps can be read off Eqs. (12), (17), (20), and (21) by making the replacement of Eq. (24). Comparing with the fourth- and sixth-order terms of Eq. (1) we find

$$a'=1,\quad b'=-1,\quad c'=\tfrac{1}{2}. \qquad (25)$$

The eighth- and tenth-order $I=2$ contributions to Eq. (1) are correctly given by RFT when Eq. (25) is used.

The above formulas hold when a Higgs doublet is coupled to the gauge mesons. When three Higgs triplets are used instead, the only changes are $\Delta^2+\frac{5}{4}\lambda^2\to\Delta^2$ in the $I=0$ expressions, and $\Delta^2+2\lambda^2\to\Delta^2+3\lambda^2$ in the $I=2$ expressions.[3]

As we mentioned in the Introduction, Bartels has suggested that as a first approximation the $I=0$ and $I=2$ channels can be described in terms of Regge poles with trajectories given by Eq. (5), and one can identify contributions to Eq. (1) which appear to be the first three terms in the $g^2\ln s$ expansion of such Regge-pole amplitudes. We will

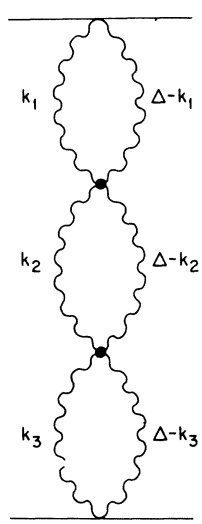

FIG. 3. Reggeon diagram with three large rapidity gaps.

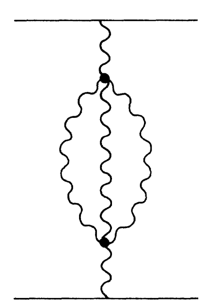

FIG. 4. Self-energy correction to Reggeon propagator. The coupling must be $O(g^3)$ or smaller.

now show that if one starts by assuming that such poles exist, one finds that they decouple from both the external particles and the two-Reggeon intermediate state.

Let us start by considering G_0. If there is an $I=0$ Regge pole with a trajectory given by Eq. (5), then it will have the $g^2\ln s$ expansion

$$\tilde{G}_0^1(s,\Delta^2)=is\beta(\Delta)[1-2(\Delta^2+\tfrac{5}{4}\lambda^2)J_1(\Delta)g^2\ln s+2(\Delta^2+\tfrac{5}{4}\lambda^2)^2J_1(\Delta)^2(g^2\ln s)^2+\cdots]. \quad (26)$$

From Eq. (1) we see that β must have the form

$$\beta(\Delta)=d^2g^4J_1(\Delta), \quad (27)$$

so the coupling of the $I=0$ Reggeon to the external particles has the rather artificial form $dg^2J_1(\Delta)^{1/2}$.

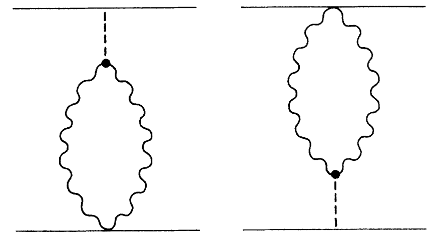

FIG. 5. Diagrams with two large rapidity gaps when there is an elementary $I=0$ Reggeon.

There are now two additional RFT diagrams with two large rapidity gaps. They are shown in Fig. 5. The dashed line represents the $I=0$ Reggeon. Their sum has the general form

$$\tilde{G}_0^2(s,\Delta^2)=2isadg^4J_1(\Delta)^{1/2}\int\frac{dE}{2\pi i}s^{-E}\int\frac{d^2k}{(2\pi)^3}D_0(k,\Delta)(k^2+\lambda^2)^{-1}[(\Delta-k)^2+\lambda^2]^{-1}[E-g^2K(k)-g^2K(\Delta-k)]^{-1}\times[E-2g^2(\Delta^2+\tfrac{5}{4}\lambda^2)J_1(\Delta)]^{-1}. \quad (28)$$

$D_0(k,\Delta)$ is the coupling function for the vertex involving one $I=0$ Reggeon and two $I=1$ Reggeons. It is clear that in order to get the transverse-momentum dependence correct in the sixth order we must take D_0 to have the form

$$D_0=eg^2(\Delta^2+\tfrac{5}{4}\lambda^2)J_1(\Delta)^{1/2}. \quad (29)$$

Then, Eq. (28) will have the expansion

$$\tilde{G}_0^2(s,\Delta^2)=-2isadeg^4\{(\Delta^2+\tfrac{5}{4}\lambda^2)J_1(\Delta)g^2\ln s-[(\Delta^2+\tfrac{5}{4}\lambda^2)^2J_1(\Delta)^3+(\Delta^2+\tfrac{5}{4}\lambda^2)J_1(\Delta)J_2(\Delta)](g^2\ln s)^2+\cdots\}. \quad (30)$$

The new diagrams with three large rapidity gaps are shown in Fig. 6. Their sum has the expansion

$$\tilde{G}_0^3(s,\Delta^2)=isg^4\{[(\tfrac{1}{2}a^2e^2+\tfrac{1}{2}d^2e^2+abde)(\Delta^2+\tfrac{5}{4}\lambda^2)^2J_1(\Delta)^3+4adce(\Delta^2+\tfrac{5}{4}\lambda^2)J_1(\Delta)J_2(\Delta)](g^2\ln s)^2+\cdots\}. \quad (31)$$

Adding the contributions of Eqs. (26), (30), and (31) to the previous terms yields

$$G_0(s,\Delta)=isg^4\{(a^2+d^2)J_1(\Delta)-[(a^2b+2d^2+2ade)(\Delta^2+\tfrac{5}{4}\lambda^2)J_1(\Delta)+a^2(4c+2)J_2(\Delta)]g^2\ln s$$
$$+[a^2(2c+1)^2(J_3(\Delta)+\tilde{J}(\Delta))+(2a^2b(2c+1)+2ade+4adce)(\Delta^2+\tfrac{5}{4}\lambda^2)J_1(\Delta)J_2(\Delta)$$
$$+(\tfrac{1}{2}a^2b^2+2d^2+2ade+\tfrac{1}{2}a^2e^2+\tfrac{1}{2}d^2e^2+abde)(\Delta^2+\tfrac{5}{4}\lambda^2)^2J_1(\Delta)^3](g^2\ln s)^2+\cdots\}. \quad (32)$$

At this point it is convenient to normalize $G_0(s,\Delta^2)$ so that the coefficient of $(\ln s)^0$ is exactly $isg^4J_1(\Delta)$. We now have five parameters which must satisfy the following six equations if we are to obtain agreement with the Feynman-diagram calculation through eighth order:

$$\text{(i) } a^2+d^2=1, \quad \text{(ii) } a^2b+2d^2+2ade=2,$$
$$\text{(iii) } a^2(4c+2)=-2, \quad \text{(iv) } a^2(2c+1)^2=1,$$
$$\text{(v) } 2a^2b(2c+1)+2ade+4adce=-4, \quad (33)$$
$$\text{(vi) } \tfrac{1}{2}a^2b^2+2d^2+2ade+\tfrac{1}{2}a^2e^2+\tfrac{1}{2}d^2e^2+abde=2.$$

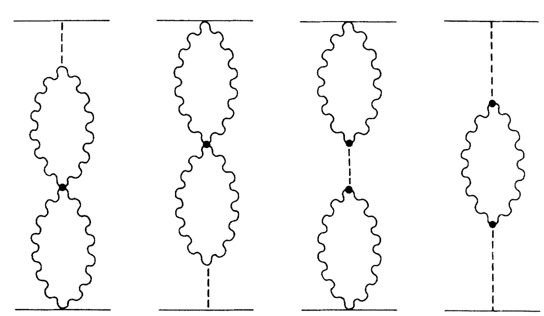

FIG. 6. Diagrams with three large rapidity gaps when there is an elementary $I=0$ Reggeon.

The unique solution to this overconstrained set of equations is

$$a=1, \quad b=2, \quad c=-1, \quad d=e=0. \tag{34}$$

So, the presumed elementary Regge pole decouples from both the external particles and the two-$(I=1)$-Reggeon state. We are simply left with our original solution. Precisely the same thing happens if we assume that there is an elementary Regge pole in the $I=2$ channel.

III. WEAK-COUPLING SOLUTION OF REGGEON FIELD THEORY

We have seen that only two-Reggeon cuts contribute in the $I=0$ and $I=2$ channels in the weak-coupling approximation to RFT. We emphasize that this does not correspond to just two vector mesons in the t channel. We have explicitly compared our predictions with perturbation theory through the tenth order, where as many as five vector mesons appear in the t channel in particular Feynman diagrams. These contributions (J_4 is an example) are reproduced in the leading-logarithm approximation when the Reggeon factors s^α are expanded in powers of g^2. In $2n$th order, the two-Reggeon cut includes as many as n vector mesons in the t channel. Therefore, the description of the dynamics is greatly simplified when written in terms of RFT.

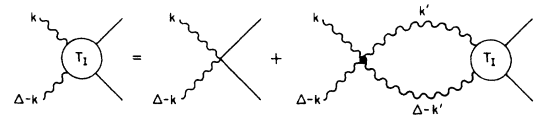

FIG. 7. Integral equation for the two-particle–two-Reggeon amplitude.

Since only two-Reggeon cuts contribute, we can write an integral equation summing all diagrams; it is illustrated in Fig. 7 for $T_I(k,\Delta,E)$, the two-particle–two-Reggeon amplitude in the isospin-I channel

$$T_I(k,\Delta,E)=1+\int\frac{d^2k'}{(2\pi)^3}\frac{C_I(k,k',\Delta)T_I(k',\Delta,E)}{(k'^2+\lambda^2)[(\Delta-k')^2+\lambda^2]}\left[E-g^2K(k')-g^2K(\Delta-k')\right]^{-1}. \tag{35}$$

The coupling functions $C_I(k,k',\Delta)$ are given by Eqs. (15), (22), (24), and (25). In terms of T_I the partial-wave amplitude is

$$f_I(\Delta,E)=g^4\int\frac{d^2k}{(2\pi)^3}\frac{T_I(k,\Delta,E)}{(k^2+\lambda^2)[(\Delta-k)^2+\lambda^2][E-g^2K(k)-g^2K(\Delta-k)]}. \tag{36}$$

Equation (35) is equivalent to equations given by Fadin, Kuraev and Lipatov,[7] and by Cheng and Lo.[1] If the kernel of the integral were of Fredholm type, we could immediately state that $f_I(\Delta,E)$ has only a two-Reggeon cut plus, possibly, poles which can be regarded as Reggeon-Reggeon bound states, and which move as Δ^2 is varied. However, the norm of the kernel, regarded as a double integral over k and k', actually diverges. This permits the partial-wave amplitude to have additional cut singularities in the E plane, and in Refs. 1 and 7, the analysis is carried far enough to argue that the leading singularity is a fixed cut at $E=-g^2(2\ln 2)/\pi^2$ or $J=1+g^2(2\ln 2)/\pi^2$. We think it is very interesting that a fixed cut can arise by summing moving Regge cuts. Fixed cuts are generally thought to lie outside the purview of RFT, but here we see how they can be incorporated naturally.

We can use Eq. (35) to calculate the leading-logarithm approximation to $F_0(s,\Delta^2)=G_0(s,\Delta^2)$ in all orders of perturbation theory for $\Delta^2\gg\lambda^2$. Our result agrees with Eq. (6), which can be regarded as further support for Eq. (35).

We begin with some restructuring of Eq. (35). Define $\psi(k,\Delta,E)$ by

$$\psi(k,\Delta,E)\equiv\frac{T_0(k,\Delta,E)}{E-g^2K(k)-g^2K(\Delta-k)}, \tag{37}$$

so that

$$F_0(s,\Delta^2)=G_0(s,\Delta^2)=isg^4\int\frac{dE}{2\pi i}s^{-E}\times\int\frac{d^2k\,\psi(k,\Delta,E)}{(2\pi)^3(k^2+\lambda^2)[(\Delta-k)^2+\lambda^2]}. \tag{38}$$

In order to compare with Eq. (6), it is convenient to expand $F_0(s,\Delta^2)$ in powers of $\ln s$. This is done by expanding

$$\psi(k,\Delta,E)=\sum_{n=1}^{\infty}\frac{\psi_n(k,\Delta)}{E^n} \tag{39}$$

so that

$$F_0(s,\Delta^2)=isg^4\sum_{n=0}^{\infty}\frac{(-\ln s)^n}{n!}\times\int\frac{d^2k\,\psi_{n+1}(k,\Delta)}{(2\pi)^3(k^2+\lambda^2)[(\Delta-k)^2+\lambda^2]}. \tag{40}$$

Equation (35) now can be recast as a recursion relation for the ψ_n's.

$$\psi_1(k,\Delta)=1\,,$$
$$\psi_{n+1}(k,\Delta)=[K(k)+K(\Delta-k)]g^2\psi_n(k,\Delta)+\int\frac{d^2k'C_0(k,k',\Delta)\psi_n(k',\Delta)}{(2\pi)^3(k'^2+\lambda^2)[(\Delta-k')^2+\lambda^2]}\,.\quad(41)$$

Now we must exploit the relation $\Delta^2\gg\lambda^2$. There is no longer a cut in the E plane which dominates the picture because we have expanded in powers of E^{-1}. However, note that when Δ^2 is large, the integral operator in Eq. (41) is large only when k' is near zero or Δ. As a result, ψ_n is going to be needed only in this region in order to evaluate either ψ_{n+1}, or $F_0(s,\Delta^2)$ in Eq. (40). Since $\psi_n(k,\Delta)=\psi_n(\Delta-k,\Delta)$, let us concentrate on evaluating $\psi_n(k,\Delta)$ for $\lambda^2\ll k^2\ll\Delta^2$. (It is unimportant to calculate ψ_n accurately for $0<k^2\lesssim\lambda^2$ because the phase space of this region is small.) Now we can approximate

$$C_0(k,k',\Delta)\sim g^2\Delta^2\left[1-\frac{k^2+k'^2+2\lambda^2}{(k-k')^2+\lambda^2}\right],$$
$$K(k)\sim\frac{1}{4\pi^2}\ln(k^2/\lambda^2)\,,$$
$$K(\Delta-k)\sim\frac{1}{4\pi^2}\ln(\Delta^2/\lambda^2)\,,\quad(42)$$
$$\frac{1}{(\Delta-k')^2+\lambda^2}\sim\frac{1}{\Delta^2}\,.$$

In connection with the last approximation we observe that after substitution the integral operator of Eq. (41) becomes logarithmically divergent when applied to typical terms in ψ_n. To logarithmic accuracy we can made the substitution provided we choose $k'^2<\Delta^2$ as the region of integration. Altogether, Eq. (41) reads, for $\lambda^2\ll k^2\ll\Delta^2$,

$$\psi_1(k,\Delta)=1\,,$$
$$\psi_{n+1}(k,\Delta)=\frac{g^2}{4\pi^2}\left(\ln\frac{k^2}{\lambda^2}+\ln\frac{\Delta^2}{\lambda^2}\right)\psi_n(k,\Delta)+2g^2\int_{k'^2<\Delta^2}\frac{d^2k'\psi_n(k',\Delta)}{(2\pi)^3(k'^2+\lambda^2)}\times\left[1-\frac{k^2+k'^2+2\lambda^2}{(k-k')^2}\right].\quad(43)$$

The factor two in front of the integral is to include the region $k'\sim\Delta$ in Eq. (41).

In the Appendix we establish the result

$$\int_{k'^2<\Delta^2}\frac{d^2k'[\ln(k'^2/\lambda^2)]^\rho}{(2\pi)^3(k'^2+\lambda^2)}\left[1-\frac{k^2+k'^2+2\lambda^2}{(k-k')^2+\lambda^2}\right]\sim\frac{1}{4\pi^2}[\ln(k^2/\lambda^2)]^{\rho+1}\,,\quad\lambda^2\ll k^2\ll\Delta^2\,.\quad(44)$$

We immediately see that the general term in the recursion relation has the form

$$\psi_n(k,\Delta)=\sum_{l=0}^{n-1}A_{n,l}\left(\ln\frac{k^2}{\lambda^2}\right)^l\left(\ln\frac{\Delta^2}{\lambda^2}\right)^{n-l-1}\,,\quad(45)$$

and the recursion relation for the $A_{n,l}$'s is

$$A_{1,0}=1\,,$$
$$A_{n+1,0}=\frac{g^2}{4\pi^2}A_{n,0}\,,$$
$$A_{n+1,n}=-\frac{g^2}{4\pi^2}A_{n,n-1}\,,\quad(46)$$
$$A_{n+1,j}=\frac{g^2}{4\pi^2}A_{n,j}-\frac{g^2}{4\pi^2}A_{n,j-1}\,,\quad 1<j<n\,.$$

Since Eq. (43) obviously has a unique solution, so must Eq. (46). (It is not hard to see how the terms are calculated successively.) Therefore it suffices to exhibit a single solution of Eq. (46) to know it is unique,

$$A_{n,j}=(-1)^j\left(\frac{g^2}{4\pi^2}\right)^{n-1}\frac{(n-1)!}{j!(n-j-1)!}\,.\quad(47)$$

The partial-wave amplitude can be calculated from Eqs. (40), (45), (47), and the integral

$$\int_{k^2<\Delta^2}\frac{d^2k[\ln(k^2/\lambda^2)]^\rho}{(2\pi)^3(k^2+\lambda^2)}\sim\frac{1}{8\pi^2(\rho+1)}[\ln(\Delta^2/\lambda^2)]^{\rho+1}\,,\quad(48)$$

which we derive in the Appendix. The result is

$$F_0(s,\Delta^2)=\frac{isg^4}{\Delta^2 4\pi^2}\sum_{n=0}^{\infty}\frac{(-\ln s)^n}{n!}[\ln(\Delta^2/\lambda^2)]^{n+1}\times\sum_{j=0}^{n+1}\frac{A_{n+1,j}}{j+1}\,.\quad(49)$$

This includes a factor of two for the region $k\sim\Delta$ in Eq. (40). From the binomial theorem

$$\sum_{j=0}^{n+1}\frac{A_{n+1,j}}{j+1}=\frac{(g^2/4\pi^2)^n}{n+1}\,,$$

so for each value of n the coefficient of $\ln(\Delta^2/\lambda^2)$ is in agreement with that found in Refs. (4) and (5). The sum of these leading-logarithm contributions is

$$F_0(s,\Delta^2)=\frac{isg^2}{\Delta^2\ln s}\left[1-\exp\left(-\frac{g^2}{4\pi^2}\ln s\ln\frac{\Delta^2}{\lambda^2}\right)\right]\,,\quad(50)$$

which agrees with Eq. (6).

We point out again that the calculation of the leading powers of $\ln(\Delta^2/\lambda^2)$ at each order in perturbation theory provides a significant consistency check on our assumptions. On the other hand, the resulting sum does not have a singularity at $J=1+g^2(2\ln2)/\pi^2$, which presumably shows that the sum of leading powers of $\ln(\Delta^2/\lambda^2)$ does not give the leading behavior of the amplitude for $\Delta^2\gg\lambda^2$.

IV. ASYMPTOTIC BEHAVIOR OF THE RFT KERNEL

We now want to determine whether the norm of the kernel of Eq. (35) is divergent when higher orders of perturbation theory are taken into account. We shall use the renormalization group as the main tool in our analysis.

When higher terms in perturbation theory are included, Eq. (35) will become

$$T_I(k,\Delta,E)=1+\int\frac{d^2k'C_{I,R}(k,k',\Delta)T_I(k',\Delta,E)}{(2\pi)^3\Gamma^{(2)}(-k^2)\Gamma^{(2)}(-(\Delta-k)^2)}\times[E+\alpha(-k^2)+\alpha(-(\Delta-k)^2)-2]^{-1}. \quad (51)$$

The correction of the Reggeon denominator is obvious, where $\alpha(t)$ is the complete gluon trajectory function. The replacement of $(k^2+\lambda^2)$ by $-\Gamma^{(2)}(-k^2)$ is consistent with the vanishing of $\sin\pi\alpha(-k^2)$ at $\alpha=1$, and can be motivated directly by considering the effect of self-energy insertions on gluon lines in the NAGT diagrams. $C_{I,R}$ includes corrections to the weak-coupling four-Reggeon interaction, and the effects of intermediate states of more than two Reggeons. Equation (51) can, of course, be thought of as defining $C_{I,R}(k,k',\Delta)$.

The functions of Eq. (51) are all renormalized quantities, as they must be when one goes beyond the leading-logarithm approximation. They therefore depend upon a normalization momentum, μ^2, at which normalization conditions are placed on propagators and vertices. In addition, they depend upon the Higgs-meson self-coupling which is required to spontaneously break the gauge symmetry, the Yukawa coupling of the Higgs mesons to the fermions, and the gauge parameter, ξ, of the vector mesons. We will suppress the dependence on these additional couplings since they do not appear in the weak-coupling expressions. Under these conditions, the functions in Eq. (51) depend upon renormalized parameters g, λ^2, ξ, and μ^2.

We can immediately dispose of the gauge dependence by recalling that S-matrix elements are gauge independent.[12] We can therefore imagine having done our initial leading-logarithm calculation in the Landau gauge, instead of the Feynman gauge used in Refs. 1 and 3. Equation (35) would have again appeared, and in Eq. (51) the effect of high-order corrections would not change the Landau gauge condition. The important property of the Landau gauge under a renormalization transformation is that one remains in it after the transformation. We can therefore set $\xi=0$ in the rest of the discussion.

When the normalization point, μ^2, is changed to $\overline{\mu}^2$, the NAGT n-meson amplitudes undergo a renormalization

$$\overline{\Gamma}^{(n)}(p_i,g,\lambda^2,\overline{\mu}^2)=Z_3^{n/2}\Gamma^{(n)}(p_i,Z_1Z_3^{-3/2}g,Z_4Z_1^{-1}\lambda^2,\mu^2), \quad (52)$$

where the renormalization constants are finite. We now want to determine what transformation this renormalization induces on Eq. (51). The Regge trajectories are invariant since the asymptotic behavior cannot depend on the renormalization scheme, and the effect on the inverse meson propagator can be read off Eq. (52). If we consider contributions like those of Figs. 1–3, it is evident that

$$\overline{C}_{I,R}(k,k',\Delta,g,\lambda^2,\overline{\mu}^2)=Z_3^2C_{I,R}(k,k',\Delta,Z_1Z_3^{-3/2}g,Z_4Z^{-1},\lambda^2,\mu^2). \quad (53)$$

From this it follows that $C_{I,R}$ satisfies the same renormalization group equation as $\Gamma^{(4)}$,

$$\left(\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial g}+\kappa\lambda\frac{\partial}{\partial\lambda}-2\gamma\right)C_{I,R}(k,k',\Delta,g,\lambda^2,\mu^2)=0. \quad (54)$$

By dimensional analysis this can be converted to

$$\left[\rho\frac{\partial}{\partial\rho}-\beta\frac{\partial}{\partial g}+(1-\kappa)\lambda\frac{\partial}{\partial\lambda}-2+2\gamma\right]\times C_{I,R}(\rho k,\rho k',\rho\Delta,g,\lambda^2,\mu^2)=0. \quad (55)$$

At the one-loop level,

$$\gamma=a_0g^2,\quad \beta=-\tfrac{1}{2}b_0g^3,\quad \kappa=c_0g^2, \quad (56)$$

with $b_0>0$, unless there is an exceptionally large number of fermions. The solution of Eq. (55) is

$$C_{I,R}(\rho k,\rho k',\Delta,g,\lambda^2,\mu^2)=\rho^2C_{I,R}(k,k',\Delta/\rho,\hat{g}(-\ln\rho),\hat{\lambda}^2(-\ln\rho),\mu^2)\times\exp\left(2\int_{-\ln\rho}^{0}d\tau\gamma(\hat{g}(\tau))\right), \quad (57)$$

where

$$\frac{d\hat{g}(\tau)}{d\tau}=-\beta(\hat{g}(\tau)),\quad \hat{g}(0)=g,$$
$$\frac{d\ln\hat{\lambda}(\tau)}{d\tau}=1-\kappa(\hat{g}(\tau)),\quad \hat{\lambda}(0)=\lambda. \quad (58)$$

Using Eq. (56),

$$\hat{g}^2(-\ln\rho)=\frac{g^2}{1+b_0g^2\ln\rho},$$
$$\hat{\lambda}(-\ln\rho)=\frac{\lambda}{\rho}(1+b_0g^2\ln\rho)^{c_0/b_0}. \quad (59)$$

From this it follows that both g and λ decrease monotonically to 0 as ρ increases above 1.

Similar results can be derived for the other quantities appearing in Eq. (51),

$$\left[\rho\frac{\partial}{\partial\rho}-\beta\frac{\partial}{\partial g}+(1-\kappa)\lambda\frac{\partial}{\partial\lambda}-2+\gamma\right]\Gamma^{(2)}(\rho k,g,\lambda^2,\mu^2)=0\,, \tag{60}$$

$$\left[\rho\frac{\partial}{\partial\rho}-\beta\frac{\partial}{\partial g}+(1-\kappa)\lambda\frac{\partial}{\partial\lambda}\right]\alpha(\rho k,g,\lambda^2,\mu^2)=0\,,$$

$$\Gamma^{(2)}(\rho k,g,\lambda^2,\mu^2)=\rho^2\Gamma^{(2)}(k,\hat{g}(-\ln\rho),\lambda^2(-\ln\rho),\mu^2)\exp\left(\int_{-\ln\rho}^{0}d\tau\gamma(\hat{g}(\tau))\right), \tag{61}$$

$$\alpha(\rho k,g,\lambda^2,\mu^2)=\alpha(k,\hat{g}(-\ln\rho),\hat{\lambda}^2(-\ln\rho),\mu^2)\,.$$

It is convenient to symmetrize the kernel of Eq. (51). Recalling that $C_{I,R}$ is $O(g^2)$ we write the symmetrized kernel in the form

$$g^2K_I(k,k',\Delta,g,\lambda^2)=C_{I,R}(k,k',\Delta)\{\Gamma^{(2)}(-k^2)\Gamma^{(2)}(-(\Delta-k)^2)\Gamma^{(2)}(-k'^2)\Gamma^{(2)}(-(\Delta-k')^2)$$
$$\times[E+\alpha(-k^2)+\alpha(-(\Delta-k)^2)-2][E+\alpha(-k'^2)+\alpha(-(\Delta-k')^2)-2\}^{-1/2}\,. \tag{62}$$

The norm of the kernel is

$$N=\int\frac{d^2k}{(2\pi)^3}\frac{d^2k'}{(2\pi)^3}\left|g^2K_I(k,k',\Delta,g,\lambda^2)\right|^2$$
$$=\int_0^\infty d\rho^2\rho^2\int\frac{d^2q}{(2\pi)^3}\frac{d^2q'}{(2\pi)^3}\delta^2(q^2+q'^2-1)\left|g^2K_I(\rho q,\rho q',\Delta,g,\lambda^2)\right|^2$$
$$=\int_0^\infty\frac{d\rho^2}{\rho^2}\hat{g}^4(-\ln\rho)\int\frac{d^2q}{(2\pi)^3}\frac{d^2q'}{(2\pi)^3}\delta(q^2+q'^2-1)\left|K_I(q,q',\Delta/\rho,\hat{g}(-\ln\rho),\hat{\lambda}^2(-\ln\rho))\right|^2. \tag{63}$$

Superficially it would appear that N is finite provided $g^2(-\ln\rho)\to 0$ as $\rho\to\infty$. This is the result found by Cardy[9] and Lovelace[10] in $\phi^3_{D=6}$ theory. However, before we can draw any such conclusion we must investigate the infrared behavior of the kernel since $\lambda(-\ln\rho)\to 0$ as $\rho\to\infty$. We can only do this in the weak-coupling approximation, where we know K_I explicitly. Making use of Eqs. (4), (15), and (59) we see that in the weak-coupling approximation

$$K_0(q,q',\Delta/\rho,g(-\ln\rho),\lambda^2(-\ln\rho))\underset{\rho\to\infty}{\sim}\left(E-\frac{1}{2\pi^2b_0}\right)^{-1}\{2[\Delta^2/\rho^2+\tfrac{5}{4}\hat{\lambda}^2(-\ln\rho)][q^2+\hat{\lambda}^2(-\ln\rho)]^{-1}[q'^2+\hat{\lambda}^2(-\ln\rho)]^{-1}$$
$$-[(q-q')^2+\hat{\lambda}^2(-\ln\rho)]^{-1}-[(q+q')^2+\hat{\lambda}^2(-\ln\rho)]^{-1}\}\,. \tag{64}$$

with an analogous expression for K_2. If one now substitutes Eq. (64) into Eq. (63) the regions $q\approx\pm q'$ make a contribution to the q and q' integrals of order $\hat{\lambda}^{-2}$, so at least in the weak-coupling approximation the ρ^2 integration diverges linearly. The factor $\hat{g}(-\ln\rho)$ cannot come close to removing so strong a divergence. We would again expect to find fixed singularities in the angular momentum plane.

Of course we cannot say whether N continues to diverge if we go beyond the weak-coupling approximation. What we have shown is that the mechanism identified by Cardy[9] and Lovelace[10] for ensuring Regge behavior in asymptotically free $\phi^3_{D=6}$ theory is not applicable in NAGT in any simple way. The difference is that in $\phi^3_{D=6}$ theory the infrared problem is much milder, partly because the transverse-momentum integrals are four dimensional and partly because the kernel has been smoothed by the partial-wave projection.

V. SUMMARY AND CONCLUSIONS

In previous sections we have seen that in the Regge limit the scattering amplitudes of NAGT with a spontaneously broken gauge symmetry can be generated by summing a moving pole and moving cuts in the angular momentum plane. The pole is the Reggeized vector meson, and the cuts are produced from the pole by RFT. The couplings in RFT were deduced in Sec. II by comparing the general RFT expressions with the perturbation-theory expansion of the NAGT amplitudes. The couplings were highly overconstrained when the comparison was made through the tenth order in perturbation theory, which we took as confirmation of our hypothesis that RFT generates the asymptotic amplitude. In Sec. III this comparison was extended to all orders in perturbation theory in the limit $\Delta^2\gg\lambda^2$ by making use of the results of Refs. (4) and (5). All of the perturbation-theory calculations with which we compared RFT were carried out in the leading-logarithm approximation, so only the weak-coupling limits of the trajectories and couplings in RFT were obtained.

In the weak-coupling limit, the structure of Regge singularities in the t channel is very simple. In the $I = 1$ exchange amplitudes there is a single pole, the Reggeized vector meson. In the $I = 0$ and $I = 2$ exchange amplitudes there are two vector-meson Reggeons, which can elastically scatter off each other with a momentum-dependent coupling we deduced. Since there are only two Reggeons, we were able to write an integral equation for their interaction. We think it is remarkable that although any number of vector mesons can appear in the t channel in the leading-logarithm approximation to NAGT amplitudes, only one or two Reggeons appear in the RFT formulation of the problem.

When the integral equation for Reggeon-Reggeon scattering is solved in the $I = 0$ channel, the leading singularity turns out to be a fixed cut in the angular momentum plane. Of course there are only moving cuts order-by-order in Reggeon perturbation theory, and there is also a moving Regge-Mandelstam cut in the sum whose discontinuity is controlled by Reggeon unitarity.[13] The fixed cut is additional and only appears when all RFT graphs are summed. Here we see explicitly that under some circumstances RFT is capable of generating singularities other than the moving poles and cuts that are envisioned in its formulation.

In Sec. IV we considered the possibility that the fixed cut becomes a moving singularity if asymptotic freedom is taken into account.[7] We found that this was not the case, at least in the weak-coupling approximation. The mechanism identified by Cardy[9] and Lovelace[10] in $\phi^3_{D=6}$ theory is not applicable in NAGT because the kernel of the t-channel integral equation is not sufficiently well behaved in the infrared limit.

It is tempting to speculate about generalizations of our results. Our explicit calculations have been carried out for a theory with an SU(2) gauge group, but it seems clear from the work of Yeung[3] that our approach can be straightforwardly extended to theories with other gauge groups. It would also be interesting to study backward meson-fermion scattering in the same spirit to see if there is a RFT formulation of meson-fermion scattering.

The calculations we have made are for spontaneously broken NAGT with Higgs generation of the vector-meson mass. We have said nothing about the interesting case where the symmetry is unbroken and $\lambda = 0$. Our expressions diverge as $\lambda \to 0$. This in itself is not alarming because the particles we have been scattering, the fermions and vector mesons, are then believed to be confined. They should no longer appear as asymptotic states. Neither should the vector-meson Reggeon continue as an ordinary Reggeon, at least in the sense of generating bound states and resonances at right-signatured integers. It is possible that the broken symmetry theory with massive gluons is relevant to hadron scattering,[14] but the relevance of our results to self-confining quantum chromodynamics must await the elucidation of QCD.

APPENDIX

In this appendix we derive the result quoted in Eq. (44). The integral involves three terms. The first integral is

$$I_1 = \int_{k'^2<\Delta^2} \frac{d^2k'[\ln k'^2/\lambda^2)]^\rho}{(2\pi)^3(k'^2+\lambda^2)} \sim \frac{[\ln(\Delta^2/\lambda^2)]^{\rho+1}}{8\pi^2(\rho+1)}, \quad \text{(A1)}$$

where all the contribution comes from the upper limit. This is also the integral cited in Eq. (48). The second integral is

$$I_2 = \int_{k'^2<\Delta^2} \frac{d^2k'[\ln(k'^2/\lambda^2)]^\rho}{(2\pi)^3[(k-k')^2+\lambda^2]}. \quad \text{(A2)}$$

Let $k' = qk$. Then there are contributions to I_2 from $q \sim \hat{k}$, $q \sim \Delta/k$. Separating these terms ($\epsilon \ll 1$),

$$\begin{aligned} I_2 &= \frac{1}{8\pi^2}\int_0^\epsilon \frac{d(q^2)[\ln(k^2/\lambda^2)]^\rho}{q^2+\lambda^2/k^2} + \frac{1}{8\pi^2}\int_\epsilon^{\Delta^2/k^2}\frac{d(q^2)}{q^2}[\ln(q^2k^2/\lambda^2)]^\rho \\ &\sim \frac{[\ln(k^2/\lambda^2)]^{\rho+1}}{8\pi^2} + \frac{1}{8\pi^2(\rho+1)}\left\{[\ln(\Delta^2/\lambda^2)]^{\rho+1} - \left[\ln\left(\frac{k^2\epsilon}{\lambda^2}\right)\right]^{\rho+1}\right\} \\ &\sim \frac{\rho}{\rho+1}\frac{[\ln(k^2/\lambda^2)]^{\rho+1}}{8\pi^2} + \frac{[\ln(\Delta^2/\lambda^2)]^{\rho+1}}{8\pi^2(\rho+1)}. \end{aligned} \quad \text{(A3)}$$

The third integral is

$$I_3 = (k^2+\lambda^2)\int_{k'^2<\Delta^2}\frac{d^2k'[\ln(k'^2/\lambda^2)]^\rho}{(2\pi)^3(k'^2+\lambda^2)[(k-k')^2+\lambda^2]}. \quad \text{(A4)}$$

Again we scale the integral, and now the contributions come from $q\sim 0$, $q\sim\hat{k}$. Separating these terms,

$$I_3 \sim \frac{1}{8\pi^2}\int_0^{\epsilon}\frac{d(q^2)[\ln(k^2q^2/\lambda^2)]^{\rho}}{q^2+\lambda^2/k^2} + \frac{1}{8\pi^2}\int_0^{\epsilon}\frac{d(p^2)[\ln(k^2/\lambda^2)]^{\rho}}{p^2+\lambda^2/k^2}, \quad \text{(A5)}$$

where $p=q-\hat{k}$, and $\epsilon \ll 1$. In the first term we can replace the lower limit by λ^2/k^2 and then drop λ^2/k^2 in the denominator. As a result,

$$I_3 \sim \frac{[\ln(k^2/\lambda^2)]^{\rho+1}}{8\pi^2(\rho+1)} + \frac{[\ln(k^2/\lambda^2)]^{\rho+1}}{8\pi^2}. \quad \text{(A6)}$$

The integral in Eq. (44) is now $I_1 - I_2 - I_3$.

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