Derivation of a quark-confinement equation in the Hamiltonian formalism of gauge field theories*

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Using the Hamiltonian formalism of gauge field theories I derive a quark-confining wave equation for a gauge-invariant amplitude describing a system of a quark and an antiquark connected by a linear electric flux. I obtain an exact potential in the form of the Brillouin-Wigner series, which I make finite and well defined by introducing a finite radius to the tube of the electric flux as a dynamical parameter, to be determined from the stability of the solution. The potential is shown to vanish at the origin and become linear at a large distance. The confinement solution is compared with the normal positronium solution to the wave equation with the Coulomb potential, and the latter is shown the stabler of the two for $g_r^2/4\pi < 2$ where g_r is the renormalized coupling constant. For $g_r^2/4\pi > 2$, the confinement solution is the only possible one. Essential differences between Abelian and non-Abelian gauge fields are pointed out. A possibility of g_r being replaced by g_{eff} in the sense of asymptotic freedom is pointed out also.

I. INTRODUCTION AND SUMMARY.

There has been a large amount of literature on the possibility of confinement of quarks in field theories. The existence of such confinement was shown in particular by Wilson[1] and also by Kogut and Susskind[2] using the lattice model of the gauge fields. In this paper I will not introduce a lattice. Instead I introduce a gauge-invariant two-body amplitude corresponding to a stringlike electric flux. My approach to the problem of confinement is analogous in many respects to the BCS treatment[3] of the superconductive ground state. There the Fermi sphere, appropriate as a zeroth-order approximation to the ground state in the normal phase, is no longer useful to describe the superconductive ground state. One has to introduce the BCS ground state, which is related to the Fermi sphere not by perturbation but by collective excitation. Now in considering a quark-antiquark system interacting via an Abelian or non-Abelian gauge field an appropriate zeroth-order amplitude to describe the normal Coulomb solutions (or we may call them positronium solutions), which should certainly obtain for a small coupling constant g, will be the Bethe-Salpeter (BS) amplitude

$$\chi_{BS}(1,2) = (\Psi_0, (q(1), q^\dagger(2))_+ \Psi) , \tag{1.1}$$

where Ψ is the positronium state under consideration and Ψ_0 is the vacuum. $q(x)$ is the quark field and the symbol $(\,)_+$ stands for the time ordering of operators inside. In the above, the Coulomb gauge is assumed, where no gauge freedom is left, so that the amplitude (1.1) is gauge invariant by itself. In the confinement solutions, on the other hand, the electric flux is squeezed into a string connecting the quark and antiquark, according to the lattice model mentioned above. The stringlike electric flux must involve both the longitudinal and the transverse parts of the field, and therefore must involve the creation of an infinite number of photons from the vacuum. An appropriate zeroth-order amplitude to describe such a state of collective excitation turns out to be, in the case of quantum electrodynamics (QED),

$$\chi(1,2) = (\Psi_0, q(1,2)\Psi) , \tag{1.2}$$

where the gauge-invariant bilocal operator $q(1,2)$ is defined by

$$q_{\alpha\beta}(1,2) = \exp\left[ig\int_1^2 \vec{A}(\vec{x})\cdot d\vec{x}\right] q_\alpha(1) q_\beta^\dagger(2) . \tag{1.3}$$

$q(1,2)$ and $\chi(1,2)$ should then be considered as 4×4 matrices. In the Coulomb gauge, $\vec{A}(\vec{x})$ in the above expression would express the transverse part. In this paper, however, the Coulomb gauge will not be adopted. Assuming that the state Ψ represents specifically the system at rest, I choose in this rest frame a "spatial" gauge

$$A_0(\vec{x}) = 0 . \tag{1.4}$$

In this gauge all three components of $\vec{A}(\vec{x})$ are independent dynamical variables and one is still left with the freedom of a local gauge transformation for which the amplitude (1.3) is invariant.

In the following I consider $q(1,2)$ and $\chi(1,2)$ only on the equal-time plane

$$t_1 = t_2 .$$

This is appropriate in the Hamiltonian formalism. Also, it was shown in a previous paper by the author[4] that the amplitude $\chi(1,2)$ is nonvanishing only on the equal-time plane in a theory of absolute quark confinement. The amplitude (1.2) still depends on the path of the line integral. The known meson spectrum, on the other hand, seems to be

consistent with the two-body description with no extra degree of freedom needed. Thus we have to eliminate the path dependence of the amplitude. To obtain a path-independent amplitude which depends only on the coordinates $\vec{x}_1$ and $\vec{x}_2$, I may either restrict the path to a special one determined uniquely by $\vec{x}_1$ and $\vec{x}_2$, or average the amplitude over all possible paths with a certain measure. In this paper, I will restrict the path to a straight line connecting points 1 and 2.

That amplitude (1.3) is appropriate for the description of the confinement states can be seen by commuting the Hamiltonian with the biocal operator (1.3). We find that the commutator of the electric field energy with the exponential factor in (1.3) yields a potential term,

$$V_0 = k^2 r \quad (r = |\vec{x}_1 - \vec{x}_2|), \tag{1.5}$$

expressing the concentration of the electric-field energy along the straight line connecting points 1 and 2. In the above

$$k^2 = \tfrac{1}{2} g^2 \delta^2(0), \tag{1.6}$$

and $\delta^2(0)$ can be interpreted as the inverse of the cross section of the string.

It should be noted that the amplitude (1.2), appropriate to describe a quark-confined state Ψ, vanishes if Ψ is a positronium state. This follows from an observation that

$$\exp\left(ig\int_1^2 \vec{A}(\vec{x})\cdot d\vec{x}\right) = :\exp\left(ig\int_1^2 \vec{A}(\vec{x})\cdot d\vec{x}\right): \times \exp(-g^2\Lambda r/8\pi), \tag{1.7}$$

where the symbol $:O:$ represents the Wick-product expansion of an operator O. The cutoff parameter Λ should go to infinity, so that any matrix element of (1.7) between states involving only a finite number of photons vanishes. Since the positronium states involve only a finite number of photons on the average, amplitude (1.2) would vanish for such states. Conversely, the BS amplitude (1.1) would vanish for confinement state Ψ, as can be inferred by setting

$$\Psi \sim \exp\left(-ig\int_1^2 \vec{A}(\vec{x})\cdot d\vec{x}\right) q^\dagger(1) q(2) \Psi_0,$$

and using (1.7) again.

In the above discussion I took the case of QED purely for the sake of simplicity of presentation. The case of non-Abelian fields can be discussed in an almost parallel way. Whether or not QED yields a valid confinement solution is an entirely different matter which will be discussed later.

The equation for $\chi(1,2)$ involves a coupling to other channels represented by an infinite set of amplitudes:

$$\chi_n(1,2) = (\Psi_n, q(1,2)\Psi), \tag{1.8}$$

where the state Ψ_n represents, for example, one-gluon states, two-gluon states, two-pair states, and so forth. These couplings to other channels can be eliminated successively and one obtains the wave equation for $\chi(1,2)$ in a closed form:

$$(-i\vec{\alpha}\cdot\vec{\nabla}_1 + \beta m)\chi(1,2) + \chi(1,2)(-i\vec{\alpha}\cdot\vec{\nabla}_2 - \beta m) = [M - V(\vec{r})]\chi(1,2). \tag{1.9}$$

Here $\vec{\alpha}$ and β are the Dirac matrices. m is the quark mass and M is the energy eigenvalue of the system which is taken to be at rest. The potential $V(r)$ which may involve the Dirac matrices in general consists of the confinement potential (1.5) and a contribution coming from the coupling to other amplitudes (1.8). The latter contribution can be given in a form of the Brillouin-Wigner series.[5] [See Eq. (2.25) in Sec. II.] Equation (1.9) was proposed and investigated in a previous paper by the author,[6] who showed that in spite of the Klein paradox associated with an ever increasing potential such as (1.5), one can still obtain a meaningful solution to the equation which meets all the physical requirements. The equation has many novel features which were explored in an expanded form by Geffen and the author[7] (referred to as GS in the following). The wave equation (1.9) with *nonconfining* potential V is nothing new. In fact, the Breit equation for positronium[8] has exactly this form, allowing V to depend on the Dirac matrix $\vec{\alpha}$. The equation has also been applied for the study of the meson spectrum,[9] but with no confining potential. The potential $V(r)$ in QED will be evaluated in the static limit. However, the series (2.25) is not well defined because of the ultraviolet divergences. Of these divergences, those associated with the vacuum polarization can easily be isolated and absorbed into the charge renormalization. The other renormalizations could possibly be done also, but the renormalization based on the perturbation expansion may not be appropriate here because of the strong electric field already present in the zeroth-order approximation. Therefore, I introduce a cutoff by assigning finite radius a to the string of the electric flux. This is realized by changing a straight-line integral in (1.3) into a bundle of line integrals in a sausage-like tube connecting points 1 and 2 [see Eq. (3.8)]. It is important to recognize that the cutoff is *not* the modification of the gauge field itself, but is the modification of the zeroth-order approximation to the state Ψ. Whether or not such a tube-like electric flux is realizable is a matter of dynamics, which can be determined in principle as discussed later. The introduction of the finite width of the string can be done in a gauge-invariant way. However, for the

sake of simplicity, I shall employ a square cutoff in the momentum component perpendicular to the direction of the string. This particular cutoff does violate gauge invariance and as a result the gauge transformation on a state produces a local phase transformation. Such a degeneracy is again reminiscent of the BCS solution, which also violates gauge symmetry superficially.

With the introduction of the cutoff corresponding to a finite string width, every term of series (2.25) for the potential is now finite. In the static limit the series can be summed, and for a large distance r I find, to all orders in the coupling constant g^2 (there is no charge renormalization in the static limit of QED) that

$$V(r) = k^2 r \quad (r \to \infty). \tag{1.10}$$

Here k^2 is given in terms of the unrenormalized constant g^2 and the radius of the string a ($1/a = \Lambda$ = cutoff momentum) by

$$k^2 = g^2/8\pi a^2, \tag{1.11}$$

which replaces relation (1.6). If I pick out just terms of order g^2, on the other hand, I find that

$$V(r) = -\frac{g^2}{4\pi}\left(\frac{1-e^{-r/a}}{r} - \frac{1}{a}\right), \tag{1.12}$$

where the last term represents the electrostatic self-energy of the quarks. The potential series (2.25) involves the interaction kernel $I(2,1)$ which is given by the line integral of the fields between points 1 and 2. Hence $I(2,1) \to 0$ as $r \to 0$, and we must have

$$V(0) = 0. \tag{1.13}$$

This condition on the potential was derived previously[6] from the requirement of the axial-vector divergence relation. Here it is more generally a consequence of the finite width of the string. Piecing together relations (1.10), (1.12), and (1.13), with some additional considerations, I arrive at an electrostatic potential valid for all ranges of r:

$$V(r) = k^2 r - \frac{g_r^2}{4\pi}\,\frac{C(r/a)}{r}, \tag{1.14}$$

where the function $C(x)$ is assumed to behave like $C(x) \sim \text{const} \times x^2$ for $x \ll 1$ and bounded for large x. g_r is the renormalized coupling constant, which should replace g in (1.12) if the vacuum polarization is taken into account. In guessing at (1.14) I have assumed that the perturbation expansion in g_r is valid at a short distance, and that the self-energy in (1.12) is reduced by taking into account the magnetic interaction.

Once the potential and the wave equation are given as in (1.14) and (1.9), the relative stability of the confinement solution versus the positronium solution can now be settled as a dynamical problem. In the positronium states we have a pure Coulomb potential [$k^2 = 0$ and $C(x) = 1$] in (1.14), and the equation has no solution for $g_r^2/4\pi > 2$. This is due to the well-known Klein paradox at the origin, which arises for a large Coulomb potential. In the confinement solution, g_r^2 can take any value as long as a is finite so that the condition (1.13) is satisfied. Hence, for $g_r^2/4\pi > 2$, the confinement solution is the only possibility. It is interesting to note that the numerical fit of the low-lying $I = 1$ meson spectrum given in GS favors the value of $g_r^2/4\pi$ slightly above the critical value 2. If we know the explicit a dependence of the potential from (1.11) and (1.14), and if we assume that neither g_r^2 nor g^2 is dependent on a, then we should in principle be able to determine whether or not a finite optimal value of a that makes the energy of the confinement-state minimum exists. The affirmative anser would give a justification for introducing the finite width of the string. I will show using the numerical analysis of GS that indeed such a minimum exists for states other than the π meson. Although the analysis is based on a particular choice of $C(x)$ in (1.14), the conclusion may be considered more general if we note that because $k^2 = g^2/8\pi a^2$ (1.11), the confining potential becomes infinite as $a \to 0$. The π meson is an exceptional case because in GS it was treated as the Nambu-Goldstone boson. For $g_r^2/4\pi < 2$ both types of solutions coexist, so that one has to compare their energies. For this, I consider the limit of the vanishing quark mass. Then, I know that all positronium levels coalesce into the zero-energy state $M = 0$. I will show that for a particular model potential of the form (1.14), there exists no confinement solution with $M = m = 0$. Thus, for $g_r^2/4\pi < 2$ the Coulomb solutions are the stable ones. According to the asymptotic-freedom argument, the renormalized coupling constant g_r in the above, which corresponds to subtraction on the gluon mass shell, would be replaced by the effective coupling constant g_{eff}, which is a function of the distance r, or average momentum exchange between the quark and the antiquark. For heavy quarks then $g_{\text{eff}}^2/4\pi$ can be less than 2 even if $g_r^2/4\pi > 2$. Then the preceding argument on the relative stability would have to be revised. The derivation of g_{eff} in the present formalism will be discussed elsewhere.

Although the formal derivation of the confinement equation is carried for the case of QED, the actual quark confinement cannot possibly be expected for this case. According to the asymptotic-freedom argument[10] the perturbation expansion is not valid at a short distance in QED, so that the potential, such as (1.14), cannot be expected to be

valid. Furthermore, the ratio g^2/g_r^2 should be greater than 1 or possibly infinite in QED,[11] leading to too large a value of k^2 for a given constant g_r^2.

The derivation of the confinement equation will be repeated for the case of a non-Abelian gauge field [color SU(3) group for example], and it can be carried almost in parallel to the case of QED. However, there appears an extra complication in this case, arising from the contributions of color nonsinglet states. Only in the static limit and in the classical limit of the color spin does one obtain the same result as in QED.

In Sec. II, I derive the confinement equation in QED. In Sec. III, I introduce the radius of the string, derive the confinement potential, and discuss the stability of the solutions relative to the Coulomb solutions. In Sec. IV, I treat the color SU(3) gauge field. Appendix A gives the summation of the Brillouin-Wigner series for the static potential. In Appendix B I treat the confinement equation for $m=0$.

II. DERIVATION OF THE EQUAL-TIME TWO-BODY CONFINEMENT EQUATION

For the sake of simplicity in presentation I will take QED in this section and demonstrate a formal derivation of the wave equation satisfied by the amplitude $\chi(1,2)$ given in (1.2). As will be discussed in the following section, however, one cannot expect a real confinement solution in QED. The derivation for the case of a non-Abelian gauge field will be given in Sec. IV. In the spatial gauge $A_0=0$, all three components of the vector potential $\vec{A}$ are independent dynamical variables. The Lagrangian is given by

$$L=\int d^3x\left\{\tfrac{1}{2}[\vec{E}^2(\vec{x})-\vec{B}^2(\vec{x})] + q^\dagger(\vec{x})\left[i\frac{\partial}{\partial t}+\vec{\alpha}\cdot(i\vec{\nabla}+g\vec{A})-\beta m\right]q(\vec{x})\right\}, \tag{2.1}$$

where

$$\vec{E}(\vec{x})=-\dot{\vec{A}}(\vec{x}),\quad \vec{B}(\vec{x})=\vec{\nabla}\times\vec{A}(\vec{x}). \tag{2.2}$$

The canonical momentum conjugate to $\vec{A}(\vec{x})$ is $-\vec{E}(\vec{x})$, and we have a commutation relation,

$$[A_i(\vec{x}),E_j(\vec{x}')]=-i\delta_{ij}\delta^3(\vec{x}-\vec{x}'). \tag{2.3}$$

The Hamiltonian is then given by

$$H=\frac{1}{2}\int d^3x[\vec{E}^2(\vec{x})+\vec{B}^2(\vec{x})]+H_{\text{quark}}, \tag{2.4}$$

with

$$H_{\text{quark}}=\int d^3x\, q^\dagger(\vec{x})[\vec{\alpha}\cdot(-i\vec{\nabla}-g\vec{A})+\beta m]q(\vec{x}). \tag{2.4'}$$

The equations of motion guarantee that

$$G(\vec{x})=gq^\dagger(\vec{x})q(\vec{x})-\vec{\nabla}\cdot\vec{E}(\vec{x}) \tag{2.5}$$

is a constant of motion and

$$\dot{G}(\vec{x})=i[H,G(\vec{x})]=0. \tag{2.6}$$

$G(\vec{x})$ is the generator of the local gauge transformations. Namely, the unitary operator

$$U[\omega]=\exp\left(i\int\omega(\vec{x})G(\vec{x})\,d^3x\right), \tag{2.7}$$

where $\omega(x)$ is a c-number function, generates the gauge transformations

$$\vec{A}'(\vec{x})=U[\omega]\vec{A}(\vec{x})U^{-1}[\omega]=\vec{A}(\vec{x})-\vec{\nabla}\omega(\vec{x}),$$

and

$$q'(\vec{x})=U[\omega]q(\vec{x})U^{-1}[\omega]=e^{-ig\omega(\vec{x})}q(\vec{x}). \tag{2.8}$$

The two-body operator $q(1,2)$ defined in (1.3) is obviously gauge invariant under (2.8). All physical states must satisfy

$$G(\vec{x})\Psi=0, \tag{2.9}$$

because otherwise $U[\omega]\Psi$ will give an infinite set of degenerate states.

Now I am in a position to derive formally the wave equation satisfied by $\chi(1,2)$. For the general amplitude $\chi_n(1,2)$ defined in (1.8) we have

$$(W-W_n)\chi_n(1,2)=(\Psi_n,[q(1,2),H]\Psi), \tag{2.10}$$

where W and W_n are the energy eigenvalues of the states Ψ and Ψ_n, respectively. From (1.3) and (2.4') I obtain

$$\begin{aligned}[q(1,2),H_{\text{quark}}]=&(-i\vec{\alpha}\cdot\vec{\nabla}_1+\beta m)q(1,2)\\ &-q(1,2)(i\vec{\alpha}\cdot\vec{\nabla}_2+\beta m)\\ &+U(2,1)\vec{\alpha}\cdot[i\vec{\nabla}_1-g\vec{A}(1)]q(1)q^\dagger(2)\\ &+q(1)q^\dagger(2)\vec{\alpha}\cdot[i\vec{\nabla}_2+g\vec{A}(2)]U(2,1),\end{aligned} \tag{2.11}$$

where

$$U(2,1)=\exp\left(ig\int_1^2\vec{A}(\vec{x})\cdot d\vec{x}\right). \tag{2.12}$$

The modification of the line integral to accommodate the finite radius of the electric flux will be made in the next section. Since the line integral is always defined along the straight path connecting the end points, $U(2,1')$ involves a new straight path $1'\to 2$, if we displace 1 into $1'$. Thus, using Stokes's theorem, one finds

$$\vec{\nabla}_1 U(2,1) = -ig U(2,1)\left[\vec{A}(1) - \frac{1}{r}\int_1^2 |\vec{x}-\vec{x}_2|\,\vec{B}\times d\vec{x}\right] \tag{2.13}$$

and the corresponding equation for $\nabla_2 U(2,1)$. Introducing (2.13) into (2.11), I obtain

$$\begin{aligned}[q(1,2), H_{\text{quark}}] = {} & (-i\vec{\alpha}\cdot\vec{\nabla}_1 + \beta m) q(1,2) \\ & - q(1,2)(i\vec{\alpha}\cdot\vec{\nabla}_2 + \beta m) \\ & + \vec{b}_L(2,1)\cdot\vec{\alpha}\, q(1,2) \\ & + q(1,2)\vec{b}_R(2,1)\cdot\vec{\alpha}\,,\end{aligned} \tag{2.14}$$

where

$$\vec{b}_L(2,1) = \frac{g}{r}\int_1^2 d\vec{x}\times\vec{B}(\vec{x})|\vec{x}-\vec{x}_2|\,, \tag{2.15}$$

$$\vec{b}_R(2,1) = \frac{g}{r}\int_1^2 d\vec{x}\times B(\vec{x})|\vec{x}-\vec{x}_1|\,.$$

The magnetic field energy commutes with $q(1,2)$. For the electric field energy, I obtain

$$\begin{aligned}[U(2,1), \tfrac{1}{2}\textstyle\int \vec{E}^2(\vec{x})\,d^3x] = {} & \tfrac{1}{2}\textstyle\int d^3x\,[[\,U(2,1), \vec{E}(\vec{x})], \vec{E}(\vec{x})] \\ & + \textstyle\int d^3x\,\vec{E}(\vec{x})\cdot[U(2,1), \vec{E}(\vec{x})]\,.\end{aligned} \tag{2.16}$$

The commutation relation (2.3) gives

$$[U(2,1), \vec{E}(\vec{x})] = g\int_1^2 d\vec{x}'\,\delta^3(\vec{x}'-\vec{x})U(2,1)\,. \tag{2.17}$$

The double commutator term then gives

$$V_0 = \frac{g^2}{2}\int_1^2\int d\vec{x}\cdot d\vec{x}'\delta^3(\vec{x}-\vec{x}') = \frac{g^2}{2}\delta^2(0)r = k^2 r\,. \tag{2.18}$$

Thus from (2.16), (2.17), and (2.18) I obtain

$$[q(1,2), \tfrac{1}{2}\textstyle\int\vec{E}^2(\vec{x})\,d^3x] = [k^2 r + e(2,1)]\,q(1,2)\,, \tag{2.19}$$

where

$$e(2,1) = g\int_1^2 d\vec{x}\cdot\vec{E}(\vec{x})\,. \tag{2.20}$$

Introducing (2.14) and (2.19) into (2.10), I obtain

$$\begin{aligned}(W - W_n - k^2 r)\chi_n(1,2) = {} & \mathcal{H}_D\chi_n(1,2) \\ & + \sum_{n'} I_{nn'}(2,1)\chi_{n'}(1,2)\,,\end{aligned} \tag{2.21}$$

where I have defined operators $\mathcal{H}_D$ and $I(2,1)$ by

$$\mathcal{H}_D = (-i\vec{\alpha}_L\cdot\vec{\nabla}_1 + \beta_L m) - (i\vec{\alpha}_R\cdot\vec{\nabla}_2 + \beta_R m) \tag{2.22}$$

and

$$I(2,1) = e(2,1) + \vec{b}_L(2,1)\cdot\vec{\alpha}_L + \vec{b}_R(2,1)\cdot\vec{\alpha}_R\,. \tag{2.23}$$

In the above I have introduced an operator convention that all operators with suffix L (R) are to operate on $\chi(1,2)$ from the left (right). The simultaneous set of equations (2.21) can be solved for $\chi(1,2) \equiv \chi_0(1,2)$ ($\Psi_n = \Psi_0$), eliminating successively the other amplitudes $\chi_n(2,1)$ ($n \neq 0$) that enter into the interaction term. The result is

$$(W - k^2 r - \mathcal{H}_D)\chi(1,2) = \mathcal{K}\chi(1,2)\,, \tag{2.24}$$

where $\mathcal{K}$ has the form of the Brillouin-Wigner series

$$\mathcal{K} = {\sum_n}' I_{0n}\frac{1}{D}I_{n0} + {\sum_{n,n'}}' I_{0n}\frac{1}{D_n}I_{nn'}\frac{1}{D_{n'}}I_{n'0} + \cdots\,, \tag{2.25}$$

with

$$D_n = W - W_n - k^2 r - \mathcal{H}_D\,. \tag{2.26}$$

The summation symbol $\sum'$ means to exclude the vacuum state Ψ_0 in the summation. Equation (2.24) is exactly the wave equation (1.9) and the potential $V(r)$ is now identified with

$$V(\vec{r}) = k^2 r + \mathcal{K}(\vec{r})\,. \tag{2.27}$$

The interaction kernel $I(2,1)$ is a line integral involving $\vec{E}$ and $\vec{B}$ as seen from (2.15), (2.20), and (2.23), so that $\mathcal{K}(1,2) \to 0$ as $r = |\vec{x}_2 - \vec{x}_1| \to 0$, provided the series (2.25) is well defined in this limit. With this qualification we have

$$V(0) = 0 \tag{1.13}$$

as mentioned in Sec. I.

III. CONFINEMENT POTENTIAL AND THE STABILITY OF THE CONFINEMENT SOLUTIONS

To obtain an insight into the physical meaning of the higher-order corrections to the potential $\mathcal{K}$, I will take the static limit in this paper. This means that in the energy denominator (2.26) $\mathcal{H}_D \to 2m$ and $W - \mathcal{H}_D \to 0$ so that

$$D_n \to -(W_n + k^2 r)\,. \tag{3.1}$$

I will also set $\vec{\alpha} \to 0$ and the magnetic field no longer contributes. Hence $I(2,1) \to e(2,1)$. (The magnetic interaction will be evaluated to order g^2 later to compare with the Breit potential.) In this limit I also neglect all quark-pair states in the intermediate-state summation $\sum_n'$. Then in the first-order correction

$$\mathcal{K}^{(1)} = {\sum_n}' I_{0n}\frac{1}{D_n}I_{n0}\,, \tag{3.2}$$

only one-photon states contribute. (In the non-

Abelian case, the situation is different because of the contribution from the gluon-pair states.) Although the longitudinal part of $\vec{A}$ is a dynamical variable in the present gauge, the subsidiary condition (2.9) eliminates the logitudinal degree of freedom. [Since $I(2,1)$ is gauge invariant, all the intermediate states should be gauge invariant and satisfy (2.9)]. Thus, in the summation (3.2), I sum over only transverse photons. Noting that all the physical quantities introduced so far are unrenormalized, one obtains

$$\langle \vec{l}, \vec{\epsilon}|\vec{E}(\vec{x})|0\rangle = iZ_3{}^{1/2}(l/2)^{1/2}\vec{\epsilon}(\vec{l})e^{i\vec{l}\cdot\vec{x}}, \tag{3.3}$$

where Z_3 is the renormalization constant. To be consistent with neglecting the pair states in the summation (3.2), one should put $Z_3 = 1$ in the case of QED. However, the effect of the charge renormalization will be taken into account at a later stage. Note that in a non-Abelian gauge theory, gluon loop diagrams do contribute to Z_3 even in the static limit. Using (3.1), (3.3), and (2.20), I obtain

$$\begin{aligned}\mathcal{K}^{(1)} &= -\frac{g^2}{2}\sum_l \frac{l}{l+k^2r}\left(\delta_{ij}-\frac{l_il_j}{l^2}\right) \\ &\quad\times\int_1^2\int_1^2 dx_i\,dx_j' e^{-i\vec{l}\cdot(\vec{x}-\vec{x}')} \\ &= -\frac{g^2}{2}\delta^2(0)r - \frac{g^2}{4\pi}\left(\frac{1}{r}-\frac{1}{0}\right)+\mathcal{K}'^{(1)}.\end{aligned} \tag{3.4}$$

The first two terms of the last line are obtained by replacing the energy denominator $l+k^2r$ by l. It is then obvious that $\mathcal{K}'$ is of order g^4. Thus to order g^2, I obtain the following from (1.6), (2.27), and (3.4):

$$V(r) = -g^2/4\pi r, \tag{3.5}$$

where I have dropped the electrostatic self-energy. The fact that the same potential as for the BS amplitude was obtained in this limit should not be generalized. It happens only to order g^2 and in the static limit. For large r, the higher-order terms are crucial to determine the potential. Also, as will be shown later, the magnetic interaction gives a potential entirely different from the one for the BS amplitude even to order g^2. There is no reason to expect the same potential for the two entirely different amplitudes.

The perturbation series (2.25) is not completely well defined because of ultraviolet divergences such as the electrostatic self-energy term in (3.4). Except for divergences associated with the vacuum-polarization diagrams which can be incorporated into the charge renormalization it is not obvious how the other divergences can be renormalized in the present formulation. The difficulty is avoided by introducing a finite width to the string of the electric flux, which will serve as a cutoff in momentum space. Each term of the series (2.25) is now finite; this is achieved without ever modifying the gauge field itself, which would destroy the gauge invariance. Instead, I am assuming that in the actual state of the system of the confined quark and antiquark the electric flux is concentrated in a tube of radius a, which is then a physical parameter to describe the system. The potential will now be a function of a, the value of which could, in principle, be determined by solving the confinement equation (2.24) and examining the stability of the solution against the change of a.

The above program can be realized by modifying the line integral in (1.3) and (2.12) into a bundle of line integrals. For instance I may choose as the modified line integral

$$U(2,1) = \exp\left[ig\int d^2\xi_\perp \sigma(\vec{\xi}_\perp)\int_0^1 \vec{A}(\vec{x}(s;\vec{\xi}_\perp))\cdot\frac{d\vec{x}}{ds}ds\right], \tag{3.6}$$

where

$$\vec{x} = \vec{x}(s;\vec{\xi}_\perp), \quad s=[0,1]$$

is a parametric representation of a path connecting points 1 and 2 for a fixed two-dimensional parameter $\vec{\xi}_\perp$. The latter is a parameter representing the path's position from the straight line connecting the two points. $\sigma(\vec{\xi}_\perp)$ is the density of the paths, normalized to unity

$$\int \sigma(\vec{\xi}_\perp)d^2\xi_\perp = 1. \tag{3.7}$$

Under the gauge transformation (2.8), $U(2,1)$ transforms into $e^{ig[\omega(1)-\omega(2)]}U(2,1)$, so that $q(1,2)$ is still gauge invariant. The derivation of the wave equation given in Sec. II can again be carried through, with appropriate changes in (2.18), $e(2,1)$, and $b_{R,L}(2,1)$. Namely, instead of (2.18), one will have

$$\begin{aligned}V_0 = \frac{g^2}{2}\int\int d^2\xi_\perp d^2\xi_\perp' \sigma(\vec{\xi}_\perp)\sigma(\vec{\xi}_\perp') \\ \times\int\int_0^1 ds\,ds'\,\frac{d\vec{x}(s,\vec{\xi}_\perp)}{ds}\cdot\frac{d\vec{x}'(s',\vec{\xi}_\perp')}{ds} \\ \times\delta^3(\vec{x}(s,\vec{\xi}_\perp)-\vec{x}'(s',\vec{\xi}_\perp')).\end{aligned} \tag{3.8}$$

Instead of (2.20) one has

$$\begin{aligned}e(2,1) = -g\int d^2\xi_\perp\sigma(\vec{\xi}_\perp) \\ \times\int_0^1 ds\,\vec{E}(\vec{x}(s,\vec{\xi}_\perp))\cdot\frac{d\vec{x}(s,\vec{\xi}_\perp)}{ds}.\end{aligned} \tag{3.9}$$

Similar changes must be made for $b_{R,L}(2,1)$ also.

For a practical reason of computability, however, I will replace all the paths by straight lines parallel to $\vec{n}=(\vec{x}_2-\vec{x}_1)/r$. Then I obtain

$$\vec{\xi}_\perp=\vec{x}_\perp=\vec{x}-(\vec{n}\cdot\vec{x})\cdot\vec{n},$$

and

$$\frac{d\vec{x}(s,\vec{\xi}_\perp)}{ds}=\vec{n}\frac{dx_\parallel}{ds}, \tag{3.10}$$

with $x_\parallel=\vec{n}\cdot\vec{x}$. Furthermore, I will introduce a straight cutoff in momentum space:

$$\sigma(\vec{l}_\perp)=\theta(\Lambda^2-l_\perp^2), \tag{3.11}$$

where the Fourier transform $\tilde{\sigma}(l)$ is defined by

$$\tilde{\sigma}(\vec{l}_\perp)=\int d^2x_\perp e^{-i\vec{l}_\perp\cdot\vec{x}_\perp}\sigma(\vec{x}_\perp). \tag{3.12}$$

It should be stressed that approximation (3.10) is being made in order to be able to evaluate integrals such as (3.8) and (3.9) in a simple way. The approximation of replacing a curved path leading from 1 to 2 by a straight line parallel to the one connecting the two points should become better for a large separation of the two points. If this approximation were made at the stage of defining $U(2,1)$, it would correspond to

$$U(2,1)=\exp\left[ig\int d^2x_\perp\sigma(\vec{x}_\perp)\int_1^2\vec{A}(\vec{x}_\perp,x_\parallel)\cdot\vec{n}\,dx_\parallel\right]. \tag{3.13}$$

which would destroy the gauge invariance of $q(1,2)$. Under the gauge transformation (2.8), $q(1,2)$ would now transform such that

$$q(1,2)\to\exp\left\{ig\int d^2x_\perp\sigma(\vec{x}_\perp)[\omega(\vec{x}_1+\vec{x}_\perp)-\omega(\vec{x}_2+\vec{x}_\perp)-\omega(\vec{x}_1)+\omega(\vec{x}_2)]\right\}q(1,2).$$

Hence the state Ψ could no longer satisfy the subsidiary condition (2.9) because otherwise $\chi(1,2)$ would vanish. Thus $U[\omega]\Psi$ would produce an infinite set of degenerate states. The situation would be very similar to the BCS solution for the superconductive ground state, where the gauge symmetry is destroyed and the electron number is superficially nonconserved.

With the approximation (3.10) and a particular choice (3.11), I can now evaluate (3.8) and (3.9). For the confinement potential (3.8), I obtain

$$V_0=\frac{g^2}{2}r\int d^2x_\perp\sigma(x_\perp)^2=\frac{g^2\Lambda^2}{8\pi}r=k^2r. \tag{3.14}$$

Hence

$$k^2=\frac{g^2\Lambda^2}{8\pi}=\frac{g^2}{8\pi a^2}, \tag{3.15}$$

where $a=1/\Lambda$ by definition. Also, instead of (2.20) I obtain

$$e(2,1)=g\int d^2x_\perp\sigma(\vec{x}_\perp)\int_{x_{1\parallel}}^{x_{2\parallel}}\vec{E}(\vec{x})\cdot\vec{n}\,dx_\parallel. \tag{3.16}$$

Introducing (3.16) into (3.2), I obtain

$$\mathcal{K}^{(1)}=-\frac{g^2}{2}\sum_l\theta(\Lambda^2-l_\perp^2)\frac{l}{l+k^2r}(\delta_{ij}-l_il_j/l^2)n_in_j\times\iint_{x_{1\parallel}}^{x_{2\parallel}}dx_\parallel dx'_\parallel\exp[-il_\parallel(x_\parallel-x'_\parallel)], \tag{3.17}$$

where $l_\parallel$ and $l_\perp$ are components of $\vec{l}$ parallel and perpendicular to $\vec{n}$. Comparing with (3.4) I find that the only change is the cutoff in $l_\perp$. Again splitting $(l+k^2r)^{-1}$ into l^{-1} and $-k^2r[l(l+k^2r)]^{-1}$, I obtain

$$\mathcal{K}^{(1)}=-\frac{g^2\Lambda^2r}{8\pi}-\frac{g^2}{4\pi}\left(\frac{1-e^{-\Lambda r}}{r}-\Lambda\right)+\mathcal{K}^{(1)\prime}, \tag{3.18}$$

which replaces (3.4).

In order to find the behavior of $\mathcal{K}$ for large r, the series (2.25) must be summed. The summation of the series (2.25) in the static limit is done in Appendix A, where I find that

$$\frac{\mathcal{K}}{k^2r}\to 0 \quad (\Lambda r\gg 1) \tag{3.19}$$

so that

$$V(r)\to k^2r \quad (r\gg\Lambda^{-1}). \tag{3.20}$$

The potential to order g^2 is obtained by adding (3.14) and (3.18), which yield (1.12). This potential is positive because of the Coulomb self-energy term, whereas the nonrelativistic treatment of the charmonium spectrum[12] indicates a potential consisting of the confinement and the attractive Coulomb potentials. In order to infer the overall behavior of the potential, two additional factors which have been neglected so far must be considered. One is the renormalization of the coupling constant g^2 and the other is the magnetic interaction. As seen from (3.3) the coupling constant defining the interaction kernal $I(2,1)$ must be the renormalized one $g_r=Z_3^{1/2}g$, whereas the confinement potential k^2r should remain to be defined in terms of g as in (3.15). According to the asymptotic-freedom argument,[10] the perturbation series will be valid at a short distance in the case of the non-Abelian gauge field, so we may expect that in the static limit

$$V(r)=\frac{(g^2-g_r^2)\Lambda^2r}{8\pi}-\frac{g_r^2}{4\pi}\left(\frac{1-e^{-\Lambda r}}{r}-\Lambda\right) \tag{3.21}$$

instead of (1.12). The perturbation calculation indicates that $g^2 \ll g_r^2$ for the non-Abelian case and $g^2 \gg g_r^2$ in QED. In either case, the potential rises like $V(r) \sim k^2 r$ for $r \ll 1/\Lambda$. In the former case it turns negative for $r \gtrsim 1/\Lambda$ and turns positive again for $r \gg 1/\Lambda$ because of (3.20).

The magnetic interaction changes the short-distance behavior significantly. It can be evaluated by keeping $b_{R,L}(2, 1)$ terms in (2.23), and then neglecting the recoil and $k^2 r$ term in the energy denominator of (3.2), just as was done before. This is the same approximation as was made to derive the Breit potential for the BS amplitude (1.1). It gives

$$\mathcal{K}_m^{(1)} = -\frac{g_r^2}{2r^2} \sum_{D,D'} \vec{\alpha}_{D\perp} \cdot \vec{\alpha}_{D'\perp} \times \sum_l \theta(\Lambda^2 - l_\perp^2) \frac{l^2 - \frac{1}{2} l_\perp^2}{l^2} \times \iint_{z_1}^{z_2} dz dz' D(z) D'(z') e^{i l_z (z - z')}, \tag{3.22}$$

where I have taken $\vec{n} = (\vec{x}_2 - \vec{x}_1)/r$ as the z axis; D and D' stand for either R or L; $D(z) = |z_2 - z|(|z - z_1|)$ if $D = L$ (R); also $\vec{\alpha}_\perp = \vec{\alpha} - (\vec{n} \cdot \vec{\alpha})\vec{n}$. Using $\vec{\alpha}_{R\perp}^2 = \vec{\alpha}_{L\perp}^2 = 2$ and rearranging the terms I obtain

$$\mathcal{K}_m^{(1)} = -\mathcal{K}_e^{(1)} - \frac{g_r^2 \Lambda^2}{4\pi} r + (1 - \tfrac{1}{2} \vec{\alpha}_{L\perp} \cdot \vec{\alpha}_{R\perp}) \frac{g_r^2}{4\pi} \times \left[\frac{\Lambda^2}{6} r + \frac{1 + e^{-\Lambda r}}{r} - \frac{2}{\Lambda r^2} (1 - e^{-\Lambda r}) \right], \tag{3.23}$$

where $\mathcal{K}_e^{(1)}$ is (3.18) without the $\mathcal{K}^{(1)'}$ term and with g replaced by g_r. Hence to order g_r^2,

$$V(r) = \frac{(g^2 - 2g_r^2)}{8\pi} \Lambda^2 r + (1 - \tfrac{1}{2} \vec{\alpha}_{L\perp} \cdot \vec{\alpha}_{R\perp}) \frac{g_r^2}{4\pi} \times \left[\frac{\Lambda^2}{6} r + \frac{1 + e^{-\Lambda r}}{r} - \frac{2}{\Lambda r^2} (1 - e^{-\Lambda r}) \right]. \tag{3.24}$$

Comparing with the purely electrostatic potential (3.21), there are important differences. For an extremely small r we obtain

$$V(r) \sim \frac{(g^2 - 2g_r^2)\Lambda^2 r}{8\pi} + (1 - \tfrac{1}{2} \vec{\alpha}_{L\perp} \cdot \vec{\alpha}_{R\perp}) \frac{g_r^2 \Lambda^2 r}{12\pi} \quad (r \ll \Lambda). \tag{3.25}$$

Since $1 - \frac{1}{2} \vec{\alpha}_{L\perp} \cdot \vec{\alpha}_{R\perp}$ is effectively positive with a maximum value of 2, the slope of V at the origin $V'(0)$ is negative if $g_r^2 > g^2$ (non-Abelian). For $r \gtrsim \Lambda^{-1}$ we obtain

$$V(r) \sim \frac{(g^2 - 2g_r^2)}{8\pi} \Lambda^2 r + (1 - \tfrac{1}{2} \vec{\alpha}_{L\perp} \cdot \vec{\alpha}_{R\perp}) \frac{g_r^2}{4\pi} \left(\frac{\Lambda^2 r}{6} + \frac{1}{r} \right). \tag{3.26}$$

Compared with (3.21) one finds that the Coulomb self-energy has disappeared. Furthermore, the $1/r$ term has changed sign. However, because of the large negative linear term $-g_r^2 \Lambda^2 r/4\pi$, the potential is negative as a whole. For $r \gg \Lambda^{-1}$ the potential should approach (3.20). It should be noted that potential (3.24) is totally different from the Breit potential.[8] There is no reason to expect that they are the same, as stated earlier.

Summarizing the results of the above analysis, I may require as the key features of an effective static potential: (1) $V(0) = 0$, (2) $V'(0) \sim -g_r^2 \Lambda^2/4\pi$, (3) $V(\Lambda^{-1}) \sim -g_r^2 \Lambda/4\pi$, and (4) $V(r) \sim k^2 r$ $(r \gg \Lambda^{-1})$ with $k^2 = g^2 \Lambda^2/8\pi$. It is not clear at exactly what point the last stage sets in. In the analysis of (1.9) by GS, a model potential of the form

$$V(r) = k^2 r - \frac{g_r^2 r}{4\pi(r^2 + a^2)} \tag{3.26}$$

was employed. It certainly satisfies all the conditions above if $g_r^2/4\pi a^2 > k^2$. A numerical fit of the $I = 1$ meson spectrum by (3.26) determined the value of $g_r^2/4\pi$ to be about 2.3, slightly larger than the critical value 2 discussed below. The value of k^2 is determined by the asymptotic slope of the trajectories and is about 0.1 (GeV)^2. The parameter a was determined to be extremely small, $a \lesssim 10^{-2}$ GeV^{-1} for the case of the π meson. The solution is certainly consistent with $g^2 \ll g_r^2$. The analysis made above also clearly shows that even though one can derive the confinement equation and the potential formally in QED one could not expect any real confinement to happen. The perturbation series are not expected to converge according to the asymptotic-freedom argument, and also $g^2 > g_r^2$.

The stability of the confinement states obtained by solving (1.9) with a potential such as (3.26) can be determined by comparing the energy eigenvalues with those of the positronium states for the same coupling constant g_r^2. For this purpose I take a state of total angular momentum $J = 0$, charge-conjugation parity $C = 1$, and parity $P = -1$ (π meson). The reduction of the wave Eq. (1.9) for this state was done previously[5,7] and we obtained the eigenvalue equation

$$\frac{d^2 F}{dr^2} + \left(\frac{2}{r} + \frac{V'}{m - V} \right) \frac{dF}{dr} + [\tfrac{1}{4}(M - V)^2 - m^2] F = 0, \tag{3.27}$$

subject to the boundary conditions

$$F(r)=\text{const}\quad (r=0),$$
$$F(r)\sim (r-R)^2\quad (r\sim R), \tag{3.28}$$

where R is defined by

$$V(R)=M. \tag{3.29}$$

For the pure Coulomb potential $V(r)=-g_r^2/4\pi r$, the indicial equation for a regular solution $F=r^s(a_0+a_1r+\cdots)$ is

$$s(s+2)+\frac{1}{4}\left(\frac{g_r^2}{4\pi}\right)^2=0. \tag{3.30}$$

Hence the regular solution exists only for $g_r^2/4\pi<2$. This limit on the strength of the Coulomb potential is similar to the well-known Klein-paradox phenomenon at the origin in the Dirac equation for the hydrogen atom where $Z\alpha<1$ is required. Thus, the positronium states do not exist for $g_r^2/4\pi>2$. In the confinement equation, however, the potential satisfies $V(0)=0$, so that there is no limit to the magnitude of g_r^2. For $g_r^2/4\pi>2$ the confinement states are the only possible states, and as mentioned above the numerical fit of the meson spectrum does give the value of g_r^2 in this range. For $g_r^2/4\pi<2$ both types of solutions are possible, so that their energy eigenvalues must be compared. I take specifically the limit of the vanishing quark mass $m=0$ in Eq. (3.27). Then, for the Coulomb case, I know that all the positronium levels coalesce into the zero-energy state $M=0$. I will show in Appendix B that Eq. (3.27) with potential (3.21) does not yield a solution $m=0$. Thus I infer that the confinement solutions have higher eigenvalues than the Coulomb solutions in general. According to the remark made about Eq. (1.7), any matrix element between a confinement state and a Coulomb state with a finite number of photons should vanish. This means that for $g_r^2/4\pi<2$, it is impossible to create a confinement state from the normal states by perturbation. If somehow a confinement state is created, it will decay into a normal state emitting an infinite number of photons.

In the analysis made in Ref. 7 the potential parameters k^2, g_r^2, and a in (3.26) were regarded as universal constants independent of the states. Now I have shown that $k\propto 1/a^2$ and I have also proposed that the radius of the strong a be regarded as a dynamical parameter whose value should be determined so as to make the eigenvalue minimum. I will show below that it is indeed a workable program for potential (3.26). If we introduce a dimensionless variable

$$z=r/R,$$

where R is defined by (3.29), than Eq. (1.9) or the reduced wave equation (3.27) involve parameters in two forms, $R(M-V)$ and mR. Eliminating M in terms of R by (3.29) and using (3.26), I obtain (see Appendix B for details)

$$R(M-V)=Q(1-z)\left(1+\eta\,\frac{z-\gamma^2}{z+\gamma^2}\right) \tag{3.31}$$

and

$$m^2R^2=\frac{m^2}{k^2}Q\,, \tag{3.32}$$

where

$$Q=k^2R^2,$$
$$\eta=\frac{g_r^2}{4\pi(k^2a^2+Q)}\,,$$

and

$$\gamma^2=\frac{a^2}{R^2}=\frac{k^2a^2}{Q}\,. \tag{3.33}$$

Hence by solving the eigenvalue equation we determine the eigenvalue Q in terms of three independent parameters as

$$Q=Q(g_r^2,k^2a^2,m^2/k^2). \tag{3.34}$$

For fixed values of g_r^2, k^2, and a^2, Q will in general increase as $m^2\to\infty$. It will have a finite value for $m=0$ except for the case of the π meson, where one imposes the Nambu-Goldstone condition $M=m=0$. Now replacing k^2 by

$$k^2=g_0^2/a^2$$

where g_0^2 is a constant related to g^2, I obtain

$$Q=Q(g_r^2,g_0^2,m^2a^2/g_0^2). \tag{3.35}$$

Therefore, the dependence of Q on m^2 for fixed g_r^2, k^2, and a^2 is the same as the dependence of Q on a^2 for fixed g_r^2, g_0^2, and m^2. Now I know the dependence of M on a^2 because

$$M=V(R)=k^2R(1-\eta)$$
$$=\frac{g_0}{a}\sqrt{Q}\left(1-\frac{g_r^2}{4\pi}\,\frac{1}{g_0^2+Q}\right). \tag{3.36}$$

The numerical solutions obtained in GS show that $\sqrt{Q}$ increases linearly in m^2. Hence there must exist an optimal value of a for which M is the minimum. The extremely small value of a obtained to satisfy the Nambu-Goldstone condition applies only for the π-meson state.

Equation (3.36) suggests a scaling law $M/m=$ independent of m. This follows from the assumption that g_r^2 is independent of a or M. As the asymptotic-freedom argument suggests, however, g_r^2 would be replaced by g_{eff}^2 if one takes into account the higher-order terms of $\mathcal{K}$ (for instance, two-gluon intermediate states). The effective coupling-constant g_{eff} would in general depend on a and also on the average momentum exchanged, which in turn

would depend on M. Therefore the scaling mentioned above will break down. $g_{eff}^2/4\pi$ could be less than 2 even if $g_r^2/4\pi > 2$. The relative stability of the two kinds of solutions given above will have to be reexamined in this case.

IV. THE CASE OF A NON-ABELIAN GAUGE FIELD

I will sketch the derivation of the confinement equation in the case of the non-Abelian fields. For example, I take the standard Lagrangian for an octet of color gauge fields ($a = 1, 2, \ldots, 8$) as follows:

$$L = \int d^3x \left[-\tfrac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \bar{q}\gamma_\mu \left(i\partial^\mu - g \frac{\lambda_a}{2} A^{a\mu} \right) q - m\bar{q}q \right], \tag{4.1}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f_{abc} A^b_\mu A^c_\nu. \tag{4.2}$$

$\{\lambda_a\}$ is a set of color spin matrices and f_{abc} is the structure constant. I shall use a convention such that

$$\sum_a \frac{\lambda_a}{2} O_a = \underline{O}. \tag{4.3}$$

Again I take the spatial gauge

$$A^a_0 = 0 \tag{4.4}$$

in the rest frame of the hadron. The canonical momentum conjugate to $\vec{A}^a(\vec{x})$ is $\vec{\dot{A}}(\vec{x}) = -\vec{E}^a(\vec{x})$ and I have the commutation relation

$$[A^a_i(\vec{x}), E^b_j(\vec{x})] = -i\delta_{ij}\delta_{ab}\delta^3(\vec{x} - \vec{x}'), \tag{4.5}$$

and the Hamiltonian

$$H = \int d^3x \, \mathrm{tr}[\underline{\vec{E}}^2(\vec{x}) + \underline{\vec{B}}^2(\vec{x})] + H_{\text{quark}}, \tag{4.6}$$

with

$$H_{\text{quark}} = \int d^3x \, q^\dagger(\vec{x}) \{ \vec{\alpha} \cdot [-\vec{\nabla} - g\underline{\vec{A}}(\vec{x})] + \beta m \} q(\vec{x}). \tag{4.7}$$

The magnetic field $\vec{B}^a$ is defined by

$$\vec{B}^a(\vec{x}) = \vec{\nabla} \times \vec{A}^a(\vec{x}) + \tfrac{1}{2} g f_{abc} \vec{A}^b(\vec{x}) \times \vec{A}^c(\vec{x}). \tag{4.8}$$

The equations of motion guarantee that

$$G^a(\vec{x}) = J^a_0(\vec{x}) - \vec{\nabla} \cdot \vec{E}^a(\vec{x}) \tag{4.9}$$

is a constant of motion. Here the total charge density is given by

$$J^a_0(\vec{x}) = g q^\dagger(\vec{x}) \tfrac{1}{2} \lambda_a q(\vec{x}) + g f_{abc} \vec{A}^b(\vec{x}) \cdot \dot{\vec{A}}^c(\vec{x}). \tag{4.10}$$

$\{G^a(\vec{x})\}$ are a set of generators for the local gauge transformations. For a set of infinitesimal c-number functions $\{\omega^a(\vec{x})\}$ the transformations are

$$\vec{A}^a(\vec{x})' = U[\omega] \vec{A}^a(\vec{x}) U^{-1}[\omega] = \vec{A}^a(\vec{x}) - \vec{\nabla}\omega^a(\vec{x}) + g f_{abc} \omega^b(\vec{x}) \vec{A}^c(\vec{x}), \tag{4.11}$$

and

$$q'(\vec{x}) = U[\omega] q(\vec{x}) U^{-1}[\omega] = [1 - ig\underline{\omega}(\vec{x})] q(\vec{x}), \tag{4.12}$$

where the unitary operator $U[\omega]$ is defined by

$$U[\omega] = \exp\left(i \int \omega^a(\vec{x}) G^a(\vec{x}) d^3x \right). \tag{4.13}$$

As a bilocal operator replacing (1.3) for QED, I take

$$q(1,2) = \left[\exp\left(ig \int_1^2 \underline{\vec{A}}(\vec{x}) \cdot d\vec{x} \right) \right]_+ q(1) q^\dagger(2). \tag{4.14}$$

The quark field operator $q(x)$ has three sets of indices, corresponding to spinor, color spin, and flavor components. In (4.13) $q(1)q^\dagger(2)$ should be considered a matrix in these three sets of indices. The ordering symbol $(\cdots)_+$ means the ordering along the integration path, which I again take as a straight-line connection of points 1 and 2. Explicitly I define the ordered product by

$$U(2,1) = \left(\exp\left[ig \int_1^2 \underline{\vec{A}}(\vec{x}) \cdot d\vec{x} \right] \right)_+ = \lim_{N \to \infty} \prod_{n=0}^{N-1} \exp[ig\underline{\vec{A}}(\xi_n) \cdot (\vec{\xi}_{n+1} - \vec{\xi}_n)], \tag{4.15}$$

where $\vec{\xi}_n = (n/N)(\vec{x}_2 - \vec{x}_1) + \vec{x}_1$ with $\vec{\xi}_0 = \vec{x}_1$ and $\vec{\xi}_N = \vec{x}_2$. The product is ordered from right to left with increasing n. The bilocal operator is *not* gauge invariant. Under the infinitesimal gauge transformation (4.11), $U(2,1)$ transforms as follows:

$$U(2,1) \to e^{-ig\underline{\omega}(2)} U(2,1) e^{ig\underline{\omega}(1)}. \tag{4.16}$$

It will be recognized that the ordering of $U(2,1)$ is necessary to obtain this transformation property because of the color rotation of $\vec{A}^a(\vec{x})$ represented by the third term of Eq. (4.11). $q(1,2)$ transforms as follows:

$$q(1,2) \to e^{-ig\underline{\omega}(2)} q(1,2) e^{ig\underline{\omega}(2)} = q(1,2) - ig[\underline{\omega}(2), q(1,2)]. \tag{4.17}$$

$q(1,2)$ consists of the color-singlet part and the octet part linear in λ_a. The color-singlet part, which is $\frac{1}{3}\mathrm{tr}^c q(1,2)$ (tr^c = the trace with respect to color spin), is invariant, but not the octet part. I keep the octet part of $q(1,2)$ because the intermediate states necessarily involve the color-nonsinglet states. Now consider a set of amplitudes

$$\chi_n(1,2) = (\Psi_n, q(1,2)\Psi). \tag{4.18}$$

The state Ψ under consideration is a color singlet and satisfies

$$G^a(\vec{x})\Psi = 0, \tag{4.19}$$

but Ψ_n need not be a color singlet and hence need

not satisfy (4.19).

Now I evaluate the commutator of $q(1,2)$ with the Hamiltonian H (4.6). As in Sec. II, Eq. (2.11) still holds if $\vec{A}$ is replaced by $\underline{A}$ and if it is understood that $U(2,1)$ stands always to the left of $q(1)q^{\dagger}(2)$. The evaluation of the gradient of $U(2,1)$ is very involved but can be done. Here I shall give only the results. Instead of (2.13) I obtain

$$\vec{\nabla}_1 U(2,1) = -igU(2,1)\underline{A}(1) - i\frac{g}{r}\int_1^2 d\vec{x}\times U(2,\vec{x})\underline{\vec{B}}(\vec{x})U(\vec{x}_2 1)\,|\vec{x}-\vec{x}_2| \tag{4.20}$$

and a similar expression for $\vec{\nabla}_2 U(2,1)$. Inserting these into (2.11), in place of (2.14) I find

$$[q(1,2),H_{\text{quark}}] = (-i\vec{\alpha}\cdot\vec{\nabla}_1+\beta m)q(1,2) - q(1,2)(i\vec{\alpha}\cdot\vec{\nabla}_2+\beta m) + \vec{b}_L(2,1)\cdot\vec{\alpha}q(1,2) + \vec{b}_R(2,1)\cdot q(1,2)\vec{\alpha} + [\underline{\vec{A}}(2),q(1,2)]\cdot\vec{\alpha}. \tag{4.21}$$

Here

$$\vec{b}_L^{(2,1)} = \frac{g}{r}\int_1^2 d\vec{x}\times\vec{B}^a(\vec{x})\tfrac{1}{2}\Lambda_a(\vec{x})\,|\vec{x}-\vec{x}_2|,$$
$$\vec{b}_R(2,1) = \frac{g}{r}\int_1^2 d\vec{x}\times\vec{B}^a(\vec{x})\tfrac{1}{2}\Lambda_a(\vec{x})\,|\vec{x}-\vec{x}_1|, \tag{4.22}$$

where $\Lambda_a(\vec{x})$ is defined by

$$\Lambda_a(\vec{x}) = U(2,\vec{x})\lambda_a U^{-1}(2,\vec{x}). \tag{4.23}$$

Note that the last term of (4.21) drops if $q(1,2)$ is a color singlet. The magnetic field energy again commutes with $q(1,2)$. For the electric-field energy, I obtain

$$[U(2,1),\vec{E}^a(\vec{x})] = g\int_1^2 d\vec{x}'\,\delta^3(\vec{x}-\vec{x}') \times U(2,\vec{x}')\tfrac{1}{2}\lambda_a U(\vec{x}',1). \tag{4.24}$$

Hence for the double commutator term in (2.16) I obtain

$$\tfrac{1}{2}\int [\,[U(2,1),\vec{E}^a(\vec{x})],\vec{E}^a(\vec{x})]\,d^3x = \frac{g^2}{2}\int_1^2\int d\vec{x}\cdot d\vec{x}'\delta^3(\vec{x}-\vec{x}')\sum_a\left(\frac{\lambda_a}{2}\right)^2 U(2,1) = k^2 r U(2,1), \tag{4.25}$$

with

$$k^2 = \frac{4}{3}\frac{g^2}{2}\delta^2(0). \tag{4.26}$$

The factor $\frac{4}{3}$ is $\sum_a(\lambda^a/2)^2$. I again obtain (2.19) with $e(2,1)$ now defined by

$$e(2,1) = g\int_1^2 d\vec{x}\cdot\vec{E}^a(\vec{x})\tfrac{1}{2}\Lambda_a(\vec{x}). \tag{4.27}$$

Introducing (4.21) and (2.19) into (2.10) I obtain

$$(W-W_n-k^2r)\chi_n(1,2) = \mathcal{K}_D\chi_n(1,2) + \sum_{n'} I_{nn'}(2,1)\chi_{n'}(1,2), \tag{4.28}$$

where $\mathcal{K}_D$ is given by (2.22); but for the interaction kernel $I(2,1)$ I obtain

$$I(2,1) = e(2.1) + \vec{b}_L(2,1)\cdot\vec{\alpha}_L + \vec{b}_R(2,1)\cdot\vec{\alpha}_R + (\tfrac{1}{2}\lambda_{aL} - \tfrac{1}{2}\lambda_{aR})\vec{\alpha}_R\cdot\vec{A}^a(2). \tag{4.29}$$

The last term will drop in (4.28) if Ψ_n is a color singlet, because then $\chi_n(1,2)$ is an identity in color spin and $[\lambda_a,\chi_n(1,2)]=0$. However, in the series (2.25), this last term in I will contribute unless all the intermediate states are limited to the color-singlet states for which $G^a(\vec{x})\Psi=0$. The latter situation will arise if the energy of the color-color-nonsinglet states are infinite so that the energy denominator is infinite. Then, for instance, a one-gluon state will not be present in the intermediate states and one would obtain a completely different behavior of the potential at a short distance compared to the case of QED. The analysis given in Sec. III would be valid in this case only if the color spins are replaced by constant c numbers, the classical limit of the color spin. An extensive study of the non-Abelian case is necessary before making any concrete statement. I have not given the prescription of how to define a bundle of line integrals to replace (4.15). It will be discussed elsewhere.

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APPENDIX A

I try to sum the Brillouin-Wigner series (2.25),

$$\mathcal{K}=\sum_n{}'I_{0n}\frac{1}{D_n}I_{n0}+\sum_{n,n'}{}'I_{0n}\frac{1}{D_n}I_{nn'}\frac{1}{D_{n'}}I_{n'0}+\cdots, \tag{A1}$$

in the static limit of QED where

$$D_n=-(W_n+k^2r) \tag{A2}$$

and

$$I(2,1)=e(2,1). \tag{A3}$$

State n then must represent a certain number of photons. In (A1), $\sum_n'$ excludes the vacuum state by definition. Instead of (A1) I will evaluate

$$\tilde{\mathcal{K}}=\sum_n I_{0n}\frac{1}{D_n}I_{n0}+\sum_{n,n'}I_{0n}\frac{1}{D_n}I_{nn'}\frac{1}{D_{n'}}I_{n'0}+\cdots, \tag{A4}$$

where $\sum_n$ includes the vacuum state. Obviously,

$$\tilde{\mathcal{K}}=\mathcal{K}\left(1+\frac{1}{D_0}\tilde{\mathcal{K}}\right), \tag{A5}$$

where $D_0=-k^2r$. The matrix element $\tilde{\mathcal{K}}$ is related to the vacuum expectation value of an operator $1/(D-I)$ by

$$\left\langle\frac{1}{D-I}\right\rangle_0=\frac{1}{D_0}\left(1+\frac{1}{D_0}\tilde{\mathcal{K}}\right), \tag{A6}$$

where

$$D=-k^2r-\tfrac{1}{2}\int:(\vec{\mathbf{E}}_{tr}{}^2+\vec{\mathbf{B}}^2):d^3x, \tag{A7}$$

and

$$I=g_r\int d^2x_\perp\sigma(x_\perp)\int_{x_{1\parallel}}^{x_{2\parallel}}\vec{\mathbf{E}}(x_\perp,x_\parallel)\cdot\vec{\mathbf{n}}dx_\parallel. \tag{A8}$$

The operator $D-I$ can easily be diagonalized.

Introducing creation and annihilation operators $a_i(\vec{l})$ and $a_i^\dagger(\vec{l})$ as usual by expansions

$$\vec{\mathbf{A}}(x)=\sum_{l,i}\frac{1}{(2l)^{1/2}}\vec{\epsilon}_i(\vec{l})[a_i(\vec{l})e^{i\vec{l}\cdot\vec{x}}+\text{H.c.}]$$

and (A9)

$$\vec{\mathbf{E}}(x)=\sum_{l,i}i\left(\frac{l}{2}\right)^{1/2}\vec{\epsilon}_i(\vec{l})[a_i(\vec{l})e^{i\vec{l}\cdot\vec{x}}-\text{H.c.}];$$

where $i=1,2$ refer to polarization states, I obtain

$$-D+I=k^2r+\sum_{l,i}la_i^\dagger(\vec{l})a_i(\vec{l})+\sum_{l,i}[f_{l,i}a_i(\vec{l})+\text{H.c.}]. \tag{A10}$$

The last term is I and from (A8) and (A9)

$$f_{l,i}=g_r\theta(r-l_\perp)\left(\frac{l}{2}\right)^{1/2}[\vec{\epsilon}_i(\vec{l})\cdot\vec{\mathbf{n}}]\times\frac{e^{il_zx_1}-e^{il_zx_2}}{l_z}, \tag{A11}$$

where I have taken $\vec{\mathbf{n}}$ as the z axis. The modes $l_\perp>\Lambda$ are not excited and can be neglected. Now I diagonalize (A10) in the standard way:

$$-D+I=k^2r+\sum_{l,i}\theta(r-l_\perp)\left[la_i^\dagger(\vec{l})a_i(\vec{l})-\frac{|f_{l,i}|^2}{l}\right], \tag{A12}$$

where

$$\alpha_i(\vec{l})=a_i(\vec{l})+f_{l,i}^*/l. \tag{A13}$$

The last term of (A12) can easily be evaluated to give

$$\begin{aligned}w(r)&\equiv-\sum_{l,i}\frac{|f_{l,i}|^2}{l}\\&=-\frac{g_r{}^2}{2}\sum_l\theta(r-l_\perp)\left(\delta_{ij}-\frac{l_il_j}{l^2}\right)n_in_j\frac{2(1-\cos l_zr)}{l_z{}^2}\\&=-g_r{}^2\frac{\Lambda^2}{8\pi}r-\frac{g_r{}^2}{4\pi}\left(\frac{1-e^{-\Lambda r}}{r}-\Lambda\right),\end{aligned} \tag{A14}$$

which is equal to the electrostatic value of $\mathcal{K}^{(1)}$ (3.18) (replacing g by g_r). Thus we obtain

$$-D+I=k^2r+w(r)+\sum_{l,i}\theta(r-l_\perp)la_i^\dagger(\vec{l})a_i(\vec{l}).$$

The vacuum state $|0\rangle$ in (A6) is defined by $a_i(\vec{l})|0\rangle=0$. The vacuum state $|0\rangle$ is related to the vacuum $|\tilde{0}\rangle$ defined by

$$\alpha_i(\vec{l})|\tilde{0}\rangle=0 \tag{A15}$$

through

$$|0\rangle=N\exp\left[\sum_{l,i}\frac{f_{l,i}^*}{l}\alpha_i^\dagger(\vec{l})\right]|\tilde{0}\rangle, \tag{A16}$$

where

$$N=\exp\left(-\frac{1}{2}\sum_{l,i}|f_{l,i}|^2/l^2\right). \tag{A17}$$

Here both $|0\rangle$ and $|\tilde{0}\rangle$ are normalized to 1. The matrix element (A6) can be evaluated using (A14), (A16), and a relation

$$\langle\tilde{0}|e^{b\alpha}e^{ix\alpha^\dagger\alpha}e^{b^*\alpha^\dagger}|\tilde{0}\rangle=e^{b^*be^{ix}}, \tag{A18}$$

which holds for any pair of operators α and $\alpha^\dagger$ satisfying $[\alpha,\alpha^\dagger]=1$ and $\alpha|\tilde{0}\rangle=0$. Namely,

$$\left\langle \frac{1}{D-I} \right\rangle_0 = +i\int_0^\infty ds \left\langle 0 \left| \exp\left[is\left(k^2 r + w(r) + \sum_{l,i} l\alpha_i^\dagger(\vec{l})\alpha_i(\vec{l}) \right) \right] \right| 0 \right\rangle = +i\int_0^\infty ds \exp\{is[k^2 r + w(r)] + P(s)\}, \tag{A18'}$$

where

$$P(s) = \sum_{l,i} \frac{|f_{l,i}|^2}{l^2}(e^{isl} - 1) = \frac{g_r^2}{4\pi^2}\int_0^\Lambda dl_\perp l_\perp \int_{-\infty}^\infty dl_z \frac{l_\perp^2}{l^3}\frac{1-\cos l_z r}{l_z^2}\{\exp[is(l_\perp^2 + l_z^2)^{1/2}] - 1\}$$

$$= \frac{g_r^2}{4\pi^2}\int_0^{\Lambda r}\frac{dy}{y}\int_{-\infty}^\infty \frac{1-\cos yx}{x^2(1+x^2)^{3/2}}\left[\exp\left(\frac{isy}{r}(1+x^2)^{1/2}\right) - 1\right]. \tag{A19}$$

The last expression was obtained by a change of the variables $l_z = xl_\perp$ and $l_\perp r = y$. Introducing a new variable $t = s/r$, I obtain

$$\left\langle \frac{1}{D-I} \right\rangle_0 = ir\int_0^\infty dt \exp\{it[k^2 r^2 + rw(r)] + Q(t)\}, \tag{A20}$$

with

$$Q(t) = P(rt)$$

$$= \frac{g_r^2}{4\pi^2}\int_0^{\Lambda r}\frac{dy}{y}\int_{-\infty}^\infty dx \frac{1-\cos yx}{x^2(1+x^2)^{3/2}} \times\{\exp[ity(1+x^2)^{1/2}] - 1\}. \tag{A21}$$

For any finite t, the real part of $Q(t)$ behaves for large Λr as follows:

$$Q_k(t) \to \frac{-g_r^2}{4\pi}\Lambda r \quad (\Lambda r \gg 1). \tag{A22}$$

Also $|k^2 r + rw(r)| \gg 1$ for $\Lambda r \gg 1$ so that the phase of the integrand oscillates very rapidly. Thus the contribution to the integral in (A21) comes only from $t \lesssim (\Lambda r)^{-1}$. I can then replace $Q(t)$ by

$$Q'(0)t = rP'(0)t = ir\sum_{l,i}\frac{|f_{l,i}|^2}{l} = -irw(r), \tag{A23}$$

and obtain

$$\left\langle \frac{1}{D-I} \right\rangle_0 \to \frac{1}{k^2 r} = \frac{1}{D_0} \quad (\Lambda r \gg 1). \tag{A24}$$

This means from (A6) that $\tilde{\mathcal{K}}/k^2 r \to 0$ as $\Lambda r \to \infty$ and hence from (A5) that $\mathcal{K}/k^2 r \to 0$. I have established that $V(r) \to k^2 r$ for $\Lambda r \gg 1$. The imaginary part of $Q(t)$ can be evaluatee explicitly. For completeness, it is given below:

$$\mathrm{Im}[Q(t)] = \frac{g_r^2}{4\pi}\left\{ t(1 - e^{\Lambda r}) + \frac{1-\cos(\Lambda r t)}{t} + 2[J_0(\Lambda r t) - 1] - \Lambda r t\int_{\Lambda r t}^\infty dx \frac{1}{x} J_1(x) \right\}. \tag{A23}$$

APPENDIX B

I will show that the confinement equation (3.27) for the $J=0$, $C=1$, and $P=-1$ states with potential (3.26) cannot give an eigenvalue $M=0$ for the vanishing quark mass $m=0$ if $g_r^2/4\pi < 2$. It was shown already in GS that the solution $M=m=0$ exists (Goldstone-Nambu condition) if $g_r^2/4\pi > 2$. Thus I conclude that the confinement solution gives eigenvalues of $M>0$ for $g_r^2/4\pi < 2$, making it unstable relative to the positronium states. Introducing a new variable z by

$$r = Rz, \tag{B1}$$

where R is defined by (3.29) or

$$M = k^2 R - \frac{g_r^2 R}{4\pi(a^2 + R^2)} = k^2 R(1-\eta), \tag{B2}$$

with

$$\eta = \frac{g_1^2/4\pi}{k^2 a^2 + Q}. \tag{B3}$$

Here

$$Q = k^2 R^2 \tag{B4}$$

replaces the eigenvalue M. Using (B2) and (3.26), I obtain

$$M - V = k^2 R(1-z)D(z), \tag{B5}$$

with

$$D(z) = 1 + \eta\frac{z-\gamma^2}{z^2+\gamma^2}. \tag{B6}$$

Here

$$\gamma^2 = a^2/R^2 = k^2 a^2/Q, \tag{B7}$$

and the wave equation (3.27) becomes

$$\frac{d^2F}{dz^2} + \left(\frac{2}{z} + \frac{1}{1-z} - \frac{D'}{D}\right)\frac{dF}{dz} + \tfrac{1}{4}Q^2(1-z)^2 D^2 F = 0. \tag{B8}$$

This is the eigenvalue equation for Q for given po-

tential parameters g_r^2 and k^2a^2. Suppose the solution gives $M=0$ for certain values of the parameters. From (B2) it corresponds to

$$\eta=1, \quad \text{or} \quad Q=g_r^2/4\pi - k^2a^2 \equiv Q_0.$$

For this value of η, I have

$$D(z)=\frac{z(1-z)}{z^2+\gamma^2}.$$

Introducing (B9) into (B8) and using a new variable $x=z^2$, the equation reduces to

$$F''+\left(\frac{1}{x}+\frac{1}{1-x}+\frac{1}{x+\gamma^2}\right)F'+\tfrac{1}{16}Q_0^2\frac{(1-x)^2}{(x+\gamma^2)^2}F=0. \tag{B10}$$

I shall prove that there exists no solution of the equation for $Q_0<2$ and hence certainly none for $g_r^2/4\pi<2$ from (B9). Put

$$F(x)=(x+\gamma^2)^q f(x),$$

so that $f(x)$ satisfies

$$f''+\left(\frac{1}{x}+\frac{1}{1-x}+\frac{2q+1}{x+\gamma^2}\right)f'+P(x)f=0, \tag{B.11}$$

where

$$P(x)=\frac{q^2}{(x+\gamma^2)^2}+\frac{q}{(x+\gamma^2)x}+\frac{q}{(1-x)(x+\gamma^2)}+\frac{Q_0^2}{16}\frac{(1-x)^2}{(x^2+\gamma^2)^2}. \tag{B12}$$

Choose q so that $P(x)$ has no $1/x^2$ term in the limit $\gamma^2\to 0$,

$$q^2+q+\frac{Q_0^2}{16}=0, \tag{B13}$$

or

$$q=-\tfrac{1}{2}+\tfrac{1}{2}(1-\tfrac{1}{4}Q_0^2)^{1/2}<0.$$

q is real since $Q_0<2$. Multiply (B11) by

$$\rho(x)=\frac{x(x+\gamma^2)^{2q+1}}{1-x}$$

and f, and integrate over $[0,1]$. Using the boundary conditions $f(x)\sim(1-x)^2$ for $x\sim 1$ and $f(0)=\text{const}$ corresponding to (3.28), one obtains

$$\int_0^1 dx\,\rho(x)[-f'^2(x)+P(x)f^2(x)]=0. \tag{B14}$$

Using the relation (B13), I find that

$$P(x)=q\left[\frac{\gamma^2}{x(x+\gamma^2)^2}+\frac{1}{(1-x)(x+\gamma^2)}\right]-\frac{Q_0^2}{16}\frac{1-(1-x)^2}{(x+\gamma^2)^2},$$

which shows $P(x)<0$ since $q<0$. Hence (B14) cannot be satisfied.

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