Two-component Pomeron and hadron total cross sections and real parts

Harry J. Lipkin*
Argonne National Laboratory, Argonne, Illinois 60439
and Fermi National Accelerator Laboratory, Batavia, Illinois 60510 †

(Received 11 July 1977)

The predictions from a five-parameter formula obtained from a two-component Pomeron model and fit to hadron-nucleon total cross sections from 2 to 200 GeV/c are in remarkable agreement with new CERN ISR data on total proton-proton cross sections and real parts up to an equivalent $P_{\rm lab}$ of 2000 GeV/c and with total-cross-section data from cosmic rays up to 40000 GeV/c. This oversimplified formula with a Regge term varying as $s^{-1/2}$, two Pomeron-like terms with slightly increasing and slightly decreasing power behavior, and dependence upon quantum numbers given by simple quark-counting rules is adequate to fit all available data and can be useful for analysis of future data. Predictions for the ratios of real to imaginary parts of $\pi^{\pm}p$, $K^{\pm}p$, and $p^{\pm}p$ forward amplitudes are given.

The total proton-proton cross section and the real part of the forward scattering amplitude has been recently measured[1] at CERN ISR. Table I and Fig. 1 show that the new data in the energy range equivalent to $P_{\rm lab}$ =500 to 2000 GeV/c are in excellent agreement with the predictions from a five-parameter formula based on a two-component Pomeron model,[2] with no adjustment of the values of these parameters from already published values[2] fixed by fits to data below 200 GeV/c. Table I also lists predictions for higher energies and shows remarkable agreement with results from cosmic-ray experiments[3] up to $P_{\rm lab}$ =40 000 GeV/c. Whether these agreements confirm the validity of the oversimplified two-component model is unclear. However, the formula can certainly be used as a simple parametrization of the data and a guide to the physics of further experiments. The ISR group fitted their data with a seven-parameter formula.[1] Since data for the ratio ρ of the real to imaginary parts of the forward amplitudes for all hadron-proton scattering processes should soon be available, predictions are given in Table II. These are uniquely determined by the values of the five parameters already fixed.

The two-component Pomeron model describes hadron-nucleon total cross sections as the sum of a Regge term and two Pomeron-type components, one increasing slowly with energy and one decreasing slowly. The decreasing Pomeron component was introduced to describe the difference between pion-nucleon and kaon-nucleon cross sections, which shows this otherwise unexplained slowly decreasing behavior, and also the otherwise unrelated observation that exactly the same decreasing behavior is shown by the deviation of baryon-baryon cross sections from quark-model predictions based on meson-baryon cross sections. The total cross section for a hadron H on a proton target in this model is given by

$$\sigma_{\rm tot}(Hp)=C_1\sigma_1(Hp)+C_2\sigma_2(Hp)+C_R\sigma_R(Hp)\,, \quad (1)$$

where

$$\sigma_1(Hp)=N_q^H(P_{\rm lab}/20)^{\epsilon}\,, \quad (2a)$$

$$\sigma_2(Hp)=N_q^H N_{ns}^H(P_{\rm lab}/20)^{-\delta}\,, \quad (2b)$$

TABLE I. Theoretical predictions and experimental data for $\sigma_{\rm tot}(pp)$ and $\rho(pp)$.

$P_{\rm lab}$ (GeV/c)	$\sqrt{s}$ (GeV)	$\sigma_{\rm tot}(p\bar{p})$ Theory (mb)	$\sigma_{\rm tot}(pp)$ Theory (mb)	$\sigma_{\rm tot}(pp)$ Experiment (mb)	$\rho(pp)$ Theory	$\rho(pp)$ Experiment
498	30.6	41.8	40.0	40.1 ± 0.4	0.025	0.042 ± 0.011
1064	44.7	42.8	41.6	41.7 ± 0.4	0.064	0.062 ± 0.011
1491	52.9	43.5	42.5	42.4 ± 0.4	0.079	0.078 ± 0.010
2075	62.4	44.3	43.5	43.1 ± 0.4	0.092	0.095 ± 0.011
4600	92.9	46.8	46.2	47.0 ± 0.8	0.118	
10 000	137	49.8	49.5	50.6 ± 1.2	0.138	
25 000	217	54.3	54.0	53.8 ± 2.2	0.156	
40 000	274	56.9	56.7	55.0 ± 3.0	0.163	
100 000	433	62.7	62.6		0.174	

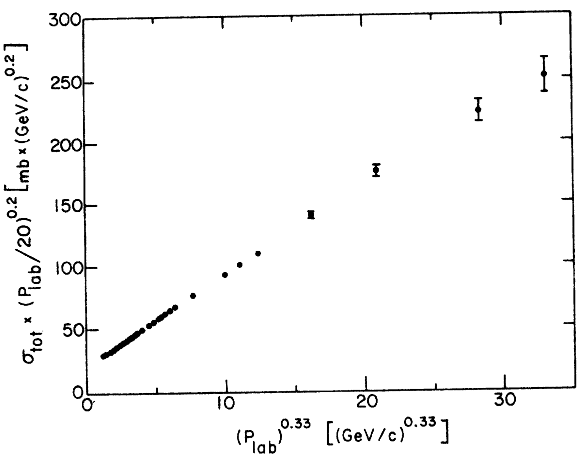

FIG. 1. $\sigma_{tot}(pp) \times (P_{lab}/20)^{0.2}$ plotted against $(P_{lab})^{0.33}$. Formula (1) predicts that the data should lie on a straight line. The four cosmic-ray points (Ref. 3) above $(P_{lab})^{0.33} = 15$ and the four ISR points (Ref. 1) in the interval $7 < (P_{lab})^{0.33} < 15$ are seen to lie on a straight line with a slope determined by fits to the lower-energy data.

$$\sigma_R(Hp) = (N_{\bar{n}}^H + 2N_{\bar{p}}^H)(P_{lab}/20)^{-1/2}; \tag{2c}$$

N_q^H is the total number of quarks and antiquarks in hadron H ($N_q^H = 2$ for mesons and 3 for baryons); N_{ns}^H is the total number of nonstrange quarks and antiquarks in hadron H and $N_{\bar{n}}^H$ and $N_{\bar{p}}^H$ are the total number of $\bar{n}$ and $\bar{p}$ antiquarks in hadron H.

The dependence of the individual terms in Eqs. (2a) and (2b) on the quantum numbers of H are determined by the model and discussed in Ref. 2. The explicit form for the energy dependence is chosen to minimize the number of free parameters. Thus, power behavior is chosen rather than logarithmic behavior for the two components of the Pomeron, because two parameters are sufficient to describe a power and at least three are needed to describe logarithmic behavior. The Regge term was chosen to minimize the number of free parameters by assuming exact duality and exchange degeneracy for the leading trajectories with the conventional intercept of $\frac{1}{2}$.

A convenient graphical test of the formula (1) for $\sigma_{tot}(pp)$ is shown in Fig. 1. Since $\sigma_R(pp) = 0$, by Eq. (2c), a plot of $\sigma_{tot}(pp) \times (P_{lab}/20)^{\delta}$ vs $(P_{lab})^{\epsilon+\delta}$ is predicted to give a straight line. Figure 1 shows that the ISR and cosmic-ray data fit very well on a straight line with a slope determined by the fit to the lower-energy data. Similar plots of $\sigma_{tot}(K^+p)$ and linear combinations of cross sections for which there is no Regge contribution also show straight lines for the momentum range below 200 GeV/c, where data are available. Similar plots with slightly different values for the parameters show straight lines over a range of values of δ, but slight changes in ϵ destroy the straight line. It is difficult to determine the "best value" of δ because there is no clear criterion for what is a "best fit" without a model which defines

TABLE II. Theoretical predictions for $\rho(Hp)$.

P_{lab} (GeV/c)	$\rho(pp)$	$\rho(p\bar{p})$	$\rho(K^+p)$	$\rho(K^-p)$	$\rho(\pi^-p)$	$\rho(\pi^+p)$
2	−0.76	−0.098	−0.68	−0.0092	−0.230	−0.47
10	−0.40	−0.07	−0.24	0.037	−0.120	−0.24
15	−0.33	−0.059	−0.17	0.051	−0.095	−0.19
20	−0.28	−0.05	−0.13	0.060	−0.076	−0.16
25	−0.25	−0.043	−0.098	0.068	−0.061	−0.13
30	−0.23	−0.036	−0.075	0.074	−0.050	−0.11
35	−0.21	−0.031	−0.057	0.079	−0.040	−0.10
40	−0.19	−0.026	−0.043	0.083	−0.032	−0.087
45	−0.18	−0.022	−0.031	0.087	−0.025	−0.077
50	−0.16	−0.018	−0.020	0.09	−0.019	−0.068
70	−0.13	−0.006	0.010	0.10	0.000	−0.040
100	−0.094	0.007	0.037	0.11	0.019	−0.014
120	−0.077	0.014	0.05	0.11	0.029	−0.001
150	−0.058	0.023	0.064	0.12	0.039	0.013
170	−0.048	0.027	0.071	0.12	0.045	0.021
200	−0.036	0.033	0.08	0.13	0.053	0.030
240	−0.022	0.04	0.089	0.13	0.061	0.041
280	−0.011	0.046	0.096	0.14	0.067	0.049
500	0.025	0.066	0.12	0.15	0.089	0.076
1000	0.061	0.088	0.14	0.16	0.11	0.10
1400	0.076	0.098	0.15	0.16	0.12	0.11
2000	0.090	0.11	0.16	0.17	0.13	0.12

the energy range and quantum numbers for which the model is expected to be valid. It is interesting that a good fit is obtained for $\delta=-0.185=\frac{1}{2}(\epsilon-\frac{1}{2})$. This value makes the energy behavior of σ_2 like that of $(\sigma_1\sigma_R)^{1/2}$ and might suggest that σ_2 is due to an interference term between amplitudes responsible for σ_1 and σ_R.

The extension of the formula (1) to the real part of the amplitude is a straightforward application of analyticity and crossing, which is particularly simple for terms with power behavior.[4]

The first two components have even signature and the ratios ρ of their real parts to their imaginary parts are simply given by the expressions

$$\rho_1=\tan(\pi\epsilon/2), \tag{3a}$$

$$\rho_2=-\tan(\pi\delta/2). \tag{3b}$$

The Regge term must be separated into its even- and odd-signature parts as follows:

$$\sigma_R(Hp)=\sigma_{Re}(Hp)+\sigma_{Ro}(Hp), \tag{4a}$$

where

$$\sigma_{Re}(Hp)=[\sigma_R(Hp)+\sigma_R(\bar{H}p)]/2, \tag{4b}$$

$$\sigma_{Ro}(Hp)=[\sigma_R(Hp)-\sigma_R(\bar{H}p)]/2. \tag{4c}$$

The corresponding ratios of the real to the imaginary parts of these components are given by

$$\rho_{Re}(Hp)=-1, \tag{5a}$$

$$\rho_{Ro}(Hp)=+1. \tag{5b}$$

Combining these equations gives the following expression for the real to the imaginary part of the Hp amplitude:

$$\rho(Hp)=\frac{C_1\sigma_1(Hp)\tan(\frac{1}{2}\pi\epsilon)-C_2\sigma_2(Hp)\tan(\frac{1}{2}\pi\delta)-C_R\sigma_R(\bar{H}p)}{\sigma_{\text{tot}}(Hp)}. \tag{6}$$

This expression shows the expected qualitative behavior for the real part, a positive contribution from the increasing component, and negative contributions from the two decreasing components. Thus ρ is negative at low energies and goes through zero and becomes positive at high energies, in agreement with experiment.

The good fits obtained to very-high-energy data indicate that these rather crude approximations are nevertheless adequate up to these energies. As long as this reasonable fit continues, models containing more detailed assumptions will not be easily tested by the available data. For example, as long as a good fit is obtained with power behavior for the first component, the necessity for logarithmic terms will be difficult to demonstrate since a considerably better fit is required to justify the use of additional parameters. The same is true for more detailed or realistic descriptions of the Regge component, since breaking exchange degeneracy or choosing a value different from $\frac{1}{2}$ for the intercept necessarily requires more parameters. However, as soon as data appear which fail to fit this formula, the underlying assumptions are so simple that the physics of the disagreement should be readily apparent. The nature of the disagreement might suggest, for example, that the rise of the cross sections is logarithmic rather than a power, that exchange degeneracy is breaking down, or that the Regge intercept is not $\frac{1}{2}$. There may also be a breakdown of the two-component-Pomeron picture if the dependence on the quantum numbers of hadron H no longer satisfies the simple relations of the model. Thus, regardless of the validity of the two-component-Pomeron description, formula (1) should be a valuable guide to the analysis of data on high-energy total cross sections and real parts of scattering amplitudes.

Stimulating discussions with G. Cocconi, C. Quigg, and G. Yodh are gratefully acknowledged.

*On leave from Weizmann Institute of Science, Rehovot, Israel.

†Operated by Universities Research Association Inc. under contract with the Energy Research and Development Administration.

[1]U. Amaldi *et al.*, Phys. Lett. 66B, 390 (1977).

[2]H. J. Lipkin, Phys. Rev. D 11, 1827 (1975); see also Nucl. Phys. B78, 381 (1974); Phys. Lett. 56B, 76 (1975); in *High Energy Physics*, Proceedings of the 1975 EPS Conference, Palermo, Italy, edited by A. Zichichi (Editrice Compositori, Bologna, 1976), p. 1000.

[3]G. B. Yodh, in the Proceedings of the Brookhaven Symposium on Prospects of Strong Interaction Physics at Isabelle, 1977 (unpublished); also F. Siohan, Ph.D. thesis, University of Maryland, Report No. PP77611, 1976 (unpublished); and T. K. Gaisser, G. B. Yodh, V. Barger, and F. Halzen, in *Proceedings of the Fourteenth International Conference on Cosmic Rays, Munich, 1975*, edited by Klaus Pinkau (Max-Planck-Institute, München, 1975), Vol. 7, p. 2161.

[4]R. J. Eden, *High Energy Collisions of Elementary Particles* (Cambridge Univ. Press, London, 1967), p. 194.