## Pole terms in the sum rules for single-pion-observed inclusive reactions

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Pole terms in the sum rules for single-pion-observed inclusive reactions induced by electromagnetic currents or hadronic neutral weak currents are estimated in the deep-inelastic region.

Recently, this author studied the physical application of the algebra of bilocal currents<sup>1,2</sup> based on null-plane quantization.<sup>3</sup> Among the commutators

$$
[J_a^{\mu}(x|y), J_b^{\nu}(u|v)]|_{x^+=y^+=u^+=v^+=0},
$$

the parts  $\mu = \nu = +$ ,  $x = y$ , and  $\mu = +$ ,  $\nu = i$ ,  $x = y$  and  $\mu$ =+,  $\nu$  = –, and those where the vector currents are replaced by the axial-vector currents were mainly used, since the neutral-vector-gluon model gave the same results as those of the free-quark

model in the above cases.<sup>4</sup> In the previous work,<sup>2</sup> we encountered various pole terms due to a pion bremsstrahlung, and the sum rules in the case of the nucleon target had been derived only in the single- $\pi$ <sup>0</sup>-observed inclusive reaction. The purpose of this paper is to discuss the above pole terms in other reactions (reactions in Sec. II and in Sec.  $\text{IIC}$  in the previous papers<sup>2</sup>).

First we consider the inclusive reactions  $\gamma_{v}$  $+N-\pi$ <sup>+</sup> + X. According to a usual technique<sup>5</sup> the hadronic part of the process will be given as

$$
T_{abcd}^{\mu\nu} = \frac{(m_{\pi}^2 - q^2)^2}{2m_{\pi}^4 f_{\pi}^2} \int d^4x d^4y d^4z \exp[-iq \cdot (x - z) + ik \cdot y]
$$
  
\n
$$
\times \{q_{\lambda}q_{\rho}(N(p) \|T^* (J_{a}^{5\lambda}(x), J_{b}^{\mu}(y)), T(J_{c}^{5\rho}(z), J_{d}^{\nu}(0))\} |N(p)\rangle
$$
  
\n
$$
- iq_{\rho}\delta(x^* - y^*)\langle N(p) | [J_{a}^{5\lambda}(x), J_{b}^{\mu}(y)], T(J_{c}^{5\rho}(z), J_{d}^{\nu}(0))]|N(p)\rangle
$$
  
\n
$$
- iq_{\lambda}\delta(z^*)\langle N(p) | [T^* (J_{a}^{5\lambda}(x), J_{b}^{\mu}(y)), [J_{c}^{5\lambda}(z), J_{d}^{\nu}(0)]]|N(p)\rangle
$$
  
\n
$$
-\delta(x^* - y^*)\delta(z^*)\langle N(p) | [[J_{a}^{5\lambda}(x), J_{b}^{\mu}(y)], [J_{c}^{5\lambda}(z), J_{d}^{\nu}(0)]]|N(p)\rangle
$$
  
\n(1)

where  $a^* = a' = c = 1 - i2$ ,  $b = d$  specifies electromagnetic currents, the partially conserved axial-vector current (PCAC) relation is used, and a spectral condition is used to obtain the commutator. Since  $T_{abcd}^{\mu\nu}$  satisfies gauge invariance, and the average over spin is taken, it will be given as

$$
T_{abcd}^{\mu\nu} = G^{\mu\nu}V_L + \left[k^2 P^{\mu} P^{\nu} + \frac{(\dot{p} \cdot k)^2}{k^2} G^{\mu\nu}\right] V_2 + (P^{\mu} K^{\nu} + P^{\nu} K^{\mu}) V_3 + K^{\mu} K^{\nu} V_4,
$$
\n(2)

where

$$
G^{\mu\nu} = k^{\mu}k^{\nu} - k^2g^{\mu\nu}, \quad P^{\mu} = p^{\mu} - \frac{p \cdot k}{k^2}k^{\mu}, \text{ and } K^{\mu} = q \cdot k k^{\mu} - k^2 q^{\mu}, \tag{3}
$$

and  $V_i$  (i=1,..., 4) is a function of p q, q<sup>2</sup>, q · k, k<sup>2</sup>, and p · k. Now we take  $q^+ = 0$ ,  $\dot{q}^{\perp} = 0$ , and after that  $q = 0$  at the right-hand side of Eq. (1). At this limit there appear many pole terms as shown in Figs. 1–5. Then Eq. (1) will be written as

$$
T_{abcd}^{\mu\nu} = \sum_{i=1}^{3} \left( A_i^{\mu\nu} + B_i^{\mu\nu} \right) + C_1^{\mu\nu} + D_4^{\mu\nu} , \qquad (4)
$$

where

$$
A_1^{\mu \nu} = \frac{1}{8f_\pi^2 (p^+)^2} \int d^4 y \exp(i k \cdot y) [\langle N(p) | J_a^{5+}(0) | N'(p) \rangle \langle N'(p) | J_b^{\mu}(y) J_a^{\nu}(0) | N'(p) \rangle \langle N'(p) | J_c^{5+}(0) | N(p) \rangle
$$
  
 
$$
- \langle N(p) | J_c^{5+}(0) | N'(p) \rangle \langle N'(p) | J_a^{\nu}(0) J_b^{\mu}(y) | N'(p) \rangle \langle N'(p) | J_a^{5+}(0) | N(p) \rangle ] , \qquad (5)
$$

$$
\frac{17}{10} \qquad \qquad 361
$$

 $\gamma_{\mathbf{v}}(\mathbf{k})$  $x^{N(p)}$  $\pi$ N  $\gamma_{\mathbf{v}}(\mathbf{r})$ 

FIG. 1. Representation of pole term  $A_1^{\mu\nu}$ ; the dashe line denotes the bremsstrahlung pion.



FIG. 2. Representation of pole term  $A_2^{\mu\nu}+A_3^{\mu\nu}$ ; the dashed line denotes the bremsstrahlung pion.

$$
A_{2}^{\mu\nu} + A_{3}^{\mu\nu} = -\frac{1}{4f_{\pi}^{2}p^{+}} \int d^{4}x d^{4}y \exp(ik \cdot y) \delta(x^{+} - y^{+}) \{ \langle N(p) | [J_{a}^{5+}(x), J_{b}^{\mu}(y)] J_{a}^{\nu}(0) | N'(p) \rangle \langle N'(p) | J_{c}^{5+}(0) | N(p) \rangle \right. \\ \left. + \frac{1}{4f_{\pi}^{2}p^{+}} \int d^{4}y d^{4}z \exp(ik \cdot y) \delta(z^{+}) \{ \langle N(p) | J_{a}^{5+}(0) | N'(p) \rangle \langle N'(p) | J_{a}^{\mu}(0) [J_{a}^{5+}(x), J_{b}^{\mu}(y)] | N(p) \rangle \} \right. \\ \left. + \frac{1}{4f_{\pi}^{2}p^{+}} \int d^{4}y d^{4}z \exp(ik \cdot y) \delta(z^{+}) \{ \langle N(p) | J_{a}^{5+}(0) | N'(p) \rangle \langle N'(p) | J_{b}^{\mu}(y) [J_{b}^{5+}(z), J_{a}^{\nu}(0)] | N(p) \rangle \right. \\ \left. + \langle N(p) | [J_{c}^{5+}(z), J_{a}^{\nu}(0)] J_{b}^{\mu}(y) | N'(p) \rangle \langle N'(p) | J_{a}^{5+}(0) | N(p) \rangle \right\}, \tag{6}
$$

$$
B_1^{\mu\nu} = \frac{1}{2f_{\pi}^2} \int d^4 y \exp(ik \cdot y) \sum_X \int \frac{d^3 X}{(2\pi)^3 2X^*} \int \frac{d^3 \eta}{(2\pi)^3 (2n^*)^3} \times \left[ \langle N(p) | J_{\phi}^{\mu}(y) | N'(n) \rangle \langle N \rangle \langle n \rangle \right] \int \frac{1}{2} \langle N(p) | J_{\phi}^{\mu}(y) | N'(n) \rangle \langle N'(p) | J_{\phi}^{\mu}(0) | N'(n) \rangle \langle N''(n) | J_{\phi}^{\mu}(0) | N'(n) \rangle \langle N''(n) | J_{\phi}^{\mu}(0) | N'(n) \rangle \langle N'(n) | \langle X | J_{\phi}^{\mu}(0) | N(\phi) \rangle \right], (7)
$$

$$
B_{2}^{\mu\nu} + B_{3}^{\mu\nu} = -\frac{1}{2f_{\pi}^{2}} \int d^{4}y d^{4}z \exp(ik \cdot y)\delta(z^{+}) \sum_{X} \int \frac{d^{3}X}{(2\pi)^{3}2X^{+}} \int \frac{d^{3}n}{(2\pi)^{3}(2n^{+})^{2}} \times \left\{ \langle N(p)|J_{\nu}^{\mu}(y)|N'(n)\rangle \langle X \rangle \langle N'(n)|J_{\alpha}^{\xi_{+}}(0)|N''(n)\rangle \langle N''(n)|\langle X|[J_{\sigma}^{\xi_{+}}(z), J_{\nu}^{\nu}(0)]|N(p)\rangle \right. \\ \left. + \langle N(p)|[J_{\sigma}^{\xi_{+}}(z), J_{\nu}^{\nu}(0)]|N'(n)\rangle \right|X\rangle \langle N'(n)|J_{\alpha}^{\xi_{+}}(0)|N''(n)\rangle \langle N''(n)|\langle X|J_{\nu}^{\mu}(y)|N(p)\rangle \right\} + \frac{1}{2f_{\pi}^{2}} \int d^{4}x d^{4}y \exp(ik \cdot y)\delta(x^{+} - y^{+}) \sum_{X} \int \frac{d^{3}X}{(2\pi)^{3}2X^{+}} \int \frac{d^{3}n}{(2\pi)^{3}(2n^{+})^{2}} \times \left\{ \langle N(p)|[J_{\alpha}^{\xi_{+}}(x), J_{\nu}^{\mu}(y)]|N'(n)\rangle |X\rangle \langle N'(n)|J_{\sigma}^{\xi_{+}}(0)|N''(n)\rangle \langle N''(n)|\langle X|J_{\alpha}^{\nu}(0)|N(p)\rangle \right. \\ \left. + \langle N(p)|J_{\alpha}^{\nu}(0)|N'(n)\rangle |X\rangle \langle N'(n)|J_{\nu}^{\xi_{+}}(0)|N''(n)\rangle \langle N''(n)|\langle X|[J_{\alpha}^{\xi_{+}}(x), J_{\nu}^{\mu}(y)]|N(p)\rangle \right\}, \tag{8}
$$

$$
C_{1}^{\mu\nu} = -\frac{1}{2f_{\pi}^{2}} \int d^{4}y \exp(ik \cdot y) \sum_{X} \int \frac{d^{3}X}{(2\pi)^{3}2X^{+}} \int \frac{d^{3}n}{(2\pi)^{3}(2n^{+})^{2}2p^{+}} \times \left[ \left\{ \langle N(p) | J_{b}^{\mu}(y) | N'(n) \rangle \mid X \rangle \langle N'(n) | J_{a}^{5*}(0) | N''(n) \rangle \langle N''(n) \rangle \langle X | J_{a}^{\nu}(0) | N'''(p) \rangle \langle N'''(p) | J_{c}^{5*}(0) | N(p) \rangle \right. \\ \left. \left. - \langle N(p) | J_{a}^{\nu}(0) | N'(n) \rangle \mid X \rangle \langle N'(n) | J_{c}^{5*}(0) | N''(n) \rangle \langle N''(n) | \langle X | J_{b}^{\mu}(y) | N'''(p) \rangle \langle N'''(p) | J_{a}^{5*}(0) | N(p) \rangle \right] \right. \\ \left. + \left\{ \langle N(p) | J_{a}^{5*}(0) | N'(p) \rangle \langle N'(p) | J_{b}^{\mu}(y) | N''(n) \rangle \mid X \rangle \langle N''(n) | J_{c}^{5*}(0) | N''(n) \rangle \langle N'''(n) \rangle \langle N'''(n) \rangle \langle X | J_{b}^{\nu}(0) | N(p) \rangle \right. \\ \left. - \langle N(p) | J_{c}^{5*}(0) | N'(p) \rangle \langle N'(p) | J_{a}^{\nu}(0) | N''(n) \rangle \mid X \rangle \langle N''(n) | J_{a}^{5*}(0) | N'''(n) \rangle \langle N'''(n) | \langle X | J_{b}^{\mu}(y) | N(p) \rangle \right], \tag{9}
$$

N(p)

FIG. 3. Representation of pole term  $B^{uv}_1$ ; the dashed line denotes the bremsstrahlung pion.



FIG. 4. Representation of pole term  $B_2^{\mu\nu}+B_3^{\mu\nu}$ , the dashed line denotes the bremsstrahlung pion.

$$
D_4^{\mu\nu} = -\frac{1}{2f_*^2} \int d^4x \, d^4y \, d^4z \, \exp(ik \cdot y) \delta(x^+ - y^+) \delta(z^+) \langle N(\rho) | \left[ \left[ J_a^{5+}(x), J_b^{\mu}(y) \right], \left[ J_c^{5+}(z), J_d^{\nu}(0) \right] \right] |N(\rho) \rangle \tag{10}
$$

where  $N'$  or  $N''$  or  $N'''$  is a suitable nucleon and is determined in each term according to the internalsymmetry index,  $\sum_{x}$  denotes the summation over the intermediate states X, and  $d^3p$  denotes  $d\vec{p}^{\perp}d\phi^*$ . Now we apply the method of Dicus, Jackiw, and Teplitz<sup>7</sup> by using the fact that expressions  $(5)-(10)$ are odd under exchange of  $a \leftrightarrow c$ ,  $b \leftrightarrow d$ ,  $\mu \leftrightarrow \nu$ ,  $k \rightarrow -k$ : We integrate over  $k^-$ , change the variable from  $k^-$  to  $\nu = p \cdot k$ , and assume the interchange of  $\nu$  integration and set  $k^+=0$ . In the previou papers, the contributions from expressions (5), (5), and (10) were estimated. Therefore we concentrate on the pole terms  $(7)-(9)$ . First we take  $\mu = \nu = +$ , and the target as the proton. After the  $\nu$ integration, the pole term  $B_1^{++}$  will be considered as the difference between the multiplicity in the reaction  $\gamma_Y$  + proton  $\rightarrow$  proton + X and that in the reaction  $\gamma_{V}$  + proton  $\rightarrow$  neutron + X. At high energy the multiplicity of the proton or the neutron mill be expected to be the same since mainly pions are produced. Thus  $B_1^{++}$  will contribute only in the low-energy region. But in this region, we always get the factor  $[G(-\overline{k}^{12})]^2$ , the form factor for the reaction nucleon + currents  $\rightarrow$  nucleon + (a suitable low-energy state or the vacuum). Then, if we take a large  $k^2 = -k^2$  (for example, above 5 GeV<sup>2</sup>).

these contributions will be strongly suppressed, and will be neglected. Therefore  $B_1^+$  will be neglected in the above kinematical region. Next we consider  $B_2^{++} + B_3^{++}$ . Since we consider the order of this term we set the second and fourth terms equal to zero, because they are zero in the rest system df the nucleon. The first and third terms are Hermitian-conjugate to each other, therefore they will be given as

$$
\int_{-\infty}^{\infty} dk^{-} \sum_{X} (2\pi)^{4} \delta^{4} (p+k-X-n)
$$

$$
\times \int \frac{d^{3}X}{(2\pi)^{3} 2X^{+}} \int \frac{d^{3}n}{(2\pi)^{3} 2n^{+}} \operatorname{Re}(EF), \qquad (11)
$$

where

 $\overline{a}$ 

$$
E = \frac{1}{\sqrt{2} f_{\pi} 2n^+}
$$
  
× $\langle N(p) | J_b^+(0) | N'(n) \rangle | X \rangle \langle N'(n) | J_a^{5+}(0) | N''(n) \rangle$   
(12)

$$
F = \frac{i f_{ode}}{\sqrt{2} f_{\pi}} \langle N''(n) | \langle X | J_e^{5+}(0) | N(p) \rangle , \qquad (13)
$$

(14)

and  $Re(EF)$  denotes the real part of  $EF$ . We apply the Schwarz inequality of the following form:

$$
\left| \int_{-\infty}^{\infty} dk^{-} \sum_{X} \int \frac{d^{3}X}{(2\pi)^{3} 2X^{+}} \int \frac{d^{3}n}{(2\pi)^{3} 2n^{+}} (2\pi)^{4} \delta^{4}(p+k-X-n) \text{Re}(EF) \right|
$$
  
\n
$$
\leq \left| \int_{-\infty}^{\infty} dk^{-} \sum_{X} \int \frac{d^{3}X}{(2\pi)^{3} 2X^{+}} \int \frac{d^{3}n}{(2\pi)^{3} 2n^{+}} (2\pi)^{4} \delta^{4}(p+k-X-n) |E|^{2} \right|^{1/2}
$$
  
\n
$$
\times \left| \int_{-\infty}^{\infty} dk^{-} \sum_{X} \int \frac{d^{3}X}{(2\pi)^{3} 2X^{+}} \int \frac{d^{3}n}{(2\pi)^{3} 2n^{+}} (2\pi)^{4} \delta^{4}(p+k-X-n) |F|^{2} \right|^{1/2}
$$

Since

$$
\int_{-\infty}^{\infty} d\mathbf{k} \, \sum_{\mathbf{x}} \int \frac{d^3 X}{(2\pi)^3 2X^+} \int \frac{d^3 n}{(2\pi)^3 2n^+} (2\pi)^4 \delta^4 (p + k - X - \mathbf{x}) |\mathbf{F}|^2 \left| \frac{\mathbf{x}}{2} \right| \int_{-\infty}^{\infty} d\mathbf{k} \, D_4^{++} \qquad (15)
$$

and

$$
\left|\int_{-\infty}^{\infty} d\boldsymbol{k} \right| \sum_{k=1}^{\infty} \int \frac{d^3 X}{(2\pi)^3 2X^*} \int \frac{d^3 n}{(2\pi)^3 2n^*} (2\pi)^4 \delta^4 (p+k-X-n) |\boldsymbol{k}|^2
$$

is proportional to  $\int_{-\infty}^{\infty} d\mathbf{k} \cdot \mathbf{B}_{1}^{+}$ , the right-hand side of Eq.  $(14)$  will be estimated to be zero, if we take of Eq. (14) will be estimated to be zero, if we take<br>a sufficiently large  $k^2 = -\vec{k}^{12}$ . Therefore we con-<br>clude that  $R_b^{++} + B_b^{++}$  will be neglected also in this clude that  $B_2^{++}$  +  $B_3^{++}$  will be neglected also in this kinematical **region.** Finally we consider  $C_1^{++}$ , and find it to be zero by the same kind of discussion as above. Now we get<sup>8</sup>

$$
\int_0^{\infty} d\nu \, \vec{k}^{12} [V_2^+(\nu, -\vec{k}^{12}) - V_2^-(\nu, -\vec{k}^{12})]
$$
  
=  $\frac{4\pi}{f_{\pi}^2} [1 - g_A^2(0)] I_3 + \frac{g_A^2(0)}{6f_{\pi}^2} (10I_3 - 1) P \int \frac{d\alpha}{\alpha} A(\alpha, 0)$   
+  $\frac{g_A(0)}{3f_{\pi}^2} P \int \frac{d\alpha}{\alpha} [A^5(\alpha, 0) + \alpha \bar{A}^5(\alpha, 0)]$ , (16)

where

$$
\langle p | (1/2i) [q(x)\gamma^{\mu} q(0) - \overline{q}(0)\gamma^{\mu} q(x)] | p \rangle
$$
  
\n
$$
= p^{\mu} A(p \cdot x, x^{2}) + x^{\mu} \overline{A}(p \cdot x, x^{2}) ,
$$
  
\n
$$
\langle p | (1/2i) [q(x)\gamma^{\mu} \gamma^{5} q(0) - \overline{q}(0)\gamma^{\mu} \gamma^{5} q(x)] | p \rangle
$$
  
\n
$$
= S^{\mu} A^{5}(p \cdot x, x^{2}) + p^{\mu} (s \cdot x) \overline{A}^{5}(p \cdot x, x^{2})
$$
  
\n
$$
+ x^{\mu} (x \cdot s) \widetilde{A}^{5}(p \cdot x, x^{2}) , \qquad (17)
$$

and  $g_{\mathbf{A}}(0)$  is the nucleon axial-vector coupling constant. The same kind of discussion cannot be applied to the  $\mu = + \nu = i$  case, since the contribution from the high-energy region may not be neglected. But in the case of  $\mu = + \nu = -$ , it will be applied and we get

$$
\int_0^\infty d\nu \left[ V_L^+(\nu, -\vec{k}^{\perp^2}) - V_L^-(\nu, -\vec{k}^{\perp^2}) \right] = 0 \ , \tag{18}
$$

where

$$
\frac{1}{\mathbf{k}^{12}} \left\{ \frac{g_A^2(0)}{6f_\pi^2} (10I_3 - 1) \int d\alpha \, \overline{A}(\alpha, 0) + \frac{4\pi m^2}{f_\pi^2} g_A^2(0) I_3 - \frac{g_A(0)}{3f_\pi^2} P \int \frac{d\alpha}{\alpha} \left[ m^2 A^5(\alpha, 0) - \alpha^2 \overline{A}^5(\alpha, 0) \right] \right\} \tag{19}
$$

is neglected, since  $\tilde{k}^{\perp^2}$  is large.

It is straightforward to discuss the reaction in the case of neutral currents. We take the currents as'

$$
V_{\text{neutral}}^{\mu}(x) = \overline{q}(x)\gamma^{\mu}C_{\gamma}q(x) + \overline{q}(x)\gamma^{\mu}\gamma^{5}C_{A}q(x), \quad (20)
$$

where

$$
C_V = \text{diag}(\frac{1}{2} - \frac{4}{3}\sin^2\theta_W, -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W, -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W, \frac{1}{2} - \frac{4}{3}\sin^2\theta_W)
$$

and  $C_A = \text{diag}(-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$ . By setting  $\mu = + \nu = +$ , we get

$$
\int_{0}^{\infty} d\nu \left[ W_{2}^{0*}(\nu, -\tilde{k}^{2}) - W_{2}^{0-}(\nu, -\tilde{k}^{2}) \right] = \left[ 1 + (1 - 2 \sin^{2} \theta_{W})^{2} \right] \left[ 1 - g_{A}^{2}(0) \right] \frac{4\pi}{f_{\pi}^{2}} I_{3}
$$
  
+ 
$$
\frac{g_{A}^{2}(0)}{3f_{\pi}^{2}} \left[ \sin^{2} \theta_{W} (1 - 2 \sin^{2} \theta_{W}) + (20 \sin^{4} \theta_{W} - 18 \sin^{2} \theta_{W} + 9) I_{3} \right] P \int \frac{d\alpha}{\alpha} A(\alpha, 0)
$$
  
- 
$$
\frac{2g_{A}(0)}{3f_{\pi}^{2}} \sin^{2} \theta_{W} (1 - 2 \sin^{2} \theta_{W}) P \int \frac{d\alpha}{\alpha} \left[ A^{5}(\alpha, 0) + \alpha \overline{A}^{5}(\alpha, 0) \right],
$$
 (21)

and by setting  $\mu = + \nu = -$ , we get

$$
\int_0^\infty d\nu [W_1^0-(\nu, -\vec{\mathbf{k}}^{\perp 2}) - W_1^{0+}(\nu, -\vec{\mathbf{k}}^{\perp 2})] = 0 , \qquad (22)
$$

where

$$
T^{\mu\nu} = -g^{\mu\nu}W_1 + \not{p}^{\mu}\not{p}^{\nu}W_2 - i\epsilon^{\mu\nu\alpha\beta}p_{\alpha}k_{\beta}W_3 + \cdots,
$$
\n(23)

corresponding to Eq.  $(2)$ .

Finally we notice that there is another possibil-

ity. If we assume the dominance of the Born term **(when**  $|X\rangle$  is the vacuum) in  $\mathbf{F}_{i}^{\mu\nu}$  and  $C_{i}^{\mu\nu}$ , sum rules slightly different from those listed above will be obtained. These sum rules will be useful at low  $k^2 = -\vec{k}^{\perp 2}$ . If the above approximation is not **valid,** we can take into account the contributions from the low-energy region. These contributions will be measured through the exclusive processes. If such an analysis is done, there arises an interesting application: First, as noted in the previous  $\mathbf{p}$ apers<sup>2</sup> we can apply the results to the reaction

 $\gamma + N \rightarrow \pi^+ + X$ , and second, to the reaction  $\pi + N$  $-\pi+X$ .

Note added. After this paper was submitted, the target spin was taken into account, and, in the course of the work, some errors in the previous paper (Ref. 2) have been found. First, the calculation of  $A_1^{\mu\nu}$  or  $A_2^{\mu\nu}+A_3^{\mu\nu}$  is misleading due to inadequate treatment of internal symmetry, and second, the sum rule in the case of  $\mu = +$ ,  $\nu =$ is wrong due to inadequate treatment of symmetrybreaking terms (see Ref. 7). These two points are corrected in this paper. Further, the proof of cancellation of  $C_1^*$  at high energy due to internal symmetry may fail, but there is another cancellation mechanism at high energy: a positive or negative helicity state of the final nucleon gives a contribution opposite in sign in the case of  $B_2^{\mu\nu}$ +  $B_3^{\mu\nu}$  or  $C_1^{\mu\nu}$ , which has been already pointed out in Ref. 6 in the case of  $\pi^0$  production. This fact makes the sum rules meaningful even in the case of  $\mu = +$ ,  $\nu = i$ , and makes the calculation of the form factor possible in the deep-inelastic region if we use the light-cone current algebra. These

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FIG. 5. Representation of pole term  $C^{yy}_1$ ; the dashed line denotes the bremsstrahlung pion.

points, together with the correction of other trivial errors in the previous paper, will be made clear in the near future.

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- <sup>8</sup>Here (10 $l_3$ -1)/3 means diag ( $\frac{8}{3}, \frac{2}{3}, \frac{2}{3}$ ) and that the matrix element by the neutron should be taken in the case of the proton target and vice versa with consideration for the appropriate sign. Concerning the part proportional to  $g_A(0)$ , the internal-symmetry index  $1 \pm i 2$  should be understood. The same kind of rule should be understood in the case of neutral currents. <sup>9</sup>S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev.
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