Comments and Addenda

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Measurement of the antiproton magnetic moment and mass*

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We report a reanalysis of our $\bar{p}$ x-ray data from Pb and U for fine-structure splitting and have included perturbations to the $\bar{p}$-atomic energy levels arising from radiative corrections, relativistic effects, electron screening, finite nuclear size, and nuclear recoil. The value for the antiproton magnetic moment is -2.817 ± 0.048 nuclear magnetons which is 0.1% lower than our previous result and in agreement with the *CPT* prediction of -2.793. In addition we have obtained a value for the $\bar{p}$ mass from the measured $\bar{p}$-Pb x-ray transition energies from $n = 15$ to $n = 10$. The resulting mass 938.30 ± 0.13 MeV is in good agreement with the *CPT* prediction that the $\bar{p}$ mass should equal the proton mass. We find $m_p - m_{\bar{p}} = -0.02 \pm 0.13$ MeV.

In a recent paper[1] we reported measurements of the magnetic dipole moments of the antiproton and the Σ^- hyperon using the exotic-atom method. Subsequently another group[2] reported the results of similar measurements which agreed with our results, but the value of the $\bar{p}$ magnetic moment reported in Ref. 2 was more precise by a factor of 2. Both analyses used a relationship between the fine-structure splitting of a $\bar{p}$-atomic level and the magnetic moment of the orbiting $\bar{p}$, which was derived in the Pauli approximation.[3] In this approximation, the fine-structure splitting of a level with principal-quantum number n and orbital angular momentum l is given by

$$\Delta E_{n,l} = (g_0 + 2g_1)\, \frac{(Z\alpha)^4}{2n^3}\, \frac{m}{l(l+1)}\,,$$

where Z is the nuclear charge, α the fine-structure constant, and m is the reduced mass of the hadron-nucleus system. The magnetic moment is composed of two parts: the Dirac moment $g_0 = -1$ for antiprotons and g_1 the anomalous (Pauli) moment.

It has recently been proposed by Pilkuhn[4] that a nuclear-recoil effect would increase the contribution of the Pauli moment to a measured fine-structure splitting. Pilkuhn points out that the factor g_1 should be replaced by $(m/m_{\bar{p}})g_1$ which is approximately a 0.4% effect for $\bar{p}$-U. He claims that the fact that g_1 enters in the combination $g_1/m_{\bar{p}}$ simply reflects the *ad hoc* role of the mass $m_{\bar{p}}$ in the anomalous moment interaction V_p which is discussed below. The corresponding recoil correction to the Dirac moment is of the order $(m/m_{\bar{p}})^2$ and can be neglected. Since the error on the combined value of the $\bar{p}$ magnetic moment from Refs. 1 and 2 is 0.7%, the recoil correction to g_1 is worth noting.

However, one should also note that there are other corrections of the same order which should be included if a complete analysis is to be performed. In our reanalysis we included corrections to the $\bar{p}$-atomic Dirac energy levels arising from the reduced-mass approximation, finite nuclear size, electron screening, nuclear polarization, and corrections to the Pauli moment. All corrections were carried out to second order in perturbation theory. Vacuum-polarization corrections to order $\alpha(Z\alpha)^7$ and the reduced-mass corrections were carried out using the potentials described by Blomqvist.[5] The finite-size correction was made by assuming a uniform nuclear-charge distribution. The electron-screening correction was calculated using the parametrized potential described by Vogel,[6] and the nuclear polarization was calculated following Ericson and Hüfner.[7]

The Pauli-moment term was included as a perturbation using a potential[3,8] $V_p = (g_1/2m_{\bar{p}})\beta\,\vec{\alpha}\cdot\vec{E}$, where $\vec{\alpha}$ and β are the standard Dirac operators, g_1 is the Pauli moment, and $m_{\bar{p}}$ is the antiproton mass. The electric field E is determined from the Coulomb potential. One should note that since this potential is evaluated using Dirac wave functions, the nuclear-recoil correction calculated by Pilkuhn[4] is not automatically included. In order to include the nuclear-recoil effect in the calculation without using the approximation given by Pilkuhn, it would be necessary to use wave functions obtained from a relativistic two-body theory, such as solutions of the Bethe-Salpeter equation, rather than of the Dirac equation, which is a relativistic *one*-body equation.

The least-squares fitting was carried out as described previously.[1] The relative amplitudes of noncircular transitions included were taken from a cascade program as described previously.[1] For the higher transitions, the first and second noncircular fine-structure doublets were included if needed. The five $\bar{p}$ transitions, containing statistically the most significant data: $(n=13\rightarrow n=12)$, $(12\rightarrow 11), (11\rightarrow 10)$ in U and $(12\rightarrow 11), (11\rightarrow 10)$ in Pb, were used in the determination of the magnetic moment. The values obtained for each transition were averaged to obtain the final value. Neglecting the recoil correction to the Pauli moment we obtain $\mu(\bar{p}) = (-2.809\pm 0.048)\mu_N$, where μ_N is the nuclear magneton $e\hbar/2m_pc$. If we include the recoil correction as given by Pilkuhn, we obtain

$$\mu(\bar{p}) = (-2.817\pm 0.048)\mu_N .$$

This value is 0.1% lower than that previously reported[1] by us but in good agreement with both that result and with the *CPT* prediction of $-2.793\mu_N$.

Since the value reported in Ref. 2 for the $\bar{p}$ mass was 1.7 standard deviations below the proton mass, we have also analyzed our $\bar{p}$-Pb x-ray data to determine a value for the $\bar{p}$ mass. The earlier calibration difficulties (see Ref. 1) were overcome by using the e^+ 511-keV line and the Pb $K_{\alpha 1}$ electronic x ray as calibration lines. These lines appeared as background in the $\bar{p}$-Pb x-ray spectra and, because their source was, in part, the Pb shielding, they also appeared in data accumulated with Au in a kaon beam. These lines were not subject to the zero shift encountered using radioactive sources. The linearity of the system under beam conditions was checked from the K^--Au data which were accumulated under exactly the same experimental conditions as the $\bar{p}$-Pb. It was possible to use the kaonic x-ray data as a check because the K^--Au transition energies have been previously measured.[9]

The measured and calculated energies for the $\bar{p}$-Pb, $\Delta n=-1$ transitions from $n=15$ to $n=10$ are given in Table I. The calculated energies were obtained to second order in perturbation theory using hydrogenlike Dirac wave functions as the unperturbed system and the corrections mentioned above as perturbations. The Pauli moment was taken to be equal in magnitude and opposite in sign to that of the proton. The measured energies were obtained by fitting the complex x-ray line shape to a functional form which included the circular doublet and first and second noncircular doublets if necessary. The energy of the spin-up to spin-up circular transition was determined in this manner, and the corresponding mass was computed using the relationship

$$m_{\bar{p}} = \frac{E_m}{E_c}\, m_c ,$$

where E_m was the measured energy, E_c the calculated energy, and m_c the proton mass used in the

TABLE I. Calculated and measured energies for antiprotonic x-ray transitions in Pb. The calculated energies include the Dirac energy and corrections due to vacuum polarization (VP), reduced-mass approximation, finite nuclear size, electron screening, nuclear polarization, and corrections to the Pauli moment (see text). The energies listed are for the lower-energy circular transitions ($l=n-1$, $j=l+\frac{1}{2}$). The uncertainties on the calculated energies are ± 10 eV. The calculated values were obtained using $m_{\bar{p}}=938.2796$, Pauli moment $g_1=-1.792\,845\,6$, and a Pb atomic mass of 207.19 amu. All energies are given in keV. Finite-nuclear-size and strong-interaction corrections contribute less than 1 eV.

Transition $n_i\rightarrow n_f$	Point Coulomb energy	First-order VP	Higher-order VP	Pauli moment	Electron screening	Other corrections[a]	Total calculated energy, E_c	Measured energy, E_m
$15\rightarrow 14$	110.023	0.449	-0.009	-0.060	-0.032	0.000	110.371	110.431 ± 0.110
$14\rightarrow 13$	136.397	0.615	-0.011	-0.094	-0.028	0.001	136.880	136.935 ± 0.067
$13\rightarrow 12$	171.927	0.856	-0.014	-0.149	-0.025	0.002	172.597	172.578 ± 0.060
$12\rightarrow 11$	220.967	1.213	-0.019	-0.247	-0.021	0.004	221.897	221.909 ± 0.060
$11\rightarrow 10$	290.602	1.761	-0.021	-0.429	-0.018	0.008	291.903	291.890 ± 0.061

[a]Nuclear recoil, finite size, nuclear polarization.

TABLE II. Values for the mass of the $\bar{p}$ obtained from the experimentally measured x-ray transition energies [$m_{\bar{p}} = (E_m/E_c)m_c$, see text].

Transition ($n_i \to n_f$)	Antiproton mass (MeV)
$15 \to 14$	938.773 ± 0.939
$14 \to 13$	938.636 ± 0.464
$13 \to 12$	938.182 ± 0.326
$12 \to 11$	938.330 ± 0.254
$11 \to 10$	938.241 ± 0.196
Weighted average	938.30 ± 0.13 MeV

calculation. The error on the mass was determined from the error on the experimental energy taken in quadrature with the error on the calculated energy. This scaling of the mass is valid to within the accuracy of this experiment since the measured energies are very close in value to the calculated ones.

The $\bar{p}$ mass obtained from each of the transitions is listed in Table II. The weighted average is

$$m_{\bar{p}} = 938.30 \pm 0.13 \text{ MeV},$$

which is in good agreement with the proton mass of 938.2796 ± 0.0027 MeV. Because of the large uncertainty on the $\bar{p}$ mass, our result also agrees with the value of Ref. 2, 938.18 ± 0.06 MeV. Using our value of $m_{\bar{p}}$, we obtain the difference

$$m_p - m_{\bar{p}} = -0.02 \pm 0.13 \text{ MeV}.$$

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