

## Even-wave harmonic-oscillator theory of baryonic states. III. Decay analysis of baryon resonances

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A new classification of  $\underline{70}$  states, i.e.,  $(l, u)$  for  $L^P = 1^-$  and  $(l, m, u)$  for  $L^P = 2^+$ , as well as a relativistic formulation of orbital matrix elements for their decays, derived in two earlier papers (I and II of the series), are employed in this paper to make an extensive analysis of various pseudoscalar decay modes in the language of "direct" versus "recoil" quark couplings. A  $\underline{56}$ - $\underline{70}$  octet mixing angle ( $\cot\theta = -\sqrt{2}$ ), which was needed in II to produce several important low-energy fits, is also found to play a crucial role in the present analysis which makes *a priori* quantum assignments for the various states on the basis of their mass positions. By making one exception to the rule of SU(6) unsplit states in this theory, i.e., an upward mass shift of  $l(\underline{8}_q)$  states so as to overlap with their  $u$  counterparts, it is not only possible to understand the earlier successes of this model on the anomaly implicit in  $p\gamma$  and  $n\gamma$  photoproduction of  $D_{13}$ , but also to account for a few other similar anomalies [especially the  $N\bar{K}$  versus  $\Sigma\pi$  mode of  $D_{03}(1830)$ ], none of which are amenable to the usual harmonic-oscillator-type description without elaborate mixing assumptions. On the other hand, most other  $u$ -type states which cause the bulk of the proliferation of the low-lying states in this theory do not pose a serious problem of immediate detection in view of their generally low partial widths compared to their  $l$ -type counterparts.

### I. INTRODUCTION

The analysis of decay data of baryon resonances, in particular their partial decay widths, has long been regarded as an extremely reliable guide to their classification within any symmetry scheme, especially, SU(6)<sub>w</sub> × O(3). It has also been used as a first check on various dynamical theories of couplings. Thus in the language of single quark transitions, decay patterns have helped identify the distinct roles of the so-called direct and recoil terms,<sup>1</sup> and with the extra input of a comprehensive phenomenological model (e.g., the relativistic harmonic-oscillator (h.o) model of Feynman *et al.*<sup>2</sup>) these patterns seem to harmonize with the spectroscopy of states arrived at on the basis of their mass values.

The sustained interest in the h.o. model<sup>2-5</sup> has been primarily due to some of its attractive features, especially linear Regge trajectories and, of course, its general simplicity. However, its inability to provide an adequate understanding of several observational features of the mass spectra motivated a proposal<sup>6</sup> for a modification of the conventional h.o. model so as to suppress the odd partial waves relative to their even counterparts. While this mechanism retains the basically straight-line behavior of (mass)<sup>2</sup> with excitation, it also predicts a ground state of  $(\underline{70}, 0^+)$  which thus lies below the  $(\underline{70}, 1^-)$ . This facilitates (i) an interesting alternative to the conventional<sup>3,5</sup>  $(\underline{56}, 0^+)_2$  for the Roper octet and (ii) a natural candidate for mixing between the  $\underline{8}_d$  members of  $(\underline{56}, 0^+)$  and  $(\underline{70}, 0^+)$ . Indeed, as suggested

some time ago<sup>7</sup> and investigated quantitatively in a recent paper,<sup>8</sup> an ideally mixed nucleon in the even-wave h.o. model<sup>6</sup> is found to harmonize with three important low-energy parameters, i.e., the pion-nucleon coupling constant  $G_{N\pi}$ , the neutron  $\beta$ -decay constant  $-G_A/G_V$ , and the  $P_{33}(1236) \rightarrow N\pi$  width. A  $(\underline{56}$ - $\underline{70})$ -mixed nucleon, but with a smaller mixing angle, has also been suggested by Le Yaouanc *et al.*<sup>9</sup> within the conventional h.o. model, where the  $(\underline{70}, 0^+)$  state belongs to the  $N = 2$  excitation. In the even-wave model on the other hand this very state is significantly depressed in energy so as to appear as an effective ground state of the  $\underline{70}$  series, thus providing a more natural candidate for mixing than in the conventional h.o. model.

The classification of the various baryonic resonances and the matrix elements describing various supermultiplet transitions in the even-wave model have already been dealt with in some detail in I and II, respectively. Its applications to resonance photocouplings,<sup>10</sup> using the simple quark transition picture for magnetic and charge couplings, has yielded results in good accord with the latest data,<sup>11</sup> including some "difficult" cases<sup>2</sup> such as  $P_{11}$ ,  $D_{15}(p\gamma)$ , and  $F_{35}$ , which are unexplained in the conventional h.o. model and which seem to tally with some special feature of the orbital classification of states brought about by the even-wave model.<sup>6</sup>

It is the purpose of this paper to present a systematic analysis of the pseudoscalar partial widths predicted by the even-wave theory in relation to the different states involved and discuss their rel-

evance to experiment. In doing so we shall find it necessary to take account of the following aspects of the even-wave model (EWM) in relation to the data:

(a) What are the effects of the quantum classification ( $l, u$ , etc.) of the various baryon resonances on the various partial decay widths? Since the model is primarily concerned with orbital degrees of freedom, we shall have to go beyond the traditional SU(6) content of various states in this analysis.

(b) What experimental indications are available on the apparent proliferation of low-lying states predicted by the EWM and where are the latter located? Since it is not within the scope of this paper to offer any theory of spin, etc. splittings, we shall merely use general considerations like distinguishable decay patterns (preferably without mass shifts) for the location of these states. We shall see that certain photoproduction and hadronic decay modes of a *complementary* nature are useful tools for such identification.

(c) A general feature of the EWM lies in the uniformly low partial widths predicted for the  $u$  states because of the significantly smaller overlaps with the ground states in these cases. Yet this aspect of the model is important because it enables us to account for certain decays (e.g.,  $D_{05} - N\bar{K}$ ) which appear impossible to explain otherwise. We shall show that, in conjunction with (a) and (b), this feature enables us to estimate some mass shifts from the data.

While the detailed arguments are discussed in the sections to follow, it is useful to draw attention to some of our main conclusions:

(i) There are strong indications from photoproduction and hadronic decay data that even for extended angular momentum states which should be more easily visible than their  $J$  satellites, especially spin quartets, several resonance effects observed via phase-shift analysis are amenable to interpretation as two or more overlapping states with complementary experimental signatures. For example, we predict two distinct  $D_{15}$  states at 1670 MeV, one of which ( $D_{15}^I$ ) is photoproduced via  $n\gamma$  only, and the other ( $D_{15}^u$ ) via  $p\gamma$  only, without violating the Moorhouse rule,<sup>12</sup> which applies only to the latter. Similarly we predict two  $D_{05}$  states near 1830 MeV of which  $D_{05}^u$  decays via  $N\bar{K}$  (and not  $\Sigma\pi$ ) and  $D_{05}^I$  decays via  $\Sigma\pi$  (and not  $N\bar{K}$ ).

(ii) Recalling that the EWM proposed in I was a theory of *orbital splitting only* and that for  $(70, 1^-)l$  vs  $u$  states the mass difference was of the order of 0.37 GeV<sup>2</sup> the decay data effectively implies an upward mass shift of  $l(q)$  states with respect to  $l(d)$  states, so as to push the latter up to the mass of a  $u(q)$  state. No mass shift of such magnitude

seems to be required for any other state, including  $u$  as well as  $L \geq 2$  states and radial excitations, in conformity with our general philosophy of unsplit states due to spin, so that  $l(q)$  states are more an exception than the rule.

(iii) A similar conclusion on trajectories is indicated despite the 70-56 mixing hypothesis for nucleons. Basically we assume the trajectories, including the higher recurrences, to be unmixed, but an exception is made empirically for the lowest member to meet the compulsions of some low-energy data ( $G_A/G_V$ , etc.).<sup>8</sup>

In carrying through this analysis of the  $(L \pm 1)$ -wave decays, we shall follow the customary language of direct and recoil couplings with separate reduced coupling constants.<sup>1</sup> Of course for the  $(L-1)$ -wave modes it is still an open question as to whether the interference between the direct and recoil terms is constructive or not.<sup>13,14</sup> Our numerical results favor the former at least for the  $1^-$  states.

There are two free parameters in this analysis, namely the strengths of the direct and recoil couplings. The former has been fixed from the  $L=0$  decay  $P_{33}(1236) - N\pi$  and works out to

$$c^{(+2)} = 4\pi \times 3.5431. \quad (1)$$

The ratio ( $\rho$ ) of the recoil to direct couplings has been estimated from  $S$ -wave decays to work out to

$$\rho^2 \equiv c^{(-2)}:c^{(+2)} = 0.213. \quad (2)$$

For easy reference we display one explicit example of an entire coupling Lagrangian involving both direct and recoil terms, say  $S_{11}(1535) - N\pi$ , using the phase conventions and normalizations given in II, and in the notation of Dirac spinors given elsewhere<sup>15</sup>:

$$\begin{aligned} \mathcal{L}(\bar{N}_+ \pi N_-) = & \frac{2\sqrt{2}}{3} c^{(+)} \left( \frac{2\tau}{\sqrt{3}\epsilon} \right)^3 f_1 \left( \frac{4i}{3} \right) \\ & \times \bar{\psi}^{(+)} \left[ \frac{k^2}{\sqrt{3}} \gamma_p + \sqrt{3} \left( \frac{1}{2} \rho \right) (M-m) \right] \psi^{(-)}. \quad (3) \end{aligned}$$

We shall also display below the orbital overlaps for the  $(56, 3^-) - (56, 0^+)$  and  $(70, 3^-) - (56, 0^+)$  transitions, which were not listed in II but are required for some of the cases analyzed in this paper (see Sec. IV).

$(56, 3^-) - (56, 0^+)$ :

$$(i/\sqrt{3}) k_{i_1} k_{i_2} k_{i_3} (\sigma \cdot k) \gamma_p^3 f_0; \quad (4)$$

$(70, 3^-) - (70, 0^+)$

$$\begin{aligned} \text{direct: } & (2/3\epsilon)^{1/2} (2\tau/3)^3 f_1 (\sigma \cdot k) k_{i_1} k_{i_2} k_{i_3} \gamma_p^3, \\ \text{recoil: } & \rho (7/2\epsilon)^{1/2} (2\tau/3)^3 f_1 k_{i_1} k_{i_2} \gamma_p^3. \end{aligned} \quad (5)$$

The outline of this paper is as follows. In keep-

ing with the nature of the program, i.e., to check the even-wave assignments ( $l$  or  $u$ ) from a study of baryonic decays, we shall display our results for *both* ( $L \pm 1$ )-wave decays *together* in the numerical tables. In Sec. II we propose to argue qualitatively (anticipating, of course, our results presented later) that the generally depressed spectrum of  $\underline{70}$  states in the EWM, leading to an effective proliferation of low-lying states, opens up a fresh approach to baryon spectroscopy. We also indicate with the help of a "level diagram," the important experimental indications that could enable us to distinguish these states and their mass levels. In Secs. III and IV we discuss the results obtained in Tables I, II and III, IV for the even- and odd-parity resonances, respectively, drawing attention to some characteristic features of the EWM in relation to the suggested SU(6) assignments. In particular, Table IV contains most of the results that we shall need for our discussion in Sec. II. Comparison with the standard h.o. results, as prototyped by FKR, will be given in Secs. III and IV in the appropriate contexts.

The  $\Lambda$  states of the ( $\underline{70}, 1^-$ ) have been separately discussed in Sec. V to highlight the inadequacy of present thinking in the standard h.o. model on the SU(3)  $\times$  SU(2) assignments for these states, and to suggest a plausible reassignment within the context of the EWM. Finally Sec. VI summarizes some of the main conclusions of this paper.

## II. GENERAL CONSIDERATIONS ON $l$ VERSUS $u$ STATES

From the physical point of view the main burden on this model is in accounting for the apparent clustering of low-lying states in relation the observed resonances that have been reported through various phase-shift analyses in the Particle Data Group tables,<sup>16</sup> but whose picture in the conventional HO theory leaves several cases unaccounted for. We would like to illustrate the nature of the problems involved in the EWM alternative, as well as the possibilities of solution, with the help of a few specific examples. Let us consider the  $D_{13}, D_{15}$  states, analyzed in I as  $l$  and  $u$ , respectively, each of which has a doublet ( $d$ ) and a quartet ( $q$ ) assignment. To a first approximation we could assume that spin-orbit, spin-spin effects, etc., are small so that, e.g.,  $D_{13}^l(q)$  and  $D_{15}^u(d)$  should lie close to  $D_{13}(1520)$  and  $D_{15}(1670)$ , respectively, in accordance with the principle of *average* mass assignments for ( $l, u$ ) states.<sup>6</sup> The question now arises: Do the available data on decays support this point of view, and if so, how far?

We first consider photoproduction. The experimental observation of  $D_{15}(p\gamma)$  and  $D_{15}(n\gamma)$  tells us

that there are two distinct states which can be identified as  $u(q)$  and  $l(q)$ , respectively, in this model for the following reasons:

(i) The Moorhouse selection rule<sup>12</sup> says that  $D_{15}(p\gamma)$  and  $D_{15}(n\gamma)$  have zero and nonzero amplitudes, respectively. This checks with the assignment of  $D_{15}$  to an  $l(q)$  state, remembering that the nonzero overlap comes from the  $\underline{56}$  component of the nucleon in this case.<sup>8</sup>

(ii) If on the other hand  $D_{15}$  is regarded as a  $u$  state, then  $D_{15}(p\gamma)$  becomes visible because of its nonzero overlap with the  $\underline{70}$  component of the nucleon.<sup>8</sup> However, this matrix element is proportional to  $(1 + \tau_3)$  so that this state cannot be photoproduced<sup>10</sup> from  $n\gamma$ . Apparently *both* states are needed to explain *both*  $n\gamma$  and  $p\gamma$  photoproduction in this region. Note that the usual h.o. model predicts only one state here and is therefore unable to explain  $p\gamma$  photoproduction without a nontrivial mixing hypothesis with higher supermultiplets ( $L=3$ ?). In the EWM description, however, there are indeed *two* states in this mass region, so that in principle there is no difficulty in explaining *both*  $n\gamma$  and  $p\gamma$  photoproduction. However, the  $l(q)$  state would have to lie somewhat lower in mass than the  $u(q)$  state without some extra assumption on mass shifts since the theory proposed in I is basically one of orbital ( $l, u$ ) splitting only, and we are not *a priori* entitled in this model to make further splitting assumptions. However, if we make one single exception to this rule, namely an *ad hoc* upward mass shift of only  $l(q)$  states by an amount roughly comparable to the  $l-u$  (mass)<sup>2</sup> splitting, i.e.,  $\alpha(\sqrt{3}-1)/2 \approx 0.31 \text{ GeV}^2$ , for which we do not offer any theory, then we are able to explain in a very simple way the occurrence of  $D_{15}(p\gamma)$  and  $D_{15}(n\gamma)$  photoproduction.<sup>10</sup>

We also find very similar anomalies in some  $N\bar{K}$  vs  $\Sigma\pi$  decays of  $D_{05}(1830)$  since the  $l$  counterpart of this state (which is the closest analog of the  $D_{05}$  state of the usual h.o. theory<sup>2</sup>) predicts zero for  $N\bar{K}$ , in qualitative disagreement with experiment. However, in the present model, there is a natural facility for a nonzero  $N\bar{K}$  mode, via the  $u$  assignment to this state. Curiously, the  $\Sigma\pi$  mode of the latter turns out to be zero, thus bringing out once again the need for both  $l(q)$  and  $u(q)$  states in this region.

As in the  $D_{15}$  case, the same assumption of an upward mass shift of  $l(q)$  seems to reconcile both these modes without any further mixing assumptions at the supermultiplet level (which the usual theory is unable to avoid). In the case of the  $D_{25}(1765)$  for which the  $l$  assignment alone seems to suffice, a similar assumption of an  $l(q)$  mass shift with respect to the  $D_{23}(1670)$  helps reconcile its quantum status with that of a  $u$  state, which is

the appropriate description for  $D_{25}(1765)$  in this model.<sup>6</sup> The corresponding decay predictions of the associated  $u$  state are too low to be easily observable in the more prominent  $l$  background.

The foregoing examples illustrate the need for an extra assumption of an  $l(q)$  mass shift as a *single exception* to the basic philosophy of unsplit states in this model. What we are really saying, in other words, is that states like  $D_{15}$ ,  $D_{25}$ , etc. observationally appear like one, but are in reality two states designated in the EWM as  $D_{15}^l(q)$  and  $D_{15}^u(q)$ , both of which are situated close together in mass. The more important thing to recognize, irrespective of the merits of this model, is that the above anomalies are not easily accountable within the conventional h.o.-type description which needs mixing (understandably) with a higher supermultiplet. In our model this has already been brought about through the *exchange component* of the Serber-type  $qq$  interaction,<sup>6</sup> which effectively brings down some of the high- $N$  *cum* low- $L$  states to the level of the observed region of resonances.<sup>8</sup>

We shall continue to adhere to basically unsplit states for other quantum numbers  $l(d)$ ,  $u(d)$ , and  $u(q)$ . In a similar spirit we shall ignore splittings or mass shifts of  $L \geq 2$  resonances as well as those of the radially excited states. The last assumption is in conformity with the unsplit character of  $u(d)$  and  $u(q)$  states which really belong to the  $N=3$  excitation in the HO language.

The  $F_{35}(1890)$  is an example of an interplay of a  $(70, 2^+)$  state out of three such states  $(l, m, u)$  predicted by the EWM,<sup>6</sup> with its more familiar  $(56, 2^+)$  assignment.<sup>2,5</sup> The latter accounts for its hadronic (especially  $N\pi$ ) widths, which are predicted to be too small in terms of the former assignment. On the other hand, the photoproduction data strongly favor a  $(70, 2^+)_m$  assignment.<sup>10</sup> Note that a  $(70, 2^+)$  in this vicinity is also indicated by considerations more general<sup>2,5</sup> than the EWM, and its near degeneracy to the  $(56, 2^+)$  need not give the illusion of a single state. Rather, their diverse modes of couplings in various strengths should help distinguish between their more detailed quantum members. Now the EWM predicts two more  $(70, 2^+)$  states  $(l, u)$ ,<sup>6</sup> but most of their modes are too small to be easily detectable.

As a natural corollary to the philosophy of basically unsplit states, we shall keep the Regge recurrences of the nucleon octet unmixed. Thus the trajectory itself is pure except for the ground state, the mixing in the latter case being motivated by certain low-energy compulsions<sup>7,8</sup> that need not necessarily linger over the entire trajectory. (This assumption, which seems to check with  $L=2$  data, does not bear out a conjecture made in II on the persistence of mixing for the

entire  $N$  trajectory.)

The foregoing discussion represents our basic strategy to reconcile the problem of clustering of low-lying states in the EWM with the ones actually observed. As indicated in above arguments, this should suffice to a large extent in accounting for at least the "stretched" angular momentum states, e.g.,  $\frac{5}{2}^-$ , which already seem to pose non-trivial problems of baryon spectroscopy. Their  $J$  partners are generally expected to be less observable, unless certain modes are particularly large, as in the case of  $S_{11}(1535) \rightarrow N\eta$  and  $S_{11}(1700) \rightarrow N\pi$ . We would like to conclude this section on an optimistic note by observing that the only assumption of an  $l(q)$  mass shift as a single exception to our general rule against SU(6) mass mixing facilitates an easy resolution of several contradictory modes of decay, and hence makes a strong *a priori* case for its experimental test, possibly through a more careful search for the "dual partners" of some of the better-established quartet states.

### III. EVEN-PARITY DECAYS

The results for the even-parity resonances are collected in Tables I and II for the unmixed and mixed cases, respectively. The former fall into two broad categories:

(a) The  $\underline{56}$  states, the mass positions and form of the hadronic matrix elements of which remain unchanged in the EWM relative to the FKR framework.

(b) The  $\underline{70}$  states, for which there are few new states whose positions in the EWM are indicated in Fig. 1. Table II contains the decay systematics of the Roper octet consisting of  $P_{11}(1470)$  and  $P_{21}(1620)$  as the mixture  $-\sin\theta|56, 0^+\rangle + \cos\theta|70, 0^+\rangle$  so as to make it orthogonal to the nucleon octet.

We shall be interested not only in verifying the quantum assignments  $(l, m, u)$  for the higher  $\underline{70}$  states, but also to compare, for some important cases, our results with those obtained by FKR. The direct coupling governing these matrix elements is computed using the  $P_{33}(1236) \rightarrow N\pi$  as input (110 MeV) and is given in Eq. (1).

Let us consider the  $\underline{56}$  states first. The overall pattern of theoretical predictions is quite impressive when evaluated with the experimental data. This is especially true of  $P_{23}(1385)$ ,  $F_{37}(1950) \rightarrow (\Sigma K, \Delta\pi)$ ,  $F_{27}(2030)$ ,  $F_{05}(1815)$ ,  $F_{25}(1915)$ , and  $F_{15}(1688)$ . Apart from some mismatches as in  $\Xi_{13}^*(1530) \rightarrow \Xi\pi$ ,  $F_{37}(1950) \rightarrow N\pi$ , and  $F_{05}(1815) \rightarrow N\bar{K}$ , our results more or less tally with those obtained by FKR. The differences can be ascribed to the following factors: (a) the argument of FKR's exponential factor is an *ad hoc* structure unrelated

TABLE I. Partial widths of the even-parity resonances predicted by the EWM. The full h.o. predictions prototyped by FKR are included for comparison only.

Decay	(SU(6), L <sup>P</sup> ) <sub>N</sub>	Partial width in MeV		
		Expt.	EWM	FKR
$P_{33}(1236) \rightarrow N\pi$	(56, 0 <sup>+</sup> )	113	Input	94
$P_{23}(1385) \rightarrow \Lambda\pi$		32	42	35
$\rightarrow \Sigma\pi$		6	6	4
$P_{13}(1530) \rightarrow \Xi\pi$		9	17	12
$F_{15}(1690) \rightarrow N\pi$	(56, 2 <sup>+</sup> )	84	50	64
$\rightarrow N\eta$		<0.4	0.3	0.27
$\rightarrow \Lambda K$		?	0.09	0.07
$\rightarrow \Delta\pi$		16	22	
$F_{05}(1815) \rightarrow N\bar{K}$		~51	27	35
$\rightarrow \Sigma\pi$		~10	12	13
$\rightarrow \Sigma^*\pi$		13-17	15	
$F_{25}(1915) \rightarrow N\bar{K}$		5-15	3	3
$\rightarrow \Lambda\pi$		20	14	15
$\rightarrow \Sigma\pi$		?	22	24
$F_{35}(1890) \rightarrow N\pi$		~38	60	
$\rightarrow \Sigma K$		<8	3	
$F_{37}(1950) \rightarrow N\pi$		~88	42	109
$\rightarrow \Sigma K$		<2	4	7
$\rightarrow \Delta\pi$		~44	63	
$F_{27}(2030) \rightarrow N\bar{K}$		~36	21	28
$\rightarrow \Lambda\pi$		~36	18	37
$\rightarrow \Sigma\pi$		9-18	11	17
$P_{31}(1910) \rightarrow N\pi$	(56, 2 <sup>+</sup> ) <sub>2</sub>	30-70	152	151
$\rightarrow \Sigma K$		4-40	54	59
$P_{11}(1780) \rightarrow N\pi$	(70, 0 <sup>+</sup> ) <sub>2</sub>	~40	10	0.5
$\rightarrow \Lambda K$		~14	0.3	
$\rightarrow N\eta$		4-40	1	
$P_{01}(1750) \rightarrow N\bar{K}$			3	
$\rightarrow \Sigma\pi$			2	
$P_{31}(1910) \rightarrow N\pi$		30-70	2	
$\rightarrow \Sigma K$		4-40	0.2	
$P_{13}(1810) \rightarrow N\pi$	(70, 2 <sup>+</sup> ; $\underline{l}$ ) <sub>2</sub>	~40	58	75
$\rightarrow \Lambda K$		~10	16	22
$\rightarrow N\eta$		<10	17	32
$\bar{F}_{15}(1860) \rightarrow N\pi$	(70, 2 <sup>+</sup> ; $m$ ) <sub>2</sub>		11	
$\rightarrow N\eta$			0	
$F_{35}(1890) \rightarrow N\pi$	(70, 2 <sup>+</sup> ; $m$ ) <sub>2</sub>	~38	2	
$\rightarrow \Sigma K$		<8	0.1	
$F_{17}(1990) \rightarrow N\pi$	(70, 2 <sup>+</sup> ; $u$ ) <sub>2</sub>	2	15	
$\rightarrow N\eta$			0.1	

to their input dynamics, unlike in our case, and (b) the neglect in FKR of the Licht-Pagnamenta-type Lorentz-contraction effects on the multiple derivative coupling structures. Both these features are crucially connected with the better fits obtained by FKR for some cases, especially  $F_{37}(1950) \rightarrow N\pi$  and  $\Xi_{13}^*(1530) \rightarrow \Xi\pi$ . The  $F_{15}$ ,  $F_{25}$ , and  $F_{05}$  are well represented in our calculation as pure 56 states, while a continuation of the 56-70 mixing hypothesis to the Regge recurrences would

TABLE II. Partial widths of the Roper octet which is taken as a mixture of the octets of (56, 0<sup>+</sup>) and (70, 0<sup>+</sup>) with  $\cot\theta = -\sqrt{2}$ . In the FKR calculation the Roper octet belongs to (56, 0<sup>+</sup>)<sub>2</sub>.

Decay	(SU(6), L <sup>P</sup> )	Partial width in MeV		
		Expt.	EWM	FKR
$P_{11}(1470) \rightarrow N\pi$		120	466	8
$\rightarrow \Delta\pi$		38	93	
$P_{21}(1620) \rightarrow N\bar{K}$	(70, 0 <sup>+</sup> )		48	
$\rightarrow \Lambda\pi$			103	
$\rightarrow \Sigma\pi$			54	

have suppressed the latter widths by factor 3-4 from the quoted figures. The 56 assignments are also supported by the  $N\pi$  and  $\Sigma K$  modes of  $F_{35}(1890)$  and  $P_{31}(1910)$ , respectively. However, these states have also been considered with alternative 70 assignments (see below) to check the corresponding decay predictions of the EWM for some relevant (70, 0<sup>+</sup>)<sub>2</sub> and (70, 2<sup>+</sup>) states in this mass region.

Let us consider the 70 states. Some of the well-established resonances here are  $P'_{11}(1780)$ ,  $P_{13}(1810)$ , and  $F_{17}(1990)$ , the last with a two-star rating. The EWM predicts the  $P_{11}(1780)$  as a radially excited (70, 0<sup>+</sup>), some of whose other members are conceivably  $P_{31}(1910)$  and  $P_{01}(1750)$ . Our predicted width of ~10 MeV for  $P'_{11} \rightarrow N\pi$  falls rather short of the data. However, the latter are generally ambiguous, e.g., the mass of  $P_{11}(1780)$  ranges from 1.7 to 1.8 GeV and its  $N\pi$  width varies from 20 MeV to 50 MeV. While a clean judgement on the widths is clearly difficult, one of the important triumphs of the EWM over the usual h.o. model lies in the predicted mass values of these states. For a more meaningful test of the EWM, it would be desirable to make a better determination of these parameters ( $M_L, \Gamma$ ) of the states in the relevant mass region. However, notice that the small widths, e.g.,  $N\pi$  of  $P_{31}(1910)$  under this model would necessarily make the detection of such states somewhat more difficult, thus, e.g., enhancing the visibility of the more prominent  $P_{31}(1910)$  member of (56, 2<sup>+</sup>) in relation to its (70, 0<sup>+</sup>)<sub>2</sub> counterparts.

Among the (70, 2<sup>+</sup>) listed in Table I, the  $F_{17}(1990)$ ,  $F_{15}(1860)$ , and  $P_{13}(1810)$  belong to the  $u$ ,  $m$ , and  $l$  quantum classifications, respectively.<sup>6</sup> The general tendency<sup>8</sup> for  $u$ - (and  $m$ -) type states to predict small partial widths (compared to  $l$  states), as already explained in Sec. II,<sup>8</sup> should make their observation difficult. This accounts for the lack of easy observation of these two states as well as the  $F_{35}(1890)$ . Indeed, the latter can show up only via  $p\gamma$  photoproduction because of the convective cur-

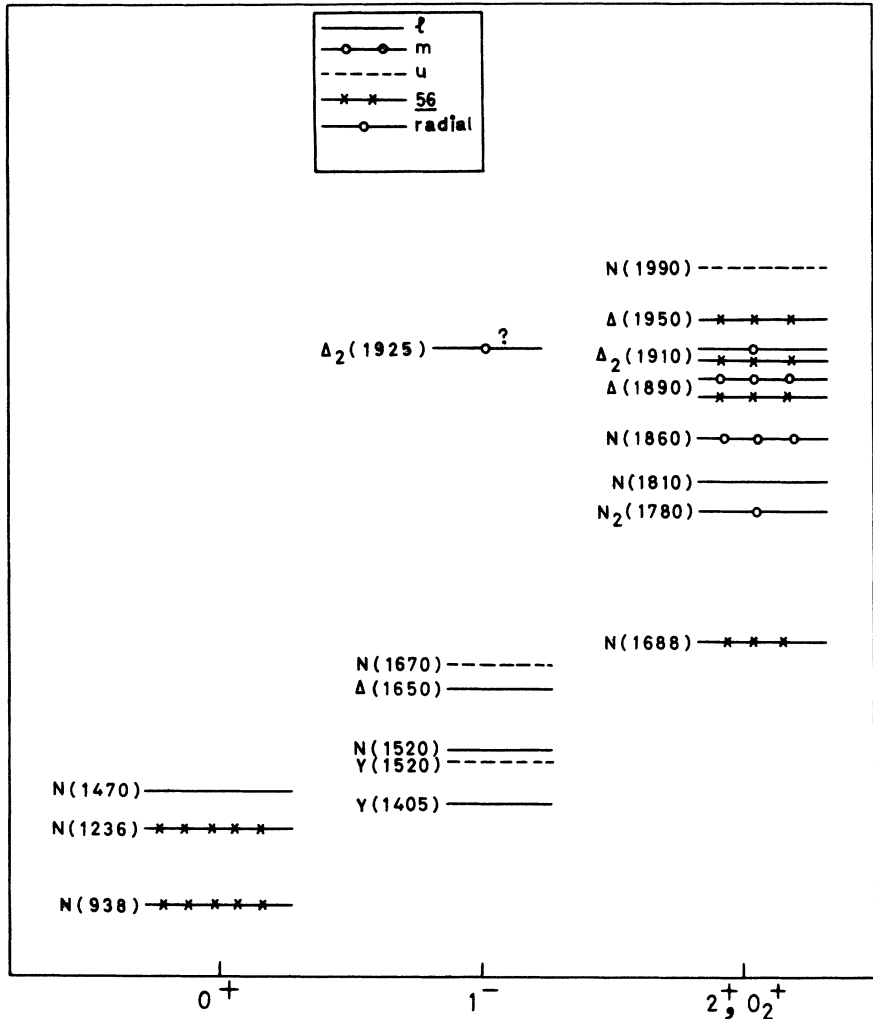


FIG. 1. Schematic representation (not to scale) of the quantum assignments ( $l, m, u$ ) for various  $70$  resonances in the EWM. A few clear  $56$  states have been included for comparison. The subscript 2 on some resonances indicates their radial status in this model. The SU(3) contents, namely 1, 8, and 10, are understood from  $Y, N$ , and  $\Delta$ , respectively.

rent available here,<sup>10</sup> and could more hopefully be looked for in such experiments. In terms of mass, the  $P_{13}(1810)$  fits in rather well as an  $l$  state ( $P_{13}^2 - P_{11}^2 \approx 1 \text{ GeV}^2$ ) and its width also checks with experiment (with the *opposite* interference between recoil and direct couplings). Its  $m$  counterpart  $P_{13}(1860)$  would again be less easy to observe, because of small  $70$ - $70$  overlap effects.<sup>8</sup> As to the results of the usual h.o. model, prototyped by FKR, we offer the following comments:

(a) Our  $2^+$  states with two units of  $l$  excitation are the nearest counterpart of the  $2^+$  states in the FKR model. However, neither  $F_{15}^u$  nor  $F_{35}^m(1890)$  are predicted as components of the  $(70, 2^+)$  in that theory. It is probable that  $F_{15}$  as a  $(70, 2^+)$  was not considered in FKR, presumably because of its small  $N\pi$  width as in our case. However, the

$F_{35}^m$  is an entirely new state in our model. Its small hadronic widths, e.g.,  $N\pi$  will no doubt tend to obscure its visibility from the more prominent  $F_{35}$  member of  $(56, 2^+)$ . Nevertheless, its supermultiplet assignment  $(70, 2^+)_m$  seems to check rather well with resonance photocoupling data,<sup>10</sup> unlike the  $56 F_{35}$ , thus lending some credibility to its formal existence (see also Sec. II).

(b) Though the  $P_{13}$  partial widths have been calculated by FKR with a  $(70, 2^+)$  assignment, our  $l$  assignment for this state seems to check somewhat better with the data, while its  $\bar{F}_{15}$  counterpart has a very small  $N\pi$  width and hence causes no further problems.

Finally, let us consider the results for the Roper octet given in Table II. The effect of mixing on  $P_{11}(1470)$  is clearly felt through the more reason-

able order of magnitude ( $\approx 466$  MeV) of its  $N\pi$  width, compared to the unacceptable 8 MeV predicted by FKR with the  $(56, 0^+)_2$  assignment. The mode also checks in order of magnitude, but both  $N\pi$  and  $\Delta\pi$  indicate a smaller mixing angle<sup>9</sup> than assumed in II. For the  $P_{21}$  and  $P_{23}$  resonances, however, the available partial widths are too tentative to warrant a clean comparison with experiment.

In conclusion, we believe that the results obtained in this section for the even-parity resonances emphasize the following main aspects:

(i) The need to make a cleaner determination of the masses and widths of states such as (a)  $P_{11}(1780)$  and its partners  $P_{31}(1910)$ ,  $P_{01}(1750)$  belonging to the radially excited  $(70, 0^+)$  and (b)  $F_{35}(1890)$ ,  $F_{15}(1860)$ , and  $F_{17}(1990)$ , each of which has a sufficiently small hadronic width so as to make its observation difficult. In particular, an experimental detection of the different modes of manifestation of an apparently single  $F_{35}(1890)$ , viz., via  $p\gamma$  photoproduction for a  $(70, 2^+)$  and via  $N\pi$  and  $\Delta\pi$  modes for a  $(56, 2^+)$ , as already explained in Sec. II, should help to discriminate between these resonances.

(ii) A gradual disappearance of mixing effects as one goes up the Regge trajectories of the nucleon and Roper octets, as a plausible ansatz providing agreement with the observed partial widths for  $F_{15}$ -type states.

TABLE III. Partial widths predicted by the EWM for the  $l$ -type states of  $(70, 1^-)$  which are the nearest analogs to that of the full h.o. model. Both  $S$  and  $D$  wave decays have been included.

Decay	$(SU(6), L^P)_N$	Partial width in MeV		
		Expt.	EWM	FKR
$D_{13}(1520) \rightarrow N\pi$	$(70, 1^-)_1$	$\sim 70$	73	102
$\rightarrow \Delta\pi$		$\sim 31$	33	
$D_{23}(1670) \rightarrow N\bar{K}$		5-13	2	3
$\rightarrow \Lambda\pi$		$< 10$	4	6
$\rightarrow \Sigma\pi$		10-30	41	49
$D_{33}(1670) \rightarrow N\pi$		$\sim 30$	20	30
$\rightarrow \Delta\pi$		$\sim 90$	87	
$S_{11}(1535) \rightarrow N\pi$		$\sim 30$	66	220
$\rightarrow N\eta$		$\sim 65$	39	71
$\rightarrow \Delta\pi$		$\sim 1$	2	
$S_{31}(1650) \rightarrow N\pi$		$\sim 49$	10	25
$\rightarrow \Delta\pi$		$\sim 70$	37	
$D_{23}(1940) \rightarrow N\bar{K}$	$(70, 1^-)_3$	$> 46$	7	
$\rightarrow \Lambda\pi$		$\sim 9$	10	
$\rightarrow \Sigma\pi$		$\sim 15$	9	
$D_{33}(1925) \rightarrow N\pi$		$\sim 40$	28	

#### IV. ODD-PARITY DECAYS (EXCEPT $\Lambda$ STATES OF $70$ )

The odd-parity resonances we shall consider in this paper (Tables III, IV) belong to the  $(70, 1^-)$ ,  $(56, 3^-)$ , and  $(70, 3^-)$  supermultiplets, typical members of which are  $D_{13}(1520)$ ,  $G_{07}(2100)$ , and  $D_{35}(1925)$ , respectively, on the basis of their mass positions. We have two purposes to achieve in this section:

(a) Check the  $(l, u)$  quantum assignments already inferred in II for the various states through their mass levels. As much as possible we shall keep to the usual SU(6) assignments for these states except where alternatives appear much more convincing.

(b) Examine the decay prediction for the less-established states like  $D_{35}(1925)$ , the supermultiplet assignments for which have again been arrived at on the basis of their masses in the even-wave model. These predictions are compared and commented upon in relation to the available data.

As to the  $\Lambda$  states of  $L^P = 1^-$ , we have found it necessary to consider all of them except  $G_{07}(2100)$  separately in Sec. V because of some special features of these resonances associated with wide mass splittings which are not fully resolved.

Let us consider the  $(70, 1^-)$  states listed in Table IV, all of which are  $l$ -types states in the EWM and are the nearest analogs of the  $L = 1$

TABLE IV. Partial widths predicted by the EWM for the  $u$ -type states, which have no analogs in the full h.o. model. The calculated partial widths for both  $l$  ( $q$ ) and  $u$  ( $q$ ) assignments have been displayed (see Sec. II of the text). For the  $S$  states, the numbers in parentheses denote the  $u$  ( $d$ ) predictions.

Decay	Expt.	Partial width in MeV		
		$l$	$u$	FKR
$D_{15}(1670) \rightarrow N\pi$	$\sim 70$	12	12	36
$\rightarrow N\eta$	$< 0.8$	4	4	7
$\rightarrow \Delta\pi$	$\sim 78$	50	0	
$D_{25}(1765) \rightarrow N\bar{K}$	$\sim 43$	31	4	66
$\rightarrow \Lambda\pi$	$\sim 18$	12	0	25
$D_{25}(1765) \rightarrow \Sigma\pi$	$\sim 1$	6	17	
$D_{05}(1830) \rightarrow N\bar{K}$	$< 10$	0	19	0
$\rightarrow \Sigma\pi$	33-71	35	0	73
$\rightarrow \Lambda\eta$	$< 4$	5	0.2	6
$S_{11}(1760) \rightarrow N\pi$	$\sim 83$	25	5 (13)	45
$\rightarrow \Lambda K$	$\sim 6$	0	14 (10)	
$\rightarrow \Sigma K$	$\sim 3$	114	7 (5)	
$\rightarrow N\eta$		88	29 (0)	112
$\rightarrow \Delta\pi$	$\sim 6$	23	0	
$S_{21}(1750) \rightarrow N\bar{K}$	8-30	39	10 (7)	14
$\rightarrow \Lambda\pi$	4-15	25	0	9
$\rightarrow \Sigma\eta$	$< 6$	15	0.5 ( $\sim 0$ )	
$\rightarrow \Sigma\pi$	12-42	9	11 (2)	4

states in the FKR (usual h.o.) model. These consist of the decays of the  $D$ - and  $S$ -type states, of which only the latter receives comparable contributions from "direct" plus "recoil" couplings, while the former goes entirely via the direct term. Since the strength of the direct coupling has already been estimated [see Eq. (1)] from  $L=0$  decays, the only free parameter is the ratio of the "recoil" to "direct" coupling strengths, the magnitude of which [see Eq. (2)] has been adjusted to obtain optimal agreement with experiment for important decays like  $D_{13}(1520) \rightarrow \Delta\pi$ ,  $D_{33}(1670) \rightarrow \Delta\pi$ , and  $S_{11}(1535) \rightarrow N\pi$ .

*l-type D states.* The clean cases here are  $D_{13}(1520)$ ,  $D_{15}(1670)$ , and  $D_{33}(1670)$ , the partial widths of which (in both  $S$ - and  $D$ -waves) compare very well with experiment, thus confirming the  $l$  assignment for these states in the even-wave model. (Note that a  $\Delta\pi$  width for  $D_{13}$  and  $D_{33}$  would vanish with a  $u$  assignment.) We have also included in our results the  $D_{23}(1940)$ , the mass of which fits in excellently as an  $l$ -type *radially* excited<sup>17</sup>  $D_{23}(1670)$  in the EWM and is not easy to explain in the usual h.o. model (because of its high mass for  $N=1$  excitation, but too low for  $N=3$ ). While its  $N\bar{K}$  mode is merely comparable within the experimental limits, its other partial widths check very well with our assignment for this resonance.

*l-type S states.* The only clean cases available are  $S_{11}(1535)$  and  $S_{31}(1650)$ . From Table IV we see that our results for  $S_{11}(1535) \rightarrow N\pi$ ,  $N\eta$  compare excellently with experiment (in contrast to FKR). For both these decays the Gell-Mann-Oakes-Renner (GMOR) factor of  $(M_L - m)$  associated with the recoil term has been useful in bringing about better fits to the data. The somewhat smaller figure obtained for  $S_{31}(1650) \rightarrow N\pi$  is the price of destructive interference between the direct and recoil terms, which, however, gives better results for most other cases. On the other hand the  $D$ -wave  $\Delta\pi$  decays of  $S_{31}(1650)$  and  $S_{11}(1535)$  again check very well with the  $l$  assignments for these resonances.

$G_{07}(2100)$ . The mass of this state suggests that it could easily be regarded in this model as the  $L=3$  Regge recurrence of the  $\Lambda(1116)$  with a  $(\underline{56}, 3^-)$  assignment. This may be contrasted with the standard interpretation of this state<sup>15</sup> as the first Regge recurrence of the  $D_{05}(1520)$ , belonging to the  $(\underline{70}, 3^-)$ . We cannot afford to keep this latter assignment in the EWM because the mass of this resonance would be predicted too low with three  $l$  excitations (or even *one*  $l$  and *two*  $u$  excitations). Moreover, this assignment also leads to negligible partial widths, so that the corresponding states will be hardly observable. With the  $\underline{56}$  assignment, the decay ratio  $N\bar{K}/\Sigma\pi$  is of course the same as

would be calculated in the conventional HO model and works out to  $\sim 2.5$ , which is in much less disagreement with the experimental value of  $\sim 5$  than the  $(\underline{70}, 3^-)$  assignment which yields  $\sim 1$  for this ratio.

$D_{35}(1935)$ . In a recent analysis of the resonant states in the  $\sim 1900$ -MeV region, Cutkosky *et al.*<sup>18</sup> report on a clear evidence for a  $D_{35}(1925)$  with a rather prominent  $N\pi$  width of about 40 MeV. We offer two possible interpretations of this resonance in the EWM, both of which have been arrived at on the basis of its mass position:

(i) as a Regge recurrence of  $S_{31}(1650)$ , belonging to  $(\underline{70}, 3^-)$ ,

(ii) as a radial excitation of  $D_{33}(1670)$  which would keep it in company with  $D_{23}(1940)$ .

With the first alternative, the  $N\pi$  width comes out about two orders of magnitude too low from the quoted value. This means that in our model, while this state exists in principle, it is unlikely to be easily observable. On the other hand, the decay prediction for the second alternative matches rather well ( $\approx 28$  MeV) with the experimental value. On the basis of this comparison (which predicts  $J = \frac{3}{2}$ ) we would be strongly inclined to urge a more satisfactory determination of the  $J$  value of this resonance. We of course agree with Cutkosky *et al.*<sup>18</sup> that the usual h.o. model does not have a natural place for this resonance. However, our EWM seems to account for this state rather naturally in terms of both the mass position and width without having to resort to yet another model.<sup>18</sup>

*u-type states.* Finally, we consider the  $u$  states that are included in Table IV. In displaying the results for these resonances, we have kept to the spirit of the discussion in Sec. II by regarding each of these resonances as manifestations of distinct states  $l(q)$  and  $u(q)$ . This last point of view represents our essential reinterpretation of the usual picture of single states in the conventional picture<sup>15</sup> and, as already emphasized earlier, appears to be a practical way of handling the genuine problem of incompatible decay modes in the usual description<sup>2</sup> without resorting to elaborate mixing hypotheses.<sup>18</sup>

For the  $D$ -type hadronic widths given in Table IV, the above description does quite well except for some odd cases with comparable widths in the  $(l, u)$  descriptions, e.g., in  $D_{15} \rightarrow N\pi$ , each of which is small compared to experiment. For  $S_{11}$ ,  $D_{25}$ , and  $S_{21}$ , it is the  $l(q)$  state that shows up clearly in most decay channels since the neighboring  $u(q)$  or  $u(d)$  counterparts have rather small widths for easy detection in several decay channels. Some notable exceptions are  $D_{05}(1830) \rightarrow N\bar{K}$  and  $S_{11}(1700) \rightarrow \Lambda K$ , where the  $u$  modes come to



the rescue without involving assumptions like mixing, etc., since the  $l$  mode is inhibited by selection rules.

One problem is with the  $N\pi$  mode which is rather small but comparable in both  $l$  and  $u$  assignments. In this respect, a simple qualitative consideration that may have relevance is the overlap effect of these individual partial widths. A detailed discussion of such a picture merits a separate analysis in terms of the theory of overlapping resonances which seem to have been rather scantily explored in the literature.<sup>20,21</sup> Presumably overlapping resonances with comparable total widths would exhibit amplitudes quite different in structure (dipole shaped?)<sup>20</sup> from the familiar Breit-Wigner forms. A similar point of view appears to be indicated for the  $D_{03}(1520)$  resonance that is the subject of the next section. However, a quantitative discussion of this issue is beyond the scope of this paper.

#### V. $\Lambda$ STATES OF $(70, 1^-)$

The states we shall discuss in this section comprise the  $\Lambda$  states in the 1400–1800 region. We believe that these states merit separate consideration from those dealt with in the earlier sections because of the complex nature of the  $\Lambda$ 's both in terms of their mass positions (Fig. 2) and the predicted decay widths in the existing literature. To quote some examples, what is the  $D_{03}(1520)$  vs  $S_{01}(1405)$  mass difference due to? If it is a spin-orbit effect, why is the same not observed for other resonances like  $D_{13}(1520)$  vs  $S_{11}(1535)$ ? Again, is SU(3) mixing absolutely essential between  $S_{01}(1670)$  and  $S_{01}(1405)$ , or  $D_{03}(1690)$  and  $D_{03}(1520)$  and, if so, where is the third pair of  $S$  and  $D$  states which is also predicted to take part in an elaborate but phenomenological mixing ansatz<sup>22</sup> in order to provide fits to the decay patterns of the observed ones? Finally, is the  $D_{03}(1690)$  vs  $D_{03}(1520)$  mass difference necessarily of SU(3) origin, i.e., between singlets and octets, as suggested by FKR?

We wish to systematically study in this section the interplay of these effects in the context of the even-wave model. We remember of course, that the EWM purports to understand the bulk of the baryon mass spectrum from the point of view of orbital splitting alone. This is not to deny the existence of other mass effects like SU(3), spin orbit, etc., but merely to suggest that these are possibly smaller effects. While this seems to be reasonable for the states  $(N, \Sigma)$  considered in earlier sections, the corresponding ansatz does not seem to be adequate for the  $\Lambda$  states. Certainly the "unsplit" philosophy is not easily borne out for

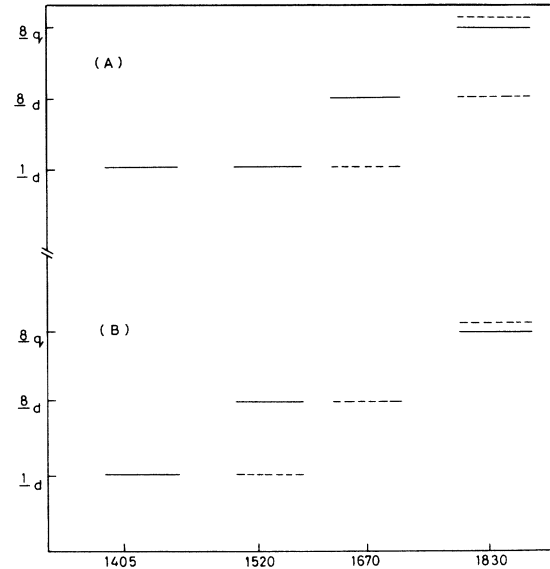


FIG. 2. Two possible quantum assignments for the  $\Lambda$  states of  $(70, 1^-)$  in the EWM. The notation for  $l$  and  $u$  are solid line  $l$  and  $u$ , dashed line.

resonances like  $D_{03}(1520)$  vs  $S_{01}(1405)$ . At the level of SU(6) we also find that the predictions on the partial widths that follow from the EWM assignments in I for the  $\Lambda$  states do not readily check with the experimental data. Before presenting a concrete picture, let us first review the status of the  $\Lambda$ 's as indicated in I from the point of view of orbital structures only:

(a) The  $D_{03}(1690)$  is an  $l$ -type state in company with  $D_{13}(1520)$  and  $D_{23}(1670)$ . Its  $u$  counterpart is located at 1.8 GeV and is identified with the  $D_{05}(1630)$  in the absence of any other state in this region. In an exactly similar manner, the  $S_{01}(1670)$  is an  $l$ -type state together with the  $S_{11}(1535)$  and  $S_{21}(1670)$ . Notice that in contrast to the usual HO description, the effective doubling of states in the EWM gives an extra option of associating a  $1_d$  and  $8_{d,q}$  with each of the  $l$ - and  $u$ -type  $\Lambda$ 's. This extra degree of freedom no doubt generates more states but at the same time provides additional opportunities for interpretation of the observed states in the 1680 region. Of course, following the discussion in Sec. II we should expect the  $l(8_q)$  resonances to lie close to the  $u(8_q)$  states in the region of 1830 MeV.

(b) Two resonances whose formal assignments are still to be made in the EWM (because they are believed to be  $1_d$  states) are  $D_{03}(1520)$  and  $S_{01}(1405)$ . One possibility that has of course been already suggested by FKR as well as tacitly assumed in I, is to regard the (mass)<sup>2</sup> difference between these states as due to spin-orbit effects. However, the EWM provides an interesting alternative, in as

much as their (mass)<sup>2</sup> difference (=0.34 GeV<sup>2</sup>) is nearly equal to that between  $u$  and  $l$  states, viz., 0.37 GeV<sup>2</sup>. The latter point of view is more in keeping with the basic spirit of this paper, namely, to keep all splitting other than  $l$ - $u$  as minimal as possible, except for an upward shift of  $l(q)$  vs  $l(d)$  states. But this still does not explain why  $D_{\infty}(1690)$  and  $D_{\infty}(1520)$  are situated so far apart. In this regard, we shall try to accommodate the FKR observation of a significant mass difference between  $\underline{1}$  and  $\underline{8}$   $\Lambda$  states with the help of decay predictions, but without recourse to a mixing hypothesis. The partial decay widths for the  $\Lambda$  states are displayed in Tables IV and V. Of these the  $D_{\infty}(1830)$  has essentially been covered in the discussion in Sec. II, which brings out the complementary roles of  $l(q)$  and  $u(q)$  states in predicting nonzero widths for  $\Sigma\pi$  and  $N\bar{K}$  decays, respectively, without any further mixing hypotheses.

As to the rest, we offer two possible pictures (A and B) of the  $\Lambda$  spectroscopy in the EWM covering the region from 1405 MeV to 1830 MeV, and these are represented in Fig. 2. In A the unsplit  $l$  states start from 1520 MeV, while a spin-orbit splitting is responsible for pushing down the  $S_{01}(1405)$  state as in FKR. In B however, the unsplit  $l$  state starts at the 1405-MeV level, so that  $D_{\infty}(1520)$  is a  $u$  state relative to  $S_{01}(1405)$ . Three common features of these two pictures are the following:

(a) The presence of  $\underline{1}$  vs  $\underline{8}$  splitting which operates separately for  $l$  and  $u$  states in both A and B. Thus the  $l(\underline{8}_d)$  which is expected to lie in the 1520-MeV region gets pushed upward to  $D_{\infty}(1690)$  because of this mechanism.

(b) The  $u$  vs  $l$  mass difference also imitates the above mechanism and is responsible for pushing up the  $u(\underline{1}_d)$  state from its  $l(\underline{1}_d)$  counterpart located at 1520 MeV to 1680 MeV.

(c) Finally, the  $l, u(\underline{8}_q)$   $\Lambda$ 's are located at 1830 MeV with the  $l(\underline{8}_d)$  states taken to be 1690 MeV.

Let us now separately consider the decay-width predictions for the assignments given in the A and B versions.

*Picture A.* If the  $D_{\infty}(1520)$  is taken to be a  $l(\underline{1}_d)$  state as in FKR, the predicted decay widths are in accord with experiment for both  $N\bar{K}$  and  $\Sigma\pi$  modes. However, the presence of an  $l(\underline{8}_d)$  and  $u(\underline{1}_d)$  state in the 1680-MeV region disturbs the resulting fits to the decay data. Indeed, the extremely large  $N\bar{K}$  widths predicted with the  $l(\underline{8}_d)$  assignment for both  $D_{\infty}(1680)$  and  $S_{01}(1670)$ , in company with the FKR prediction, tend to overshadow the modest figures with the  $u(\underline{1}_d)$  assignment for these states whose formal numerical tally with experiment is thus of no avail.

TABLE V. Partial widths predicted by the EWM for the  $\Lambda$  states of  $(70, 1^-)$  each with  $l$ - and  $u$ -type assignments. See Fig. 2 for the two possible pictures that have been offered in this paper for these  $\Lambda$  states.

Decay	Mass type	SU(6) Expt.	Partial width in MeV			FKR
			$\underline{1}_d$	$\underline{8}_d$	$\underline{8}_q$	
$D_{\infty}(1690) \rightarrow N\bar{K}$	$l$	12-18	...	63	0	102
	$u$		6	24	...	
$\rightarrow \Sigma\pi$	$l$	9-24	...	10	7	11
	$u$		7	28	...	
$S_{01}(1670) \rightarrow N\bar{K}$	$l$	6-14	...	163	0	415
	$u$		5	21	...	
$\rightarrow \Sigma\pi$	$l$	8-24	...	57	10	22
	$u$		2	8	...	
$D_{\infty}(1520) \rightarrow N\bar{K}$	$l$	7	5	5	...	7
	$u$		0.7	...	...	
$\rightarrow \Sigma\pi$	$l$	6.3	11	1.2	...	12
	$u$		1.5	...	...	
$S_{01}(1405) \rightarrow \Sigma\pi$	$l$	40	20	2	...	56

*Picture B.* By taking an  $l(\underline{8}_d)$  state to lie close to the  $u(\underline{1}_d)$  state of  $D_{\infty}(1520)$ , we see that the  $N\bar{K}$  mode receives dominant contributions from the former and is also better placed in relation to the data. The  $\Sigma\pi$  mode of  $D_{\infty}(1520)$ , on the other hand, has comparable widths from both  $u(\underline{1}_d)$  and  $l(\underline{8}_d)$  but each is small compared to the data. Again, we conjecture a sort of  $\Sigma\pi$  enhancement arising from overlapping resonance effects<sup>20</sup> (see also the end of Sec. IV). On the positive side, the  $u(\underline{8}_d)$  assignment for  $S_{01}(1670)$  and  $D_{\infty}(1690)$  is excellently favored by the data for both  $N\bar{K}$  and  $\Sigma\pi$  modes, thus remedying a long-standing discrepancy<sup>2</sup> in the relative orders of magnitude of these widths resulting from an  $l(\underline{8}_d)$  assignment to these states.

There are, however, problems with both these versions, as shown in the following:

For A, we do not know what the  $D_{\infty}(1520)$  vs  $S_{01}(1405)$  mass difference is caused by (other than spin-orbit effects), nor do we understand why the  $l(\underline{8}_d)$  resonances at 1690 and 1670 with such large  $N\bar{K}$  widths ( $\approx 100$  MeV for each) should go unreported in the PDG (Particle Data Group) tables.<sup>6</sup>

While neither of the preceding difficulties is present in B, we are unable to offer a clear explanation as to why an *octet*  $\Lambda$  state of  $D_{\infty}(1520)$  should lie so closely to the  $N$  state of the  $D_{13}(1520)$ , even more so when the corresponding  $\Sigma$  is taken to lie at  $D_{23}(1670)$ .

Before concluding this analysis of  $\Lambda$  states, it is of interest to consider still another possibility within the EWM framework for  $D_{\infty}(1830)$  in view of its unusually high mass. Indeed, the relation

$D_{05}^2 - D_{03}^2 \approx 1 \text{ GeV}^2$  strongly suggests considering the  $L=3$  assignment with three  $l$ -type excitations ( $\Delta m^2 \approx 0.5 \text{ GeV}^2$ ). Such a state with quark spin- $\frac{1}{2}$  has both  $N\bar{K}$  and  $\Sigma\pi$  modes via the  $(L-1)$  wave without any mixing hypothesis, in principle. However, the partial widths are much too small to make this state easily detectable despite its formal existence in this model.

To conclude this EWM analysis of the  $\Lambda$  decays vis-a-vis the usual h.o. picture, we believe that the quantum classification ( $l$  vs  $u$ ) in the EWM has provided an extra physical dimension to help reduce the heavy parametric dependence of mass splittings on unknown dynamical effects. We have already seen in Sec. III that this picture works reasonably well for the  $N$  and  $\Sigma$  (albeit with one assumption), and we now find that it solves at least some of the issues concerning  $\Lambda$  states which the conventional h.o. model does not, yet it seems to leave enough to be desired for the EWM to develop.

## VI. SUMMARY AND CONCLUSIONS

It has been the basic aim of this paper to use the hadronic decay data to check the quantum assignments for the various resonances inferred in the EWM on the basis of their mass positions. In carrying through this analysis, we have kept in view some of the basic problems of the standard (h.o.-oriented) approach manifest in the Particle Data Group tables and elsewhere, and the formal opportunities which the EWM provides for remedying at least some of them.

The usual problems have been mainly in the nature of the following: (i) large mass splittings requiring elaborate parametric representations of the mass operator,<sup>23</sup> and (ii) several incompatible modes of decay which demand elaborate configuration mixing programs often involving more than one supermultiplet. The EWM accounts for a major portion of these splittings through the  $(l, u)$  and  $(l, m, u)$  assignments for  $\underline{70}$  states of  $L^P = 1^-, 2^+$ , respectively, even before involving spin-orbit, etc. effects and thus provides *a priori* identification of their extended quantum numbers for further tests via decay predictions. In calculating decay prediction via EWM, we have made use of its unique facility for a nontrivial  $\underline{56}$ - $\underline{70}$  octet mixing through the availability of the ground state of  $(\underline{70}, 0^+)$ , and a new set of selection rules for  $u$ -state transitions which occur only to the  $\underline{70}$  component of the nucleon octet. These  $u$  transitions are generally characterized by small widths, a fact which can be understood by recognizing that the  $u$  states, though relatively low-lying in the EWM, correspond to fairly high ( $\geq 3$ ) values of the

total quantum number  $N$  of the usual HO description.<sup>2</sup> On the other hand, the  $l$  states of the EWM, despite the low magnitudes ( $\Delta M^2 = 0.5 \text{ GeV}^2$ ) of their excitations, are the nearest analogs of the  $(\underline{70}, 1^-)$ ,  $(\underline{70}, 2^+)$ , etc. states of the conventional description.

Indeed, the generally depressed spectrum of  $\underline{70}$  states compared with the full h.o. model is mainly caused by the exchange part of the  $q$ - $q$  interaction in the EWM, and gives rise to an apparent proliferation of states in the low-lying region, with several interesting possibilities for understanding some difficult data in relation to the standard h.o. description, though not entirely without problems. Examples of such data include the photoproduction of  $D_{15}$  via  $p\gamma$  and  $n\gamma$ , photoproduction of  $F_{35}(1890)$ , which is not easy to understand with a  $\underline{56}$  assignment to this state, and hadronic decays like  $S_{11}(1535) \rightarrow N\pi, N\eta$  and  $D_{05}(1830) \rightarrow N\bar{K}$ . In the course of the analysis made in this paper, we have attempted, apparently with some success, to tie up these photoproduction data and the low partial widths suggested for  $u$  states with the help of the extra low-lying states referred to above through a single additional assumption of a mass shift between quartets versus doublets for only the  $l$  states of  $(\underline{70}, 1^-)$ , over and above the orbital mass splitting between  $l$  and  $u$ , which is already present as a dynamical attribute of the model. The extent of this mass shift is such that there are really *two* quartet ( $l$  and  $u$ ) states instead of *one* in a given mass region.

We have then found that photoproduction data for  $D_{15}(n\gamma)$  and  $D_{15}(p\gamma)$  are due to the  $l$  and  $u$  states of  $D_{15}$ , respectively, and not vice versa. This complementary aspect is again evident for  $D_{05}(1830)$ , whose  $l$  state decays via  $\Sigma\pi$  only (and not  $N\bar{K}$  as in the standard h.o. description) and whose  $u$  counterpart decays via  $N\bar{K}$  only (and not via  $\Sigma\pi$ ). This is also observed at the  $2^+$  level for the  $F_{35}(1890)$ , whose decay widths support a  $\underline{56}$  assignment only (and not a  $\underline{70}$ ), while the photoproduction data support a  $\underline{70}$  assignment only (and not a  $\underline{56}$ ). Some other results obtained in this paper are the following:

(a) The  $D_{23}(1940)$  fits in very well as a radial mode of the  $D_{23}(1670)$ . Similarly, we have ventured to suggest that the  $D_{35}(1925)$  observed by Cutkosky *et al.*<sup>18</sup> is really a  $\frac{3}{2}^-$  state and a radial excitation of  $D_{33}(1670)$ .

(b) The pure  $\underline{56}$  states have good fits in almost all cases, with predictions that tally quite favorably with FKR. The  $G_{07}(2100)$  has been found to give better overlap with the decay data as an  $L=3$  recurrence of a  $\underline{56}$   $\Lambda(1116)$  instead of as a Regge recurrence of  $D_{05}(1520)$ , having the  $(\underline{70}, 3^-)$  assignment.

(c) We have also suggested that the effect of mixing on the nucleon octet should not be continued further up on its Regge trajectory. Interestingly enough, this suggestion appears to be in company with similar ideas relating to mixing effects on vector-meson ( $\phi$ ) trajectories, proposed in a deeper theoretical context.<sup>24</sup>

Finally, in comparing our results in this paper to those of the conventional h.o. model we have found that the EWM does at least as well, and frequently better, than the former in most of the cases, in addition to providing the requisite understanding on the mass position of the states. Also, its problems do not appear to be serious enough to inhibit further investigations, one of which, bearing on a rather sensitive role of 70 versus 56 mixing in explaining the sign and magnitude of the

neutron charge radius in relation to the shape of the  $n, p$  structure functions,<sup>9</sup> has just been formally completed in this model.<sup>25</sup> We would therefore urge a closer-search program for detection of states, especially in the  $1^-$  region, preferably within the framework of an overlapping-resonance formalism so as to provide a cleaner background for comparison with the more usual theory.

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