Some remarks on the Green's function formalism of Klauder's augmented quantum field theory: ϕ^4 model

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Two systems of equations for Green's functions of augmented quantum field theory proposed by Klauder are rewritten in terms of irreducible many-point functions. Then the equations become nonlinear and some of them "degenerate" to constraints. The problem of finding lower many-point functions for given higher many-point functions and renormalization of the first system are discussed.

I. INTRODUCTION

Recently, Klauder[1,2] proposed augmented quantum field theory as an alternative to the canonical quantum field theory. He formulated the theory in an operator formalism as well as in a Green's function formalism. In this paper we restrict ourselves to the Green's function formalism and consider the structure of the systems of equations without referring to perturbation theory. (As will be seen later on, expansions of Green's functions in powers of the coupling constant with known functions are impossible anyway.)

Though the equations proposed by Klauder are apparently linear, they become nonlinear if rewritten in terms of irreducible Green's functions. Nevertheless, the first system of equations has some nice features, as we will see in Sec. IV. Here we do not repeat the derivation of the equations.

II. TWO SYSTEMS OF EQUATIONS

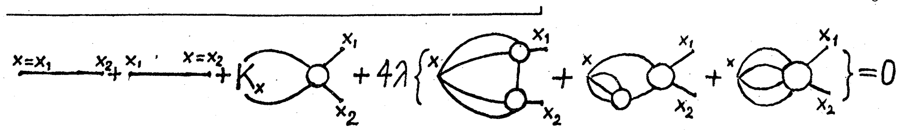
The first system of relevant equations reads

$$i\sum_{r=1}^{m} \delta(x-x_r)G_m(x_1,\ldots,x_m) + \lim_{x'\to x} K_x G_{m+2}(x,x',x_1,\ldots,x_m) + 4\lambda G_{m+4}(x,x,x,x,x_1,\ldots,x_m)=0, \quad (2.1)$$

where G_m is the connected m-point function. K_x is the Klein-Gordon operator at x. Important features of this system are apparent linearity and homogeneity as well as "degeneracy."

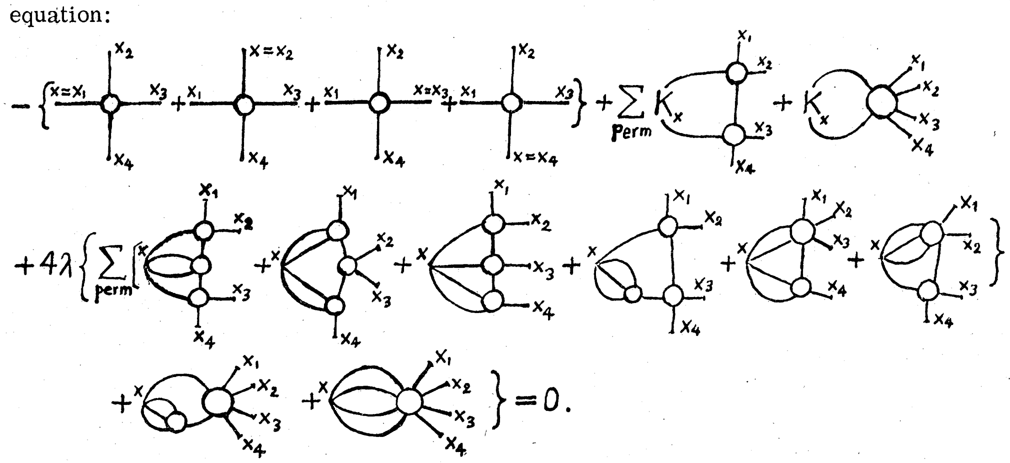
It can be easily seen, however, that the linearity is superficial if one looks at the reducibility (pole structures in the momentum representation) of G_m ($m \geq 6$).

Let us put $m=2$ in Eq. (2.1). Then, assuming the pole structure of G_6, one finds



(2.2)

while in the ordinary (canonical) theory the Schwinger-Dyson equation for G_2 does not involve the six-point function and is linear in the (unamputated) four-point function G_4. Similarly one finds for $m=4$ the following equation:



(2.3)

Owing to the "degeneracy," we have only two equations involving four unknown functions G_2, G_4, G_6 and G_8. Here, any circle with six or more legs stands for the irreducible (in any channel) part. On the other hand, in the canonical theory, the first two equations involve only up to G_6.

In an attempt to get rid of the "degeneracy," Klauder[1] proposed his second system of equations, which reads

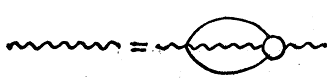
$$i\sum_{r=1}^{m}\delta(x-x_r)G'_{m-1,n}(x_1,\ldots,\hat{x}_r,\ldots,x_m;y_1,\ldots,y_n)+K_xG'_{m+1,n}(x,x_1,\ldots,x_m;y_1,\ldots,y_n)$$
$$+G'_{m+1,n+2}(x,x_1,\ldots,x_m;x,x,y_1,\ldots,y_n)+4\lambda G'_{m+3,n}(x,x,x,x_1,\ldots,x_m;y_1,\ldots,y_n)=0 \qquad (2.4)$$

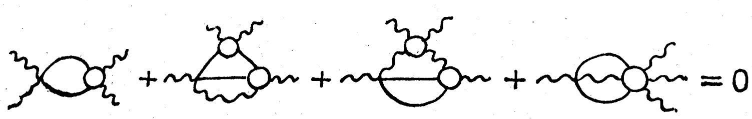
(for $m\geq 1$ odd, $n\geq 0$ even) and

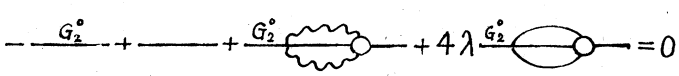
$$i\sum_{s=1}^{n}\delta(x-y_s)G'_{m,n-1}(x_1,\ldots,x_m;y_1,\ldots,\hat{y}_s,\ldots,y_n)+G'_{m+2,n+1}(x,x,x_1,\ldots,x_m;x,y_1,\ldots,y_n)=0 \qquad (2.5)$$

(for $m\geq 0$ even, $n\geq 1$ odd).

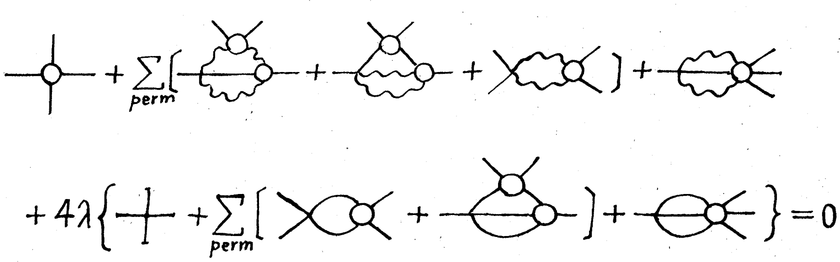
Again, it can be easily seen that the "linearity" and "homogeneity" of this system are superficial. After the separation of disconnected graphs and the subtraction of tadpoles, one gets

 $(m=0,\ n=1)$, (2.6)

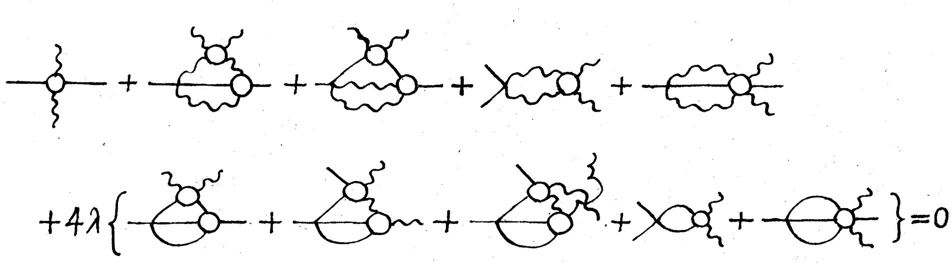
 $(m=0,\ n=3)$, (2.7)



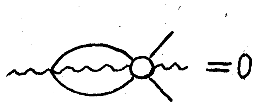
$(m=1,\ n=0)$, (2.8)



$(m=3,\ n=0)$, (2.9)



$(m=1,\ n=2)$, (2.10)

 $(m=2,\ n=1)$. (2.11)

Here combinatorial factors are omitted and diagrams with different assignments of external lines are not shown explicitly.

III. DEGENERACY OF THE SECOND SYSTEM OF EQUATIONS

In this section we show that the system of Eqs. (2.4) and (2.5) does not remove the "degeneracy" of the system (2.1).

First, let us count the number of equations and unknown functions. With $m+n \leq 3$, we have six equations (2.6)–(2.11) involving eight unknown functions G_{02}, G_{04}, G_{20}, G_{22}, G_{24}^{ir}, G_{40}, G_{42}^{ir} and G_{60}^{ir}, so that the system is indeterminate.

Eq. (2.6), with $m=0$, $n=1$, is not an equation for G_{02} at all, but a constraint on G_{22}:

$$G_{22}(x,x;x,y) = -\delta(x-y). \tag{3.1}$$

Therefore we have no clue as to how to determine G_{02}.

On the other hand, in Eq. (2.8) for G_{20} (two-point function in the ordinary sense), we have inhomogeneous terms

$$G_2^0(y_1,y_2) + \int dy_3 G_2^0(y_1,y_3) G_{22}(y_3,y_3;y_3,y_2)$$

instead of the G_2^0 in the canonical theory. The lack of an appropriate equation for G_{02} is irrelevant so far. But the situation is not so simple in Eq. (2.9) for the four-point function G_{40}, because $(G_{02})^{-1}$ appears in the second term on the left-hand side. Moreover, the first term in the curly bracket has an undesirable asymptotic behavior in the momentum representation. Equation (2.10) for G_{22}, involves G_{04}, but we do not have an equation of the form $G_{04}(\cdots)+\cdots=0$, because the equation with $m=0$, $n=3$ degenerates [see (2.7)].

As has been seen above, the system (2.4), (2.5) is no better than the system (2.1) at all, because the former not only does not remove the degeneracy of the latter, but also involves auxiliary many-point functions G_{mn} $(n \neq 0)$ which are subject to strange constraints. So, let us return to the system (2.1).

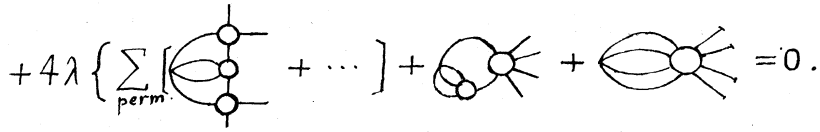
IV. FURTHER DISCUSSION OF THE FIRST SYSTEM OF EQUATIONS

In this section we consider the system of Eq. (2.1) more closely in the context of "descending" and "ascending" problems[3] as well as of renormalization. By "descending" problem we mean a problem of finding lower many-point functions corresponding to given higher many-point functions, and by "ascending" problem we mean the reverse.

For this purpose, we take Fourier transforms of the first two equations of the system (2.1):

$$\begin{aligned} &G_2(k_1)+G_2(k_2)+\int d^4p(p^2-m^2)G_4(p,-p-k_1+k_2,k_1,-k_2) \\ &\quad +4\lambda\int d^4p_1 d^4p_2 d^4p_3 \{G_4(p_1,p_2,k,-p_1-p_2-k)[G_2(p_1+p_2+k)]^{-1}G_4(p_3,-p_1-p_2-p_3-k_1+k_2,p_1+p_2+k_1,-k_2) \\ &\qquad +G_4(p_2,p_3,p_1-p_2-p_3+k_1-k_2,-p_1-k_1+k_2)[G_2(p_1+k_1-k_2)]^{-1}G_4(p_1,k_1,-k_2,-p_1-k_1+k_2) \\ &\qquad +G_6^{\text{ir}}(p_1,p_2,p_3,-p_1-p_2-p_3-k_1+k_2,k_1,-k_2)\}=0, \end{aligned} \tag{4.1}$$

$$\begin{aligned} &G_4(p_1-q,p_2,p_3,-p_1-p_2-p_3+q)+G_4(p_1,p_2-q,p_3,-p_1-p_2-p_3+q) \\ &\quad +G_4(p_1,p_2,p_3-q,-p_1-p_2-p_3+q)+G_4(p_1,p_2,p_3,-p_1-p_2-p_3) \\ &\quad +\int d^4k(k^2-m^2)\Big\{G_6^{\text{ir}}(k,q-k,p_1,p_2,p_3,-p_1-p_2-p_3-q) \\ &\qquad \times\sum_{\text{perm}} G_4(p_1,p_2,-k,-p_1-p_2+k)[G_2(p_1+p_2-k)]^{-1}G_4(p_1+p_2-k,p_3,-p_1-p_2-p_3+q,k-q)\Big\} \end{aligned}$$



(4.2)

Now, we find that there appear linear combinations of G_2's and G_4's with different arguments in (4.1) and (4.2), respectively. Putting $k_1=k_2=k$ into Eq. (4.1), one gets a "nonlinear integral equation" for G_2:

$$G_2(k)+\tfrac{1}{2}\int d^4p(p^2-m^2)G_4(p,-p,k,-k)$$

$$+2\lambda\int d^4p_1d^4p_2d^4p_3\{G_4(p_1,p_2,k,-p_1-p_2-k)[G_2(p_1+p_2+k)]^{-1}G_4(p_3,-p_1-p_2-p_3,p_1+p_2+k,-k)$$

$$+G_4(p_1,-p_1,k,-k)[G_2(p_1)]^{-1}G_4(p_2,p_3,-p_1-p_2-p_3,p_1)+G_6^{1r}(p_1,p_2,p_3,-p_1-p_2-p_3,k,-k)\}$$

$$\equiv\Lambda[G_2;G_4,G_6^{1r};k]=0. \quad (4.3)$$

Or, in terms of amputated many-point functions, one gets

$$G_2(k)+\tfrac{1}{2}\int d^4p[G_2(p)]^2(p^2-m^2)\Gamma_4(p,-p,k,-k)[G_2(k)]^2$$

$$+2\lambda\int d^4p_1d^4p_2d^4p_3G_2(p_1)G_2(p_2)G_2(p_3)G_2(p_1+p_2+p_3)$$

$$\times[\Gamma_4(p,p_2,k,-p_1-p_2-k)G_2(p_1+p_2+k)\Gamma_4(p_3,-p_1-p_2-p_3,p_1+p_2+k,-k)$$

$$+\Gamma_4(p_1,-p_1,k,-k)G_2(p_1)\Gamma_4(p_2,p_3,-p_1-p_2-p_3,p_1)+\Gamma_6(p_1,p_2,p_3,-p_1-p_2-p_3,k,-k)][G_2(k)]^2$$

$$\equiv\Omega[G_2;\Gamma_4,\Gamma_6^{1r}](k)=0. \quad (4.4)$$

In these equations, the first terms contain one G_2 while other terms apparently have a double pole at $k^2=m^2$. This implies that the integrals in those terms ought to be renormalized so that the integrals behave as $(k^2-m^2)^{-1}$ in (4.3) and (k^2-m^2) in (4.4), respectively. The following formulas give the recipe of renormalization:

$$\left[\int d^4p(p^2-m^2)G_4(p,-p,k,-k)\right]^{\text{ren}}$$

$$\equiv\left[(k^2-m^2)^{-1}\int_{m^2}^{k^2}d(p'^2)\frac{d}{d(p'^2)}\int d^4p(p^2-m^2)G_4(p,-p,p',-p')(p'^2-m^2)^2\right](k^2-m^2-i\epsilon)^{-1}, \quad (4.5)$$

etc.,

$$\left[\int d^4p(p^2-m^2)[G_2(p)]^2\Gamma_4(p,-p,k,-k)\right]^{\text{ren}}\equiv\int_{m^2}^{k^2}d(p'^2)\frac{d}{d(p'^2)}\int d^4p(p^2-m^2)[G_2(p)]^2\Gamma_4(p,-p,p',-p'), \quad (4.6)$$

etc.

It should be noticed, however, that this is not a full renormalization scheme. In other words, the residue of the pole of $G_2(k)$ at $k^2=m^2$ is not yet normalized, though the position of the pole is fixed. This implies that the system of equations under consideration does not give a possibility of evaluating dynamically generated mass.

Now, let us consider the question whether one can pose the problem of finding G_2 as a fixed-point problem in a Banach space if G_4 and G_6 are given. This is a descending problem.[3] One can easily see that the map Λ is not bounded near the origin of any Banach space of candidates for G_2. So we have to look for a more suitable unknown function. Let us postpone this problem for the time being, and consider the descending problem from amputated four-point and six-point functions. Then the equation to be considered is (4.4). It is now obvious that this equation has a trivial solution, $G_2=0$. If the integrals are interpreted as renormalized, the map Ω is Fréchet differentiable with respect to G_2. Therefore, if a nontrivial solution exists at all, it must have a norm (in a suitable Branch space) large enough to violate the necessary conditions for local uniqueness. But such a domain of candidates for G_2 in any Banach space is not convex, so that we cannot apply any known method to search for such a solution.

If Γ_4 only is given, one can choose G_2 arbitrarily (as long as causality, etc., are satisfied) and Eq. (4.4) becomes a linear constraint on Γ_6. If Γ_6 only is given, Eq. (4.4) becomes a nonlinear constraint on Γ_4 after an arbitrary choice of G_2.

Curtailing further discussion of these contraints, let us return to the problem with given G_4 and G_6. Assuming that $G_2(k)$ is of the form

$$G_2(k)=[\zeta+\eta(k)]^{-1}(k^2-m^2-i\epsilon)^{-1}$$
$$=[\zeta+\eta(k)]^{-1}G_2^0(k), \quad (4.7)$$

with $\sup|\eta(k)|<|\xi|$, one can rewrite Eq. (4.3) in the following form:

$$[\zeta+\eta(k)]^{-1}+\{\tfrac{1}{2}[\![G_2^0*\mathfrak{G}]\!]+2\lambda[\![(G_2^0)^{**4}*(\mathfrak{G}_4*G_2^0(\zeta+\eta)*\mathfrak{G}_4)]\!]$$

$$+2\lambda[\![((G_2^0)^{**3}*\mathfrak{G}_4)G_2^0(\zeta+\eta)*G_2^0*\mathfrak{G}_4]\!]$$

$$+2\lambda[\![(G_2^0)^{**4}*\mathfrak{G}_4]\!]\}[\zeta+\eta(k)]^2(k^2-m^2)^{-1}$$

$$\equiv\Theta[\eta;\mathfrak{G}_4,\mathfrak{G}_6](k)=0, \quad (4.8)$$

where $[\![*]\!]$ is a short-hand for renormalized convolution, and

$$\mathfrak{G}_4=KKKKG_4, \quad \mathfrak{G}_6=KKKKKKG_6, \quad K=(G_2^0)^{-1}.$$

Let us take ζ as a root of the quadratic equation

$$+1+\zeta\{\tfrac{1}{2}[\![G_2^0*\mathfrak{G}_4]\!]+2\lambda[\![(G_2^0)^{**4}*\mathfrak{G}_6]\!]\}G_2^0\big|_{k^2=m^2}$$

$$+\zeta^2\tfrac{1}{2}(2)[\![(G_2^0)^{**5}*\mathfrak{G}_4^{**2}]\!]G_2^0\big|_{k^2=m^2}=0, \quad (4.9)$$

and the 0th approximation $\eta_{(0)}(k)=0$. Now we can apply Jankő's theorem,[4] provided $\mathfrak{G}_4$ and $\mathfrak{G}_6$ satisfy certain conditions, to be specified below. If

$$|\mathfrak{G}_4(k_1,\ldots,k_4)|\le c_4\frac{|k_1^2|^\alpha\cdots|k_4^2|^\alpha}{(|k_1^2|+\cdots+|k_4^2|)^{4\alpha+\beta}},$$

$$\alpha>0, \ \beta>2 \quad (4.10)$$

$$|\mathfrak{G}_6(k_1,\ldots,k_6)|\le c_6\frac{|k_1^2|^\gamma\cdots|k_6^2|^\gamma}{[|k_1^2|+\cdots+|k_6^2|]^{6\gamma+\delta}}$$

$$\gamma>0, \ \delta>2 \quad (4.11)$$

and

$$\|[\![G_2{}^0*\mathfrak{G}_4]\!]\|\ge B>0, \quad (4.12)$$

with sufficiently small c_4 and c_6, then the Fréchet derivative $\Theta'[0; \ ;\mathfrak{G}_4,\mathfrak{G}_6;k]$ is invertible, so we define

$$\Phi\equiv\Theta'^{-1}[0; \ ;\mathfrak{G}_4,\mathfrak{G}_6].$$

Moreover,

$$\|\Theta'[\eta'; \ ;\mathfrak{G}_4,\mathfrak{G}_6]-\Theta'[\eta''; \ ;\mathfrak{G}_4,\mathfrak{G}_6]\|<\lambda M\|\eta'-\eta''\|$$

$$\eta',\eta''\in S(0,2N) \quad (4.13)$$

$$\|\Phi(0,\Theta(0;\mathfrak{G}_4,\mathfrak{G}_6);\mathfrak{G}_4,\mathfrak{G}_6)\|\le N, \quad (4.14)$$

with positive numbers M and N. If the inequality

$$\lambda BMN<\tfrac{1}{2} \quad (4.15)$$

is satisfied, there exists a unique solution $\eta^*\in S(0,2N)$. However, this does not mean that the solution of Eq. (4.3) is unique, because we have two choices of ζ in general. Of course, $\eta^*(k)|_{k^2=m^2}$ need not be equal to 0, so that the residue of G_2 is affected,

$$G_2^*(k)=[\zeta+\eta(k)|_{k^2=m^2}]^{-1}(k^2-m^2-i\epsilon)^{-1}+\sigma(k)$$

$$=\frac{z}{k^2-m^2-i\epsilon}+\sigma(k). \quad (4.16)$$

So we renormalize G_2 as follows:

$$G_2^R(k)=G_2^0(k)+z^{-1}\sigma(k)$$

$$=z^{-1}G_2^*(k). \quad (4.17)$$

Then Eq. (4.3) is rewritten as follows:

$$G_2^R(k)+\{\tfrac{1}{2}[\![G_2^R*\mathfrak{G}_4^R]\!](k)$$

$$+2\lambda[\![(G_2^0)^{**4}*\mathfrak{G}_4^R*(G_2^R)^{-1}(G_2^0)^2*\mathfrak{G}_4^R]\!](k)$$

$$+2\lambda[\![(G_2^0)^{**4}*\mathfrak{G}_6^R]\!](k)\}(G_2^0(k))^2=0, \quad (4.18)$$

with

$$\mathfrak{G}_4^R=KKKKG_4^R=z^{-1}\mathfrak{G}_4, \quad (4.19)$$

$$\mathfrak{G}_6^R=KKKKKKG_6^R=z^{-1}\mathfrak{G}_6, \quad (4.20)$$

where the renormalization of G_4 and G_6 is so chosen that the second term on the right-hand side of (4.18) does not acquire an extra factor. An interesting feature of this renormalization is that the coupling constant λ is not affected while G_2, G_4, and G_6 all get the same renormalization factor z^{-1}. So far we have dealt with the problem as though there is no condition (4.1). If condition (4.1) is taken into account, G_6 with general arguments must satisfy the following constraint:

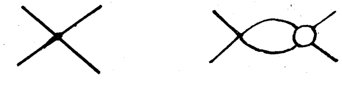
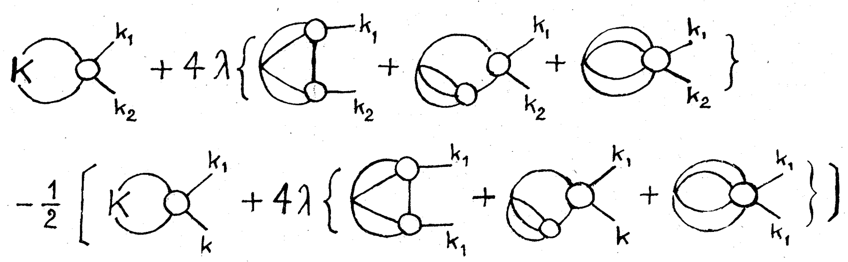


FIG. 1. Diagrammatic representation of terms with undesirable asymptotic behavior, which appear in the Schwinger-Dyson equation for the four-point function in the canonical quantum field theory. These terms do not appear in the augmented theory.



$$-\frac{1}{2}(k_1 \to k_2) = 0. \tag{4.21}$$

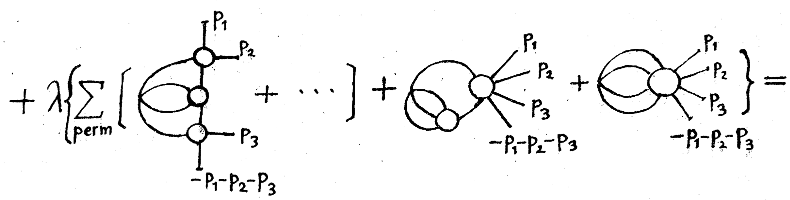
Therefore, what can be given as input to the descending problem is not G_6 with general arguments but $G_6(p_1, p_2, p_3, -p_1-p_2-p_3, k, -k)$.

Now the next question is whether one can determine G_2 and G_4 if G_6 and G_8 are specified as input. One favorable feature of Eq. (4.2) compared to the Schwinger-Dyson equation for the four-point function in the canonical theory is the absence of the terms shown in Fig. 1 which have undesirable asymptotic behavior. Putting $q=0$ in Eq. (4.2), one gets the following equation for G_4:

$$-G_4(p_1,p_2,p_3,-p_1-p_2-p_3)$$

$$+\tfrac{1}{4}\int d^4k\,(k^2-m^2)\left\{G_6^{ir}(k,-k,p_1,p_2,p_3,-p_1-p_2-p_3)\right.$$

$$\left.+\sum_{\text{perm}} G_4(k,p_1,p_2,-p_1-p_2-k)[G_2(p_1+p_2+k)]^{-1}G_4(p_1+p_2+k,p_3,-p_1-p_2-p_3,-k)\right\}$$



$$= 0. \tag{4.22}$$

As has been the case in the previous problem, G_6 with general arguments cannot be given as input. Similarly, one cannot give

$$\int d^4k_1 d^4k^2 d^4k^3 G_8^{ir}(k_1,k_2,k_3,-k_1-k_2-k_3,p_1,p_2,p_3,-p_1-p_2-p_3)$$

as input, because Eqs. (4.1) and (4.2) become constraints, but there is no guarantee that these constraints are satisfied by the solution G_4^*, G_6^* of the coupled equations [(4.3) and (4.22)] with input as mentioned above.

If one begins with $G_6(p_1,p_2,p_3,-p_1-p_2-p_3,k,-k)$ and the sum of those terms in Eq. (4.21) that involve G_6 or G_8 as input, Eqs. (4.3) and (4.21) can be regarded as coupled nonlinear integral equations for G_2 and G_4 or abstractly as an operator equation in the direct sum $\mathfrak{B}_2 \oplus \mathfrak{B}_4$ of Banach spaces of two-point functions, $\mathfrak{B}_2$, and the Banach space of four-point functions, $\mathfrak{B}_4$. Then Eqs. (4.1) and (4.2) can be regarded as linear constraints involving the solution of Eqs. (4.3) and (4.21) on G_6 and G_8 with general arguments. These constraints, however, do not determine G_6 and G_8 uniquely, as can be easily seen from the structure of Eqs. (4.1) and (4.2).

Let us write the system of Eqs. (4.3) and (4.22) abstractly,

$$\Xi(G_2, G_4; G_6, G_8) = 0\,. \tag{4.23}$$

Then the map Ξ is Fréchet differentiable provided the input has a nice asymptotic behavior. If the input and the zeroth approximation $G_2^{(0)}, G_4^{(0)}$ are such that the Fréchet derivative $\Xi'(G_2^{(0)}, G_4^{(0)};\ G_6, G_8)$ has right (or left) inverse and some other conditions are satisfied, one can prove the existence of a solution. Alternative sets of sufficient conditions for the existence of a solution can be found in Altman's work.[5]

V. IS PSEUDOPERTURBATION THEORY POSSIBLE?

Curtailing further discussion of the descending problem, let us ask the following question: Can one construct a solution of the system (2.1) in terms of pseudofree Green's functions, i.e., a solution of the system of equations with $\lambda=0$?

The first two equations of the pseudofree theory read

$$G_2(k)+\tfrac{1}{2}\int d^4p\,(p^2-m^2)G_4(p,-p,k,-k)=0\,, \tag{5.1}$$

$$-G_4(k_1,k_2,k_3,-k_1-k_2-k_3)+\tfrac{1}{4}\sum_{\text{perm}}\int d^4p\,(p^2-m^2) \times\{G_4(p,k_1,k_2,-p-k_1-k_2)[G_2(p+k_1+k_2)]^{-1}G_4(p,-p-k_1-k_2,-k_3,k_1+k_2+k_3) + G_6(p,-p,k_1,k_2,k_3,-k_1-k_2-k_3)\}=0\,. \tag{5.2}$$

In the pseudofree theory we have the following constraint on G_4:

$$\tfrac{1}{2}\int d^4p\,(p^2-m^2)G_4(p,-p,k_1,-k_1)+\tfrac{1}{2}\int d^4p\,(p^2-m^2)G_4(p,-p,k_2,-k_2) -\int d^4p\,(p^2-m^2)G_4(p,-p-k_1+k_2,k_1,-k_2)=0\,, \tag{5.3}$$

and a similar one for G_6.

Suppose that G_2 and G_4 can be expanded in powers of λ:

$$G_2=\sum_{n=0}^{\infty}\lambda^nG_2^{(n)},\quad G_4=\sum_{m=0}^{\infty}\lambda^nG_4^{(n)}\,. \tag{5.4}$$

Then in the first order in λ one gets the following condition:

$$G_2^{(1)}(k)+\tfrac{1}{2}\int d^4p\,(p^2-m^2)G_4^{(1)}(p,-p,k,-k) +2\int d^4p_1d^4p_2d^4p_3\{G_4^{(0)}(p_1,p_2,k,-p_1-p_2-k)[G_2^{(0)}(-p_1-p_2-k)]^{-1}G_4^{(0)}(p_1+p_2+k,p_3,-p_1-p_2-p_3,-k) +G_6^{(0)}(p_1,p_2,p_3,-p_1-p_2-p_3,k,-k)\}=0\,. \tag{5.5}$$

Because of the second term on the left-hand side of Eq. (5.5) one cannot express $G_2^{(1)}$ in terms of $G_n^{(0)}$'s. The situation does not improve even if one takes the power-series expansion of Eq. (4.22) into account because it involves G_6 in integrated form. Therefore, one cannot formulate a Feynman rule in terms of $G_n^{(0)}$'s.

VI. CONCLUDING REMARKS

We have found that some interesting features can be derived from the first system of equations (Sec. IV) while the second system is hopelessly degenerate after the separation of disconnected parts. Because of homogeneity, both the first and the second systems of equations have a trivial solution, i.e., $G_n=0$, $\forall n\in Z^+$, and $G_{mn}=0$, $\forall m,n\in Z^+$ is a solution. On the other hand, as has already been seen, the "linearity" is superficial, and the equations become nonlinear after taking into account the reducibility (pole structure in the momentum representation) of G_n ($n\geq 6$) in the first system and of G_{mn} ($m+n\geq 6$) in the second system.

Interesting features of the augmented quantum field theory in the Green's function formalism are (1) that the coupling constant is not affected by renormalization, (2) the absence of terms with undesirable asymptotic behavior in the equation for G_4, (3) trivial solutions of descending problems with given amputated many-point functions.

The next task in the augmented quantum field theory in the Green's function formalism is to develop techniques of successive approximation (algorithm) to construct functions so as to satisfy (approximately) linear and nonlinear constraints arising in the descending and ascending problems of the first system of equations.

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