Instantaneous approximation for a gauge theory with dressed vertices

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We consider a Bethe-Salpeter kernel consisting of one gluon propagating between two dressed quark-gluon vertices. In order to derive the corresponding nonrelativistic potential to $O((v/c)^2)$ we find that it is necessary to choose a special gauge in which the time-time component of the propagator is instantaneous (to that order). The resulting Fermi-Breit potential including an anomalous magnetic moment is presented and compared with previous results.

In recent years, models of quark binding have led to a revival of interest in the problem of deriving a nonrelativistic potential from field theory. The technique is well known in QED where positronium and muonium[1] potentials to order $(v/c)^2$ are derived, yielding the familiar Fermi-Breit potential. It has not yet been possible to carry out a similar program for quantum chromodynamics (QCD), although work in this direction has been started by a number of groups.[2] Nevertheless, much success in calculating the meson spectrum has been achieved by simply assuming that quarks are bound by a nonrelativistic potential whose form is "suggested" by QCD.[3]

In order to do still better phenomenology, it becomes important to postulate the form of fine-structure (v/c) corrections to this potential (we need this to compute, for instance, the mass differences of the χ states of charmonium). In this paper, we will focus our attention on one model for relativistic corrections[4,5] which is presented as follows: assume that the quarks are bound in mesons by a "dressed gluon." That is, in the Bethe-Salpeter kernel we replace the ladder propagator, $P_{\mu\nu}(k^2)/k^2$, by $P_{\mu\nu}(k^2)g(k^2)$. Then the full kernel is given by

$$\mathcal{V} = \left(\gamma^{(1)\mu} - \frac{\kappa}{2m}\sigma^{(1)\mu\nu'}q_{\nu'}\right) D_{\mu\nu}(q^2) \times \left(\gamma^{(2)\nu} + \frac{\kappa}{2m}\sigma^{(2)\nu\nu''}q_{\nu''}\right), \tag{1a}$$

where

$$D_{\mu\nu}(q^2) = P_{\mu\nu}(q^2)\,[g(q^2)\,], \tag{1b}$$

$P_{\mu\nu}$ is dimensionless, and we have included an anomalous gluon-quark magnetic moment (which might be a function of q^2). $D_{\mu\nu}$ will be referred to as the propagator, although it is correct to think of it as the product of vertices and propagator. The motivation for this ansatz has been given by Schnitzer[4,5] and by Pumplin *et al.*[6] We will simply assume it and discuss the derivation of the corresponding nonrelativistic potential to $O((v/c)^2)$.

We first note that such derivations do exist in the literature but do not all lead to the same result. For example (for $\kappa=0$), Schnitzer[4] finds that spin-independent terms in the potential are of a slightly different form from those derived by Pumplin, Repko, and Sato.[6] We trace this disparity to the fact that those two calculations start from two separate assumptions. In Ref. 4 our Eq. (1) is not the starting point—but rather, our $D_{\mu\nu}(q^2)$ is replaced by $P_{\mu\nu}(q^2)[g(\vec{q}^{\,2})\vec{q}^{\,2}/q^2]$, where $P_{\mu\nu}$ is then chosen to be the Coulomb gauge projection. With this assumption, the calculation of order $(v/c)^2$ corrections is unambiguous and is given in Ref. 4. On the other hand, in the work by Pumplin *et al.*,[6] our Eq. (1) is used and $P_{\mu\nu}=g_{\mu\nu}$. This choice seems to us to lead to an ambiguity in computing $O((v/c)^2)$ corrections. This is due to the energy dependence in the time-time component of the propagator, and in what follows we will resolve that ambiguity by introducing a new "instantaneous gauge." We should point out here that we will assume throughout this work that [in line with the nature of our ansatz of Eq. (1)] the dressed gluon couples to a conserved current. This, of course, would ultimately have to be derived from the underlying field theory. Before carrying out the derivation $O((v/c)^2)$ corrections we must first review the basic technique for finding a nonrelativistic potential for the Bethe-Salpeter kernel (a slightly different method is to be found in Akhiezer and Berestetskii[7]).

The Bethe-Salpeter equation[8] is used to find the bound state

$$(\not{p}_1 - m)(\not{p}_2 - m)\chi_{12}(p) = \frac{i}{(2\pi)^4}\int d^4k\,\mathcal{V}(k^2)\chi_{ab}(p+k), \tag{2}$$

where $k=(2m-E,\vec{0})$(in the center-of-mass frame). We perform an expansion of (2) to $O((v/c)^2)$. If it becomes a Schrödinger equation,[9] then the potential V can be read from that equation. As we can see, the problem is to derive such a Schrödinger equation. It will soon be seen that this can be done only in a special instantaneous gauge.

As discussed below, the recipe for determining whether Eq. (2) allows a Schrödinger equation through $O(v^2/c^2)$ is the following: (1) The energy transfer $k_0/|\vec{k}|$ is $O(v/c)$ because it vanishes in the nonrelativistic limit. (2) Note that because "lower components" of the Dirac spinors are $O(v^2/c^2)$, terms of $\mathcal{V}$ which mix upper and lower components [e.g., D_{ij} of Eq. (8)] need only be written to zeroth order in $(v/c)^2$, whereas "upper-upper" terms must be written to order $(v/c)^2$. If to this order k_0 does not appear, the instantaneous approximation $\mathcal{V}(q^2)=\mathcal{V}(\vec{q}^2)$ is justified. (3) Perform a similarity transformation of Eq. (2) to separate particle and antiparticle. (4) From this, check to see that the transformed kernel has a separated form [Eq. (6) below]. If so, then V is determined and a Schrödinger equation can be written.

The transformation of Eq. (2) to a Schrödinger-type equation is done for QED by Schwinger on pages 330–343 of *Particles, Sources and Fields*, Vol. II,[1] and for ease of presentation we simply follow this derivation, reviewing only the salient details.[10] We begin by making the instantaneous approximation to be justified in detail later. We rewrite Eq. (2) as

$$[(\gamma\cdot p+m)_1(\gamma\cdot p+m)_2-I_{12}]G_{12}=1, \tag{3a}$$

where

$$I_{12}(x_1,x_2)=i\,\delta(x_1^0-x_2^0)\gamma_1^0\gamma_2^0V(\vec{x}_1-\vec{x}_2), \tag{3b}$$

with $V=-\mathcal{V}^T$.[11] From this equation, Schwinger derives a condition for establishing a Schrödinger equation. He first defines, for a bimatrix $A=A^{(1)}A^{(2)}$, the submatrix $A_{ij}=A_{ij}^{(1)}A_{ij}^{(2)}$, where i and j can take the values + and −. A representation is chosen in which $\gamma_{(a)}^0$ are diagonal. Then $A_{ij}^{(a)}$ is the submatrix whose row and column indices have $\gamma_{(2)}^0$ eigenvalues i and j, respectively. Having defined A_{ij}, he introduces

$$U_i=\exp[\tfrac{1}{2}(\gamma_i\cdot p_i/|p_i|)\phi_i], \tag{4a}$$

where

$$\sin\phi=|p|/(\vec{p}^2+m^2)^{1/2},\quad \cos\phi=m/(\vec{p}^2+m^2)^{1/2}, \tag{4b}$$

and

$$\overline{V}=U_1U_2VU_1^{-1}U_2^{-1}. \tag{5}$$

He then shows that the condition for establishing a Schrödinger equation is

$$\overline{V}_{++}=\overline{V}_{--},\quad \overline{V}_{+-}=\overline{V}_{-+}. \tag{6}$$

The Schrödinger potential turns out to be $V_{\rm NR}=\overline{V}_{++}$.

We see that in order to find $V_{\rm NR}$ to order $(v/c)^2$, we must establish (6) to that order. It is straightforward to show that for $\mathcal{V}$[given as in Eq. (1)], in the instantaneous approximation (IA), (6) is indeed satisfied [to $O((v/c)^2)$.] So the only step left to complete is to establish the validity of $\mathcal{V}(q^2)\cong\mathcal{V}(\vec{q}^2)$ (IA). After that, the calculation of $V_{\rm NR}$ is straightforward.

Thus, we turn now to the question of the instantaneous approximation. We will show the existence of a gauge which justifies IA. When the interaction is Coulomb, the gauge will be the Coulomb gauge. For a more general interaction, a more general gauge will be found. To begin with, we look at the special case of (1), where $\kappa=0$ and $D_{\mu\nu}(k^2)$ is the photon propagator in the Coulomb gauge

$$D_{\mu\nu}(k^2)=\left[g_{\mu\nu}-\frac{k_\mu k_\nu+(n\cdot k)(k_\mu n_\nu+n_\mu k_\nu)}{k^2+(n\cdot k)^2}\right]\left(\frac{1}{k^2}\right), \tag{7}$$

where n is the unit vector in the time direction (1, 0, 0, 0):

$$\begin{aligned}D_{00}&=-1/\vec{k}^2,\\ D_{0i}&=D_{i0}=0,\\ D_{ij}&=\left(\delta_{ij}-\frac{k_ik_j}{|\vec{k}|^2}\right)\frac{1}{k^2}.\end{aligned} \tag{8}$$

We see that the only contribution to the noninstantaneous part comes from D_{ij}. Taking $k_0/|\vec{k}|=O(v/c)$, we make the expansion

$$1/k^2=1/\vec{k}^2+(k_0^2/\vec{k}^2)(1/\vec{k}^2)+O((v/c)^4),$$

i.e.,

$$D_{ij}=(\delta_{ij}-k_ik_j/\vec{k}^2)[(1/\vec{k}^2)+O((v/c)^2)]. \tag{9}$$

Now notice that $(\gamma^{(1)i}D_{ij}\gamma^{(2)j})_{++}=0$. Hence, this term contributes to $\overline{V}_{++}$ only when multiplied by the γ matrices in the expressions for U_1U_2 and $U_1^{-1}U_2^{-1}$,

$$\begin{aligned}U_1U_2&=1-\frac{1}{8}\left(\frac{\vec{p}_1^{\,2}}{m_1^2}+\frac{\vec{p}_2^{\,2}}{m_2^2}\right)+\frac{1}{4}\left(\frac{\vec{\gamma}\cdot\vec{p}}{m}\right)_1\left(\frac{\vec{\gamma}\cdot\vec{p}}{m}\right)_2\\&\quad+\frac{1}{2}\left[\left(\frac{\vec{\gamma}\cdot\vec{p}}{m}\right)_1+\left(\frac{\vec{\gamma}\cdot\vec{p}}{m}\right)_2\right]+O\left(\left(\frac{v}{c}\right)^3\right),\\ U_1^{-1}U_2^{-1}&=1-\frac{1}{8}\left(\frac{\vec{p}_1^{\,2}}{m_1^2}+\frac{\vec{p}_2^{\,2}}{m_2^2}\right)+\frac{1}{4}\left(\frac{\vec{\gamma}\cdot\vec{p}}{m}\right)_1\left(\frac{\vec{\gamma}\cdot\vec{p}}{m}\right)_2\\&\quad-\frac{1}{2}\left[\left(\frac{\vec{\gamma}\cdot\vec{p}}{m}\right)_1+\left(\frac{\vec{\gamma}\cdot\vec{p}}{m}\right)_2\right]+O\left(\left(\frac{v}{c}\right)^3\right).\end{aligned} \tag{10}$$

We see those γ matrices multiply factors of v/c [e.g. $(\vec{\gamma}\cdot\vec{p}/m)=O(v/c)$]. Thus, only the 0th order (in v/c) term of D_{ij} [in (9)] contributes to $\overline{V}_{++}$. This establishes the validity [to order $(v/c)^2$] of the IA in the case of QED in the Coulomb gauge.

For a general propagator [in (1)], the situation is different. It is still the case that D_{ij} can be taken to be instantaneous. However, if on the right-hand side of Eq. (7) we replace $1/k^2$ by $g(k^2)$, we find that

$$D_{00} = -\frac{k^2}{\vec{k}^2} g(k^2) . \tag{11}$$

This time, in order $(v/c)^2$ there is a noninstantaneous (and therefore time-dependent) piece, and it is not immediately obvious that a Schrödinger equation can be written. We resolve the problem by introducing a gauge in which to $O((v/c)^2)$ D_{00} is instantaneous. The assumption we make is, of course, that the Bethe-Salpeter equation derived from (1) is gauge invariant so that induced ghosts and gauge transformations in higher orders of QCD conspire to change $P_{\mu\nu}$ [in Eq. (1)] only subject to the following condition.

If $k_\mu J^\mu = 0$, then

$$J_\mu P^{\mu\nu} J_\nu = J_\mu g^{\mu\nu} J_\nu . \tag{12}$$

To find the instantaneous gauge (instantaneous with respect to g) we take

$$D^{\mu\nu} = [g^{\mu\nu} + A(k^2) k^\mu k^\nu + B(k^2)(k^\mu n^\nu + k^\nu n^\mu)] g(k^2),$$

and determine A and B by demanding

$$D^{0i} = 0,$$

$$D_{00} = -g(\vec{k}^2) + O((v/c)^3).$$

We thus find that in the instantaneous gauge

$$P_{\mu\nu} = g_{\mu\nu} + \frac{g'(\vec{k}^2)}{g(\vec{k}^2)} k_\mu k_\nu - \frac{g'(\vec{k}^2)}{g(\vec{k}^2)} k_0 (k_\mu n_\nu + k_\nu n_\mu) . \tag{13}$$

(In the Appendix it is shown that this instantaneous gauge propagator can also be derived from a Faddeev-Popov[12] gauge condition.) Notice that for the special case $g(k^2) = 1/k^2$, Eq. (13) is equivalent to Eq. (7), i.e., the $(1/k^2)$ instantaneous gauge is the Coulomb gauge.

In the instantaneous gauge, Eq. (13) implies

$$D_{ij} = g\delta_{ij} + g' k_i k_j . \tag{14}$$

So in calculating $\overline{V}_{++}$, we need to know the Fourier transform of $g' k_i k_j$. To find this, use

$$\overleftrightarrow{kk} g' = \overleftrightarrow{kkk} \cdot \frac{\partial_{\vec{k}} g}{2\vec{k}^2}$$

and[13]

$$\frac{1}{\partial^2} f(r) = -\int_0^r dr' r'^2 \frac{f(r')}{r} - \int_r^\infty r' dr' f(r') .$$

These lead to

$$(\overleftrightarrow{kk} g')^T = -\frac{\overleftrightarrow{1}}{2} g^T - \frac{\vec{r}\vec{r}}{2|\vec{r}|} g'^T . \tag{15}$$

Finally, from Eqs. (1), (5), (10), (13), (14), and (15), it is perfectly straightforward to derive $V_{\rm NR} (= \overline{V}_{++})$ to $O((v/c)^2)$ (now only spatial quantities appear in the calculation):

$$V_{\rm NR}(r) = g(r) + \frac{1}{2m^2}(3+4\kappa)\frac{1}{r}\frac{dg}{dr}(\vec{s}_1 + \vec{s}_2)\cdot\vec{L} + \frac{2}{3m^2}(1+\kappa)^2 \vec{s}_1\cdot\vec{s}_2 \nabla^2 g - \frac{1}{3m^2}(1+\kappa)^2 T_{12}\left(\frac{d^2g}{dr^2} - \frac{1}{r}\frac{dg}{dr}\right)$$
$$+ \frac{1}{4m^2}\left(2p_i(g - rg')p_i + \tfrac{1}{2}\nabla^2(3g + rg') + \frac{2g'}{r}\vec{L}^2 + 4\kappa\nabla^2 g\right) , \tag{16}$$

where $T_{12} = 3(\vec{s}_1\cdot\hat{r})(\vec{s}_2\cdot\hat{r}) - (\vec{s}_1\cdot\vec{s}_2)$. We can compare this expression to those derived by Pumplin *et al.*[6] and Schnitzer.[4,5] Schnitzer uses the IA in the Coulomb gauge and disagrees with us for the spin-independent part. On the other hand, Pumplin *et al.* do the calculation in the covariant gauge $P_{\mu\nu} = g_{\mu\nu}$, but they express k_0 in terms of k_i by using the equations of motion on the mass shell. It turns out that their answer for $V_{\rm NR}(\kappa = 0)$ agrees with ours. A little further thought shows that this curious "gauge invariance" should indeed be the case.

The difference between $J_1^\mu P_{\mu\nu} J_2^\nu$ in two different gauges must vanish if the currents are conserved. Thus each term must have a factor $J_i^\mu k_\mu$, $i = 1$ or 2. We consider such factors in this difference between the instantaneous gauge and an arbitrary gauge. The expression in the arbitrary gauge reduces to that in the instantaneous gauge if k_0 is replaced by anything which makes $J_i^\mu k_\mu$ automatically vanish. In the pure Dirac case, $J^\mu k_\mu = e\gamma_0(k_0 - \vec{\alpha}\cdot\vec{k})$, so the replacement $k_0 \to \vec{\alpha}\cdot\vec{k}$ reduces the expression to the instantaneous gauge form. One can be more sloppy and still obtain the correct answer. Assuming that the convective part of the current is the only important part, $J_i/J_0 \approx v_i = P_i/m$. The symmetric part $P = \frac{1}{2}(P_{\rm out} + P_{\rm in})$ should be used for the current, for example, coming from $(\varphi^\dagger \partial_\mu \varphi - \partial_\mu \varphi^\dagger \varphi)/2i$ in the scalar theory. Therefore, the replacement

$$k_0 \to \frac{P_{\rm out} + P_{\rm in}}{2m}\cdot k = \frac{P_{\rm out}{}^2 - P_{\rm in}{}^2}{2m} = \frac{E_{K\rm out} - E_{K\rm in}}{2m}$$

can be used. Explicit use of this replacement in the Coulomb or Lorentz gauge gives the same answer as in the instantaneous gauge.

Finally, we point out that the κ dependence we derive is at variance with Schnitzer's,[4] although it is in agreement with that of Chan[14] and others.[15] This may be relevant in Schnitzer's phenomenological comparison of hyperfine and spin-orbit splitting in the charmonium system. It is worthwhile noting that the difference in answers is not due to the fact that we work in the instantaneous gauge. We can easily show[16] that the answer should be the same in the Coulomb gauge (IA).

Note added. After writing this paper it was brought to our attention that Gromes[18] has done a study of the $O((v/c)^2)$ corrections for theories with dressed vertices. In that work he discusses many of the questions raised in this paper, but from a different viewpoint. In particular, he does not introduce a new gauge.

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APPENDIX: THE INSTANTANEOUS GAUGE AS DERIVED FROM A FADDEEV-POPOV GAUGE CONDITION.

Let the action be

$$S(J) = -\frac{1}{4}\int d^4x\,[(F^a_{\mu\nu})(F_a^{\mu\nu}) + \text{terms involving other fields and } J\,],$$

where J is an external field. Then the generating functional[17] $e^{i\omega(J)}$ is taken to be

$$e^{i\omega(J)} \equiv \langle {}^+0 | 0^- \rangle_J = N\int \prod (dA)\exp(iS_{\text{eff}})\,\delta(k^\mu A_\mu + \phi(k) n^\mu A_\mu), \tag{A1}$$

where N is an overall normalization factor, functions of k are implicitly Fourier transforms [so $g(x) \equiv \int d^4x e^{ikx} g(k)$], ϕ is an arbitrary function, and $S_{\text{eff}} = S + \text{ghost terms}$.

The δ function can be written[17] as

$$\delta(k^\mu A_\mu + \phi(k) n^\mu A_\mu) = N' \exp\left(-i\int \alpha(k)\,[k_\mu A^\mu + \phi(k) n_\mu A^\mu]^2 dk\right),$$

where N' is a constant.

So we finally get

$$e^{i\omega(J)} = N\int \prod (dA)\exp(iS'_{\text{eff}}),$$

where S'_{eff} contains the quadratic form

$$\tfrac{1}{2} A^\mu (P_{\mu\nu}/k^2)^{-1} A^\nu,$$

where

$$k^2 P^{-1}{}_{\mu\nu} = k^2 g_{\mu\nu} - k_\mu k_\nu + \alpha(k)\,[k_\mu k_\nu + (k_\mu n_\nu + k_\nu n_\mu)\phi(k) + n_\mu n_\nu \phi^2(k)\,].$$

We find that if

$$\phi(k) = \frac{g' k_0 k^2}{g + k_0{}^2 g'}, \qquad \alpha(k) = \frac{g}{g + k^2 g'},$$

then $P_{\mu\nu} g(k^2)$ is the instantaneous gauge propagator.

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[7] See, for example, Akhiezer and Berestetskii, Ref. 1.

[8] H. A. Bethe and E. E. Salpeter, Phys. Rev. 84, 1232 (1951); also see Schwinger, Ref. 1.

[9] By a Schrödinger equation we mean an equation of the form

$$[(\vec{p}_1{}^2 + m_1{}^2)^{1/2} + (\vec{p}_2{}^2 + m_2{}^2)^{1/2} + V(x_1 - x_2)]\psi = E\psi.$$

[10] The metric will be

$$g^{\mu\nu}=\begin{pmatrix}-1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix},$$

and γ matrices are chosen so that $\{\gamma^\mu,\gamma^\nu\}=-g^{\mu\nu}$ [one can use, for instance, the γ^μ's given in J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964), Appendix A].

[11]We will use a superscript T to denote Fourier transformation.

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[15]H. Schnitzer, private communication.

[16]It is a simple consequence of the fact that

$$D_{\mu\nu}\text{ (instantaneous gauge)}-D_{\mu\nu}\text{ (Coulomb-gauge, IA)} \propto k_\mu k_\nu ,$$

and

$$\sigma^{\mu\nu'}k_{\nu'}k_\mu k_\nu=0$$

by the antisymmetry of σ.

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