Nonlinear gauge fields and the structure of gravity and supergravity theories

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(Received 20 March 1978)

By regarding the gauge fields which arise in theories based on nonlinear realizations of gauge symmetries as dynamical variables, we show that gravity, with or without matter, supergravity, and extended supergravity theories possess such symmetries. The model-independent transformation laws for such fields remain unaltered when extended supergravity theories are considered. For groups such as $OSp(N;4)$, the supersymmetry part of the group must be realized nonlinearly. It is pointed out that in this scheme to realize supersymmetry transformations linearly, one possibility is to consider the inhomogeneous extensions of the supergroups $OSp(N;2C)$, i.e., $IOSp(N;2C)$. We also consider the consequences of this point of view in regard to flat-space-time gauge theories and show that instead of the usual set of Yang-Mills equations supplemented with gauge conditions which break the gauge covariance completely, it is possible to formulate a system of equations which are equal in number to the above set but which maintain gauge covariance with respect to the unbroken part of the gauge symmetry.

I. INTRODUCTION

Non-Abelian gauge fields play a central role in any attempt at the unification of known interactions in particle physics. Although the importance of these fields was recognized immediately after their introduction by Yang and Mills,[1] it is relatively recently that a number of nonperturbative and novel features of gauge theories have been demonstrated.[2,3] Perhaps the most direct conclusion which could be drawn from these results is that deductions about the behavior of non-Abelian gauge fields based on extrapolations from Abelian quantum electrodynamics and perturbation theory are to be regarded with due caution. The list of novel features such as the behavior of the vacuum, the nature and the adequacy of gauge conditions, etc., is growing, and to them we may even add the structure of the field equations themselves in the sense described below.

Further insight into the structure of non-Abelian gauge theories has also been gained by studying the geometry of local gauge invariance.[4-6] In particular, it has been shown that it is possible to construct gravitation theories based on a local gauge principle,[4,6] including Einstein's theory itself.[6-8] This establishes the local gauge principle as a basis for unifying gravitation with other interactions. It also points out, however, that local gauge invariance does not completely determine the structure of equations of motions. This can be seen by contrasting Einstein's equations with the Yang-Mills equations. One is therefore led to raise two questions: (i) As theories based on local gauge principles, what distinguishes Einstein's theory from Yang-Mills theory? (ii) Is it possible to write down Einstein-type actions and field equations for gauge fields of an internal-symmetry group? It is to these questions that we address ourselves in this paper. As we shall see these questions are related and the answer to them also provides a way of looking at extended supergravity theories.

To gain insight into the features which make a gauge theory of gravity structurally different from Yang-Mills theory, we review the pertinent features in Sec. II. There we find that in gravity the essential feature is that unlike in Yang-Mills theory the gauge fields associated with a coset G/H of the symmetry group G make explicit appearance both in the action and in the equations of motion. Becuase of this, the usual linear local gauge invariance with respect to G cannot be maintained. In fact, the only possible way out is to require that some fields transform covariantly with respect to G. This means that such fields must belong to a nonlinear realization of G. So in Sec. III we review a number of known results from the theory of nonlinear realizations,[9-13] and point out that one can regard the nonlinear gauge fields as the dynamical variables of the theory without explicit reference to the so-called "pion" fields (coset parameters) or the related Yang-Mills fields of the linear theory.

As a first application of these ideas, possible forms of actions in flat space-time are considered in Sec. IV. In some cases one obtains modified field equations even for gauge fields associated with the linear subgroups of the internal-symmetry group.

In Sec. V we make a number of general remarks about the structure of gauge field equations that one should expect when part of the symmetry is realized nonlinearly. Taking our clues from how pure gravity theory may be viewed this way, we point out that it is possible to formulate a system of equations and constraints which are equal in number to those of Yang-Mills theory supplemented with gauge conditions. Our system of equations is noteworthy in that these equations are gauge covariant with respect to the unbroken part of the symmetry group. If they admit nontrivial solutions, this amounts to a way of introducing a new set of dynamical variables which play the role of the tetrad fields in gravity.

In Sec. VI, we turn to gravity and supergravity theories and show that they are compatible with the nonlinear realizations based on supergroups OSp(N; 4), $N=0, 1, \ldots$. The nonlinear transformation laws are geometrical and model independent, so that they do not change from one supergroup to the next. In this sense they are more naturally tailored to the structure of extended supergravity actions.

One drawback of working with nonlinear realizations is that one has more freedom than in unbroken Yang-Mills theories. But in the cases treated in Sec. VI, it is shown that this uniqueness can be regained by a physical requirement on the torsion tensor. If one regards gravity theory as a spontaneously broken gauge theory, then any supergravity theory such as those based on OSp(N; 4), which contain the gauge group of the gravity theory, will also be a spontaneously broken gauge theory. In all such theories, supersymmetry will be realized nonlinearly. One can then ask if within the scheme of nonlinear realizations one can construct theories in which supersymmetry is realized linearly. In Sec. VII we show that this can be done with gauge groups IOSp(N; 2C), i.e., the inhomogeneous extensions of the supergroups OSp(N; 2C).

II. GAUGE FIELD GEOMETRIES

Conventional non-Abelian gauge theories can be interpreted in terms of the geometry of a principal fiber bundle.[4,5] They are connection coefficients on a cross section of a bundle with space-time as its base and the symmetry group G as its fiber. The field strength tensors are then the components of the curvature tensor of the bundle. They are related to the gauge fields via the commutator

$$[\hat{D}_\mu, \hat{D}_\nu] = -\hat{F}^A_{\mu\nu} X_A, \tag{2.1}$$

where $\hat{D}_\mu$ is the covariant derivative with respect to the gauge group G, and X_A are the group generators.

Let H be a subgroup of G and write the algebra of G,

$$L = T \oplus S, \tag{2.2}$$

where T is the algebra of H and S contains the generators of elements homeomorphic to the quotient space G/H. Let

$$\{X_A\} = \{T_a, S_i\}, \tag{2.3}$$

where the index a runs over the elements of T and the indes i runs over those of S. Then if the splitting (2.2) between T and S corresponds to a Cartan decomposition, one has also

$$\begin{aligned} &[T_a, T_b] = f^c_{ab} T_c, \\ &[T_a, S_i] = f^j_{ai} S_j, \\ &[S_i, S_j] = f^a_{ij} T_a, \end{aligned} \tag{2.4}$$

where f^A_{BC} are the structure constants of G. In this terminology, if $\hat{D}_\mu$ is the usual covariant derivative with respect to G, then

$$\begin{aligned} \hat{D}_\mu &= \partial_\mu + h^A_\mu X_A \\ &= \partial_\mu + h^a_\mu T_a + h^i_\mu S_i, \end{aligned} \tag{2.5}$$

where the h^A_μ's are the gauge fields of the theory:

$$\hat{F}^A_{\mu\nu} = h^A_{\mu,\nu} - h^A_{\nu,\mu} + f^A_{BC} h^B_\nu h^C_\mu. \tag{2.6}$$

One can also write down a covariant derivative D_μ with respect to the subgroup H:

$$D_\mu = \partial_\mu + h^a_\mu T_a, \tag{2.7}$$

so that

$$\hat{D}_\mu = D_\mu + h^i_\mu S_i. \tag{2.8}$$

The curvature tensor with respect to the subgroup H is

$$F^a_{\mu\nu} = h^a_{\mu,\nu} - h^a_{\nu,\mu} + f^a_{bc} h^b_\nu h^c_\mu, \tag{2.9}$$

so that

$$\hat{F}^a_{\mu\nu} = F^a_{\mu\nu} + f^a_{ij} h^i_\nu h^j_\mu. \tag{2.10}$$

Under the subgroup H, the components $\hat{F}^i_{\mu\nu}$ of $\hat{F}^A_{\mu\nu}$ are not mixed with the remaining $F^a_{\mu\nu}$. From (2.7) they are given by

$$\hat{F}^i_{\mu\nu} = D_\nu h^i_\mu - D_\mu h^i_\nu. \tag{2.11}$$

In a conventional non-Abelian gauge theory with a gauge group G, the gauge fields h^A_μ satisfy the Yang-Mills equations

$$\eta^{\mu\nu} \hat{D}_\mu \hat{F}^A_{\nu\lambda} = 0. \tag{2.12}$$

These equations are manifestly covariant with respect to the group G in the sense that the dependence on the gauge fields enters only through covariant objects. Let us contrast this with the gauge theory of gravity[6,8] in which $G = \mathrm{ISO}(3,1)$ and

$H = SO(3,1)$. In that case h^i_μ and h^a_μ satisfy the equations of motion[14]

$$\epsilon^{\mu\nu\rho\lambda}\epsilon_{ab} f^b_{ij} h^i_\nu R^a_{\rho\lambda} = 0\,, \tag{2.13}$$

$$\epsilon^{\mu\nu\rho\lambda}\epsilon_{ab} f^b_{ij} h^i_\nu R^j_{\rho\lambda} = 0\,. \tag{2.14}$$

These equations are structurally different from those of Yang and Mills and the gauge fields h^i_μ explicitly appear in the field equations. They transform covariantly with respect to H but not G. So, if all h^A_μ transform according to the usual *linear* gauge transformations of Yang-Mills type, there would be no hope of having an action which is gauge invariant with respect to G. The way one normally gets around this problem is to note that the field components h^i_μ form a square matrix in which μ and i are indices of the same type and which can be inverted. Then Eq. (2.14) can be solved for h^a_μ in terms of h^i_μ. Regarded as a nonlinear function of h^i_μ the variation of h^a_μ is given by

$$\delta h^a_\mu = \frac{\delta h^a_\mu}{\delta h^i_\nu}\,\delta h^i_\nu\,. \tag{2.15}$$

As a result not all the components of h^A_μ transform linearly under G.

When G is an internal-symmetry group, the indices μ and i differ both in range and in type, so that the matrix h^i_μ is, in general, not invertible. One must therefore look for an alternative method to make the analogs of (2.13) and (2.14) covariant. The method we want to pursue is one in which the h^i_μ's transform covariantly. Since such a transformation law cannot be maintained if the h^A_μ's form a linear realization of G, we turn to nonlinear realizations irrespective of whether the symmetry group is of internal or of space-time origin.

III. NONLINEAR GAUGE FIELDS

To implement the ideas discussed in the preceding section, we must study the structure of vector fields which transform according to a nonlinear realization of the symmetry group G. The method of constructing these fields is known from the study of phenomenological field theories.[9-11] In such theories scalar and spin-$\frac{1}{2}$ fields are required to transform according to nonlinear realizations of some symmetry group G. To couple such fields to Yang-Mills fields it was found convenient to construct first nonlinear gauge fields so that invariant interactions could be written down by simple index saturation with respect to the linear subgroup H. The nonlinear gauge fields themselves were not given the status of independent dynamical variables but were, instead, expressed in terms of the usual Yang-Mills fields and complicated functions of the so-called pion fields (coset parameters) and their derivatives.

In specifying the transformation properties of the nonlinear gauge fields, we follow the method of Ref. 10 and relate them to the linear (Yang-Mills) gauge fields of the group. The point of view we adopt, however, is to give the new fields the status of independent dynamical variables and to write down actions and equations of motion for them. This is consistent with the usual axioms of field theory according to which one may replace some or all of a set of fields by appropriate functionals of these fields. One can of course attempt to relate the fields to the linear fields of unbroken theory at a later stage.

From a geometrical point of view, the construction of matter fields, linear or nonlinear, amounts to the construction of what is known as an associated fiber bundle.[12] The gauge fields themselves belong to the principal fiber bundle, where the fiber is the group space and the generators are differential operators acting on this space. In associated fiber bundles the fiber of the principal bundle, i.e., the group space, is replaced by a representation of the group which may be linear or nonlinear. A connection in the principal bundle means the specification of a covariant derivative. Then this connection "induces" connections in the bundles "associated" with a given principal bundle, i.e., it fixes their covariant derivatives. For associated bundles in which the fiber is a linear representation, this amounts to replacing the generators in the expression for the covariant derivative of the principal bundle by their appropriate matrix representations. For the associated bundle in which the fiber is the coset space G/H the connection is induced as follows.[13]

In the notation of Sec. II let H be a subgroup of G and K be the coset space G/H. Then every element $g \in G$ can be written as

$$g = kh\,, \tag{3.1}$$

where

$$h = e^{u^a T_a} \in H\,, \quad k = e^{\xi^i S_i} \in K\,. \tag{3.2}$$

Also, let

$$h_\mu = h^A_\mu X_A = V^a_\mu T_a + A^i_\mu S_i \tag{3.3}$$

be the connection in a principal fiber bundle, so that

$$\{h^A_\mu\} = \{V^a_\mu, A^i_\mu\} \tag{3.4}$$

are the usual Yang-Mills fields. Under the transformation by an element g, we have

$$h'_\mu = g h_\mu g^{-1} + g\partial_\mu g^{-1}\,. \tag{3.5}$$

Then based on these we define a connection in the associated bundle and the nonlinear fields H^a_μ and K^i_μ according to[10]

$$G_\mu \equiv H_\mu + K_\mu = H^a_\mu T_a + K^i_\mu S_i = k^{-1}(\partial_\mu + V^a_\mu T_a + A^i_\mu S_i)k\,. \tag{3.6}$$

Then one can verify that under g, H_μ and K_μ transform as

$$H_\mu = h' H_\mu h'^{-1} + h'\partial_\mu h'^{-1}\,, \tag{3.7}$$

$$K_\mu = h' K_\mu h'^{-1}\,, \tag{3.8}$$

where h' is given by the action of g on a standard $k \in K$:

$$gk = k'h'\,. \tag{3.9}$$

Notice that K^i_μ transform covariantly under g and are not mixed with H^a_μ.

Once the fields H^a_μ and K^i_μ as well as their transformation properties are specified, they can be regarded as fundmental dynamical variables without reference to the initial gauge fields. We make use of them to define covariant derivatives with respect to the group G:

$$D_\mu = \partial_\mu + H_\mu = \partial_\mu + H^a_\mu T_a\,. \tag{3.10}$$

The corresponding components of curvature two-form are

$$[D_\mu D_\nu] = -F^a_{\mu\nu} T_a\,, \tag{3.11}$$

where

$$F^a_{\mu\nu} = H^a_{\mu,\nu} - H^a_{\nu,\mu} + f^a_{\ bc} H^b_\nu H^c_\mu \tag{3.12}$$

since the fields K^i_μ do not appear in (3.10) and (3.12). It is also useful to construct, whenever possible, tensor fields which involve K^i_μ and which are structurally similar to (2.10) and (2.11). We therefore define

$$\hat{F}^a_{\mu\nu} = F^a_{\mu\nu} + C^a_{ij}(K^i_\mu K^j_\nu - K^i_\nu K^j_\mu) \equiv F^a_{\mu\nu} + K^{\ a}_{\mu\nu}\,, \tag{3.13}$$

where C^a_{ij} is an appropriate Clebsch-Gordan coefficient. We also set

$$\hat{F}^i_{\mu\nu} = D_\nu K^i_\mu - D_\mu K^i_\nu\,. \tag{3.14}$$

By making use of these fields it is possible to construct invariants which more closely follow the structure of the geometrical objects familiar from the principal bundle based on the structural group G. In analogy with the theory of gravity we refer to $\hat{F}^i_{\mu\nu}$ as the torsion tensor.

We have used the same symbols such as D_μ, $F^a_{\mu\nu}$, etc., as those used in the preceding section to show the similarity in the structures of principal and associated fiber bundles and the nature of their associations. It is to be noted, however, that there are essential differences between the gauge fields H^a_μ, K^i_μ and the Yang-Mills fields h^A_μ. This is clearly indicated by comparing the transformation law (3.5) with those given by (3.7)–(3.9). The nonlinearity of the transformation with respect to the elements of K means that the labels associated with the part of the algebra of G which generate K are no longer available as symmetry indices. In other words, the symmetry has been spontaneously broken from G to H. An irreducible representation of G will, in general, have several irreducible pieces with respect to H. Since in constructing invariant actions one only needs index saturation with respect to the subgroup H, as far as the invariance is concerned it is possible to select a subset of nonlinear fields with respect to G, which form irreducible multiplets with respect to H.

IV. POSSIBLE CLASSES OF ACTIONS IN FLAT SPACE

We now turn to the specific form of actions which could be constructed in flat space. The invariant actions for curved space will be dealt with in the next section. In the usual Yang-Mills theory, the requirement of local gauge invariance leads uniquely to invariants which are functions of the square of the curvature tensor only. This is the important advantage of Yang-Mills theory. For the nonlinear gauge fields there are more possibilities because there are more covariant objects available. We list below three principal classes of such actions. There will be obvious variations within each class. As far as the invariance alone is concerned, any linear combination of the actions from each class is also a possibility. Here we do not enter the discussion of whether or not any of these are physically relevant.

(i) *Einstein-type actions*. Consider the action

$$I_1 = \int d^4x\, \eta^{\mu\nu}\eta^{\rho\lambda}\eta_{ab} K^a_{\mu\rho} F^b_{\nu\lambda}\,, \tag{4.1}$$

where $K^a_{\mu\nu}$ is given by (3.13). Variation of this action with respect to independent dynamical variables H^a_μ and K^i_μ gives

$$\eta^{\rho\lambda} D_\lambda K^a_{\mu\nu} = 0\,, \tag{4.2}$$

$$\eta^{\mu\nu} C^i_{aj} K^j_\mu F^a_{\nu\lambda} = 0\,. \tag{4.3}$$

The "similarity" of these equations to Einstein's equations becomes apparent if we note that with $G = \mathrm{ISO}(3,1)$ or $\mathrm{SO}(3,2)$ and $H = \mathrm{SO}(3,1)$ the gauge fields K^i_μ form a square 4×4 matrix which is invertible and can be identified with the tetrad fields:

$$K^i_\mu K^\mu_j = \delta^i_j\,. \tag{4.4}$$

Then, in (4.1), making the replacements $\eta^{\mu\nu}K^j_\nu \to \eta^{ij}K^\mu_i$ and $d^4x \to K d^4x$, $K = \det(K^i_\mu)$, we get the Einstein action

$$I_E = \int K^{-1} d^4x K_i^\mu K_j^\nu F_{\mu\nu}^{ij} . \tag{4.5}$$

Since in the present formalism the K_μ^i's transform covariantly, then by (4.4) so do K_i^μ and $\det(K_\mu^i)$. Therefore, the action (4.5) is manifestly invariant under nonlinear gauge transformations of the group SO(3, 2) or ISO(3, 1) as given by Eqs. (3.7)–(3.9), and one can regard Einstein's theory, or the theory presented in Ref. 6, as a gauge theory based on a nonlinear realization of ISO(3,1). It has also been pointed out[6-8] that these theories can be regarded as linear gauge theories with constraints. We note that as far as local invariance with respect to the SO(3, 1) part of the group is concerned the two interpretations are identical. This is fortunate because only the linearity of the SO(3, 1) part of the gauge group is related to the possibility of the setting up of local light cones and is therefore of direct physical significance. As pointed out elsewhere,[8] the enlargement to ISO(3, 1) or SO(3, 2) is necessitated mainly to provide suitable independent dynamical variables. Therefore, one may wish for practical reasons to put aside the significance of the exact symmetry limit and to take advantage of the present interpretation which is more useful when matter couplings to gravity are considered.

Returning now to the flat-space action (4.1) and the field equations (4.2) and (4.3), we note that unlike in Yang-Mills theory these are coupled first-order differential equations for the potentials H_μ^a and K_μ^i. If, as in the case of gravity theory, it were possible to explicitly invert K_μ^i, one could solve (4.2) for H_μ^a and substitute the result in (4.3) to obtain a second-order equation for the independent fields K_μ^i. However, an explicit inversion of this kind is not always possible. Note that one can add to the action (4.1) an *explicit* mass term for the K_μ^i fields without altering its invariance.

(ii) *Second class of actions.* Consider next the class of actions

$$I_2 = \int d^4x \, \eta^{\mu\rho} \eta^{\nu\lambda} \eta_{ab} \hat{F}_{\mu\nu}^a \hat{F}_{\rho\lambda}^b . \tag{4.6}$$

Variation of this action with respect to H_μ^a and K_μ^i gives the Euler-Lagrange equations

$$\eta^{\nu\lambda} D_\nu \hat{F}_{\rho\lambda}^b = 0 , \tag{4.7}$$

$$\eta^{\nu\lambda} \eta_{ab} C_{ij}^a K_\nu^j \hat{F}_{\rho\lambda}^b = 0 . \tag{4.8}$$

Again, both equations are manifestly covariant with respect to the transformations (3.7) and (3.8). In this case one of the equations is Einstein type and the other Yang-Mills type.

With $G = \text{ISO}(3, 1)$ and $H = \text{SO}(3, 1)$, the direct analog of (4.6) in curved space-time is a theory of gravity[6] with a piece of the form given by (4.1) and another which is quadratic in the curvature tensor $F_{\mu\nu}^a$ of the SO(3, 1) group. Another variation of this[7] is to take $G = \text{SO}(3, 2)$ and $H = \text{SO}(3, 1)$ to obtain the Einstein action with a cosmological term:

$$I_E' = \int d^4x \, \epsilon^{\mu\nu\rho\lambda} \epsilon_{ijkl} \hat{F}_{\mu\nu}^{ij} \hat{F}_{\rho\lambda}^{kl} . \tag{4.9}$$

Under variation one obtains Eqs. (2.13) and (2.14). The point of bringing up at this stage these analogs from the curved space-time, especially the last one, is to point out that, e.g., in (4.9) general coordinate invariance can be maintained without explicit use of K_μ^i or $\det(K_\mu^i)$. This is no longer true when one considers Yang-Mills theory in curved space-time and is the source of complicated transformations which one finds in the usual treatments of extended supergravity theories and which change from Lagrangian to Lagrangian. In contrast our present transformations are geometrical and model independent.

(iii) *Yang-Mills type actions.* In this class of actions one makes use of all the available components of the curvature tensors. One possibility is to take

$$I_3 = \int d^4x \, \eta^{\mu\rho} \eta^{\nu\lambda} (\eta_{ab} \hat{F}_{\mu\nu}^a \hat{F}_{\rho\lambda}^b + \eta_{ij} \hat{F}_{\mu\nu}^i \hat{F}_{\rho\lambda}^j) . \tag{4.10}$$

Alternatively, one can take

$$I_4 = \int d^4x \, \eta^{\mu\rho} \eta^{\nu\lambda} (\eta_{ab} F_{\mu\nu}^a F_{\rho\lambda}^b + \eta_{ij} \hat{F}_{\mu\nu}^i \hat{F}_{\rho\lambda}^j) . \tag{4.11}$$

To each of these one can of course add mass terms for the covariant fields K_μ^i.

The point we want to emphasize here is that by having vector fields which transform covariantly with respect ot the symmetry group G, one can alter not only the field equations satisfied by these vector fields but also those satisfied by the gauge fields of the linear subgroup as well. It is interesting to speculate on the possible usefulness of such modifications. The only evidence we have at present is that gravitation theories must have such modifications from standard Yang-Mills theory to be experimentally viable.[15] Whether this is a desirable feature of any spontaneously broken gauge theory remains to be seen.

V. GENERAL REMARKS

It is of interest to see if there are conditions under which Einstein's or Einstein-type equations could be related to Yang-Mills equations. Suppose, for definiteness, we require that the spin-1 fields in the theory satisfy the constraint

$$\hat{F}_{\mu\nu}^i = D_\nu K_\mu^i - D_\mu K_\nu^i = 0 \tag{5.1}$$

nontrivially. For gravity this would be the condition of no torsion. This condition is satisfied if we take

$$D_\mu K^i_\nu = N^i_{\mu\nu}, \tag{5.2}$$

where $N^i_{\mu\nu}$ is a symmetric tensor:

$$N^i_{\mu\nu} - N^i_{\nu\mu} = 0. \tag{5.3}$$

Clearly (5.1) or (5.2) are covariant conditions under G only if K^i_μ transform covariantly. Thus from the start one is dealing with broken symmetry and nonlinear realizations. To simplify the notation we choose the subgroup H such that the Cartan decomposition (2.4) is satisfied. Then

$$\begin{aligned}[D_\mu, D_\lambda]K^i_\nu &= f^i_{aj} F^a_{\mu\lambda} K^j_\nu \\ &= D_\mu N^i_{\lambda\nu} - D_\lambda N^i_{\mu\nu}.\end{aligned} \tag{5.4}$$

As it stands, the set (5.4) gives too many equations. One way to reduce them is to contract two of the indices with the metric tensor of the corresponding space-time:

$$g^{\mu\nu} f^i_{aj} F^a_{\mu\lambda} K^j_\nu = g^{\mu\nu}(D_\mu N^i_{\lambda\nu} - D_\lambda N^i_{\mu\nu}). \tag{5.5}$$

For gravity

$$\begin{aligned} g^{\mu\nu} &= K^\mu_j K^\nu_k \eta^{jk}, \\ K^i_\mu K^\mu_j &= \delta^i_j, \end{aligned} \tag{5.6}$$

so with

$$D_\mu N^i_{\lambda\nu} - D_\lambda N^i_{\mu\nu} = 0 \tag{5.7}$$

we get Einstein's equations

$$K^\nu_j F^{ij}_{\mu\nu} = 0, \tag{5.8}$$

$$\hat{F}^i_{\mu\nu} = 0. \tag{5.9}$$

It is remarkable that Einstein's equations can be obtained in this way from the condition of no torsion. An important feature of Eq. (5.9) is that they are equal in number to the gauge fields H^a_μ of subgroup H and can be used to eliminate them. One can ask for what gauge groups G and their subgroups H this equality can be maintained in four-dimensional space-time. Among the unitary and orthogonal groups, aside from signature, homomorphisms, and contraction limits, this is possible only if $G = SO(5)$, $H = SO(4)$, or $G = SU(4)$ and $H = SU(3) \times U(1)$. Ruling out the second possibility on physical grounds, one is led to SO(3, 2) or ISO(3, 1) as minimal gauge groups for viewing pure gravity as a nonlinear realization. This does not imply, however, that to satisfy other requirements, one cannot embed SO(3, 2) in a larger group.

Next, suppose the indices a, j refer to internal symmetry and take the space-time to be flat. Then $g_{\mu\nu} = \eta_{\mu\nu}$, and we have instead of (5.8) and (5.9)

$$\hat{F}^i_{\mu\nu} = 0, \tag{5.10}$$

$$\eta^{\mu\nu} F^a_{\mu\lambda} f^i_{aj} K^j_\nu = 0. \tag{5.11}$$

It is easy to see how these equations are related to those of Yang and Mills. Applying the operator $\eta^{\lambda\rho} D_\rho$ to (5.11) and using (5.10) we get

$$\eta^{\lambda\rho}\eta^{\mu\nu} f^i_{aj} K^j_\nu D_\rho F^a_{\mu\lambda} = 0. \tag{5.12}$$

If $K^j_\nu \neq 0$, this is, at least for Euclidean space-time, equivalent to Yang-Mills equations for H^a_μ. Thus in the presence of vector fields K^i_μ, a solution of the system of Eqs. (5.10) and (5.11) is also a solution of Yang-Mills equations.

Let G be an n-parameter group, and set

$$n = n_H + n_K, \tag{5.13}$$

where n_H is the dimension of H and n_K the dimension of K. We note that the number of equations (5.10) and (5.11) in 4-space-time are

$$6n_K + 4n_K = 10n_K$$

and, in general this is not equal to $4(n_H + n_K)$. At first sight this may be regarded as a shortcoming. We note, however, that the Yang-Mills equations are subject to gauge conditions which are $n_H + n_K$ in number.

One is therefore dealing with $5(n_H + n_K)$ equations. Let us see under what conditions we have

$$10n_K = 5(n_H + n_K).$$

Clearly this is satisfied only if

$$n_H = n_K. \tag{5.14}$$

An important class of groups satisfying this requirement is chiral groups $SU(n) \otimes SU(n)$.

When the system of equations (5.10) and (5.11) admits nontrivial solutions, it has a definite advantage over the system "Yang-Mills equations plus a gauge condition." Gauge conditions of the usual variety, such as Coulomb or axial gauges, completely destroy the gauge covariance of the solutions. But Eqs. (5.10) and (5.11) are gauge covariant with respect to the unbroken subgroup H and will not lead to gauge-dependent conclusions. We hope to return to a further discussion of this elsewhere.

VI. GRAVITY AND SUPERGRAVITY THEORIES

The method of nonlinear realizations discussed in Sec. III are also applicable to cases in which the symmetry group is a super Lie group[16]: In the expressions such as those given by (3.6)–(3.9) one need only keep in mind that the group parameters are no longer all c numbers.[17] In this section we discuss the extent to which nonlinear gauge fields shed light on the structure of the known model's gravity and supergravity. The construction of new models will be discussed in the next section.

(i) *Pure gravity*. We have already seen in the preceding section how Einstein's theory may be constructed from the fields belonging to a nonlinear realization of SO(3, 2) or its contracted version, in which the linear subgroup H is SO(3, 1). From the physical point of view, any theory of gravity must allow for the possibility of erecting local light cones at every point of the space-time manifold. To ensure this, the gauge group of gravity must contain SO(3, 1). It has been pointed out elsewhere[8] that the gauge fields H^a_μ of SO(3, 1) are not convenient dynamical variables for the description of a massless spin-2 object. To introduce suitable dynamical variables, one can enlarge SO(3, 1) to ISO(3, 1) or SO(3, 2) [or SU(2, 2) for that matter] and then eliminate unwanted fields via a covariant constraint such as the torsion tensor

$$R^i_{\mu\nu} = T^i_{\mu\nu} . \tag{6.1}$$

For pure gravity, if we require that (a) it be torsion-free, and (b) the condition $R^i_{\mu\nu}=0$ be obtained from the variation of action, then the action given by (4.5), i.e.,

$$I_1 = \int d^4x\, \epsilon^{\mu\nu\rho\lambda}\epsilon_{ijkl} K^i_\mu K^j_\nu R^{kl}_{\rho\lambda} , \tag{6.2}$$

is unique up to a cosmological constant. In the language of Dirac,[18] requirement (b) means that the condition $R^i_{\mu\nu}=0$ is a first-class constraint. If one alters the requirement (a) or relaxes the requirement (b), then it is possible to add to I_1 the action

$$I_2 = b^2 \int d^4x K g^{\mu\rho} g^{\nu\lambda} R^a_{\mu\nu} R^b_{\rho\lambda}\eta_{ab} , \tag{6.3}$$

where $g^{\mu\nu}$ is given by

$$g^{\mu\nu} = K^\mu_i K^\nu_j \eta^{ij} . \tag{6.4}$$

The sum of the actions I_1+I_2 is just the action given in Ref. 6.

(ii) *Gravity coupled to the Yang-Mills field*. In the absence of supersymmetry, the space-time and internal symmetries are quite distinct, so that the gauge group of interest to us is of the form $SO(3,2)\otimes G_i$ or its contracted version. The subgroup H is $SO(3,1)\otimes G_i$, and the fields K^i_μ corresponding to the coset $SO(3,2)\otimes G_i/SO(3,1)\otimes G_i$ transform covariantly. So the Yang-Mills action which must be added to I_1 or I_1+I_2 above is

$$I_3 = \int d^4x K g^{\mu\rho} g^{\nu\lambda} F^a_{\mu\nu} F^b_{\rho\lambda}\eta_{ab} . \tag{6.5}$$

This action is invariant under the gauge transformations given by (3.7) as well as nonlinear transformations (3.8). Had we required that K^i_μ transform as gauge fields, there would have been no way of making (6.5) invariant under the entire group $SO(3,2)\otimes G_i$. Encouraged by the fact that the geometrical transformations (3.7) and (3.8) retain their validity in the presence of matter couplings, we now turn to the supersymmetric gauge groups.

(iii) *Simple supergravity*. For reasons which are by now well known,[7,8] let us consider the supergroup G=OSp(1; 4) or its contracted version as the relevant gauge group for simple supergravity theory.[19] Regarding supersymmetry as a broken symmetry, the linear subgroup is H =SO(3, 1), just as in pure gravity. But now the fields associated with the coset space K =OSp(1; 4)/SO(3, 1) are the sets $\{K^i_\mu\}$ and $\{\psi^\alpha_\mu\}$, where α is the spinor index specifying the supersymmetry generators. These two sets transform covariantly under G according to (3.8) and do not mix with each other. Corresponding to (3.10) the relevant covariant derivative is

$$D_\mu = \partial_\mu + H^a_\mu T_a . \tag{6.6}$$

Then

$$[D_\mu, D_\nu] = -R^a_{\mu\nu} T_a \tag{6.7}$$

gives

$$R^a_{\mu\nu} = H^a_{\mu,\nu} - H^a_{\nu,\mu} + f^a_{\ bc} H^b_\mu H^c_\nu \tag{6.8}$$

just as in pure gravity. If we were dealing with a linear realization, the components of curvature tensor would involve not just H^a_μ's but also K^i_μ's and ψ^α_μ's. So as in (3.13) we also write down covariant structures which in the linear limit would coincide with the components of curvature tensor of the full group. For the present case the corresponding curvature components have been given in Ref. 17. The nonlinear covariants we seek are thus

$$\hat{R}^a_{\mu\nu} = R^a_{\mu\nu} + C^a_{ij} K^i_\mu K^j_\nu + C^a_{\alpha\beta}\psi^\alpha_\nu\psi^\beta_\mu , \tag{6.9}$$

$$\hat{R}^i_{\mu\nu} = D_\nu K^i_\mu - D_\mu K^i_\nu + C^i_{\alpha\beta}\psi^\alpha_\nu\psi^\beta_\mu , \tag{6.10}$$

$$\begin{aligned}\hat{R}^\alpha_{\mu\nu} &= D_\nu \psi^\alpha_\mu - D_\mu\psi^\alpha_\nu + C^\alpha_{\ \beta i}(K^i_\mu\psi^\beta_\nu - K^i_\nu\psi^\beta_\mu) \\ &\equiv R^\alpha_{\mu\nu} + C^\alpha_{\ \beta i}(K^i_\mu\psi^\beta_\nu - K^i_\nu\psi^\beta_\mu) .\end{aligned} \tag{6.11}$$

The point to keep in mind in connection with these tensors is that the only way to justify their relevance is to regard $\{H^a_\mu, K^i_\mu, \psi^\alpha_\mu\}$ together as a nonlinear realization of G and to make use of those tensors which are suggested by a linear realization of G. Had we regarded H^a_μ, K^i_μ, and ψ^α_μ as separate representations of H and unrelated via a representation of G, then the variety of covariant objects would have been overwhelmingly more than what we have.

In parallel with pure gravity, if we require that (a) the generalized torsion tensor (6.10) vanish and (b) the condition $\hat{R}^{\,i}_{\mu\nu}=0$ be a first-class constraint, then one is led uniquely to the simple supergravity action

$$I_4=\int d^4x\,\epsilon^{\mu\nu\rho\lambda}[\epsilon_{ijkl}K^i_\mu K^j_\nu \hat{R}^{\,kl}_{\rho\lambda} +\chi f^{\,\beta}_{i\gamma}(C\gamma^5)_{\alpha\beta}\hat{R}^{\,\alpha}_{\mu\nu}K^i_\rho\psi^\gamma_\lambda]. \tag{6.12}$$

As emphasized before,[8] the constraint equation (6.1) plays an essential role in determining the form of the action.[20] If the condition (b) is relaxed, then it is possible to add other, in particular, quadratic terms to the action I_4, thus modifying both the gravity and the spin-$\frac{3}{2}$ equations of motion. The action (6.12) is manifestly invariant with respect to the model-independent transformations (3.7) and (3.8).

(iv) *SO(2)-extended supergravity.* The geometrical formulation of the SO(2)-extended supergravity theory[21] has been discussed elsewhere.[8,22] Here we want to point out how it can be arrived at as a nonlinear realization of OSp(2; 4) with respect to the coset space OSp(2; 4)/SO(3, 1) $\otimes$SO(2). In this case of the fields $\{H^a_\mu, A^\bullet_\mu, K^i_\mu, \psi^{\alpha,z}_\mu\}$ the sets $\{K^i_\mu\}$ and $\{\psi^{\alpha,z}_\mu\}$, $z=1,2$, transform covariantly according to (3.8), whereas $\{H^a_\mu\}$ and $\{A^\bullet_\mu\}$ transform with an inhomogeneous term as expected from gauge fields. The dot on $A^\bullet_\mu$ indicates that it is a gauge field but that for the SO(2) group it runs over a single element. Since the linear subgroup H is now SO(3, 1) $\otimes$SO(2), the relevant covariant derivative is

$$\hat{D}_\mu=\partial_\mu+H^a_\mu T_a+A^\bullet_\mu T_\bullet\,, \tag{6.13}$$

so

$$[\hat{D}_\mu,\hat{D}_\nu]=-R^{\,a}_{\mu\nu}T_a-F^\bullet_{\mu\nu}T_\bullet\,, \tag{6.14}$$

where $R^{\,a}_{\mu\nu}$ is given by (6.8), and

$$F^{\,\bullet}_{\mu\nu}=A^\bullet_{\mu,\nu}-A^\bullet_{\nu,\mu}\,. \tag{6.15}$$

As in the case of simple supergravity, we can also construct covariant structures similar in form to the curvature components of the principal bundle with structure group OSp(2; 4). The expressions for $R^{\,a}_{\mu\nu}$ and $R^{\,i}_{\mu\nu}$ are the same as those given by (6.9) and (6.10). In addition, we have

$$\hat{R}^{\,\alpha z}_{\mu\nu}=\hat{D}_\nu\psi^{\alpha z}_\mu-\hat{D}_\mu\psi^{\alpha z}_\nu+C^{\,\alpha}_{\beta i}(K^i_\mu\psi^{\beta z}_\nu-K^i_\nu\psi^{\beta z}_\mu)\,, \tag{6.16}$$

$$\hat{F}^{\,\bullet}_{\mu\nu}=F^{\,\bullet}_{\mu\nu}+C^{\,\bullet}_{(\alpha z)(\beta z')}\psi^{\alpha z}_\mu\psi^{\beta z'}_\nu\,. \tag{6.17}$$

Notice again that the knowledge of the form of the components of the curvature tensor associated with the unbroken gauge group is essential in writing down these covariants. In other words, the relevent covariants of the broken symmetry somehow "remember" their origin. Requiring as in the previous cases that (a) the torsion condition $\hat{R}^{\,i}_{\mu\nu}=0$ be satisfied and (b) this condition be a first-class constraint, one obtains in the notation of Ref. 8

$$I_5=\int d^4x\,\epsilon^{\mu\nu\rho\lambda}[\epsilon_{ijkl}\hat{R}^{\,ij}_{\mu\nu}\hat{R}^{\,kl}_{\rho\lambda}+\chi(C\gamma^5)_{\alpha\beta}R^{\alpha z}_{\mu\nu}R^{\alpha z}_{\rho\lambda} +\tfrac{1}{8}e^{\sigma\delta}_{\rho\lambda}\hat{F}^\bullet_{\mu\nu}F^\bullet_{\sigma\delta}]. \tag{6.18}$$

If condition (b) is relaxed, one can write down additional invariants. The invariance of this action with respect to the transformations (3.7) and (3.8) is again manifest, and we do not have to invent transformation laws different from those applicable to simple supergravity which leave this action invariant. It remains to be seen whether our transformation laws are in some way related to those given, e.g., in Ref. 22. This question will be dealt with elsewhere.

(v) *Generalizations.* The formalism we have set up is completely general and is applicable to any Lie of super Lie group. It applies, in particular, to the nonlinear realizations of groups OSp(N; 4) and SU(N; 4).[23] In the first case the linear subgroup of interest is H=SO(3, 1) $\otimes$SO(N). In the second case, one can directly take the linear group to be H=SO(3, 1) $\otimes$SU(N) or go through a hierarchy of symmetry breakdowns such as SU(N; 4) $\to$SU(2, 2) $\otimes$SU(N) $\to$ SO(3, 1) $\otimes$SU(N). We also note that for, e.g., OSp(N; 4) groups with $N\geq 3$, there will also be invariants of the form $Q\not{D}Q$ where Q^α is a Majorana spinor. The question of which invariants should or should not be included in the action cannot be answered by invariance arguments alone. This ambiguity is, however, no worse than what one encounters in any Lagrangian field theory. For example, in quantum chromodynamics, local gauge invariance does not exclude the generalized Born-Infeld[5] terms of the type $(1+bF^a_{\mu\nu}F^{\mu\nu}_a)^{1/2}$ or any functional of $F^a_{\mu\nu}F^{\mu\nu}_a$. It is only when one imposes other *physical* criteria such as perturbative renormalizability, positivity, etc., that one arrives at the usual action of quantum chromodynamics. As we have seen, one of the criteria which is crucial in determining the form of the action is the torsion condition (6.1) proposed in Ref. 8.

VII. NEW LOCALLY SUPERSYMMETRIC MODELS

In the models discussed in Sec. VI, supersymmetry transformations are not part of the linear subgroup H and are therefore realized nonlinearly. Since this includes OSp(N; 4) and SU(N; 4), which are the only allowed global supersymmetries of the S matrix under a broad set of conditions,[24] one may wonder if it is possible to realize supersymmetry such that local transformations are linear and given in a geometrical model independent way. That this can be done rests on the facts that (a) we are here dealing with the symmetries not of the S matrix but of the equations of motion and actions, and (b) in a nonlinear realization only the unbroken subgroup H of G can at most be a symmetry of the S matrix. Therefore, to be consistent with the conclusions of Ref. 24, it is only necessary that the subgroup H be one of the above groups.

With these points in mind, let us consider a gauge theory based on the nonlinear realization of the group $G=\mathrm{IOSp}(N;2C)$, i.e., the inhomogeneous extension of $\mathrm{OSp}(N;2C)$ which we take as the linear subgroup H.[25] The algebra of H has the generators $\{J_{ij}, S_{\alpha a}, L_{\hat{A}}\}$, where J_{ij} are generators of SO(3, 1), $S_{\alpha a}$ are generators of supersymmetry transformations, and $L_{\hat{A}}$ are generators of SO(N). Their commutators are

$$\begin{aligned}
&[J_{ij}, J_{kl}]=f^{mn}_{ijkl}\,J_{mn}, \quad [L_{\hat{A}}, L_{\hat{B}}]=f^{\hat{C}}_{\hat{A}\hat{B}}\,L_{\hat{C}},\\
&\{S_{\alpha a}, S_{\beta b}\}=f^{kl}_{\alpha a\beta b}J_{kl}+f^{\hat{C}}_{\alpha a\beta b}L_{\hat{C}},\\
&[J_{ij}, S_{\alpha a}]=f^{\beta}_{ij\alpha}S_{\beta\alpha}, \quad [L_{\hat{A}}, S_{\alpha b}]=f^{c}_{\hat{A}b}S_{\alpha c}.
\end{aligned} \tag{7.1}$$

To construct the algebra of $\mathrm{IOSp}(N;2C)$, we must add to the above generators the sets $\{T_i\}$ and $\{\theta_A\}$. The generators T_i, $i=0,1,2,3$, transform as vectors under SO(3, 1) transformations and as scalars under SO(N). The generators θ_A transform as vectors under SO(N) and as scalars under SO(3, 1). The additional commutators are

$$\begin{aligned}
&[T_i, T_j]=[\theta_A, \theta_B]=[T_i, \theta_A]=0,\\
&[T_i, J_{jk}]=f^{l}_{ijk}T_l, \quad [\theta_A, L_{\hat{B}}]=f^{C}_{A\hat{B}}\theta_C,\\
&[T_i, L_{\hat{A}}]=[\theta_A, J_{ij}]=\{T_i, S_{\alpha a}\}=\{\theta_A, S_{\alpha b}\}=0.
\end{aligned} \tag{7.2}$$

The vanishing of the last two commutators is peculiar to the singular limit of the superalgebra we consider (see Ref. 25). Note that $\mathrm{IOSp}(N;2C)$ has a Poincaré subalgebra.

To illustrate the method, we shall first confine ourselves to the case $N=1$. The nonlinear gauge fields of G which transform linearly under H are the set $\{H^{ij}_{\mu}, H^{\alpha}_{\mu}, K^{i}_{\mu}, \theta_{\mu}\}$. Of these K^{i}_{μ} and θ_{μ} transform covariantly under G according to (3.8), whereas H^{ij}_{μ} and H^{α}_{μ} transform according to (3.7) with an inhomogeneous piece. The covariant derivative is given in the usual way by

$$D_{\mu}=\partial_{\mu}+H^{ij}_{\mu}J_{ij}+H^{\alpha}_{\mu}S_{\alpha}, \tag{7.3}$$

so that

$$[D_{\mu}, D_{\nu}]=-R^{ij}_{\mu\nu}J_{ij}-R^{\alpha}_{\mu\nu}S_{\alpha}, \tag{7.4}$$

where

$$R^{ij}_{\mu\nu}=H^{ij}_{\mu,\nu}-H^{ij}_{\nu,\mu}+f^{ij}_{klmn}H^{kl}_{\nu}H^{mn}_{\mu}+f^{ij}_{\alpha\beta}H^{\alpha}_{\nu}H^{\beta}_{\mu} \tag{7.5}$$

and

$$R^{\alpha}_{\mu\nu}=H^{\alpha}_{\mu,\nu}-H^{\alpha}_{\nu,\mu}+f^{\alpha}_{ij\beta}H^{ij}_{\nu}H^{\beta}_{\mu}. \tag{7.6}$$

One can also write down tensors

$$R^{i}_{\mu\nu}=D_{\nu}K^{i}_{\mu}-D_{\mu}K^{i}_{\nu}, \tag{7.7}$$

$$R^{\cdot}_{\mu\nu}=D_{\nu}\,\theta_{\mu}-D_{\mu}\,\theta_{\nu}. \tag{7.8}$$

Under local infinitesimal gauge transformations of the subgroup H we have

$$\delta R^{ij}_{\mu\nu}=-f^{ij}_{klmn}\,\epsilon^{kl}R^{mn}_{\mu\nu}-f^{ij}_{\alpha\beta}\epsilon^{\alpha}R^{\beta}_{\mu\nu}, \tag{7.9}$$

$$\delta R^{\alpha}_{\mu\nu}=-f^{\alpha}_{ij\beta}\epsilon^{ij}R^{\beta}_{\mu\nu}-f^{\alpha}_{\beta ij}\epsilon^{\beta}R^{ij}_{\mu\nu}. \tag{7.10}$$

We can now write down an action which is manifestly invariant under local transformations of the group G as well as general coordinate transformations. Assuming that $K=\det(K^{i}_{\mu})\neq 0$, we define the inverse tetrads K^{μ}_{i} sich that

$$K^{\mu}_{i}K^{j}_{\mu}=\delta^{j}_{i}. \tag{7.11}$$

Then the simplest invariant action has the form

$$I_6=\int d^4x\sqrt{g}g^{\mu\rho}g^{\nu\lambda}[\eta_{(ij)(kl)}R^{ij}_{\mu\nu}R^{kl}_{\rho\lambda}+\eta_{\alpha\beta}R^{\alpha}_{\mu\nu}R^{\beta}_{\rho\lambda}]. \tag{7.12}$$

Next consider the case $N=2$. The subgroup H being $\mathrm{OSp}(2;2C)$, we have the nonlinear gauge fields $\{H^{ij}_{\mu}, H^{\alpha a}_{\mu}, K^{i}_{\mu}, \theta^{A}_{\mu}, A^{\cdot}_{\mu}\}$. The covariant derivative is

$$D_{\mu}=\partial_{\mu}+H^{ij}_{\mu}J_{ij}+H^{\alpha a}_{\mu}S_{\alpha a}+A^{\cdot}_{\mu}L_{\cdot}, \tag{7.13}$$

where $L_{\cdot}$ is the generator of SO(2) transformations. Then

$$[D_{\mu}, D_{\nu}]=-R^{ij}_{\mu\nu}J_{ij}-R^{\alpha a}_{\mu\nu}S_{\alpha a}-F^{\cdot}_{\mu\nu}L_{\cdot}, \tag{7.14}$$

where

$$R^{ij}_{\mu\nu}=H^{ij}_{\mu,\nu}-H^{ij}_{\nu,\mu}+f^{ij}_{klmn}H^{kl}_{\nu}H^{mn}_{\mu}+f^{ij}_{\alpha a\beta b}H^{\alpha a}_{\nu}H^{\beta b}_{\mu}, \tag{7.15}$$

$$R^{\alpha a}_{\mu\nu}=H^{\alpha a}_{\mu,\nu}-H^{\alpha a}_{\nu,\mu}+f^{\alpha a}_{ij\beta b}H^{ij}_{\nu}H^{\beta b}_{\mu}+f^{a}_{\cdot b}H^{\alpha b}_{\mu}A^{\cdot}_{\nu}, \tag{7.16}$$

$$F^{\cdot}_{\mu\nu}=A^{\cdot}_{\mu,\nu}-A^{\cdot}_{\nu,\mu}+f_{\cdot\alpha a\beta b}H^{\alpha a}_{\nu}H^{\beta b}_{\mu}. \tag{7.17}$$

The transformations of these curvature components are similar to those given by (7.9) and (7.10). We thus have the invariant action

$$I_7=\int d^4x\sqrt{g}\,g^{\mu\rho}g^{\nu\lambda}[\eta_{(ij)(kl)}R^{ij}_{\mu\nu}R^{kl}_{\rho\lambda}+\eta_{\alpha\beta}\eta_{ab}R^{\alpha a}_{\mu\nu}R^{\beta b}_{\rho\lambda}+F^{\cdot}_{\mu\nu}F^{\cdot}_{\rho\lambda}]. \tag{7.18}$$

As mentioned above, the actions I_6 and I_7 are the simplest possibilities. In particular, these actions do not involve the covariant fields θ^a_μ. This is understandable since a representation of the group G is in general reducible with respect to the linear subgroup H. Insofar as the invariants with respect to G can be constructed by index saturation with respect to H, it is not necessary that they involve all the available field. The selection of a set of fields, each irreducible with respect to H, is dictated by the specific physical application.

From the structure of non-Abelian gauge theories one expects that the dependence on gauge fields associated with the unbroken parts of the group enters the action only through the quadratic terms in the components of the curvature tensor. The actions (7.13) and (7.18) are consistent with this expectation. In these actions the fields $H^{\alpha a}_\mu$ enter as bona fide gauge fields with model-independent transformation laws which retain their form when they are part of an extended supergravity theory. We note, however, that the equations of motion for these fields are different from Rarita-Schwinger equations and may require additional conditions to describe a pure spin-$\frac{3}{2}$ field. Whether such models turn out to be of physical interest remains to be seen. What has been demonstrated here is that it is indeed possible to construct locally supersymmetric models in which (a) the spin-$\frac{3}{2}$ fields $H^{\alpha a}_\mu$ enter as gauge fields, (b) the model-independent transformation laws of $H^{\alpha a}_\mu$ are given once and for all by (3.7), and (c) the invariance of actions can be checked by direct inspection.

Note added in proof. After submission of this manuscript for publication, we received a Texas report [No. ORO-3992-333 (unpublished)] by Y. Ne'eman and T. Regge, in which a closely related point of view is developed.

ACKNOWLEDGMENTS

We would like to thank F. Gürsey, L. Marchildon, and C. Schaer for reading the manuscript and helpful comments. We would also like to thank Professor S. Drell and members of the theory group at SLAC for their hospitality during the spring and summer of 1977, where most of this work was done. This work was supported in part by the U. S. Department of Energy.

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