Testing relativistic theories of gravity with spacecraft-Doppler gravity-wave detection

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The response of a spacecraft Doppler-tracking system to the passage of a weak plane gravity wave of the most general polarization is calculated. Results show that the simultaneous tracking of several spacecraft could provide an unambiguous determination of the gravity-wave polarization, a much needed result in the continuing experimental testing of relativistic theories of gravity.

I. INTRODUCTION

In recent years, a number of authors[1] have pointed out that the next important test of theories of relativistic gravity should involve determining the polarization of gravity waves which are seen passing through the solar system. One of the main reasons for this is the fact that most of the currently viable theories of gravity have either exactly the same first-order post-Newtonian limit as general relativity or can be made arbitrarily close to general relativity by adjusting a dimensionless parameter. Therefore additional first-order solar system experiments can never hope to distinguish between these theories. Many of these theories, however, *do* differ from general relativity and from each other in their predictions of the kinds of polarization of gravitational radiation which they will allow to propagate through space. General relativity is one of the most restrictive theories, only allowing two out of a possible six different polarizations. Thus a determination that all observed gravity waves have only the two general relativistic polarizations would be powerful proof of the validity of Einstein's theory, while a single clear detection of a more general mode would indicate that the theory is wrong.

Unfortunately, most of the gravitational wave experiments which are currently operative or under construction are not sensitive to wave polarization. Eardley, Lee, and Lightman[2] have pointed out that in order to determine all six polarizations one must have a system with at least six degrees of freedom *and* the capability of independently determining the wave propagation direction. Paik[3] has shown that a resonant disk or cylinder, the most common type of detector to be implemented, will only couple three of its vibrational modes to a general gravity wave, thus providing only three degrees of freedom and no information about direction. Two perpendicular disks[4] or a resonant sphere[5] could uniquely determine polarization and direction *if* the nonexperimental assumption is made (as is often done) that only the three helicity-preserving waves (ψ_4 and Φ_{22} in the language of Sec. II) occur in nature. Without this assumption, no number of resonant antennas in a localized laboratory can unambiguously determine polarization because they all lack the time resolution necessary to indicate the direction of propagation.[6] Good time resolution (i.e., seeing the shape of the wave clearly enough to know at what time it hit each detector) is a characteristic of broad-band detectors separated by several gravity-wave wavelengths. Such a system, capable of unambiguously determining the admixture of polarizations and the direction of propagation of a gravity wave, is provided by the simultaneous Doppler tracking of six or more spacecraft.

Doppler tracking of spacecraft as a method of detecting gravity waves is a concept pioneered by Anderson[7] and Davies,[8] and developed by Estabrook and Wahlquist,[9] who discovered the three-pulse signature which a coherent gravity wave would impart to a tracking network (allowing the wave to be picked out of noise by modern methods of matched filtering). Estabrook and Wahlquist's work, however, focused only on general-relativity polarizations of gravity waves. This paper will extend their approach to a wave consisting of a linear combination of all six polarizations, showing how the polarization information may be extracted from simultaneous data from several spacecraft. Section II will introduce the polarization of weak plane waves in the framework worked out by Eardley, Lee, Lightman, Wagoner, and Will.[10] In Sec. III, we will derive the response of a Doppler-tracked spacecraft to a combination of all gravity-wave polarizations, generalizing the approach of Estabrook and Wahlquist. Finally, in Sec. IV, the prospects for such an experiment will be discussed.

II. WEAK PLANE GRAVITY WAVES

We concentrate on the problem of detecting the passage of gravity waves through a region of empty space, far removed from the source of the waves. The background metric in the absence of any wave is assumed to be constant and (with the proper choice of coordinates) equal to the Minkowski metric

$$\eta_{\mu\nu}=\begin{bmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&-1\end{bmatrix}.$$

The effect of the wave is to change the physical metric from $\eta_{\mu\nu}$ to

$$g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}.$$

Since we are far from the wave source, the waves will be weak, which is to say that they will always satisfy $|h_{\mu\nu}|\ll 1$. Also, far from the source the waves will be plane waves, allowing $h_{\mu\nu}$ to be written

$$h_{\mu\nu}=h_{\mu\nu}(k_\alpha x^\alpha \text{ only}), \tag{1}$$

where k_α is a constant propagation four-vector. Orienting the coordinate system so that the z axis points along the direction of the wave propagation, k_α may be written

$$k_\alpha=(\omega,0,0,\omega). \tag{2}$$

Note that we have required k_α to be null. This restriction may be relaxed at the cost of further complicating the algebra.

It is well known that field equations for $h_{\mu\nu}$ will typically determine $h_{\mu\nu}$ only up to a fourfold gauge freedom. Failure to specify the gauge (equivalent to a further choice of coordinates) will allow nonphysical coordinate waves to confuse the derivation, so we will choose to satisfy what we will call a "spatial gauge" condition on $h_{\mu\nu}$:

$$h_{\mu 0}=0.$$

In this gauge the tidal components of the Riemann tensor obey the simple relationship

$$R_{0i0j}=\frac{1}{2}\,\frac{\partial^2}{\partial t^2}\,h_{ij}. \tag{3}$$

Equation (3) enables us to directly relate the components of the metric waves to the Newman-Penrose[11] functions which are the basis of the gravitational wave classification scheme worked out by Eardley *et al.*[12] In their method, the six components of the Riemann tensor are written in terms of four Newman-Penrose functions, two of which are complex:

$$\begin{aligned}\Psi_2&=\tfrac{1}{6}R_{0303},\\ \Psi_3&=\tfrac{1}{2}(R_{0103}+iR_{0203}),\\ \Psi_4&=R_{0101}-R_{0202}+2iR_{0102},\\ \Phi_{22}&=R_{0101}+R_{0202}.\end{aligned} \tag{4}$$

From Eqs. (3) and (4), it may be seen that each NP function will produce a distinctive metric perturbation. A wave of general strength and general perturbation will be a linear combination of all of these pure modes; the metric is given by

$$g_{\mu\nu}=\begin{bmatrix}1&0&0&0\\0&-1+A_+&A_0&A_1\\0&A_0&-1+A_-&A_2\\0&A_1&A_2&-1+A_3\end{bmatrix}, \tag{5}$$

where

$$A_+=\frac{1}{\omega^2}[\Phi_{22}+\mathrm{Re}(\Psi_4)],$$

$$A_-=\frac{1}{\omega^2}[\Phi_{22}-\mathrm{Re}(\Psi_4)],$$

$$A_0=\frac{1}{\omega^2}[\mathrm{Im}(\Psi_4)],$$

$$A_1=\frac{1}{\omega^2}[\mathrm{Re}(\Psi_3)],$$

$$A_2=\frac{1}{\omega^2}[\mathrm{Im}(\Psi_3)],$$

$$A_3=\frac{1}{\omega^2}(\Psi_2).$$

Equation (5) is the physical metric of spacetime in the presence of a gravity wave moving at the speed of light in the z direction. It is a function of $z-t$ only.

III. DOPPLER TRACKING

A standard technique for tracking distant spacecraft is the precise monitoring of the Doppler shift of a sinusoidal electromagnetic signal, continuously transmitted to the spacecraft and coherently transponded back to earth. In the analysis to follow, we will ignore the normal orbital Doppler signal, assuming the spacecraft is stationary, and concentrate on the anomalous signal produced by the passage of a weak plane gravity wave of general polarization. It will be necessary to consider the theory of the propagation of an electromagnetic wave through a spacetime whose metric is given by Eq. (5). The method we will use was originated by Burke[13] and applied by Estabrook and Wahlquist.[14]

An electromagnetic wave with frequency ν may be described by a propagation four-vector σ_μ which is null to first order in the geometry of Eq. (5):

$$\sigma^1=\nu,$$

$$\sigma^2=\nu[\beta\eta(1+\tfrac{1}{2}A_+)+\tfrac{1}{2}\alpha A_1+\tfrac{1}{2}\beta\zeta A_0],$$

$$\sigma^2=\nu[\beta\zeta(1+\tfrac{1}{2}A_-)+\tfrac{1}{2}\alpha A_2+\tfrac{1}{2}\beta\eta A_0],$$

$$\sigma^3=\nu[\alpha(1+\tfrac{1}{2}A_3)+\tfrac{1}{2}\beta(\eta A_1+\zeta A_2)],$$

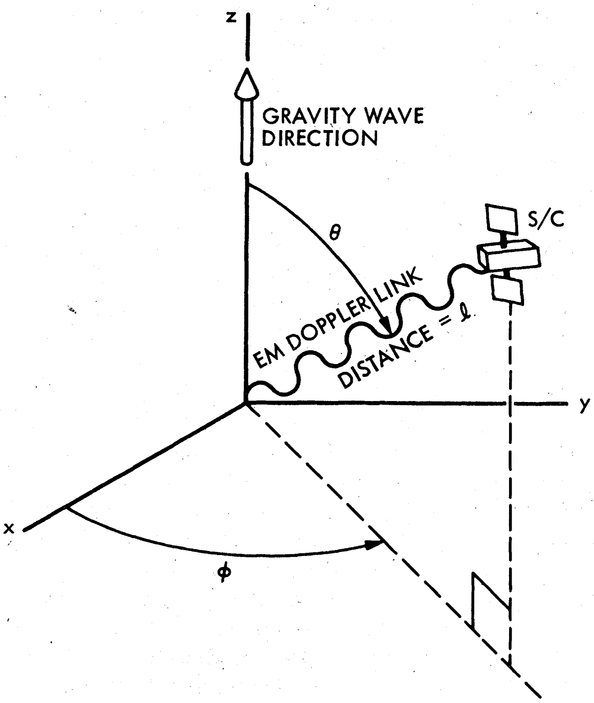

FIG. 1. Geometry of spacecraft position.

with $\alpha^2+\beta^2=1$ and $\eta^2+\zeta^2=1$. Defining the spacecraft orientation with the angles shown in Fig. 1, a consideration of the values of σ^μ in the absence of the gravity wave produces the identifications

$$\alpha=\cos\theta,\quad \beta=\sin\theta,$$

$$\eta=\cos\phi,\quad \zeta=\sin\phi.$$

All the parameters—ν, α, β, η, ζ, A_+, A_-, A_0, A_1, A_2, and A_3—are functions of space and time. At earth at transmission time they will take on some original (unprimed) values. At the spacecraft at the time of reception they may have different (primed) values. Given the gravity-wave parameters at both locations and times (A_i and A_i') and given the original frequency and direction of the signal, the problem is to determine the angles and, especially, the frequency at the time of reception. The connection between the original values and the final values is made by the requirement that the original null propagation vector must be parallel transported along itself to produce the final vector.

To say that σ^μ is parallel transported along itself is to require

$$\sigma_{\mu;\nu}\sigma^\nu=0, \tag{6}$$

where the semicolon denotes covariant differentiation. Equation (6) could be used directly to discover the change in σ^μ produced by the parallel transport, but Burke has discovered a simpler algebraic method based on the Killing vectors of the spacetime metric. Suppose that v^μ is a Killing vector and consider the inner product of v_μ and σ^μ. The change in this scalar along the null direction is given by

$$(\sigma^\mu v_\mu)_{,\nu}\sigma^\nu=(\sigma^\mu v_\mu)_{;\nu}\sigma^\nu=\sigma_{\mu;\nu}\sigma^\nu v^\mu+\sigma^\mu\sigma^\nu v_{\mu;\nu}. \tag{7}$$

The last term in Eq. (7) vanishes due to the antisymmetry of $v_{\mu;\nu}$ and the first term vanishes in the case of parallel transport. Thus the requirement of parallel transport, Eq. (6), is exactly equivalent to a requirement that

$$\sigma^\mu v_\mu=\text{constant along the null geodesic}, \tag{8}$$

where v_μ is a Killing vector. The Killing vectors of the metric in Eq. (5) are

$$\begin{aligned} {}_x v^\mu &= (0,1,0,0,):\ x\text{-translation invariance},\\ {}_y v^\mu &= (0,0,1,0):\ y\text{-translation invariance},\\ {}_n v^\mu &= (1,0,0,1):\ \text{null-translation invariance}. \end{aligned} \tag{9}$$

The requirement $\sigma^\mu v^\nu g_{\mu\nu}=\sigma^{\mu'} v^\nu g'_{\mu\nu}$ for each of the Killing vectors in Eq. (9) produces the following three equations:

$$(0,1,0,0):\ \nu[\beta\eta(1-\tfrac{1}{2}A_+)-\tfrac{1}{2}\beta\zeta A_0-\tfrac{1}{2}\alpha A_1]=\nu'[\beta'\eta'(1-\tfrac{1}{2}A_+')-\tfrac{1}{2}\beta'\zeta'A_0'-\tfrac{1}{2}\alpha'A_1'], \tag{10}$$

$$(0,0,1,0):\ \nu[\beta\zeta(1-\tfrac{1}{2}A_-)-\tfrac{1}{2}\alpha A_2-\tfrac{1}{2}\beta\eta A_0]=\nu'[\beta'\zeta'(1-\tfrac{1}{2}A_-')-\tfrac{1}{2}\alpha'A_2'-\tfrac{1}{2}\beta'\zeta'A_0'], \tag{11}$$

$$(1,0,0,1):\ \nu[1-\alpha+\tfrac{1}{2}\alpha A_3+\tfrac{1}{2}\beta\eta A_1+\tfrac{1}{2}\beta\zeta A_2]=\nu'[1-\alpha'+\tfrac{1}{2}\alpha'A_3'+\tfrac{1}{2}\beta'\eta'A_1'+\tfrac{1}{2}\beta'\zeta'A_2']. \tag{12}$$

Now we write $\alpha'=\alpha+\Delta\alpha$, $\beta'=\beta+\Delta\beta$, $\eta'=\eta+\Delta\eta$, and $\zeta'=\zeta+\Delta\zeta$ and use the definitions $\alpha^2+\beta^2=1$ and $\eta^2+\zeta^2=1$ to eliminate $\Delta\beta$ and $\Delta\zeta$. The resulting expansions are

$$\begin{aligned} \alpha'&=\alpha+\Delta\alpha, &\quad \beta'&=\beta-\frac{\alpha}{\beta}\Delta\alpha,\\ \eta'&=\eta+\Delta\eta, &\quad \zeta'&=\zeta-\frac{\eta}{\zeta}\Delta\eta. \end{aligned} \tag{13}$$

Dividing Eqs. (10) and (11) by Eq. (12), using Eq. (13), and dropping terms which are second or higher order in the Δ's and the A's, the following two equations result:

$$\frac{\eta}{\beta}(1-\alpha)\Delta\alpha+\beta(1-\alpha)\Delta\eta = \tfrac{1}{2}\beta(1-\alpha)\eta\Delta A_{+}+\tfrac{1}{2}\beta(1-\alpha)\zeta\Delta A_0 + \tfrac{1}{2}(\alpha+\eta^2\beta^2-\alpha^2)\Delta A_1 + \tfrac{1}{2}\eta\zeta\beta^2\Delta A_2+\tfrac{1}{2}\eta\alpha\beta\Delta A_3, \quad (14)$$

$$\frac{\zeta}{\beta}(1-\alpha)\Delta\alpha-\frac{\eta}{\zeta}\beta(1-\alpha)\Delta\eta = \tfrac{1}{2}\beta(1-\alpha)\zeta\Delta A_{-}+\tfrac{1}{2}\beta(1-\alpha)\eta\Delta A_0 + \tfrac{1}{2}(\alpha+\zeta^2\beta^2-\alpha^2)\Delta A_2 + \tfrac{1}{2}\eta\zeta\beta^2\Delta A_1+\tfrac{1}{2}\alpha\beta\zeta\Delta A_3, \quad (15)$$

where $\Delta A_i \equiv A_i' - A_i$. Equation (10) alone may be expanded to first order to produce

$$\frac{\nu'}{\nu}=1-\frac{\Delta\eta}{\eta}+\frac{\alpha}{\beta^2}\Delta\alpha+\frac{1}{2}\Delta A_{+} + \frac{1}{2}\frac{\zeta}{\eta}\Delta A_0+\frac{1}{2}\frac{\alpha}{\beta\eta}\Delta A_1. \quad (16)$$

Solving (14) and (15) simultaneously for $\Delta\eta$ and $\Delta\alpha$ and substituting the results into Eq. (16) produces the desired expression for ν',

$$\frac{\nu'}{\nu}=1+(1+\alpha)(A'-A), \quad (17)$$

where A is a function of space and time formed from the A_i:

$$A=\tfrac{1}{2}\eta^2A_{+}+\tfrac{1}{2}\zeta^2A_{-}+\eta\zeta A_0+\frac{\alpha}{\beta}(\eta A_1+\zeta A_2) + \frac{1}{2}\frac{\alpha^2}{\beta^2}A_3. \quad (18)$$

The signal, received at frequency ν', will be transponded at that same frequency and finally received back on earth at frequency ν''. The propagation vector during the return trip is

$$\sigma^0=\nu,$$
$$\sigma^1=-\nu[\beta\eta(1+\tfrac{1}{2}A_{+})+\tfrac{1}{2}\beta\zeta A_0+\tfrac{1}{2}\alpha A_1],$$
$$\sigma^2=-\nu[\beta\zeta(1+\tfrac{1}{2}A_{-})+\tfrac{1}{2}\beta\eta A_0+\tfrac{1}{2}\alpha A_2],$$
$$\sigma^3=-\nu[\alpha(1+\tfrac{1}{2}A_3)+\tfrac{1}{2}\beta(\eta A_1+\zeta A_2)],$$

a result which may be obtained from the uplink propagation vector by substituting $\alpha\to-\alpha, \beta\to-\beta$. The downlink solution may then be obtained simply by making this substitution in Eq. (17):

$$\frac{\nu''}{\nu'}=1+(1-\alpha)(A''-A'). \quad (19)$$

Combining Eq. (17) and (19) gives the overall shift in frequency observed at earth,

$$\frac{\Delta\nu}{\nu}=\frac{\nu''-\nu}{\nu}=(1-\alpha)A''+2\alpha A'-(1+\alpha)A. \quad (20)$$

This equation gives the predicted shift for a single photon and depends on the values of A which the photon encountered at the events of its emission, its transponse, and its reception. To see how this result gives rise to a three-pulse signature it is necessary to consider a continual string of photons emitted, transponded, and received. If the gravity wave consists of a single pulse of height h, as shown in Fig. 2(a), then that photon which is received at the instant the wave hits the earth will be shifted by an amount $\Delta\nu/\nu=(1-\alpha)h$. The photon which happened to be transponded when the wave hit the spacecraft at a later time will be seen to be shifted by $\Delta\nu/\nu=2\alpha h$ when it eventually is received at earth. Finally, the photon which was emitted when the wave struck the earth will carry that information up to the spacecraft and back, producing a shift $\Delta\nu/\nu=-(1+\alpha)h$ when it is received after its round trip. The signal $\Delta\nu/\nu$ as a function of time is shown in Fig. 2(b) for assumed values of $\theta=60°$ and one-way light time $T=4$ min. It should be noted that this three-pulse signature is only apparent when the pulse width is smaller than the light time to the spacecraft.

Figure 2 is a particlar example of how α affects

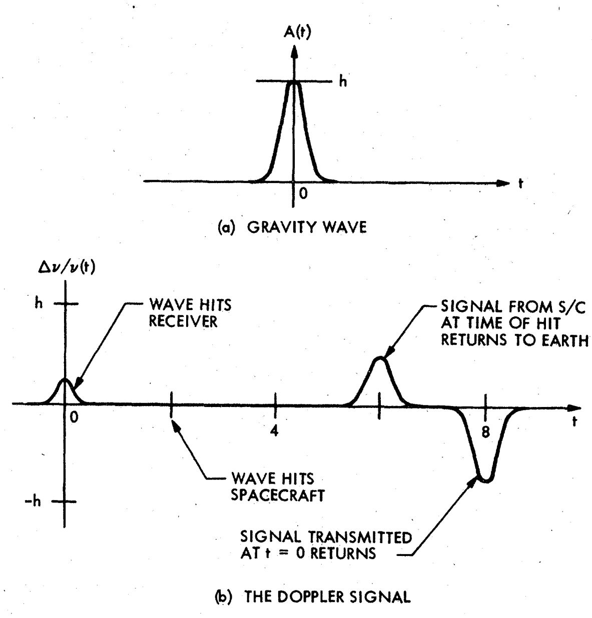

FIG. 2. (a) A typical gravity-wave pulse. (b) The Doppler signal produced by the gravity wave. $\theta=60°$, $l=4$ light minutes.

both the pulse height in $\Delta\nu/\nu$ and the time dependence of the signal. The general time dependence may be derived by writing $(t_E, 0)$ as the coordinates of the emission event, (t_T, z_T) as the coordinates of the transponse event, and $(t_R, 0)$ for the reception event. Then consideration of the geometry of Fig. 1 gives $t_E = t_R - 2l$, $t_T = t_R - l$, and $z_T = \alpha l$. $A(z-t)$ will then take on the particular values at these events given by

$$A'' = A(t_R),$$

$$A' = A(t_R - l(\alpha+1)),$$

$$A = A(t_R - 2l),$$

and Eq. (20) will become

$$\frac{\Delta\nu}{\nu} = (1-\alpha)A(t_R) + 2\alpha A(t_R - l(\alpha+1)) - (1+\alpha)A(t_R - 2l).$$

IV. EXPERIMENTAL PROSPECTS

There is much information which can be read out of a single record such as Fig. 2(b) for a gravity wave with $\lambda < l$. The relative spacing of the pulses gives the round-trip light time to the spacecraft as well as the angle between the gravity-wave propagation vector and the earth-to-spacecraft line-of-sight ($\theta = \cos^{-1}\alpha$). The height of the pulses gives the inherent strength h as well as an independent determination of α. If a single gravity wave excites two spacecraft simultaneously then the angles read out of their Doppler signals must be correlated so as to produce, at worst, a single ambiguity in the direction as shown in Fig. 3(a) and perhaps a unique direction as shown in Fig. 3(b). If there are more than two spacecraft, then all must have signal forms which correlate to give the same direction to the gravity wave, if it was indeed a single gravity-wave pulse responsible for the response.

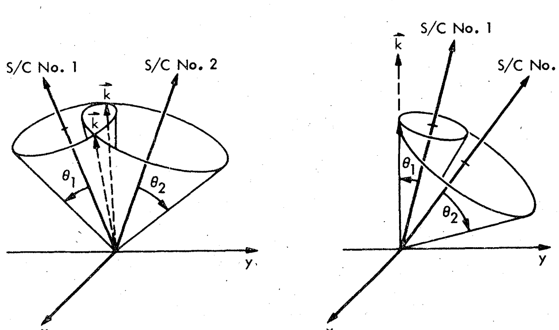

FIG. 3. Determination of gravity-wave direction via simultaneous data from two spacecraft. Two possibilities.

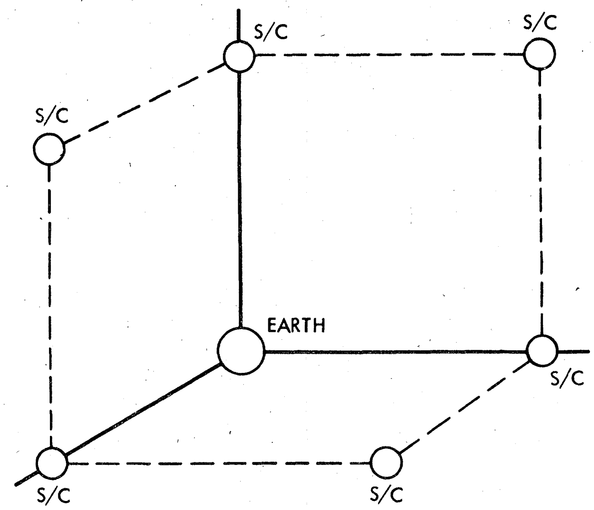

FIG. 4. An optimum orientation of six spacecraft for extracting gravity-wave polarization information.

Further comparing the records from two spacecraft, the inherent gravity-wave strength (after correcting for factors of α) will be the same for both spacecraft if the function A is independent of the spacecraft's orientation to the gravity wave, but it will not be the same for both if A is direction dependent. Using the definition of α, β, η, and ξ to write Eq. (18) as

$$A = \frac{1}{\omega^2}[\Phi_{22} + \cos2\phi\,\mathrm{Re}(\Psi_4) + \sin2\phi\,\mathrm{Im}(\Psi_4) + \cot\theta\cos\phi\,\mathrm{Re}(\Psi_3) + \cot\theta\sin\phi\,\mathrm{Im}(\Psi_3) + \tfrac{1}{2}\cot2\theta\Psi_2], \quad (21)$$

A is seen to be manifestly angle dependent for general polarizations of gravity waves. The relative amplitudes in the signal strengths will therefore allow for some indication as to the polarizations.

The most complete determination would come from the simultaneous tracking of six or more spacecraft, all at different angles as seen from earth (Fig. 4). Calling the inherent strengths observed at the ith spacecraft h_i, Eq. (21) produces a linear set of simultaneous equations,

$$h_i = \cos2\phi_i a_{4R} + \sin2\phi_i a_{4I} + a_{22} + \cot\theta_i\cos\phi_i a_{3R} + \cot\theta_i\sin\phi_i a_{3I} + \tfrac{1}{2}\cot^2\theta_i a_2, \quad (22)$$

where $a_{4R} = (1/\omega^2)\max[\mathrm{Re}(\Psi_4)]$, $a_{3R} = (1/\omega^2)\times\max[\mathrm{Re}(\Psi_3),]$, etc. Equations (22) may be solved simultaneously to uniquely determine the strengths of each mode of gravity-wave polarization. A similar determination would be available in the case of four spacecraft (one of which could be the earth) all simultaneously Doppler tracking each other. In both of these cases, a lot of redundant information exists in the signal spacing, exploitation of which could allow data filtering techniques to increase the sensitivity of the system in the same way as currently done for two Weber antennas.

Finally, it might be pointed out that important results might come by chance even in the case of two spacecraft if the gravity wave turned out to have a particularly fortuitous direction. If both spacecraft were at an angle $\theta = \pi/2$ then Eq. (22) would reduce to

$$h_1 = a_{4R} + a_{22} ,$$

$$h_2 = \cos 2\phi_2 a_{4R} + \sin 2\phi_2 a_{4I} + a_{22} ,$$

where coordinates have been rotated so that $\phi_1 = 0$. Further luck (or design) which provided $\phi_2 = \pi/2$ would produce

$$h_1 = a_{4R} + a_{22} ,$$

$$h_2 = -a_{4R} + a_{22} ,$$

a result which could be solved for (a_{4R}, a_{22}). Since general relativity insists on $a_{22} = 0$, such an event could be the method of proving general relativity wrong. Another piece of luck might be a case where both of the spacecraft and the gravity-wave propagation vector lie in a plane. A rotation would then allow us to set $\phi_i = 0$ in both equations, reducing Eq. (22) to

$$h_1 = a_{4R} + a_{22} + \cot\theta_1 a_{3R} + \tfrac{1}{2}\cot^2\theta_1 a_2 ,$$

$$h_2 = a_{4R} + a_{22} + \cot\theta_2 a_{3R} + \tfrac{1}{2}\cot^2\theta_2 a_2 .$$

A result with $h_1 \neq h_2$ would then definitely lead away from general relativity without being able to actually determine the a_i uniquely.

V. CONCLUSIONS

Doppler tracking of spacecraft is seen to be not only one of the most promising schemes for detecting gravitational radiation in the first place[15] and for employing the shape of the pulse to infer things about the source in the last place,[16] but it also is seen to be an important tool in the ongoing process of experimental tests of gravity theories. For multiple spacecraft it will be only slightly less sensitive to determining the polarization of gravity waves than it is to their outright detection (where an assumption of the truth of general relativity has to be made in order for equal pulse height to be used as an additional discrimination against noise).

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