Generalization of the electromagnetic current operator*

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The present experimental data for the radiative decays of vector mesons require a generalization of the Gell-Mann–Nishijima formula for the electromagnetic current. This generalization will affect the values of the magnetic moments but not the charges.

The results given in this communication have been arrived at using SU(4) and have already been employed in the calculations of Ref. 1. From the numerous discussions that I had on this point, however, I concluded that it requires a separate presentation which is not burdened by the complications (though inessential) of SU(4) or the concept of spectrum-generating groups.

The Gell-Mann–Nishijima formula for the electromagnetic current is, in the normalization that we use, conventionally given by

$$V_\mu^{\rm el}=V_\mu^{\pi^0}+\frac{1}{\sqrt{3}}V_\mu^\eta\,, \qquad (1)$$

where $V_\mu^{\pi^0}$ and V_μ^η are the $(I=1,\ I_3=0,\ Y=0)$ and $(I=0,\ I_3=0,\ Y=0)$ components of an SU(3)-octet operator, respectively. The F-type matrix elements of the right-hand side of (1) are proportional to $I_3+Y/2$ so that, as a consequence of (1), one obtains the Gell-Mann–Nishijima formula for the charges

$$Q=I_3+Y/2\,. \qquad (2)$$

However, (1) is not a consequence of (2).

The suggestion to generalize (1) is based upon the experimental value of $\Gamma^{\rm exp}(\rho\to\pi\gamma)/\Gamma^{\rm exp}(\omega\to\pi\gamma)\approx\frac{1}{25}$. $\Gamma^{\rm exp}(\omega\to\pi\gamma)=(870\pm61)$ keV has been measured several times and can be considered well established, and $\Gamma^{\rm exp}(\rho\to\pi\gamma)=(31\pm10)$ keV has been measured in one experiment[3] using the Primakoff effect.

The prediction for this ratio, which follows from (1) together with the usual SU(3) assignments for ρ, π, and ω is $\Gamma(\rho\to\pi\gamma)/\Gamma(\omega\to\pi\gamma)\approx\frac{1}{9}$. This prediction is only a consequence of the SU(3) group property and is largely independent of any reasonable assumption for the breaking of SU(3) symmetry, as the ρ and ω masses are almost identical. The only way to resolve the discrepancy between this prediction and the experimental value, if one wants to retain the SU(3) assignment, is to change (1) in such a way that (2) is retained. The relation to replace (1) is suggested to be

$$V_\mu^{\rm el}=V_\mu^{\pi^0}+\frac{1}{\sqrt{3}}V_\mu^\eta+V_\mu^S\,, \qquad (3)$$

where V_μ^s is an SU(3)-scalar operator. Such a generalization of (1) may also finally lead to an SU(3) explanation of the hyperon magnetic moments.[4] The discussion in this paper will only make use of the pseudoscalar and vector mesons within SU(3).

One could of course assume that

$$V_\mu^{\rm el}=V_\mu^{(+)}+V_\mu^{(-)}\,,$$

where the two terms on the right-hand side transform under charge conjugation U_c as

$$U_c^\dagger V_\mu^{(\pm)}U_c=\pm V_\mu^{(\pm)}\,.$$

Then $V_\mu^{(+)}$ could not contribute to the charges. However, in order to make as few changes as possible we will assume that

$$U_c^\dagger V_\mu^{\rm el}U_c=-V_\mu^{\rm el} \qquad (4)$$

and that the components of the octet operator V^α have the conventional charge-conjugation properties,

$$U_c^\dagger V_\mu^\alpha U_c=-V_\mu^{-\alpha}\,. \qquad (5)$$

Then

$$U_c^\dagger V_\mu^S U_c=-V_\mu^S\,. \qquad (6)$$

Taking the matrix elements of (6) between meson vectors $|M\rangle$, using

$$U_c|M\rangle=\mathcal{C}_M|\bar{M}\rangle\,,$$

where $\bar{M}$ denotes the antiparticle of M and $\mathcal{C}_M$ the charge parity, one obtains

$$\langle M|V^S|M\rangle=-\langle M|U_c^\dagger V^S U_c|M\rangle=-\bar{\mathcal{C}}_M\mathcal{C}_M\langle\bar{M}|V^S|\bar{M}\rangle\,.$$

For zero-charge mesons this leads immediately, and for charged mesons this leads after an SU(3) transformation, to

$$\langle M|V^S|M\rangle=0\,. \qquad (7)$$

By the same argument it follows that

$$\langle M_1|V^S|M_2\rangle=0 \qquad (7')$$

for any two members of two equivalent SU(3) meson multiplets with $\mathcal{C}_{M_1}=\mathcal{C}_{M_2}$.

As a consequence of (7), (3) leads again to (2).

However, if the SU(3) multiplets M_1 and M_2 have opposite charge parity, $\mathcal{C}_{M_1} \neq \mathcal{C}_{M_2}$, then one cannot conclude that $\langle M_1 | V_\mu^S | M_2 \rangle = 0$. Consequently, if M_1 are the pseudoscalar mesons P and M_2 the vector mesons V, then in general

$$\langle P | V_\mu^S | V \rangle \neq 0 .$$

Thus for the transition matrix elements $\langle P | V_\mu^{el} | V \rangle$ of the decay $V \to P\gamma$ one obtains, in addition to the reduced matrix elements of the octet operator

$$\begin{aligned} F &= \langle 0^- \, \underline{8} \| V^{(8)} \| \underline{8} \; 1^- \rangle_F , \\ D &= \langle 0^- \, \underline{8} \| V^{(8)} \| \underline{8} \; 1^- \rangle_D , \\ A &= \langle 0^- \, \underline{8} \| V^{(8)} \| \underline{1} \; 1^- \rangle , \end{aligned} \qquad (8)$$

the matrix element of the scalar operator

$$S = \langle 0^- \alpha | V^S | \alpha 1^- \rangle . \qquad (9)$$

It is easily shown, again using charge-conjugation properties, that $F = 0$. Further, D and A are related, empirically by the experimental value of $\Gamma(\phi \to \pi\gamma) \approx 0$, or theoretically by the Zweig rule or other theoretical assumptions. Thus the SU(3) part of all the matrix elements of $V_\mu^{\pi^0} + (1/\sqrt{3}) V_\mu^\eta$ can be expressed in terms of one arbitrary reduced matrix element, for which we choose

$$\tilde{d} = \langle \pi^0 | [V^{\pi^0} + (1/\sqrt{3}) V^\eta] | \omega \rangle . \qquad (10)$$

For $\langle \pi^0 | [V^{\pi^0} + (1/\sqrt{3}) V^\eta] | \rho \rangle$ one obtains then from the SU(3) Clebsch-Gordan coefficients

$$\langle \pi^0 | [V^{\pi^0} + (1/\sqrt{3}) V^\eta] | \rho \rangle = -\tfrac{1}{3} \tilde{d} . \qquad (11)$$

Equations (10) and (11) then lead to the above-mentioned prediction

$$\Gamma(\rho \to \pi\gamma) / \Gamma(\omega \to \pi\gamma) = \tfrac{1}{9} , \qquad (12)$$

usually called the quark-model prediction, in disagreement with the experimental value.

A further prediction of (1), which is independent of SU(3)-breaking assumptions, is

$$\Gamma(K^{+*} \to K^+\gamma) / \Gamma(K^{0*} \to K^0\gamma) = \tfrac{1}{4} . \qquad (13)$$

This follows from

$$\begin{aligned} \langle K^0 | [V^{\pi^0} + (1/\sqrt{3}) V^\eta] | K^{0*} \rangle &= \tfrac{2}{3} \tilde{d} , \\ \langle K^+ | [V^{\pi^0} + (1/\sqrt{3}) V^\eta] | K^{+*} \rangle &= -\tfrac{1}{3} \tilde{d} . \end{aligned} \qquad (14)$$

From the assumption (3) for the electromagnetic current it follows that all the matrix elements $\langle P | V^{el} | V \rangle$ are expressed in terms of two arbitrary parameters:

$$d = \langle \pi^0 | V^{el} | \omega \rangle , \qquad (15)$$

$$S = \langle \pi^{\pm 0} | V^{el} | \rho^{\pm 0} \rangle . \qquad (16)$$

Thus with assumption (3) one has enough freedom to explain the experimental value

$$\Gamma^{exp}(\rho \to \pi\gamma) / \Gamma^{exp}(\omega \to \pi\gamma) \approx \tfrac{1}{25} \qquad (17)$$

but one cannot predict it. A prediction that one obtains from (3) with (17) is for $\Gamma(K^{+*} \to K^+\gamma) / \Gamma(K^{0*} \to K^0\gamma)$. Using the SU(3) Clebsch-Gordan coefficients one obtains

$$\begin{aligned} \langle K^0 | V^{el} | K^{0*} \rangle &= d + S , \\ \langle K^+ | V^{el} | K^{+*} \rangle &= S . \end{aligned} \qquad (18)$$

From (17) with (15) and (16) it follows that S and d are related by

$$S = \pm \tfrac{1}{5} d . \qquad (19)$$

Consequently one obtains the predictions

$$\frac{|\langle K^0 | V^{el} | K^{0*} \rangle|}{|\langle K^+ | V^{el} | K^{+*} \rangle|} = \frac{4}{1} \quad \text{and} \quad \frac{\Gamma(K^{+*} \to K^+\gamma)}{\Gamma(K^{0*} \to K^0\gamma)} = \frac{1}{16} , \qquad (20a)$$

or

$$\frac{|\langle K^0 | V^{el} | K^{0*} \rangle|}{|\langle K^+ | V^{el} | K^{+*} \rangle|} = \frac{6}{1} \quad \text{and} \quad \frac{\Gamma(K^{+*} \to K^+\gamma)}{\Gamma(K^{0*} \to K^0\gamma)} = \frac{1}{36} , \qquad (20b)$$

which differ drastically from the prediction (13) of the conventional formula (1). The experimental data for the K^* radiative decay[5] $\Gamma(K^{0*} \to K^0\gamma) = 75 \pm 35$ keV and $\Gamma(K^{+*} \to K^+\gamma) < 80$ keV do not allow one to discriminate between the old prediction (13) and the prediction (20), so that the only empirical support for the assumption (3) which is independent of any symmetry-breaking assumption is the one experimental value for $\Gamma(\rho \to \pi\gamma)$ of Ref. 3.

Summarizing, if the only experimental value for $\Gamma(\rho \to \pi\gamma)$ is correct, then (1) cannot hold. However, there is no theoretical reason to exclude an SU(3)-scalar term V_μ^S from the electromagnetic current, because such a term will, by (7), not contribute to the charge and therefore not affect (3). The assumption (3) is consistent with the experimental data; however, it is quite possible that any other modification of (1) by a new SU(3)-tensor operator will also give agreement with experiment.

Note added. This paper is not the first to investigate the existence of an SU(3)-singlet term in the electromagnetic current. Mathur and Okubo[6] investigated the possibility of such a term for the process $V \to e\bar{e}$. The existence of a singlet contribution for these processes would mean that $\langle 0^+, C = 1, \alpha | V^S | \alpha, C = -1, 1^- \rangle \neq 0$. Though this is not excluded by (7′), it appears that the experimental data are well fitted without such a term if one uses a suitable suppression factor.[7] The appearance of such an SU(3)-singlet term in the leptonic decay process would be masked by SU(3)-breaking effects, whereas this is not the case for the quantities considered above.

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