

Effects of classical fields in meson correlations

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The existence of a classical (coherent) component in the mesonic field as suggested by recent developments in particle physics generates under certain conditions a statistical distribution corresponding to a mixture of chaotic and coherent fields. The methods of quantum optics are used to investigate the effects of this mixture in the correlations. The relevance of this to the Hanbury Brown and Twiss effect in particle physics is discussed. Preliminary comparison with the data suggests that an appreciable coherent component might be present.

I. INTRODUCTION

Several parallel developments in particle physics have focused interest on the concept of coherent states. These include the following:

(1) The discovery of stable, classical solutions in certain field theories¹; a classical field corresponds to a coherent state.

(2) Spontaneous breakdown of symmetries²; the vacuum associated with this phenomenon corresponds to a condensate which is a coherent state.

(3) The phenomenological success of the hydrodynamical model³; in this model excited hadronic matter acts coherently. Moreover it has been shown by Cooper and Sharp⁴ that ideal fluid hydrodynamics follows if the pions are produced in a coherent state.

(4) The possibility that hadronic matter might have superfluid properties has been discussed in the literature for some time^{5,6}; this would obviously imply coherence.

Besides these specific high-energy physics phenomena, at least three other nuclear physics and astrophysics effects have to be mentioned:

(5) The pion condensate⁷ assumed to occur in nuclear and stellar matter at very high densities.

(6) Abnormal nuclear matter⁸ as a consequence of a scalar field with a nonvanishing vacuum expectation value which is a classical component of the field. Moreover, in certain models like the nonlinear σ model, this scalar field leads to a pion condensate⁹;

(7) Superconductivity of nuclear matter.

Although the importance of some of these developments can hardly be overestimated, as remarked by Arnold and Barshay,¹⁰ little attention has been paid to the fact that so far there is no direct experimental proof for coherence in particle and nuclear physics. However, it is clear that many of the models mentioned above would have to un-

dergo a serious revision should coherence eventually fail to be discovered in experiments. Not only has this apparently not been realized so far, but some independent models have been put forward in which the implicit assumption is contained that coherence is actually absent from strong interactions. We have in mind the interpretation of correlation effects in meson production in terms of Bose-Einstein statistics. The considerations developed in Refs. 11-17 apply for chaotic fields, i.e., fields which have no coherence. Therefore, meaningful conclusions about the size and lifetime of hadronic fireballs¹⁴⁻¹⁷ can only be drawn from correlation experiments¹⁸⁻²⁰ after the question of coherence versus chaoticity is settled.

The purpose of this paper is to develop a theoretical scheme through which effects of coherence and classical fields (or sources) could be investigated in particle physics. We shall proceed in close analogy with quantum optics²¹ and show how a field which contains a classical and a quantum component leads to a statistical distribution which is a mixture of coherent and chaotic fields. Next we shall consider the implications for particle correlations of such a mixture.

It turns out that second-order correlations are rather insensitive to even large admixtures of coherence and that the present experimental evidence, which relies mainly on second-order correlations, is consistent with such an admixture. The implications for the determination of the fireball size are briefly discussed and an experimental program involving measurements of higher correlations is outlined, through which the existence and percentage of coherence in meson fields can be investigated.²²

II. COHERENT VERSUS CHAOTIC STATES

In quantum optics where one deals both with systems containing small numbers of particles as well

as with macroscopic systems, the use of the coherent-state representation has proved to be very useful since coherent states are eigenstates, $|\alpha_k\rangle$, of the annihilation operator a_k , and thus correspond to an undefined number of particles. The eigenvalues α_k are the amplitudes of a field which is called the coherent field. Classical fields are a particular case of coherent fields, namely coherent c -number fields.²³

In some sense, the opposites of coherent fields are chaotic fields. To see this we introduce the density operator in the coherent-state representation $\{\alpha_k\}$,

$$\rho = \int \mathcal{P}(\{\alpha_k\}) |\{\alpha_k\}\rangle \langle \{\alpha_k\}| \prod_k d^2\alpha_k, \quad (2.1)$$

where $\mathcal{P}(\{\alpha_k\})$ is the distribution function of the states treated as random variables. \mathcal{P} is assumed to be constrained to the range (0, 1). The difference between coherent and chaotic states can be defined through the distribution $\mathcal{P}(\{\alpha_k\})$.

Coherent states correspond to a well defined $\{\alpha_k\}$, i.e., to maximum "noiselessness,"

$$\mathcal{P}_c^{\beta_k}(\alpha_k) = \prod_k \delta(\alpha_k - \beta_k), \quad (2.2)$$

while the maximum "noise," i.e., chaotic states, are described by a Gaussian distribution,

$$\mathcal{P}_{\text{ch}}(\alpha_k) = \prod_k \frac{1}{\pi \langle n_k \rangle} \exp(-|\alpha_k|^2 / \langle n_k \rangle), \quad (2.3)$$

where $\langle n_k \rangle$ is the mean occupation number in mode k . A particular case of chaotic fields is fields generated by thermal sources; for identical bosons the Bose-Einstein distribution in a single mode is an example of a chaotic distribution.

The difference between coherent and chaotic states manifests itself quite strikingly in the field and intensity correlations and in the associated moments (cumulants μ). While a coherent state leads to a Poisson distribution, i.e., to

$$\mu_{(n)} = 0 \quad \text{for } n \geq 2,$$

for a chaotic distribution this is not the case. Moreover, for chaotic distributions, all higher-order correlation functions can be expressed in terms of the first-order correlation function G^1

$$G^n(x_1, \dots, x_n, x_{n+1}, \dots, x_{2n}) = \sum_P \prod_{j=1}^n G^1(x_j, x_{P(n+j)}). \quad (2.4)$$

In optics of particular importance are stationary distributions when $G^1(x_1, x_2)$ depends only on $x_1 - x_2$. This dependence is usually parametrized by an exponential,

$$G^1(x_1, x_2) = e^{-(x_1 - x_2) / \xi \langle n \rangle}, \quad (2.5)$$

where ξ is the coherence length and G^1 and thus G^n , depend only on two variables $\langle n \rangle$ and ξ .

In Sec. III we shall discuss a more general case, i.e., a mixture of coherent and chaotic distributions. In this case, \mathcal{P} is the convolution product of the two distributions,

$$\begin{aligned} \mathcal{P}^{\gamma_k}(\alpha_k) &= \int \prod_k \mathcal{P}_c^{\alpha_k}(\beta_k) \mathcal{P}_{\text{ch}}(\beta_k - \gamma_k) d\beta_k \\ &= \prod_k \frac{1}{\pi \langle n_k \rangle} \exp(-|\alpha_k - \gamma_k|^2 / \langle n_k \rangle), \end{aligned} \quad (2.6)$$

which is a Gaussian centered around the coherent-field values.

III. MIXTURE OF COHERENT AND CHAOTIC DISTRIBUTIONS

The idea that coherent or chaotic states may play some part in hadronic processes at high energies is not a new one, especially as far as π mesons are concerned. However, it seems, that with the exception of Botke, Scalapino, and Sugar,²⁴ researchers have contented themselves with the investigation of the consequences of the assumption that mesonic fields and sources are totally coherent²⁵ or totally incoherent, i.e., chaotic.^{12,13} Both extremes are obviously idealizations and in reality one should expect to find some superposition of these two cases. The simplest superposition is a mixture as defined above. This well known distribution in optics has the advantage that when comparison with experimental data is made, the result can be expressed in terms of the coherence length of the chaotic component and the ratio $\langle n_c \rangle / \langle n_{\text{ch}} \rangle$, where $\langle n_c \rangle$ and $\langle n_{\text{ch}} \rangle$ are the mean numbers of particles in the coherent and chaotic components respectively, and where

$$\langle n \rangle = \langle n_c \rangle + \langle n_{\text{ch}} \rangle \quad (3.1)$$

is the mean total number of produced particles. Furthermore, the relations between cumulants of different order which should be used to test the model and to derive the ratio $\langle n_c \rangle / \langle n_{\text{ch}} \rangle$ are already worked out and can be used in the comparison with data.

Although the model of Botke, Scalapino, and Sugar is rather general, it does not contain this special case of coherent-chaotic mixture. Indeed, in this model the density operator is

$$\rho = Z^{-1} \int \delta\pi |\pi\rangle e^{-F[\pi]} \langle \pi|, \quad (3.2)$$

where

$$Z = \int \delta\pi e^{-F[\pi]} \quad (3.3)$$

and the integrals are functional integrals. The functional $F[\pi]$ has the Landau-Ginsburg form,

$$F[\pi] = \int_0^Y dy \left[A |\pi(y)|^2 + B |\pi(y)|^4 + C \left| \frac{\partial \pi}{\partial y} \right|^2 \right]. \quad (3.4)$$

A, B, C are constants; y is the rapidity variable,

$$y = \frac{1}{2} \ln \frac{E + p_{||}}{E - p_{||}}, \quad (3.5)$$

where E the total energy of the emitted particle and $p_{||}$ is the corresponding longitudinal momentum; Y is twice the maximum rapidity compatible with energy-momentum conservation; $\pi(y)$ is the pion-field amplitude expressed as a function of rapidity y in the coherent-state representation; it is treated as a random variable. For reasons of simplicity we limit ourselves to pions only and ignore the dependence of $\pi(y)$ on transverse momentum. Furthermore, we ignore quantum number effects. It is shown in Ref. 24 how these effects can be taken care of. Appropriate choices of the parameters A, B, C in (3.4) lead to pure-chaotic or pure-coherent distributions respectively.²⁴ However, a coherent-chaotic mixture is not contained in (3.2), (3.4) because of the absence of a linear term in π in (3.4) [cf. Eqs. (2.1) and (2.6)].

As a matter of fact, because of the pseudoscalar nature of the field such a linear term could arise only if the c -number coefficient of π were also to be a pseudoscalar. The only way such a c -number pseudoscalar can appear is through a classical field and this suggests that a coherent-chaotic mixture is due to a linear superposition of a classical and quantum field. That this is indeed the case will be shown below. On the other hand, it should be stressed that a linear superposition of classical and quantum fields is exactly what the new developments in field theory, mentioned in the introduction, suggest.

The derivation of the mixture distribution pro-

ceeds as follows. We start from the well known statistical operator²¹ for a Bose distribution in terms of the creation and annihilation operators a_k^\dagger and a_k , respectively,

$$\rho = \exp\left(-\sum_k d_k a_k^\dagger a_k\right) / \text{Tr} \rho, \quad (3.6)$$

where d_k is related to $\langle n_k \rangle$, the mean number of particles in the mode k , by

$$\langle n_{\text{ch},k} \rangle = 1 / (e^{d_k} - 1). \quad (3.7)$$

In the \mathcal{P} representation described earlier, this corresponds to the chaotic case (2.3), and to emphasize this we write $\langle n_{\text{ch},k} \rangle$.

Let us consider now a quantum field $\pi(y)$ which we expand into a complete set of functions e^{iky} . In applications, the variable y can be thought of as representing rapidity, e.g.,

$$\pi(y) = \frac{1}{Y^{1/2}} \sum_k a_k e^{iky}, \quad Y = 2y_{\text{max}} \quad (3.8)$$

and

$$k = 2n\pi/Y, \quad n = 0, 1, 2, \dots$$

In terms of the field π , the density operator (3.6) reads

$$\rho = \exp\left(-\int_0^Y \pi(y) \pi^\dagger(y') D(y-y') dy dy'\right) / \text{Tr} \rho, \quad (3.9)$$

where

$$D(y) = \frac{1}{Y} \sum_k d_k e^{iky}. \quad (3.10)$$

The superposition of a Fourier-analyzed quantum field and a classical field of (c -number) intensity β_k is then given by

$$\pi(y) = \frac{1}{Y^{1/2}} \sum_k a_k e^{iky} + \frac{1}{Y^{1/2}} \sum_k \beta_k e^{iky}, \quad (3.11)$$

so that

$$\rho_M = \exp\left(-\sum_k d_k a_k^\dagger a_k - \sum_k d_k \beta_k^* a_k - \sum_k d_k \beta_k a_k^\dagger - \sum_k d_k |\beta_k|^2\right) / \text{Tr} \rho_M \quad (3.12)$$

In the \mathcal{P} representation this yields immediately (2.6).

IV. CONSEQUENCES FOR CORRELATIONS OF A COHERENT-CHAOTIC MIXTURE

The phenomenological implications of formula (2.6) may now be evaluated by taking over the re-

sults already obtained in the case of quantum optics. We leave until Sec. V a discussion of the experimental conditions which must be met for this to be permissible.

Besides the ratio $\langle n_c \rangle / \langle n_{\text{ch}} \rangle$ we also need the field-

correlation length in rapidity space for the chaotic case. This is readily obtained from (2.5) and (2.6) with $\gamma_k=0$. Thus,

$$\begin{aligned} \langle \pi(y_1)\pi(y_2) \rangle &= G^1(y_1, y_2) = \langle n_{\text{ch}} \rangle e^{-|y_1 - y_2|/\xi} \\ &= \frac{1}{Y} \sum_k \langle n_{\text{ch}, k} \rangle e^{ik(y_1 - y_2)}. \end{aligned} \quad (4.1)$$

With

$$\langle n_{\text{ch}, k} \rangle = 1/(a + bk^2), \quad (4.2)$$

and converting the sum into an integral, we have

$$\xi = (b/a)^{1/2}, \quad \langle n_{\text{ch}} \rangle = 1/2\sqrt{ab}. \quad (4.3)$$

Strictly speaking, the replacement of a summation by an integration is not valid here since Y is finite. However, in the case of a sufficiently short correlation length compared with Y our results should be qualitatively correct. The correct procedure, as used in quantum optics, is to expand the field in an orthogonal set on the interval Y chosen so as to reproduce the observed field-correlation function. In our case this would need to be done by fitting the intensity-correlation function; although for the chaotic source, which is our sole concern here, the two are closely related. In fact we shall refer to the optics results for an exponential correlation function and since these are exact we are not restricted to a short correlation length in rapidity.

In order to simplify the comparison with the optical results, we choose the single rapidity mode l of the coherent part to be the same as the central mode of the Lorentzian distribution given by (4.2) that is $l=0$. This done, the results given by Jaiswal and Mehta²⁶ may be taken over to our case by introducing the parameter $\beta = Y/\xi$.

A. Factorial cumulants

These are the most satisfactory measure of the correlation and are defined by

$$\begin{aligned} \mu_{(1)} &= \langle n \rangle, \\ \mu_{(2)} &= \langle n(n-1) \rangle - \langle n \rangle^2, \\ \mu_{(3)} &= \langle n(n-1)(n-2) \rangle - 3\langle n(n-1) \rangle \langle n \rangle + 2\langle n \rangle^3, \\ \mu_{(4)} &= \langle n(n-1)(n-2)(n-3) \rangle - 4\langle n(n-1)(n-2) \rangle \langle n \rangle \\ &\quad + 12\langle n(n-1) \rangle \langle n \rangle^2 \\ &\quad - 3\langle n(n-1) \rangle^2 - 6\langle n \rangle^4. \end{aligned} \quad (4.4)$$

For the mixture of coherent and chaotic distribu-

tions we have

$$\mu_{(l)} = (l-1)! \langle n_{\text{ch}}^l \rangle B_l + l! \langle n_{\text{ch}}^{l-1} \rangle \langle n_c \rangle \langle B_l \rangle, \quad (4.5)$$

where $\langle n_c \rangle$ and $\langle n_{\text{ch}} \rangle$ were defined in Sec. III and $B_1 = \langle B_1 \rangle = 1$,

$$\begin{aligned} B_2 &= (e^{-2\beta} + 2\beta - 1)/2\beta^2, \\ B_3 &= \frac{3}{2} [e^{-2\beta}(\beta+1) + \beta - 1]/\beta^3, \\ B_4 &= \frac{1}{8} [e^{-4\beta} + 4e^{-2\beta}(4\beta^2 + 10\beta + 7) \\ &\quad + 20\beta - 29]/\beta^4, \\ \bar{B}_2 &= 2(e^{-\beta} + \beta - 1)/\beta^2, \\ \bar{B}_3 &= [-e^{-2\beta} + 2e^{-\beta}(\beta+4) + 4\beta - 7]/\beta^3, \\ \bar{B}_4 &= [e^{-3\beta} + e^{-2\beta}(4\beta+10) + e^{-\beta}(2\beta^2 + 18\beta + 47) \\ &\quad + 16\beta - 38]/2\beta^4. \end{aligned} \quad (4.6)$$

Apart from $\mu_{(2)}$ these are not the same as the results found from a Bose-Einstein distribution over s cells when each cell is occupied on the average by $\langle n \rangle/\xi$ particles. In fact the assumption $\langle n_i \rangle = \langle n \rangle/s$, where $\langle n_i \rangle$ is the average occupation of the i th cell, corresponds to a rectangular distribution in rapidity space compared with the Lorentzian distribution we have used. The differences between the two distributions are discussed by Mandel²⁷, who shows that the quantity β may be identified with the number of occupied cells.

B. Intensity correlations

For the chaotic-coherent mixture the results may be extracted from the calculations of Lachs and Voltmer.²⁸ These authors calculate a quantity

$$g(y) = R(y)/Q(y), \quad (4.7)$$

with

$$R(y) = \langle I(0)I(y)e^{-W(y)} \rangle / \langle I \rangle \quad (4.8)$$

and

$$Q(y) = \langle I(y)e^{-W(y)} \rangle, \quad (4.9)$$

where

$$W(y) = \int_0^y I(y') dy'. \quad (4.10)$$

For $I \rightarrow 0$, $g(y)$ becomes the intensity correlation function,

$$C(y) = \langle I(0)I(y) \rangle / \langle I \rangle^2, \quad (4.11)$$

which is independent of the normalization of I .

The result for $g(y)$ for small I is given in Fig. 1

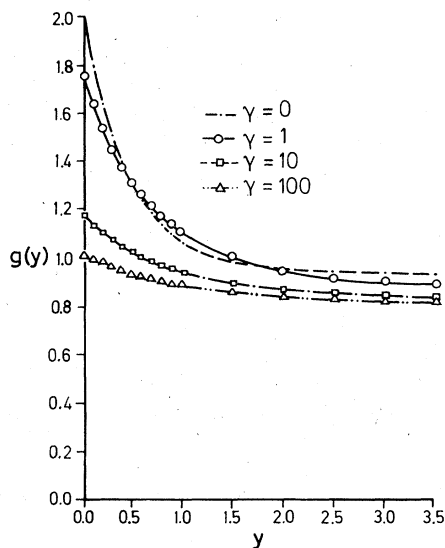


FIG. 1. Plot of the function $g(y)$ defined in Eq. (4.11); $\gamma = \langle N_c \rangle / \langle N_{ch} \rangle$ and y in units of ξ .

and this can be used for C with negligible error. Note that even a 50% coherence has a rather modest effect (10%) on the value of $C(0)$.

Higher-order correlations can be worked out along the same lines.

C. Hanbury Brown and Twiss effects

It may be seen from Fig. 1 that coherence effects increase significantly the effective coherence length and thus decrease associated size (lifetime) of the source. This implies that what one actually measures through Hanbury Brown and Twiss effects is an underestimate of the dimension of the chaotic source.

D. Rapidity-gap distributions

The rapidity-gap distribution itself is not directly accessible from Ref. 28; rather it is divided by $Q(y)$, the probability that with an arbitrarily chosen origin in rapidity the first following π meson will have a rapidity y measured from the arbitrarily chosen origin. However, the effects of a finite classical part can be clearly seen.

V. EXPERIMENTAL IMPLICATIONS

The essence of our strategy is to look for deviations from Bose-Einstein statistics attributable to coherence effects. Bose-Einstein statistics applies to identical bosons and requires the following:

(i) Stationary distributions in the relevant variable. Since the experimental distributions of $\pi^2(y)$ are flatter in pseudorapidity $\eta = \ln \tan \frac{1}{2} \theta$ than in

rapidity, the first one might be a better variable. Stability of the result for the coherent part with respect to different cutoffs in Y could serve as a criterion for the fulfillment of this condition.

(ii) Center-of-mass energies high enough so that energy-momentum-conservation constraints should not be significant. For second-order correlations, $E_{lab} > 200$ GeV is sufficient.¹⁹

(iii) Equal transverse momenta p_{\perp} . We apply Bose-Einstein statistics to rapidity only, and therefore the other variables, $p_{\perp}^{(1)}$, $p_{\perp}^{(2)}$, must be the same.

(iv) Elimination of strong-interaction effects which could mask coherence effects. This can be achieved by using small-mass pairs (below the ρ resonance) and then subtracting the residual strong interactions in the manner of Biswas *et al.*¹⁹

Since our approach requires a distribution $\rho(\alpha) > 0$, i.e., $C(y) > 1$, a necessary condition for the effectiveness of the implementation of the constraints (i), (ii), (iii), and (iv) is a positive Hanbury Brown and Twiss effect or positive cumulants.

If all these conditions are met, we expect from Fig. 1 that $C(0) - 1$ should be a measure of the amount of coherence. However, it must be pointed out that given the finite resolution of rapidity and p_{\perp} measurements, extrapolation of experimental intensity correlations $C(y)$ to $y = 0$ and $p_{\perp}^{(1)} = p_{\perp}^{(2)}$ might be difficult. In these circumstances consideration must be given to other effects which influence the source such as the motion of the emitting objects (see, e.g., Refs. 14 and 15). Higher-order correlations will be more sensitive to coherence effects.

So far there appear to be no experimental data which fulfill completely our requirements. Experiments of Refs. 11, 18, and 20 are at too low energies and the data quoted by Knox do not satisfy condition (iv); closest to our criteria are the data of Biswas *et al.*¹⁹ In this experiment, $\pi^+ \pi^-$ correlations $C_{\pi\pi}$ for $\pi^+ p$ interactions at 200 GeV/c are measured. The strong-interaction ("dynamical") contribution is estimated by subtracting $C_{\pi\pi}$ from $C_{\pi\pi}$ with the assumption that Bose-Einstein and strong-interaction correlations are independent. The result for $C_{\pi\pi}(0) - 1$ is 0.8 ± 0.1 . According to Fig. 1 this is consistent with 50% coherence. It should be stressed that in the limit $t_{12} = (p_1 - p_2)^2 \rightarrow 0$, which is considered in this experiment, the dipion mass $s_{12} = (p_1 + p_2)^2$ is also negligible, so that this subtraction procedure²⁹ eliminates (in a first approximation) the contribution of resonances to the $+-$ correlations.³⁰

The investigation of these effects in different reactions and at different energies is highly desirable since the parameters $\langle n_c \rangle / \langle n_{ch} \rangle$ and ξ are ex-

pected to depend on the initial conditions.³¹

Note added. After the completion of this work we received a report by Giovannini and Veneziano³² in which the predictions of the topological expansion model for particle correlations including the Bose-Einstein effect are discussed. Their paper exemplifies the importance of coherence for particular dynamical models. One of us (R.W.) acknowledges a useful discussion with Professor G. Veneziano along these lines.

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³¹That this is indeed the case is suggested by some preliminary experimental data on K^*p , $\bar{p}p$ (ABBCCI collaboration) and e^*e^- reactions (SPEAR). We are indebted to Profs. V. Cocconi and G. Goldhaber for private communications on this subject.

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