Mass of the axion

J. Kandaswamy, Per Salomonson,* and J. Schechter
Physics Department, Syracuse University, Syracuse, New York 13210
(Received 26 January 1978)

We derive a formula for the mass of the axion of the extended $SU(2)\times U(1)$ gauge theory in a simple but fairly general model. The axion mass depends on a new weak-interaction parameter but a definite minimum value emerges. In the four-quark model this is about 95 keV. A brief discussion of decays involving axions is included. The possibility of mock CP violation in the K_L^0 decays is shown to be untenable.

When instanton effects on the color gauge vacuum are taken into account the conservation of P and T to strong interaction order are no longer natural for the combined strong-weak-electromagnetic theory. Peccei and Quinn[1] have noted that one way to overcome this problem is to enlarge the symmetry group of the weak-electromagnetic sector of the theory from $SU(2)_W\times U(1)_W$ by an additional (global) $U(1)\times U(1)$. The simplest way to accomplish this is to introduce (besides the usual complex doublet ϕ) another Higgs doublet χ and require invariance under

$$\phi\to e^{i\alpha}\phi,\ q_{2R}\to e^{i\alpha}q_{2R},\ \ q_{3R}\to e^{i\alpha}q_{3R},\ \cdots \qquad (1a)$$

and also

$$\chi\to e^{i\beta}\chi,\ \ q_{1R}\to e^{-i\beta}q_{1R},\ \ q_{4R}\to e^{-i\beta}q_{4R},\cdots \qquad (1b)$$

where the quark fields are labeled in order of increasing mass. Now Wilczek[2] has pointed out that the existence of an additional symmetry which is spontaneously broken requires a new Goldstone boson which does not get absorbed by gauge fields. This *axion*[2] receives a *small* mass by the instanton-induced $U_A(1)$-violating term in the strong interaction. The peculiar feature is that a weak-electromagnetic symmetry is getting broken by the strong interactions, rather than the other way around. Since instanton effects are rather difficult to calculate, Wilczek gave an order-of-magnitude estimate of the axion mass, based on counting coupling constants. It is expected to be of the order of a fraction of 1 MeV but just how low is a crucial question for experimental reasons. The idea of the present paper is to find this mass by circumventing the problem of calculating instanton effects. We do this by noting that the instanton effects *also* make themselves felt in the pseudoscalar mass spectrum [U(1) problem]. We will thus proceed by *correlating* the pseudoscalar masses with the present case.

One can construct an effective term in the Lagrangian of the form

$$\mathcal{L}=\cdots -U[e^{i\theta}\det\bar{q}(1+\gamma_5)q+e^{-i\theta}\det\bar{q}(1-\gamma_5)q], \qquad (2)$$

which breaks the $U_A(1)$ symmetry. 't Hooft[3] has shown that a term like (2) can be gotten as an approximation in the color gauge theory. Such a term has also been used for a long time to resolve the U(1) problem in the context of σ models.[4] The point is that U (which contains instanton effects) is known from the pseudoscalar mass spectrum and can be used to determine the axion mass. Only for $\theta=0$ is the theory P and T invariant. Note that under the transformations (1a) and 1(b) the term in (2) goes to

$$-U[e^{Ni(\beta-\alpha)+i\theta}\det\bar{q}(1+\gamma_5)q+e^{-Ni(\beta-\alpha)-i\theta}\det\bar{q}(1-\gamma_5)q],$$

where N is the number of quark doublets. Clearly the extra symmetries of (1) enable us to rotate away any possible P- and T-violating phases which might have arisen either intrinsically or from bringing the quark-mass-type terms to γ_5-free form. We would like to stress that the problem of natural strong P and T conservation is due to the symmetry structure of the strong interaction, rather than to any specific dynamical (e.g., instanton) mechanism.

We will carry out our calculation in the framework of an extended σ model. We use scalar fields S_a^b which transform exactly as $\bar{q}_bq_a$ and pseudoscalar fields ϕ_a^b which transform as $i\bar{q}_b\gamma_5q_a$. We do not consider these spin-zero fields as fundamental but as convenient tools for deriving current-algebra-like results. The potential function of the total theory is written as

$$V=V_M+V_H+V_{MH}\,. \qquad (3)$$

In (3), V_H contains only the Higgs doublets ϕ and χ while V_M contains only the hadrons. V_{MH} is a bilinear mixing term contrived to mock up the Yukawa interactions of the weak-electromagnetic gauge theory. Specifically,

$$V_{MH}=\frac{1}{\lambda_1}\bar{\phi}(-A_2\cos\theta_CF^2+A_2\sin\theta_CG^2 -A_3\sin\theta_CF^3-A_3\cos\theta_CG^3) +\frac{i}{\lambda_2}\chi^T\tau_2(A_1F^1+A_4G^4)+\text{H.c.} \qquad (4)$$

In (4) we have introduced hadron doublets transforming under $SU(2)_W$:

$$F^a = \begin{bmatrix} S_1^a + i\phi_1^a \\ R_2^b(S_b^a + i\phi_b^a) \end{bmatrix}, \quad G^a = \begin{bmatrix} S_4^a + i\phi_4^a \\ R_3^b(S_b^a + i\phi_b^a) \end{bmatrix}, \quad \cdots$$

$$R_2^2 = R_3^3 = \cos\theta_C, \quad R_2^3 = -R_3^2 = \sin\theta_C, \qquad (5)$$

$$\text{others} = \text{zero}.$$

Also $A_1, \ldots, A_4$ are parameters exactly analogous to the quark masses $m_1, \ldots, m_4$. The nonvanishing vacuum values of the various fields in the theory are given by

$$\langle\phi_0\rangle = \langle\overline{\phi}_0\rangle = \lambda_1, \quad \langle\chi_0\rangle = \langle\overline{\chi}_0\rangle = \lambda_2, \quad \langle S_a^b\rangle = \delta_a^b \alpha_a. \qquad (6)$$

Note that the Fermi constant is

$$G_F = \frac{1}{\sqrt{2}}(\lambda_1^2 + \lambda_2^2 + \sum_a \alpha_a^2)^{-1} \simeq \frac{1}{\sqrt{2}}(\lambda_1^2 + \lambda_2^2)^{-1}$$

and that we use

$$\tan\gamma = \frac{\lambda_1}{\lambda_2} \qquad (7)$$

as a new weak-interaction parameter. The terms V_H and V_{MH} are invariant under $SU(2)_W \times U(1)_W$ as well as (1a) and (1b). The term V_M is invariant under $SU(4) \times SU(4) \times$ baryon number. It violates $U_A(1)$ and hence (1) since it is allowed to be a function of the invariant $I_5 = \det(S + i\phi) + \det(S - i\phi)$. The relevant parameters are

$$U_\sigma = \langle \partial V_M / \partial I_5 \rangle,$$
$$\tilde{U}_\sigma = 2U_\sigma \alpha_1\alpha_2\alpha_3\alpha_4. \qquad (8)$$

All our results follow from the symmetry properties of V_M and V_H regardless of the particular forms they take (although V_H should be renormalizable on general grounds). The symmetries (1a) and (1b) require V_H to satisfy

$$\sum_{a=+,0} \left(\frac{\partial V_H}{\partial \phi_a}\phi_a - \frac{\partial V_H}{\partial \overline{\phi}_a}\overline{\phi}_a\right) = \sum_{a=+,0}\left(\frac{\partial V_H}{\partial \chi_a}\chi_a - \frac{\partial V_H}{\partial \overline{\chi}_a}\overline{\chi}_a\right) = 0, \qquad (9)$$

while the $SU(2)_W$ symmetry requires

$$\frac{\partial V_H}{\partial \phi_a}\phi_b - \frac{\partial V_H}{\partial \overline{\phi}_b}\overline{\phi}_a - \tfrac{1}{2}\delta_{ab}\sum_c \left(\frac{\partial V_H}{\partial \phi_c}\phi_c - \frac{\partial V_H}{\partial \overline{\phi}_c}\overline{\phi}_c\right) + (\phi \to \chi) = 0. \qquad (10)$$

Differentiating (9) and using $\langle \partial V_H/\partial\phi_a\rangle + \langle \partial V_{MH}/\partial\phi_a\rangle = 0$, gives, for example

$$\lambda_1(\langle \partial^2 V/\partial\phi_0\partial\phi_0\rangle - \langle \partial^2 V/\partial\phi_0\partial\overline{\phi}_0\rangle) + \frac{1}{\lambda_1}(\alpha_2 A_2 + \alpha_3 A_3) = 0,$$

$$\lambda_2(\langle \partial^2 V/\partial\phi_0\partial\chi_0\rangle - \langle \partial^2 V/\partial\phi_0\partial\overline{\chi}_0\rangle) = 0.$$

In this way we can discover just the right amount of information for our purposes about the mass-squared matrix of all the neutral mesons which mix with each other. The treatment of the strong potential V_M by an analogous method has been discussed at length elsewhere.[5] The final result for the grand mass-squared matrix is

$$\begin{bmatrix}
2\frac{\alpha_2 A_2 + \alpha_3 A_3}{\lambda_1^2} & 0 & 0 & -\frac{2A_2}{\lambda_1} & -\frac{2A_3}{\lambda_1} & 0 \\
0 & 2\frac{\alpha_1 A_1 + \alpha_4 A_4}{\lambda_2^2} & \frac{2A_1}{\lambda_2} & 0 & 0 & \frac{2A_4}{\lambda_2} \\
0 & \frac{2A_1}{\lambda_2} & \frac{2A_1}{\alpha_1} - \frac{\tilde{U}_\sigma}{\alpha_1^2} & -\frac{\tilde{U}_\sigma}{\alpha_1\alpha_2} & -\frac{\tilde{U}_\sigma}{\alpha_1\alpha_3} & -\frac{\tilde{U}_\sigma}{\alpha_1\alpha_4} \\
-\frac{2A_2}{\lambda_1} & 0 & -\frac{\tilde{U}_\sigma}{\alpha_1\alpha_2} & \frac{2A_2}{\alpha_2} - \frac{\tilde{U}_\sigma}{\alpha_2^2} & -\frac{\tilde{U}_\sigma}{\alpha_2\alpha_3} & -\frac{\tilde{U}_\sigma}{\alpha_2\alpha_4} \\
-\frac{2A_3}{\lambda_1} & 0 & -\frac{\tilde{U}_\sigma}{\alpha_3\alpha_1} & -\frac{\tilde{U}_\sigma}{\alpha_3\alpha_2} & \frac{2A_3}{\alpha_3} - \frac{\tilde{U}_\sigma}{\alpha_3^2} & -\frac{\tilde{U}_\sigma}{\alpha_3\alpha_4} \\
0 & \frac{2A_4}{\lambda_2} & -\frac{\tilde{U}_\sigma}{\alpha_4\alpha_1} & -\frac{\tilde{U}_\sigma}{\alpha_4\alpha_2} & -\frac{\tilde{U}_\sigma}{\alpha_4\alpha_3} & \frac{2A_4}{\alpha_4} - \frac{\tilde{U}_\sigma}{\alpha_4^2}
\end{bmatrix}, \qquad (11)$$

where the rows and columns refer consecutively to the fields

$$\left(\frac{\phi_0 - \overline{\phi}_0}{2i}\right), \quad \left(\frac{\chi_0 - \overline{\chi}_0}{2i}\right), \quad \phi_1^1, \quad \phi_2^2, \quad \phi_3^3, \quad \text{and } \phi_4^4.$$

Our problem now is simply to find the eigenvectors and eigenvalues of (11). We find that the Goldstone boson field

$$G \propto \lambda_1\left(\frac{\phi_0-\overline{\phi}_0}{2i}\right)+\lambda_2\left(\frac{\chi_0-\overline{\chi}_0}{2i}\right)+\sum_b \epsilon_b\alpha_b\phi_b^b, \quad \epsilon_b=\begin{cases}+1, & b=2,3\\ -1, & b=1,4\end{cases} \tag{12}$$

is an eigenvector with eigenvalue zero of (11). It is the field which is eaten up by the Z meson. The axion field is the other eigenvector of (11) with small eigenvalue. The four remaining eigenvectors are π^0, η, η', and η''. We find the mass of the axion, μ by looking at the coefficient of the linear term in the secular equation of (11). After some calculation one gets the simple result

$$\mu^2=-128\bar{U}_\sigma A_1A_2A_3A_4\Big(\lambda_1{}^2+\lambda_2{}^2+\sum_a \alpha_a{}^2\Big)/\lambda_1{}^2\lambda_2{}^2m_\pi{}^2m_\eta{}^2m_{\eta'}{}^2m_{\eta''}{}^2\,. \tag{13}$$

It is convenient to express this in terms of G_F and the angle γ [Eq. (7)] and also to eliminate $m_\pi{}^2m_\eta{}^2m_{\eta'}{}^2m_{\eta''}{}^2$ by calculating the 4×4 hadronic subdeterminant from (11). Then there results the expression

$$\mu=\frac{2^{5/4}NG_F{}^{1/2}}{|\sin 2\gamma|}\left(-\frac{1}{\bar{U}_\sigma}+\sum_a\frac{1}{2A_a\alpha_a}\right)^{-1/2}. \tag{14}$$

In (14) we have also made explicit the factor $N=\frac{1}{2}$ (number of flavors). $N=2$ in the present case but (14) holds in general. Eq. (14) is our main result. It may be helpful therefore to express it in a slightly more conventional language. It was found[6] that the σ-model results amount simply to a special case of the quark-model current-algebra formulas. One simply sets $2A_a\alpha_a=-m_a\langle\overline{q}_aq_a\rangle_0$ (each a) and also

$$\bar{U}_\sigma=\bar{U}\equiv U\langle[\det\overline{q}(1+\gamma_5)q+\det\overline{q}(1-\gamma_5)q]\rangle_0\,.$$

With these substitutions,

$$\mu=\frac{2^{5/4}NG_F{}^{1/2}}{|\sin 2\gamma|}\left(-\frac{1}{\bar{U}}-\sum_a\frac{1}{m_a\langle\overline{q}_aq_a\rangle_0}\right)^{-1/2}. \tag{14'}$$

[The m_a in (14′) are the "current" values of the quark masses.] First note that when the determinant term is not present in V_M ($U_\sigma\to 0$) μ goes to zero and the axion becomes a true Goldstone boson. Note also that the result depends on the angle γ which is a new weak-interaction parameter characterizing the axion and its associated interactions. However (for $\sin 2\gamma=1$) there is a minimum value predicted—with the typical choice of parameters,[7] $\alpha_1=\alpha_2=0.5m_{\pi^0}$, $\alpha_3=\alpha_4=1.73\alpha_1$, $A_1=0.155m_{\pi^0}{}^3$, $A_2=0.345m_{\pi^0}{}^3$, $A_3=9.05m_{\pi^0}{}^3$, $A_4=187m_{\pi^0}{}^3$, $\bar{U}_\sigma=-4.9m_{\pi^0}{}^4$, $\eta''^2=440m_{\pi^0}{}^2$, one gets[8]

$$\mu_{\min}\simeq 95\ \text{keV}\,. \tag{15}$$

Our faith in the reliability of this number is enhanced by observing from (14) that the *lightest* quarks (smallest A_a's) for which the present technique is expected to be most reliable, make by far the dominant contribution. If one were to assume that current-algebra methods are somehow valid for another two heavy quarks (q_5 and q_6) (15) should be multiplied by $\frac{3}{2}$. At any rate the effect of adding more flavors is to increase the minimum value of μ.

For calculating amplitudes of physical processes involving axions it is handy to have the eigenvector of (11) representing the axion field $\mathcal{A}$. Correct to order α/λ we find

$$\mathcal{A}\simeq(\lambda_1{}^2+\lambda_2{}^2)^{-1/2}\left\{-\lambda_2\left(\frac{\phi_0-\overline{\phi}_0}{2i}\right)+\lambda_1\left(\frac{\chi_0-\overline{\chi}_0}{2i}\right)+\sum_b\left[\frac{\mu^2\lambda_1\lambda_2}{2NA_b}-\alpha_b\left(\frac{\lambda_2}{\lambda_1}\right)^{\epsilon_b}\right]\phi_b^b\right\}. \tag{16}$$

If the ϕ_b^b are expanded[5] in terms of the physical hadrons π^0, η, η', η'' (16) would tell us the fraction of each hadron present in the axion field. For example the fraction f of

$$\pi^0\simeq\frac{1}{\sqrt{2}}(\phi_1^1-\phi_2^2)$$

is

$$f=2^{-1/4}G_F{}^{1/2}\left|\alpha_2\frac{\lambda_2}{\lambda_1}-\alpha_1\frac{\lambda_1}{\lambda_2}+\frac{\mu^2\lambda_1\lambda_2}{2N}\left(\frac{1}{A_1}-\frac{1}{A_2}\right)\right|$$
$$\simeq 3.3\times 10^{-4}|\cot\gamma-0.15\tan\gamma|,\quad(\text{for } N=2)\,. \tag{17}$$

Since f is small we also have

$$\pi^0\simeq\frac{1}{\sqrt{2}}(\phi_1^1-\phi_2^2)\pm f\mathcal{A}+\cdots, \tag{18}$$

f being the fraction of axion in the π^0. Eq. (18) and its analogs can be used for estimating amplitudes according to the rule

$$|\text{amp}(\underline{X}\to Y+\mathcal{A})|\simeq f|\text{amp}(\underline{X}\to Y+\pi^0)|, \tag{19}$$

where for simplicity the η's have been neglected. A more complete discussion will be given elsewhere but we will make some brief comments on this idea now. Note that to boost the amplitude for axion production (17) shows that either $\cot\gamma$ or $\tan\gamma$ should be large. Then (14) shows that the axion mass μ will be boosted by roughly the same amount. As an application we would like to lay to rest the possibility that the observed CP violation is not real but corresponds to, for example, $K_L^0 \to \pi^+\pi^-\mathcal{a}$, with $\mathcal{a}$ somehow undetected. Using (19) we find

$$\Gamma(K_L^0 \to \pi^+\pi^-\mathcal{a}) \simeq \frac{\Phi(2\pi\mathcal{a})}{\Phi(3\pi)} f^2 \Gamma(K_L^0 \to \pi^+\pi^-\pi^0)\,,$$

where the phase-space ratio $\Phi(2\pi\mathcal{a})/\Phi(3\pi)$ is about 10. Very optimistically 1% of the light $\mathcal{a}$'s could come off with sufficiently small momenta to escape detection. Requiring

$$0.01\,\frac{\Gamma(K_L^0 \to \pi^+\pi^-\mathcal{a})}{\Gamma(K_L^0 \to \pi^+\pi^-\pi^0)} \simeq \frac{f^2}{10} = \frac{\Gamma_{\rm exp}(K_L^0 \to \pi^+\pi^-)}{\Gamma(K_L^0 \to \pi^+\pi^-\pi^0)} = \frac{1}{61}\,,$$

gives $f \simeq \frac{1}{2.5}$ which, using (17) and (14), leads to the unacceptably high axion mass, $\mu \simeq 58$ MeV. Thus the possibility of mock CP violation in the K_L^0 decays is untenable.

It seems important to recognize the correlation between the axion mass and the coupling to hadrons through the choice of angle γ. This situation extends to leptons too. The terms in the interaction Lagrangian are

$$i2^{1/4}G^{1/2}\mathcal{a} \sum_{\substack{\text{massive}\\ \text{leptons},\, l}} m_l \begin{Bmatrix} \cot\gamma \\ -\tan\gamma \end{Bmatrix} \bar{\psi}_l \gamma_5 \psi_l\,, \qquad (20)$$

where each term has either the upper or lower factor depending on whether the lepton in question transforms according to (1a) or (1b).

Note added. After this paper was typed we received a report by S. Weinberg [Phys. Rev. Lett. 40, 223 (1978)] in which a special case of our result Eq. (14′) was derived corresponding to $\tilde{U}^{-1} \to 0$. There is not much numerical difference though.

Note added in proof. In a recent report by the present authors [Phys. Lett. (to be published)], it is shown that existing experimental data on rare decays of the K^+ provide strong evidence against the existence of the axion. In this reference the derivation of our formulas by the current-algebra method is explicitly shown. We should also mention a report by W. A. Bardeen and S-H. H. Tye [Report No. Fermilab-Pub-77/110 (unpublished)] in which the $\tilde{U}^{-1} \to 0$ limit of (14′) is derived. These authors have also (private communication) investigated the decay mode $K^+ \to \pi^+ +$ axion with results similar to ours.

We would like to thank T. Kalogeropoulos for a helpful discussion. One of us (P. S.) was supported by the Swedish Atomic Research Council under Contract No. 301-026. The other two authors were supported by The U.S. Department of Energy under Contract No. EY-76-S-02-3533.

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*Permanent address: Institute of Theoretical Physics, Fack, S-402 20, Göteborg 5, Sweden.

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[7] The parameters corresponding to isospin violations were taken from line 2, Table I, of J. Schechter and Y. Ueda, Phys. Rev. D 4, 733 (1971). For other parameters and references see Ref. 5 above. Note especially that the uncertainties in the parameters involving the fourth quark have negligible numerical effect.

[8] We feel that the accuracy of this number should be better than 15%, apart from the uncertainty in number N of relevant quark doublets.