PHYSICAL REVIEW D VOLUME 17, NUMBER 11 1 JUNE 1978

Comparison of symmetry and duality constraints for radiative transitions of mesons

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(Received 4 August 1977)

Predictions for patterns of symmetry breaking in vector and tensor mesons are extracted from pole-dominated finite-energy sum rules. Kinematic factors relate deviations in couplings to the mass-spectrum splitting. For vector mesons, in addition to the usual problem of the small $\Gamma(\rho\to\pi\gamma)$, we find that $\Gamma(K^{*+}\to K^{+}\gamma)$ must almost certainly be above its present upper bound of 80 keV. Consistency arguments dictate a choice between the two possible sets of $\eta\gamma$ widths for the ρ and ω. In addition, we extract both $D\gamma$ and $D\pi$ widths for the charmed vector meson D^*. For the tensor mesons, predictions $\Gamma(A_2\to\pi\gamma) = 490\pm60$ keV, $\Gamma(K^{**+}\to K^{+}\gamma) = 125\pm60$ keV, and $\Gamma(K^{**0}\to K^{0}\gamma) = 32\pm15$ keV imply large deviations from both symmetry and naive vector-meson-dominance predictions. Additional consistency relations between η and η' reactions lead to $\Gamma_{\text{total}}(\eta') = 480\pm120$ keV.

I. INTRODUCTION

The recent revival of interest in the study of meson-decay couplings and symmetries stems from two experimental developments. One is the new measurements of vector-meson radiative-decay widths via the Primakoff effect,[1,2] which appear to be in disagreement by factors of 2 or 3 with SU(3)-symmetry predictions based on conventional measurements. The other is the discovery of charmed mesons, both hidden and overt, which naturally call for an extension to SU(4) multiplets. However, the large symmetry breaking exhibited by the mass spectrum makes any symmetry predictions for couplings somewhat suspect. It is the purpose of this work to investigate the restrictions imposed on meson couplings by duality. In particular, when finite-energy sum rules (FESR's) for two-body scattering amplitudes are saturated by low-lying poles, one obtains decay-coupling-ratio predictions. These coupling ratios *can*, but are *not required* to satisfy any symmetry constraints. Since the isospin crossing matrix is not in general diagonal (certainly not for photons as external particles), one gets constraints which relate two different ratios, thus specifying patterns of possible symmetry breaking. In addition, the coefficients of such relations depend on particle mass ratios. In this way, one relates symmetry breaking in the mass spectrum to symmetry breaking in couplings, independent of any breaking inherent in the structure of the relations. Of course, as in any narrow-resonance-saturation scheme, the main uncertainty in the results comes in the choice of cutoff point. For an estimate of this uncertainty, we use known results for strong-interaction amplitudes and decay rates.

The plan of this paper is as follows. In Sec. II the SU(3)-symmetry systematics for *VPP* and *TPP* couplings are investigated, and compared with FESR results. A generally ignored violation in the *TPP* sector is found and related to mass-breaking effects in FESR's. In Sec. III the constraints in the $VP\gamma$ sector are examined, resulting in several testable predictions. In Sec. IV these methods are extended to the charmed mesons, for both electromagnetic and strong decays. Section V deals with tensor-meson radiative decays, and some miscellaneous applications are given in Sec. VI. The main conclusions are summarized in Sec. VII.

II. STRONG MESON DECAYS

A. $V\to PP$

The decay width is given by the conventional expression

$$\Gamma(V_i\to P_jP_k)=\frac{2}{3}\,\frac{g^2}{4\pi}\,\frac{|\vec{p}|^3}{m_V^{\,2}}\,, \tag{1}$$

where $|\vec{p}|$ is the decay-product momentum in the V_i rest frame. For SU(3) symmetry, we expect

$$g_{ijk}=if_{ijk}g_{VPP}, \tag{2}$$

where f_{ijk} are the usual antisymmetric SU(3) structure coefficients. In the first three entries in Table I are shown the $g_{VPP}{}^2/4\pi$ values extracted from the measured widths $\rho\to\pi\pi$, $K^*\to\pi K$, and $\phi\to K\bar{K}$.[3] The numbers indicate that SU(3) is good to about the 5% level in amplitude. It is interesting to note that the gradual increase over the last few years[4,5] of the accepted ρ width has brought it into reasonable agreement with the SU(3) predictions based on the K^* width.

B. $T\to PP$

The decay width is given by

$$\Gamma(T_i\to P_jP_k)=\frac{16}{15}\,\frac{g^2}{4\pi}\,\frac{|\vec{p}|^5}{m_T^{\,2}}\,. \tag{3}$$

TABLE I. Decay widths for VPP, TPP, and $VP\gamma$ processes, with extracted SU(3)-symmetric coupling $g^2/4\pi$. All data are from Ref. 5, unless otherwise indicated. For ρ, $\omega \to \eta\gamma$, the two different solutions (I) and (II) come from the unknown relative phase in the production amplitudes.

Process	Γ(expt)	$g^2/4\pi$
$\rho \to \pi\pi$	152 ± 3 MeV	2.94 ± 0.06
$K^* \to K\pi$	49.4 ± 1.8 MeV	3.29 ± 0.12
$\phi \to K\bar{K}$	3.8 ± 0.26 MeV	2.84 ± 0.20
$A_2 \to K\bar{K}$	4.8 ± 0.54 MeV	1.08 ± 0.12 GeV^{-2}
$K^{**} \to K\pi$	60.6 ± 6.0 MeV	1.72 ± 0.16 GeV^{-2}
$f \to \pi\pi$	145 ± 17 MeV	1.16 ± 0.14 GeV^{-2}
$f \to K\bar{K}$	4.9 ± 1.2 MeV	0.68 ± 0.18 GeV^{-2}
$f' \to K\bar{K}$	40 ± 10 MeV	1.23 ± 0.32 GeV^{-2}
$\omega \to \pi\gamma$	880 ± 62 keV	(48.0 ± 3.0) × 10^{-3} GeV^{-2}
$\rho \to \pi\gamma$	35 ± 10 keV (Ref. 1)	(18.0 ± 5.4) × 10^{-3} GeV^{-2}
$K^{*0} \to K^0\gamma$	75 ± 35 keV (Ref. 2)	(17.4 ± 8.1) × 10^{-3} GeV^{-2}
$K^{*+} \to K^+\gamma$	< 80 keV	< 72 × 10^{-3} GeV^{-2}
$\phi \to \eta\gamma$	55 ± 12 keV (Ref. 23)	(11.8 ± 2.7) × 10^{-3} GeV^{-2}
$\omega \to \eta\gamma$ (I)	3 ± 2 keV (Ref. 23)	(31 ± 21) × 10^{-3} GeV^{-2}
$\omega \to \eta\gamma$ (II)	29 ± 7 keV (Ref. 23)	(280 ± 73) × 10^{-3} GeV^{-2}
$\rho \to \eta\gamma$ (I)	50 ± 13 keV (Ref. 23)	(63.5 ± 16.5) × 10^{-3} GeV^{-2}
$\rho \to \eta\gamma$ (II)	76 ± 15 keV (Ref. 23)	(97 ± 19) × 10^{-3} GeV^{-2}

For SU(3) symmetry, we have

$$g_{ijk} = d_{ijk} g_{TPP}, \tag{4}$$

where now d_{ijk} are the symmetric structure coefficients, allowing a singlet component. In the next entries in Table I, the $g_{TPP}{}^2/4\pi$ values are extracted from the available data. The entries for the f, f' decays are rather sensitive to the singlet-octet coupling ratio. For the numbers given, it was determined by requiring the $f'\pi\pi$ coupling to vanish. An alternative is to use the quark-model value, which is still consistent with the experimental upper limit for $f' \to \pi\pi$. Then the last three numbers are changed to 1.62 ±0.19, 1.20±0.31, and 1.65±0.42 GeV^{-2}, respectively. Thus we can conclude very little other than rough consistency from these decays. However, the $A_2 \to K\bar{K}$ and $K^{**} \to K\pi$ decays are independent of mixing, and the experimental uncertainties are small enough to reveal a sizable SU(3) violation. The ratio of extracted $g_{TPP}{}^2/4\pi$ values is 1.60 ±0.22, or almost three standard deviations from the symmetry limit. Note that this effect *cannot* be eliminated by multiplying by an $m_T{}^2$ factor to make a dimensionless coupling. Since $m_{A_2}{}^2 < m_{K^{**}}{}^2$, this moves the couplings even further apart. Also, in contrast to the VPP case, recent experimental results have tended to move these TPP couplings away from the symmetry limits. For example, a comparison of the 1972 vs 1976 editions of the Review of Particle Properties[4,5] shows that the branching ratio for $A_2 \to K\bar{K}$ moved *down* from 5.8 to 4.7 %, while the $K^{**} \to K\pi$ branching ratio moved *up* from a lower limit of 40% to a fitted value of 56.1%.

C. FESR for $PP \to PP$

To get constraints on the above couplings, we saturate pseudoscalar-meson scattering amplitudes with vector and tensor poles, and evaluate at the position of these same poles in the cross channel. For the crossing-odd vector exchange, we have

$$(4p_V{}^2 + 2t) g_{VPP}{}^2 = 2\beta_V(t) \frac{N^{\alpha_t+1}}{\alpha_t+1} \xrightarrow[\alpha_t \to 1]{} 2\alpha'_V g_{VPP}{}^2 \frac{N_V{}^2}{2} \tag{5}$$

and for the crossing-even tensor exchange we have

$$(4p_V{}^2 + 2t)\nu_B(V) g_{VPP}{}^2 = 2\beta_T(t) \frac{N^{\alpha_t+2}}{\alpha_t+2} \xrightarrow[\alpha_t \to 2]{} 4\alpha'_T g_{TPP}{}^2 \frac{N_V{}^4}{4}, \tag{6}$$

where p_V is the decay momentum in $V \to PP$, the energy variable $\nu \equiv (s-u)/2$, $\nu_B(V)$ is the s-channel vector pole position $m_V{}^2 + \frac{1}{2}t - 2m_P{}^2$, N_V is the cutoff position, taken halfway between the V and T poles, $N_V = \nu_B(V) + 0.5/\alpha'_V$, with α' the trajectory slope. The couplings g_{VPP} and g_{TPP} are normalized as in (1) and (3). If the direct-channel tensor pole is also included, one adds an additional term

$$[(4p_T^2+2t)^2-\tfrac{1}{3}(4p_T^2)^2]g_{TPP}^2$$

to the left side of (5), and the same multiplied by $\nu_B(T)$ to (6), and changes $N_V \to N_T$ on the right. In the SU(3) limit one has just three mass scales m_P, m_V, and m_T. We insert $p_V^2=\frac{1}{4}m_V^2-m_P^2$, $\alpha_V'=1/(m_T^2-m_V^2)$ into (5) at $t=m_V^2$ to obtain the mass relation

$$m_T^2=4m_V^2-4m_P^2. \tag{7}$$

This is quite well satisfied for the K-meson family, but not for the π-ρ-f sequence due to the large π-K mass splitting. When (7) is used in (6) evaluated at $t=m_T^2$, one gets the coupling ratio

$$\frac{g_{TPP}^2}{g_{VPP}^2}=\frac{16}{27(3m_V^2-4m_P^2)}. \tag{8}$$

Again, when the K-meson family is used, this expression is within the range of values obtained from Table I. An alternative method for tensor couplings is to include the tensor meson in the direct channel, and divide out the t-channel couplings by using the original FESR's with only the vector pole. In fact, this method was used (see Ref. 6) to constrain the physical masses by requiring consistency between (5) and (6) for the $PP\to PV$ amplitude. However, the $PP\to PP$ amplitude also admits a scalar-meson contribution, so that the mass constraint becomes a method for determining the scalar coupling. It contributes a term g_{SPP}^2 to the FESR's, where the normalization is

$$\Gamma(S\to PP)=\frac{g_{SPP}^2}{4\pi}\,\frac{|\vec{p}|}{2m_S^2}. \tag{9}$$

The ratio then reads

$$1+\frac{g_{TPP}^2}{g_{VPP}^2}\,\frac{(6m_V^2-8m_P^2)^2-\frac{1}{3}(4m_V^2-8m_P^2)^2}{3m_V^2-4m_P^2}+\frac{g_{SPP}^2}{g_{VPP}^2(3m_V^2-4m_P^2)}=\frac{N_T^2}{N_V^2}=4, \tag{10}$$

where we have used the mass relation (7). The insertion of (8) then gives

$$g_{SPP}^2=(3m_V^2-4m_P^2)g_{VPP}^2\times\left[\frac{17}{27}+\frac{64}{81}\left(\frac{2m_V^2-4m_P^2}{3m_V^2-4m_P^2}\right)^2\right]. \tag{11}$$

One can again use the K-meson family masses to estimate the width of the scalar meson to be 600 MeV, in reasonable agreement with the broad $\kappa(1250)$ meson (or s-wave K-π enhancement).

We now reexamine these same FESR's, using physical masses and couplings, to determine symmetry-breaking effects. As a test of (5), we use $K\pi\to K\pi$ with K^* in the direct channel and ρ exchange. The unknown coupling $g_{\rho K\bar{K}}$ is parametrized as a parameter ϵ times the SU(3) value as given by $g_{\rho\pi\pi}$. The constraint is

$$\epsilon=\frac{(4P_{K^*}^2+2m_\rho^2)}{\alpha_\rho' N_V^2}\,\frac{g_{K^*K\pi}^2}{g_{\rho\pi\pi}^2}. \tag{12}$$

This is typical of the symmetry-breaking predictions obtained by the FESR method. One coupling ratio is related to another ratio times a kinematic factor which is unity in the exact symmetry limit. In this case, we use the experimental $g_{K^*K\pi}^2/g_{\rho\pi\pi}^2=1.12\pm0.05$. The physical masses give

$$\frac{4P^2+2m_\rho^2}{\alpha_\rho' N_V^2}=0.83,$$

resulting in $\epsilon=0.93\pm0.04$, where an additional uncertainty due to the FESR saturation itself can be added. In any event, one predicts approximate (~5% level) symmetry for these VPP couplings, even when the kinematic factors include the large K-π mass difference.

For a test of tensor couplings, we consider again $\pi K\to\pi K$, and use the ratio of (5) for cutoff after the K^{**} to that after the K^* alone, to eliminate the t-channel ρ couplings. The result is[7]

$$\frac{g_{K^{**}K\pi}^2}{g_{K^*K\pi}^2}=\frac{4P_V^2+2m_\rho^2}{(4p_T^2+2m_\rho^2)^2-\frac{1}{3}(4p_T^2)^2}\left[\left(\frac{N_T}{N_V}\right)^2-1\right]=0.49\ \text{GeV}^{-2}, \tag{13}$$

which agrees well with experimental value $0.52\pm0.06\ \text{GeV}^{-2}$. Note that from (8), in the SU(3) limit this ratio would be from 0.35 to 0.42, depending on whether π or K masses are used. Another test is obtained from (6) for $K\bar{K}\to K\bar{K}$, with ϕ in the direct channel and A_2 exchanged:

$$\frac{g_{A_2K\bar{K}}^2}{g_{\phi K\bar{K}}^2}=\frac{(4P_\phi^2+2m_{A_2}^2)\nu_B(\phi)}{\alpha_{A_2}' N_V^4}=0.32\ \text{GeV}^{-2}, \tag{14}$$

again in good agreement with the experimental value $0.38\pm0.05\ \text{GeV}^{-2}$.

Note that in the SU(3) limit, (13) and (14) should be equal, whereas the use of physical masses predicts them to be unequal, and in agreement with the SU(3) breaking experimentally observed. The FESR has thus related the symmetry-breaking effect in masses to that in three-body couplings.

III. VECTOR-MESON RADIATIVE DECAYS

For the $V\to P\gamma$ decays, the couplings are defined by

$$\Gamma(V_i\to P_j\gamma)=\frac{1}{3}\,\frac{g^2}{4\pi}|\vec{p}|^3, \tag{15}$$

where for SU(3) symmetry

$$g_{ij\gamma}=d_{ij\gamma}g_{VP\gamma}, \tag{16}$$

with γ the usual $\tilde{3}+(\tilde{8}/\sqrt{3})$ u-spin-invariant charge coupling. The final entries in Table I show the measured rates and the calculated[8] $g_{VP\gamma}{}^2/4\pi$. The well known disagreements are illustrated by the first three entries. Of the many attempts[9-19] to reconcile these data with experiment, all experience trouble with the $\rho\pi\gamma/\omega\pi\gamma$ ratio. That this ratio should be troublesome is obvious in most schemes, since most attempts to deal with the symmetry breaking rely on kinematic effects, which are absent in this ratio. The conclusion is that if any semblance of SU(3) symmetry is to remain, one must hope that the first-generation Primakoff-experiment results will be replaced by subsequent measurements more in accord with model predictions. For the time being, however, we accept the new results at face value,[20] and try to see how they compare with the FESR restrictions.

For the reaction $\gamma P \to PP$, there is a single amplitude, with single spin flip,

$$A(\nu, t) \xrightarrow[\nu\to\infty]{} \nu^{\alpha} t^{-1},$$

defined in the photon helicity (λ) basis by

$$\mathfrak{M}^{\lambda} = a^{\lambda}_{\mu}(k)\epsilon_{\mu\nu\rho\sigma} k_\nu q_\rho p_\sigma A(\nu, t), \tag{17}$$

where k, q, p, are any three independent momenta. We illustrate the sum rules with an example: For the process $\gamma K \to \pi^0 K$, one can take combinations of K charge states to isolate the ω and ρ exchanges. The ϕ contribution is neglected due to its small $\pi\gamma$ coupling. The results are

$$g_{K^*K\pi^0}(g_{K^{*+}K^+\gamma} - g_{K^{*0}K^0\gamma}) = \frac{\alpha'_\omega N_V{}^2}{2\nu_B(K^*)} g_{\omega K\bar{K}} g_{\omega\pi\gamma}, \tag{18}$$

$$g_{K^*K\pi^0}(g_{K^{*+}K^+\gamma} + g_{K^{*0}K^0\gamma}) = \frac{\alpha'_\rho N_V{}^2}{2\nu_B(K^*)} g_{\rho K\bar{K}} g_{\rho\pi\gamma}, \tag{19}$$

where we have cutoff after the K^* vector pole alone.[21] In the SU(3) limit, we have

$$g_{K^*K\pi^0} = g_{\omega K\bar{K}} = g_{\rho K\bar{K}} = \tfrac{1}{2} g_{VPP},$$

$$g_{K^{*+}K^+\gamma} = \tfrac{1}{3} g_{VP\gamma},$$

$$g_{K^{*0}K^0\gamma} = -\tfrac{2}{3} g_{VP\gamma},$$

$$g_{\omega\pi\gamma} = g_{VP\gamma},$$

$$g_{\rho\pi\gamma} = \tfrac{1}{3} g_{VP\gamma},$$

and $\alpha' N_V{}^2/2\nu_B(V) = 1$, which automatically staisfy both sum rules.[22] One can write similar equations for the reactions $\gamma K \to (\eta, \eta')K$, with only the replacements $\pi^0 \to (\eta, \eta')$, and $\rho \to \omega$, the latter just from SU(2) invariance. Thus ratios (18) and (19) give the first constraints

$$\frac{g_{\omega\eta\gamma}}{g_{\rho\eta\gamma}} = \frac{g_{\omega\eta'\gamma}}{g_{\rho\eta'\gamma}} = \left(\frac{g_{\rho KK}}{g_{\omega KK}}\right)^2 \frac{g_{\rho\pi\gamma}}{g_{\omega\pi\gamma}}. \tag{20}$$

SU(3) would predict that all three ratios are $\frac{1}{3}$.

The first part can serve to discriminate between the two possible experimental results[23] for $\omega, \rho \to \eta\gamma$. From a recent experiment on η' branching ratios,[24] one uses $\Gamma(\eta' \to \rho\gamma)/\Gamma(\eta' \to \omega\gamma) = 9.9 \pm 2.0$ to deduce

$$\left|\frac{g_{\omega\eta'\gamma}}{g_{\rho\eta'\gamma}}\right| = 0.32 \pm 0.04. \tag{21}$$

For relative production phase of 0°, the results[23] $\Gamma(\omega \to \eta\gamma) = 3 \pm 2$ keV, $\Gamma(\rho \to \eta\gamma) = 50 \pm 13$ keV give

$$\left|\frac{g_{\omega\eta\gamma}}{g_{\rho\eta\gamma}}\right| = 0.23 \pm 0.07, \tag{22a}$$

and for the 180° production-phase difference, the results[23] $\Gamma(\omega \to \eta\gamma) = 21 \pm 7$ keV and $\Gamma(\rho \to \eta\gamma) = 76 \pm 15$ keV give

$$\left|\frac{g_{\omega\eta\gamma}}{g_{\rho\eta\gamma}}\right| = 0.57 \pm 0.09. \tag{22b}$$

Thus the second solution (22b) is definitely excluded, and one can even conclude that in the first solution the $\omega \to \eta\gamma$ width must be close to its 1 STD limit of 5 keV.

For the second part, we expect the $g_{\rho K\bar{K}}/g_{\omega K\bar{K}}$ ratio to be close to unity, both from SU(3) invariance in VPP and from an FESR analysis of $K\bar{K} \to K\bar{K}$ in various charge states, since $m_\omega{}^2 \approx m_\rho{}^2$. From the experimental[1,5] $\pi\gamma$ widths, one extracts

$$\left|\frac{g_{\rho\pi\gamma}}{g_{\omega\pi\gamma}}\right| = 0.20 \pm 0.03, \tag{23}$$

which overlaps (22a) but not (21). Thus we are forced to conclusions similar to those of the symmetry and dynamical models previously mentioned. Although the FESR's do not by themselves demand SU(3) ratios, they do demand equality of the various ratios. Since the η' ratio takes on the symmetry value, we are forced to demand it for the others. A "reasonable" change in the $\rho\pi\gamma$ coupling is to use the lower 1-standard-deviation point for (21), and let the $\rho K\bar{K}/\omega K\bar{K}$ ratio violate SU(3) at the 5% level *in the positive direction*, so that a ratio $g_{\rho\pi\gamma}/g_{\omega\pi\gamma}$ of 0.255 just satisfies the FESR. This implies a minimum $\Gamma(\rho \to \pi\gamma) \approx 55$ keV. Note that to satisfy the FESR ratios with the present 35-keV width would require either a violation *in amplitude* for the $\rho K\bar{K}/\omega K\bar{K}$ ratio of about 25%, which would be very unlikely in view of previous results for VPP couplings, or a change in the newly measured η' branching ratios from 9.9 ± 2.0 up to the neighborhood of 20.

We now return to an examination of (18) alone. It is obvious from Table I that to have any hope of satisfying the sum rule with the large $\omega\pi\gamma$ coupling and the small $K^*K\gamma$ couplings, we must assume the SU(3) relative signs, so that the K^* terms *add*. Then (18) can be written

$$[\Gamma(K^{*0}\to K^0\gamma)]^{1/2}+[\Gamma(K^{*+}\to K^+\gamma)]^{1/2}=f[\Gamma(\omega\to\pi\gamma)]^{1/2}, \qquad (24)$$

where f is a kinematic factor from decay phase space and other couplings in (18). This constraint is illustrated in Fig. 1 by the solid curve. The dashed curves on either side are an estimate of the uncertainty of the sum rule, and include a ±5% contribution from the VPP couplings along with a 10% contribution from the cutoff position, as well as the experimental uncertainty in the $\omega\pi\gamma$ coupling. The cross mark indicates the SU(3) prediction, taking $\omega\pi\gamma$ for normalization. The experimental value and upper limit for the K^* widths are shown by the cross-hatched area. The conclusion is obvious: If the sum rule is to be satisfied even within the generous uncertainties allowed, at least one experimental result must change. If we keep $\Gamma(K^{*0}\to K^0\gamma)=75\pm35$ keV, then one must insist that $\Gamma(K^{*+}\to K^+\gamma)\gtrsim 100$ keV. On the other hand, if $\Gamma(K^{*+}\to K^+\gamma)<80$ keV, then $\Gamma(K^{*0}\to K^0\gamma)>130$ keV, with the exact number increasing as $\Gamma(K^{*+}\to K^+\gamma)$ decreases. Thus a measurement of $\Gamma(K^{*+}\to K^+\gamma)$ is of crucial importance in either verifying or disproving the sum rule constraint. It is interesting to examine the various factors which enter the factor f. Aside from the trivial phase-space factors, there are two contributions. One is the ratio $g_{\omega K\bar{K}}/g_{K^*K\pi^0}$, which is unity in the SU(3) limit. We have used the measured value for the $K^*K\pi^0$, along with the SU(3) constraint $g_{\omega K\bar{K}}=g_{\rho K\bar{K}}=\frac{1}{2}g_{\rho\pi\pi}$, which yields 0.94 for the ratio. One can allow another 5% uncertainty as given by the ϵ factor in (12). The other kinematic factor $\alpha' N_V/2\nu_B(K^*)$ is also unity in the SU(3) limit, but with physical masses is 1.12. This is actually *away* from the direction of the observed symmetry breaking in decay width, and is of considerable importance in the constraints on the K^* decay widths.

It is also interesting to note that if we allow a large SU(3) breaking for the $g_{\omega K\bar{K}}$ so that (20) is satisfied with the small $\rho\pi\gamma$ width, this same factor will allow (24) to be satisfied in a corner of the currently allowed experimental values for the K^* widths. This is indicated by the short solid line in Fig. 1. Nevertheless, it must be repeated that this option seems highly unlikely in view of previous results in the VPP sector.

At first sight, it would appear that (19) constrains the *difference* of the K^* decay widths, and a simultaneous solution would yield unique predictions for both. That this is indeed *not* the case can be best illustrated by writing down the analogous equations for the charmed pseudoscalar D mesons:

$$g_{D^*D\pi^0}(g_{D^{*+}D^+\gamma}-g_{D^{*0}D^0\gamma})=\frac{\alpha_\omega' N_V^2}{2\nu_B(D^*)}g_{\omega D\bar{D}}g_{\omega\pi\gamma}, \qquad (25)$$

$$g_{D^*D\pi^0}(g_{D^{*+}D^+\gamma}+g_{D^{*0}D^0\gamma})=\frac{\alpha_\rho' N_V^2}{2\nu_B(D^*)}g_{\rho D\bar{D}}g_{\rho\pi\gamma}. \qquad (26)$$

From these plus the corresponding equations with

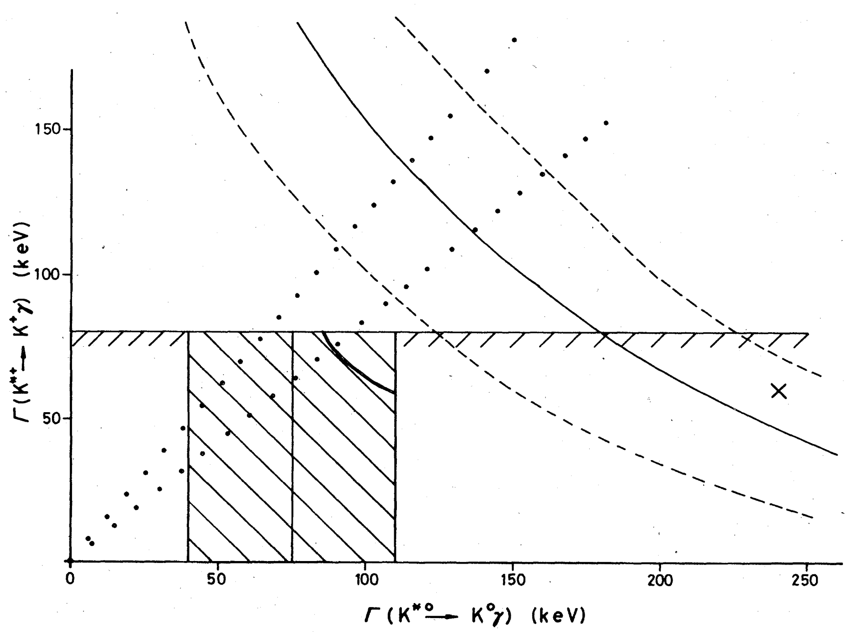

FIG. 1. Constraints on $\Gamma(K^{*0}\to K^0\gamma)$ and $\Gamma(K^{*+}\to K^+\gamma)$ which follow from FESR's for $\gamma K\to\pi K$. Curves are explained in Sec. III of the text.

$\pi^0 \to \eta, \omega \to \rho$, one can immediately deduce

$$\frac{g_{\omega D\bar{D}}}{g_{\rho D\bar{D}}} = \pm \frac{g_{\omega K\bar{K}}}{g_{\rho K\bar{K}}}, \tag{27}$$

which will be of some use later.

The SU(4) limit gives the following constraints:

$$-g_{D^*D\pi^0} = g_{\omega D\bar{D}} = -g_{\rho D\bar{D}} = \tfrac{1}{2} g_{VPP},$$

$$g_{D^{*+}D^+\gamma} = \tfrac{1}{3} g_{VP\gamma},$$

$$g_{D^{*0}D^0\gamma} = \tfrac{4}{3} g_{VP\gamma},$$

$$g_{\omega\pi\gamma} = g_{VP\gamma},$$

$$g_{\rho\pi\gamma} = \tfrac{1}{3} g_{VP\gamma},$$

and

$$\frac{\alpha' N_V^2}{2\nu_B} = 1.$$

One easily verifies that (25) is satisfied by these values but (26) is *not* satisfied, contrary to our experience with symmetry limits of FESR's.

To understand this discrepancy, it is useful to consider FESR's for the strong-interaction amplitudes $\pi K \to \omega K$ and $\pi K \to \phi K$. Both have K^* poles in the direct channel and ρ exchange in the cross channel. However, the coupling $\phi\rho\pi$ is forbidden in the quark-model [Okubo-Zweig-Iizuka (OZI)] sense, since the ideally mixed ϕ has only $s\bar{s}$ quark content and no duality diagram can be drawn. What happens to the sum rules is clear if we extend the cutoff upward to include the tensor K^{**}. For the ω reaction in the SU(3) limit, both K^* and K^{**} couplings have the same sign, and their semilocal average builds up the $\omega\rho\pi$ coupling in the usual sense. However, for the ϕ reaction, the couplings are opposite in sign, and their integral cancels to indicate a vanishing exotic $\phi\rho\pi$ coupling. Thus the small $\phi\rho\pi$ coupling does *not* imply a small $K^*K\phi$ coupling, but rather cancellation between $K^*K\phi$ and $K^{**}K\phi$. Hence cutting off the FESR after only the vector pole gives a spurious result in the cases where the cross channel is exotic.

This is precisely the thing which causes trouble in the $\gamma K \to \pi K$ and $\gamma D \to \pi D$ FESR. The photon couples to the K^*K (or D^*D) through separate parts which can be characterized by effective quantum numbers of ρ, ω, and ϕ (or ψ). However, the $\phi\rho\pi$ (or $\psi\rho\pi$) cross-channel couplings are exotic, so that using total $K^*K\gamma$ (or $D^*D\gamma$) couplings for vector pole saturation effectively sets the ϕ (or ψ) part of the photon coupling to K^*K (or D^*D) equal to zero. The solution is clear: If we must cut off the FESR after only the vector term, then we must separate the photon into its effective-quark-content parts. One then saturates separately for each part, being careful to reject the spurious conclusions which follow from exotic cross-channel couplings.

We decompose the photon according to

$$\gamma = \sum_{i=\rho,\omega,\phi,\psi} \gamma_i \tag{28}$$

and rewrite the FESR's:

$$2g_{K^*K\gamma_\rho} g_{K^*K\pi^0} = \frac{\alpha' N^2}{2\nu} g_{\omega\pi\gamma} g_{\omega K\bar{K}}, \tag{18$'$}$$

$$2g_{K^*K\gamma_\omega} g_{K^*K\pi^0} = \frac{\alpha' N^2}{2\nu} g_{\rho\pi\gamma} g_{\rho K\bar{K}}, \tag{19$'$}$$

$$2g_{D^*D\gamma_\rho} g_{D^*D\pi^0} = \frac{\alpha' N^2}{2\nu} g_{\omega\pi\gamma} g_{\omega D\bar{D}}, \tag{25$'$}$$

$$2g_{D^*D\gamma_\omega} g_{D^*D\pi^0} = \frac{\alpha' N^2}{2\nu} g_{\rho\pi\gamma} g_{\rho D\bar{D}}. \tag{26$'$}$$

For the total couplings, we have

$$g_{K^{*0}K^0\gamma} = -g_{K^*K\gamma_\rho} + g_{K^*K\gamma_\omega} + g_{K^*K\gamma_\phi} = (-\tfrac{1}{2} + \tfrac{1}{6} - \tfrac{1}{3}) g_{VP\gamma}, \tag{29}$$

$$g_{K^{*+}K^+\gamma} = g_{K^*K\gamma_\rho} + g_{K^*K\gamma_\omega} + g_{K^*K\gamma_\phi} = (\tfrac{1}{2} + \tfrac{1}{6} - \tfrac{1}{3}) g_{VP\gamma}, \tag{30}$$

$$g_{D^{*0}D^0\gamma} = -g_{D^*D\gamma_\rho} + g_{D^*D\gamma_\omega} + g_{D^*D\gamma_\psi} = (\tfrac{1}{2} + \tfrac{1}{6} + \tfrac{2}{3}) g_{VP\gamma},$$

$$g_{D^{*+}D^+\gamma} = g_{D^*D\gamma_\rho} + g_{D^*D\gamma_\omega} + g_{D^*D\gamma_\psi} = (-\tfrac{1}{2} + \tfrac{1}{6} + \tfrac{2}{3}) g_{VP\gamma}. \tag{32}$$

On the right-hand side, we have used $g_{\omega\pi\gamma_\rho} = g_{\omega\pi\gamma}$ since only the γ_ρ carries the $I=1$ part of γ, and $g_{\rho\pi\gamma_\omega} \approx g_{\rho\pi\gamma}$, since $g_{\rho\pi\gamma_\phi}$ is so small. Then it is easily seen that (19$'$) and (26$'$) both are satisfied with the symmetry limits. Also, the reason that (18) and (25) were satisfied previously is clear. Since the γ_ρ alone carries the isospin-1 part of the photon, one can write

$$g_{K^{*+}K^+\gamma} - g_{K^{*0}K^0\gamma} = 2g_{K^*K\gamma_\rho} \tag{33}$$

and a similar equation for D's. Thus the use of (18), (24), and (25) in estimating symmetry-breaking effects is valid. In addition, the ratios (20) can be rederived using (19$'$) and are also still valid. However, any further information must be extracted from the sum rules for separate photon components. Also, the η meson must be similarly divided. We write

$$\eta = \sum_{i=\omega,\phi,\psi} \alpha_i \eta_i, \tag{34}$$

$$\eta' = \sum_{i=\omega,\phi,\psi} \beta_i \eta_i, \tag{35}$$

$$X(2.8) = \sum_{i=\omega,\phi,\psi} \delta_i \eta_i, \tag{36}$$

where η_ω, η_ϕ, and η_ψ are the pure quark flavor components of the particles (normalized to unity), and α_i, β_i, δ_i are suitably normalized coefficients. We expect from physical properties of these mesons that η is primarily SU(3) octet, and η' primarily SU(3) singlet, with $X(2.8)$ primarily a pure $c\bar{c}$ charmonium state.[25] Thus we expect $\alpha_\omega \approx \beta_\phi \approx (\frac{1}{3})^{1/2}$, $\beta_\omega \approx -\alpha_\phi \approx (\frac{2}{3})^{1/2}$, $\delta_\psi \approx 1$, δ_ω, δ_ϕ, α_ψ, β_ψ small.

All possible sum rules for amplitudes $\gamma_i + P_A \to P_B + P_C$ are now listed in Table II, with numerical values for the kinematic factor $(\alpha' N^2/2\nu_B)$ (or $2\alpha' N^2$ for tensor exchanges). The α' parameters and nonstandard masses assumed are listed below Table II.

Henceforth all sum rules will be written in the form (18′), but with only subscripts written for couplings, i.e., $g_{ABC} \equiv (ABC)$, and with the kinematic factor written as μ_i, with i denoting the reaction as listed in Table II. Thus (18′) can be written

$$2(K^*K\gamma_\rho)(K^*K\pi^0) = \mu_1(\omega\pi\gamma)(\omega K\bar{K}). \qquad (18'')$$

For reactions involving the η's the physical γ couplings will be used, with explicit α, β, δ factors exhibited. Thus reaction (11) would read

$$2(K^*K\gamma_\phi)(K^*K\eta_\phi) = \frac{\mu_{11}}{\alpha_\phi}(\phi K\bar{K})(\phi\eta\gamma). \qquad (37)$$

We now proceed with the additional constraints of the FESR's, and try to estimate separately the $K^* \to K\gamma$ widths. The symbols [ ] refer to the reactions listed in Table II.

From [1] and [2],

$$\frac{(K^*K\gamma_\omega)}{(K^*K\gamma_\rho)} = \frac{(\rho K\bar{K})}{(\omega K\bar{K})}\,\frac{(\rho\pi\gamma)}{(\omega\pi\gamma)} \qquad (38)$$

or, from [13′] and [15′],

$$\frac{(K^*K\gamma_\omega)}{(K^*K\gamma_\rho)} = \frac{(\omega K\bar{K})}{(\rho K\bar{K})}\,\frac{(\omega\eta'\gamma)}{(\rho\eta'\gamma)}\,. \qquad (39)$$

From the discussion following Eq. (23) on the conflicting experimental evidence for these ratios, we can only conclude that the value is in the range 0.2–0.3. From [11] and [13],

$$\frac{(K^*K\gamma_\phi)}{(K^*K\gamma_\rho)} = \frac{\mu_{11}}{\mu_{13}}\,\frac{(\phi\eta\gamma)}{(\rho\eta\gamma)}\,\frac{\alpha_\omega}{\alpha_\phi}\,, \qquad (40)$$

where we have used SU(3) for the VPP couplings.[26] We assume that the η is mainly an SU(3) octet, so that $\alpha_\omega/\alpha_\phi \approx -1/\sqrt{2}$. Then the decay data and masses yield -0.26 ± 0.05 for the ratio (40). Note that most of this large symmetry breaking [the SU(3) value would be $-\frac{2}{3}$] comes from the small[23] $\Gamma(\phi \to \eta\gamma)$ as given in Table I. The only possibility to make this ratio SU(3) symmetric would be to increase $\alpha_\omega/\alpha_\phi$ by more than a factor of 2. This es-

TABLE II. Kinematic factors in FESR for amplitudes $\gamma_i P_A \to P_B P_C$. They are dimensionless except for tensor-exchange sum rules as indicated.

Reaction	s pole	t pole	Kinematic factor
1. $\gamma_\rho K \to \pi^0 K$	K^*	ω	1.11
1′. $\rho K \to \pi K$	K^*	ω	1.08
2. $\gamma_\omega K \to \pi^0 K$	K^*	ρ	1.11
3. $\gamma_\phi K \to \pi^0 K$	K^*	ϕ (exotic)	1.03
4. $\gamma_\rho D \to \pi^0 D$	D^*	ω	1.42
5. $\gamma_\omega D \to \pi^0 D$	D^*	ρ	1.42
6. $\gamma_\psi D \to \pi^0 D$	D^*	ψ (exotic)	1.16
7. $\gamma_{\phi,\psi} F \to \pi^0 F$	exotic	exotic	...
8. $\gamma_\omega \pi \to \pi\pi$	ρ	ρ	1.11
9. $\gamma_\psi F \to \eta_\psi F$	F^*	ψ	0.67
10. $\gamma_\psi D \to \eta_\psi D$	D^*	ψ	0.56
11. $\gamma_\phi K \to \eta_\phi K$	K^*	ϕ	0.99
11′. $\gamma_\phi K \to \eta'_\phi K$	K^*	ϕ	0.95
12. $\gamma_\phi F \to \eta_\phi F$	F^*	ϕ	1.60
12′. $\gamma_\phi F \to \eta'_\phi F$	F^*	ϕ	1.76
13. $\gamma_\rho K \to \eta_\omega K$	K^*	ρ	1.09
13′. $\gamma_\rho K \to \eta'_\omega K$	K^*	ρ	1.13
14. $\gamma_\rho D \to \eta_\omega D$	D^*	ρ	1.43
14′. $\gamma_\rho D \to \eta'_\omega D$	D^*	ρ	1.62
15. $\gamma_\omega K \to \eta_\omega K$	K^*	ω	1.09
15′. $\gamma_\omega K \to \eta'_\omega K$	K^*	ω	1.13
16. $\gamma_\omega D \to \eta_\omega D$	D^*	ω	1.43
16′. $\gamma_\omega D \to \eta'_\omega D$	D^*	ω	1.62
17. $\gamma_\phi F \to KD$	F^*	K^*	1.71
18. $\gamma_\phi K \to DF$	K^*	F^*	...
19. $\gamma_\psi F \to DK$	F^*	D^*	1.59
20. $\gamma_\psi D \to FK$	D^*	F^*	1.06
21. $\gamma_\rho K \to DF$	K^*	D^*	...
22. $\gamma_\rho D \to KF$	D^*	K^*	1.33
23. $\gamma_\omega K \to DF$	K^*	D^*	...
24. $\gamma_\omega D \to KF$	D^*	K^*	1.33
25. $\gamma_\rho K \to \pi^+ K$	K^*	A_2	7.2 GeV^2
25′. $\rho K \to \pi K$	K^*	A_2	5.2 GeV^2
26. $\gamma_\rho D \to \pi^+ D$	D^*	A_2	8.6 GeV^2
27. $\gamma_\psi D \to \eta_\psi D$	D^*	ψ	1.14
27′. $\gamma_\psi D \to \eta'_\psi D$	D^*	ψ	1.09
28. $\gamma_\psi F \to \eta_\psi F$	F^*	ψ	1.21
28′. $\gamma_\psi F \to \eta'_\psi F$	F^*	ψ	1.16
29. $\gamma\pi \to \eta\pi$	A_2	ρ	0.18 GeV^{-2}
29′. $\gamma\pi \to \eta'\pi$	A_2	ρ	0.21 GeV^{-2}

Parameters (Ref. 38)

$\alpha'_\rho = \alpha'_\omega = 0.90\ \text{GeV}^{-2}$
$\alpha'_\phi = 0.79\ \text{GeV}^{-2}$
$\alpha'_\psi = 0.33\ \text{GeV}^{-2}$
$\alpha'_{D^*} = 0.63\ \text{GeV}^{-2}$
$\alpha'_{F^*} = 0.50\ \text{GeV}^{-2}$
$\alpha'_{K^*} = 0.83\ \text{GeV}^{-2}$
$m_F^2 = 4.12\ \text{GeV}^2$
$m_{F^*}^2 = 4.57\ \text{GeV}^2$
$m_X^2 = 8.0\ \text{GeV}^2$

sentially makes the η into an SU(3) singlet, in contradiction with strong decay and mass formula analysis. This information then yields directly the ratio

$$R_K{}^2 \equiv \frac{\Gamma(K^{*+}\to K^+\gamma)}{\Gamma(K^{*0}\to K^0\gamma)} = \left(\frac{1+\dfrac{K^*K\gamma_\omega + K^*K\gamma_\phi}{K^*K\gamma_\rho}}{-1+\dfrac{K^*K\gamma_\omega + K^*K\gamma_\phi}{K^*K\gamma_\rho}}\right)^2 \approx 1.0 \pm 0.2 \tag{41}$$

from (38) and (40), compared with the SU(3)-symmetric value of $\frac{1}{4}$. We use this in conjunction with the absolute magnitude constraint (24) to restrict further the allowed $K^*K\gamma$ widths. The result is shown by the dotted radial lines in Fig. 1. It is evident again that the crucial measurement is $\Gamma(K^{*+}\to K^+\gamma)$, which must be somewhat greater than its present upper bound of 80 keV if the sum rules are to hold. Note that the uncertainties in (41) include the possibility of the $\rho\pi\gamma$ coupling to be shifted upward, so that this $K^{*+}\to K^+\gamma$ prediction is an *independent* constraint.

IV. CHARMED-MESON DECAYS

For the $D^*D\gamma$ couplings, one can repeat the analysis for the K's. From [4] and [5],

$$\frac{(D^*D\gamma_\omega)}{(D^*D\gamma_\rho)} = \frac{(\rho D\bar{D})}{(\omega D\bar{D})}\,\frac{(\rho\pi\gamma)}{(\omega\pi\gamma)}, \tag{42}$$

and using (27) and (38), we conclude the same 0.2–0.3 range for the magnitude of this ratio.

From [10] and [14],

$$\frac{(D^*D\gamma_\psi)}{(D^*D\gamma_\rho)} = -\frac{\mu_{10}}{\mu_{14}}\,\alpha_\omega\,\frac{(\psi X\gamma)}{(\rho\eta\gamma)}, \tag{43}$$

where we have used SU(4) for the VPP couplings again.[26] The small value[25] $\Gamma(\psi\to X\gamma)\leq 2$ keV, which makes trouble for nonrelativistic transition calculations in the charmonium model, provides a conservative upper limit of 0.1 for (43). This is a huge SU(4) violation away from the symmetry prediction of $-\frac{4}{3}$. While we use this small bound in what follows, we must keep open the possibility that perhaps the $X(2.8)$ is not the pseudoscalar partner of the ψ. A 0^- state at somewhat higher mass could possibly, through reduced phase space, yield a much larger $(\psi X\gamma)$ coupling even with small bounds on ψ decay into such a state.

One can now calculate

$$R_D{}^2 \equiv \frac{\Gamma(D^{*+}\to D^+\gamma)}{\Gamma(D^{*0}\to D^0\gamma)} = \left(\frac{1+\dfrac{D^*D\gamma_\omega + D^*D\gamma_\psi}{D^*D\gamma_\rho}}{-1+\dfrac{D^*D\gamma_\omega + D^*D\gamma_\psi}{D^*D\gamma_\rho}}\right)^2$$

$$= \begin{cases} 0.35 \pm 0.08 & \text{(44a)} \\ 2.9 \pm 0.7, & \text{(44b)} \end{cases}$$

where the two possible results come from the sign ambiguity for $D^*D\gamma_\omega/D^*D\gamma_\rho$. Note that this ambiguity did not occur in the calculation of R_K from (41), due to the near cancellation of the ω and ϕ terms. Note again the large SU(4) violation away from the symmetry prediction of $\frac{1}{16}$.

To get absolute magnitudes for the D^* decays, one needs information about the strong coupling $D^*D\pi$ relative to $K^*K\pi$. We do not assume SU(4) in this case, but use the method of Ref. 27. Here the $\gamma P\to\gamma P$ FESR's were used to relate radiative decays of D^* and K^* directly. As a by-product of that calculation $(A_2\gamma\gamma)$ and $(A_2KK)/(A_2DD)$ resulted. However, the method of separating the photon components described here to avoid difficulties in single-pole saturation of FESR's was not used in Ref. 27. We present here a summary of results which follow from an application of the methods developed in this paper to the reactions considered in Ref. 27.

(1) One still obtains $R_D{}^2>1$ from the $\gamma D\to\gamma D$ FESR, so that we can choose (44b) as the correct solution.

(2) There remains a sign ambiguity for the ratio $(K^*K\gamma_\omega)/(K^*K\gamma_\rho)$.

We present in Table III a compilation of numerical results for quantities defined in Ref. 27. The last entry for $\Gamma(D^{*0}\to D^0\gamma)$ follows from a comparison of (25) and (26′) with (18′) and (19′). The first entry for this same quantity is from an analysis of $\gamma D\to\gamma D$ alone. Consistency then requires that the positive-ratio assumption be adopted, so we can make the following combined estimates:

$$\Gamma(D^{*0}\to D^0\gamma)\approx 4\pm 2\ \text{keV}, \tag{45}$$

$$\Gamma(D^{*+}\to D^+\gamma)\approx 12\pm 6\ \text{keV}. \tag{46}$$

One can also use the strong-coupling ratio $D^*D\pi/K^*K\pi$ to predict strong decay widths. However, these are very sensitive to the small Q values for $D^*\to D\pi$. In any event, one expects $D^*D\pi$ widths from column 2 of Table III to be about 50% higher than the SU(4) predictions with correct phase space.

The most accurately determined Q value is the recently reported[28] 5.7 ± 0.5 MeV for $D^{*+}\to D^0\pi^+$,

TABLE III. Modifications to quantities in Ref. 27 from separation of photon components in FESR's for $\gamma P \to \gamma P$.

Quantity	Old Results	New Results $\frac{(K^*K\gamma_\omega)}{(K^*K\gamma_\rho)} > 0$	New Results $\frac{(K^*K\gamma_\omega)}{(K^*K\gamma_\rho)} < 0$
X_K	-0.20 ± 0.10	0.36 ± 0.20	-0.36 ± 0.20
$\Gamma(A_2 \to \gamma\gamma)$	1.4 ± 0.4 keV	4.1 ± 1.2 keV	0.9 ± 0.25 keV
X_D	0.23 ± 0.06	0.58 ± 0.06	0.094 ± 0.080
$\Gamma(D^{*0} \to D^0\gamma)$	3.0 ± 0.8 keV	4.8 ± 1.8 keV	0.8 ± 0.3 keV
$\Gamma(D^{*+} \to D^+\gamma)$	6.8 ± 1.8 keV	14 ± 5 keV	3.8 ± 0.9 keV
$\frac{(A_2DD)}{(A_2KK)}$	1.35	1.02	1.50
$\frac{(\omega DD)}{(\omega KK)}$	1.21	0.92	1.35
$\frac{(D^*D\pi)^2}{(K^*K\pi)^2}$	1.95	1.47	2.18
$\Gamma(D^{*0} \to D^0\gamma)$	3.2 ± 1.5 keV	3.0 ± 1.5 keV	4.4 ± 2.2 keV

which implies

$$\Gamma(D^{*+} \to D^0\pi^+) = 22 \pm 4 \text{ keV}. \qquad (47)$$

If we assume that the D^+-D^0 mass difference is almost the same as the $\pi^+ - \pi^0$, then the $D^{*+} \to D^+\pi^0$ will be approximately 11 keV, so that we can add the radiative decay width (46) to get an approximate $\Gamma_{\text{total}}(D^{*+}) \approx 45 \pm 15$ keV, where the uncertainty is just a guess at the uncertainty in the unknown Q value.

One can do a similar thing for D^{*0}, where $D^0\pi^0$ is the only kinematically allowed strong decay mode. Using as an estimate[29] $Q = 3 \pm 2$ MeV, one gets $\Gamma(D^{*0} \to D^0\pi^0) \approx 4 \pm 3$ keV, and with (45), $\Gamma_{\text{total}}(D^{*0}) \approx 8 \pm 4$ keV. Note also that branching ratio $(D^{*0} \to D^0\gamma)/(D^{*0} \to D^0\pi^0)$ should be of the order of unity, again consistent with experimental estimates.[29]

(3) One can also look at the $F^*F\gamma$ radiative decay. From a combination of [10], [11], [17], and [19], one obtains

$$(F^*F\gamma) \approx -\tfrac{1}{2}(\tfrac{3}{2})^{1/2}\mu_{11}\mu_{17}(\phi\eta\gamma), \qquad (48)$$

where we have used $(D^*D\gamma_\psi) \ll (D^*D\gamma_\rho)$ in order to get expressions in terms of the physical photon couplings. Assuming the masses in Table II, one calculates

$$\Gamma(F^* \to F\gamma) \approx 1.4 \text{ keV}. \qquad (49)$$

In addition, one can get more constraints on strong couplings. Using [1], [4], [25], and [26], we obtain

$$\frac{(A_2D\bar{D})}{(A_2K\bar{K})}\,\frac{(\omega K\bar{K})}{(\omega D\bar{D})} = \frac{\mu_{25}}{\mu_{26}}\,\frac{\mu_4}{\mu_1} = 1.11. \qquad (50)$$

This expression has been already used in Ref. 27 and in computing Table III.

(4) We must note that the consistency between $\gamma P \to \gamma P$ and $\gamma P \to PP$ FESR's forces us to solutions for coupling ratios which do not preserve the SU(3) or SU(4) relative signs. In particular, we must have (38) and (42) positive, whereas SU(4) would predict a relative minus sign. Perhaps this is not so surprising, since the magnitudes are also far from the symmetry predictions.

V. TENSOR-MESON RADIATIVE DECAYS

The normalization for the $TP\gamma$ coupling is given by

$$\Gamma(T \to P\gamma) = \frac{8}{5}\,\frac{g_{TP\gamma}^2}{4\pi}\,|\vec{p}\,|^5. \qquad (51)$$

One can isolate the $A_2\pi\gamma$ coupling immediately from [1] and [25]:

$$\frac{(A_2\pi\gamma)}{(\omega\pi\gamma)} = \frac{\mu_1}{\mu_{25}}\,\frac{(\omega K\bar{K})}{(A_2K\bar{K})}. \qquad (52)$$

We extract $(\omega\pi\gamma)$ and $(A_2K\bar{K})$ from the measured widths, and use the SU(3) symmetric $(\omega K\bar{K})$ modified by (12) as discussed previously. This leads to the prediction

$$\Gamma(A_2^\pm \to \pi^\pm\gamma) = 490 \pm 60 \text{ keV}. \tag{53}$$

The only experimental information[30] on this decay is the 500 ± 500 keV extracted from A_2 photoproduction via single-pion exchange, although new information is expected shortly.[31] It is interesting to compare this with the naive vector-meson-dominance (VMD) prediction based on $\Gamma(A_2 \to \rho\pi)$ and $\Gamma(\rho \to e^+e^-)$. As reported in Ref. 32, this predicts $\Gamma(A_2 \to \pi\gamma) = 965$ keV. Also in Ref. 32, this same width is estimated from the FESR for $\gamma\pi \to \gamma\pi$, with the result $\Gamma(A_2 \to \pi\gamma) \approx 300$ keV. Thus both of the FESR results are a substantial factor below the naive VMD prediction. One can estimate effective VMD correction factors from purely kinematic factors, without the use of symmetric VPP couplings, by considering ratios of [1] and [25] with the corresponding strong amplitudes for $\rho K \to \pi K$. One gets

$$\frac{(A_2\rho\pi)}{(\omega\rho\pi)} = \frac{\mu_{1'}}{\mu_{25'}} \frac{(\omega K\bar{K})}{(A_2 K\bar{K})}. \tag{54}$$

We define effective VMD correction factors by

$$(A_2\pi\gamma) = \mathcal{K}_A \frac{e}{g_\rho} (A_2\rho\pi), \tag{55a}$$

$$(\omega\pi\gamma) = \mathcal{K}_\omega \frac{e}{g_\rho} (\omega\rho\pi), \tag{55b}$$

where g_ρ is the usal γ-ρ coupling from $\rho \to e^+e^-$. We use (52) and (54) to get

$$\mathcal{K}_A = \mathcal{K}_\omega \frac{\mu_{25'}}{\mu_{25}} \frac{\mu_1}{\mu_{1'}} = 0.74\mathcal{K}_\omega . \tag{56a}$$

As a consistency check, we repeat the procedure with K's replaced by D's. The result is

$$\mathcal{K}_A = \mathcal{K}_\omega \frac{\mu_{26'}}{\mu_{26}} \frac{\mu_4}{\mu_{4'}} = 0.71\mathcal{K}_\omega , \tag{56b}$$

which is certainly consistent with (56a).

One can estimate $\mathcal{K}_\omega$ by the method of Ref. 33, in which $\omega \to \pi\gamma$ and $\omega \to 3\pi$ are related via simple VMD, and a correction factor $(\mathcal{K}_\omega)^2 \approx 1.3$ is found. This implies $\mathcal{K}_A \approx 0.65$, and

$$\Gamma(A_2 \to \pi\gamma) = (\mathcal{K}_A)^2 \Gamma_{\text{VMD}} = 410 \text{ keV}, \tag{57}$$

which is again consistent with the FESR estimates above. Note that a lower value for $A_2 \to \pi\gamma$ is also found by the single-quark transition method of Ref. 34, even when normalized to $A_2 \to \rho\pi$ by VMD. This is essentially due to normalization of the $TP\gamma$ amplitude so that $\Gamma \sim |A|^2|\vec{p}|^3$ rather than (51). The correction factor then comes from the two extra powers of momentum evaluated at physical ρ mass rather than for zero photon mass. One can now extend the present analysis to include K^{**} and D^{**} decays, by including them in the sum rules [1]–[6]. The general form then reads for $\gamma_i + P_A \to P_B + P_C$

$$2(VP_BP_C)(VP_A\gamma_i)\nu_B(V) + 4X_T(TP_BP_C)(TP_A\gamma_i)\nu_B(T) = \alpha_t'(V_tP_B\gamma)(V_tP_AP_C)\tfrac{1}{2}N_T^2 \tag{58}$$

with

$$X_T \equiv m_T^2 + 2m_t^2 - m_A^2 - m_B^2 - m_C^2 + \frac{m_A^2(m_C^2 - m_B^2)}{m_T^2} \tag{59}$$

a kinematic factor for spin-2 poles, and $\nu_B(V)$, $\nu_B(T)$ the position of the direct-channel poles, and m_t the mass of the t-channel vector meson. One uses the same sum rules cutoff after the vector poles alone to normalize and also eliminate the t-channel couplings. The final result is

$$\frac{(TP_A\gamma_i)}{(VP_A\gamma_i)} = \frac{1}{2X_T} \frac{\nu_B(V)}{\nu_B(T)} \frac{(VP_BP_C)}{(TP_BP_C)} \left[\left(\frac{N_T}{N_V}\right)^2 - 1\right]. \tag{60}$$

One can then use the measured[35] VPP and TPP widths to evaluate this ratio for each γ_i. For those quantum numbers which have an exotic t channel, one has simply

$$\frac{(TP_A\gamma_i)}{(VP_A\gamma_i)} = -\frac{1}{2X_T} \frac{\nu_B(V)}{\nu_B(T)} \frac{(VP_BP_C)}{(TP_BP_C)} . \tag{61}$$

This expresses the cancellation mechanism between vector and tensor poles previously discussed.

The tensor widths are then evaluated, using the measured or previously predicted $VP\gamma$ widths for normalization. The total amplitudes are sums over the γ_i, and hence the specific relative signs implied by (41), (44b), (60), and (61) are crucial. The results are shown in Table IV, along with the SU(3)- and SU(4)-symmetry predictions, based on an input VMD value for $A_2 \to \pi\gamma$. Note especially the deviation of FESR predictions from the predictions of zero for the neutral tensor K and D mesons.

TABLE IV. FESR predictions for $T \to P\gamma$ widths, compared with symmetry predictions. All values in keV.

Process	Experiment	Symmetry	FESR
$A_2^\pm \to \pi^\pm\gamma$	500 ± 500, Ref. 30	965 (input)	490 ± 60
$K^{**+} \to K^+\gamma$	. . .	790	125 ± 60
$K^{**0} \to K^0\gamma$		0	32 ± 15
$D^{**+} \to D^+\gamma$		155	38 ± 19
$D^{**0} \to D^0\gamma$		0	13 ± 7

VI. MISCELLANEOUS APPLICATIONS

We reexamine the sum rule for $\pi\eta \to \pi\gamma$, which has been used previously to estimate the decay width for $\eta \to \pi\pi\gamma$. The s-channel pole is the A_2, and the t-channel exchange is the ρ. Here we use (53) for the $A_2\pi\gamma$ coupling, plus the known $A_2\pi\eta$ and $\rho\pi\pi$ couplings to estimate the $\rho\eta\gamma$ coupling. The sum rule reads

$$(A_2\pi\eta)(A_2\pi\gamma) = \left(\frac{\alpha' N^2}{2\nu_B f}\right) \ (\rho\eta\gamma)(\rho\pi\pi) \ , \tag{62}$$

where the kinematic factor contains an additional component

$$f \equiv 4\left[\nu_B + \tfrac{3}{2} m_\rho{}^2 - m_\pi{}^2 - \tfrac{1}{2} m_\eta{}^2 - \frac{m_\pi{}^2(m_\eta{}^2 - m_\pi{}^2)}{m_{A_2}{}^2}\right] \tag{63}$$

due the the A_2 couplings. The kinematic factor is fairly sensitive to cutoff position. We take an average between the extreme cases where $N = \nu_B + 1/\alpha'$ and $N = \nu_B$ with only one-half of the resonance contribution. The result is

$$\frac{\alpha' N^2}{2\nu_B f} = 0.16 \pm 0.04 \text{ GeV}^{-2} \ ,$$

and

$$\Gamma(\rho \to \eta\gamma) = 37 \pm 9 \text{ keV} \ . \tag{64}$$

This is certainly consistent with the (50 ± 13)-keV experimental solution chosen by previous FESR constraints, and favors exclusion of the other solution 76 ± 15 keV.

We can now repeat the procedure for the η'. The kinematic factor now changes to 0.21 ± 0.05 GeV^{-2}, just from the η-η' mass difference. In this case, however, only an upper limit[5] $\Gamma(A_2 \to \pi\eta') < 1$ MeV is known. For this reason, we write the sum rule constraint as a ratio,

$$\frac{\Gamma(A_2 \to \pi\eta')}{\Gamma_{\text{total}}(\eta')} = 3.3^{+1.7}_{-1.0} , \tag{65}$$

where we have also used[5]

$$\text{BR}\left(\frac{\eta' \to \rho\gamma}{\eta' \to \text{all}}\right) = 0.304 \pm 0.017 \ .$$

From the upper limit on $A_2\pi\eta'$, we then infer $\Gamma_{\text{total}}(\eta') \lesssim 300^{+130}_{-100}$ keV, which is well below the present experimental upper bound of 1 MeV.

One can get another estimate for the $\rho\eta'\gamma$ coupling from the ratio of [13] and [13'],

$$\frac{(\rho\eta'\gamma)}{(\rho\eta\gamma)} = \frac{\mu_{13}}{\mu_{13'}} \frac{\beta_\omega}{\alpha_\omega} \ , \tag{66}$$

where we have *assumed* that the K^*K coupling to the normal quark components of η and η' are equal, independent of mass. Then the usual η-η' mixing results give $\beta_\omega/\alpha_\omega \approx \sqrt{2}$, and the standard results are modified only by the ratio of kinematic factors, giving only a 7% suppression. If we now use the preferred $\Gamma(\rho \to \eta\gamma) = 50 \pm 13$ keV, we can deduce

$$\Gamma(\eta' \to \rho\gamma) = 146 \pm 38 \text{ keV} \tag{67a}$$

or

$$\Gamma_{\text{total}}(\eta') = 480 \pm 120 \text{ keV} \ . \tag{67b}$$

This last result receives additional support for the estimate following (65), and indicates that $\Gamma(A_2 \to \pi\eta')$ cannot be very far below its present upper bound. A similar procedure can be carried out for the $\phi\eta\gamma$ and $\phi\eta'\gamma$ couplings. From the ratio [11] to [11'],

$$\frac{(\phi\eta'\gamma)}{(\phi\eta\gamma)} = \frac{\mu_{11}}{\mu_{11'}} \frac{\beta_\phi}{\alpha_\phi} \ , \tag{68}$$

and again the kinematic factors only modify the usual η-η' mixing result $\beta_\phi/\alpha_\phi \approx -1/\sqrt{2}$ by a few percent.[36] Using the $\phi \to \eta\gamma$ for normalization, one gets

$$\Gamma(\phi \to \eta'\gamma) = 0.13 \pm 0.03 \text{ keV}.$$

The small width is mainly a result of restricted phase space, and certainly consistent with the absence of this mode in experimental surveys.

As a final example, we use FESR ratios along with the surprisingly large OZI-rule-violating ψ decays,[25] $\Gamma(\psi \to \eta\gamma) = 55 \pm 12$ eV, $\Gamma(\psi \to \eta'\gamma) = 152 \pm 117$ eV, to extract α_ψ and β_ψ, the effective $c\bar{c}$ components of the η and η'. The ratio [27] to [10] gives

$$\alpha_\psi = \frac{\mu_{27}}{\mu_{10}} \frac{(\psi\eta\gamma)}{(\psi X\gamma)} = 0.068 \pm 0.032 \tag{69a}$$

and the ratio [27'] to [10] gives

$$\beta_\psi = \frac{\mu_{27'}}{\mu_{10}} \frac{(\psi\eta'\gamma)}{(\psi X\gamma)} = 0.22 \pm 0.19 \ . \tag{69b}$$

These numbers are in reasonable agreement with the estimates of Voloshin[37] based on direct $\eta, \eta'(c\bar{c})$ couplings in a static charmonium model, normalized by the leptonic decays.

VII. CONCLUSIONS

We summarize the main conclusions below.

(1) The FESR constrainsts on ρ/ω ratios in $\eta\gamma$ and $\eta'\gamma$ couplings, together with the recently measured branching ratios in η' decay, allow a choice between the two possible experimental results for the $\eta\gamma$ modes. The solution $\Gamma(\rho \to \eta\gamma) = 50 \pm 13$ keV and $\Gamma(\omega \to \eta\gamma) = 3 \pm 2$ keV is preferred.

(2) The comparison of the above with the $\pi\gamma$

decay modes, plus assumption of approximate SU(3) symmetry for the *VPP* couplings, leads to a contradiction. FESR's require that either the $\Gamma(\rho\to\pi\gamma)$ must go up from 35 ± 10 keV to at least 55 keV, or the η' ratio must change from 9.9 ± 2.0 to the neighborhood of 20.

(3) The constraint on the sum of K^{*0} and K^{*+} radiative decays indicates that one or both of them must increase from the present experimental value or upper bound, respectively. This constraint is shown in Fig. 1.

(4) Input of information on the $\phi\to\eta\gamma$ decay allows separation of the two K^* widths, and indicates that most likely the $K^{*+}\to K^+\gamma$ is the quantity which must lie above its present upper bound of 80 keV. The crucial ratio

$$\frac{\Gamma(K^{*+}\to K^+\gamma)}{\Gamma(K^{*0}\to K^0\gamma)}$$

must certainly be of the order of unity, far above the SU(3)-symmetry limit of $\frac{1}{4}$.

(5) The extension of these techniques to charmed mesons, together with input of the small $\psi\to X(2.8)\gamma$ width, allows a separation of D^{*+} and D^{*0} decays, both into $D\gamma$ and $D\pi$. We estimate $\Gamma_{\rm total}(D^{*+})=45\pm15$ keV, $\Gamma_{\rm total}(D^{*0})=8\pm4$ keV, with radiative to pionic branching ratios of about $\frac{1}{3}$ for D^{*+} and unity for D^{*0}.

(6) Extension of the FESR's to include tensor mesons yields predictions for $T\to P\gamma$ as given in Table IV. Large deviations from both naive vector-meson-dominance model and symmetry predictions are expected.

(7) Consistency between the preferred $\Gamma(\rho\to\eta\gamma)=50\pm13$ keV and the predicted $\Gamma(A_2\to\pi\gamma)=490\pm60$ keV is verified by consideration of the sum rules. The same technique applied to the η' couplings gives $\Gamma_{\rm total}(\eta')<430$ keV. The usual η-η' mixing results slightly modified by a kinematic factor also come out of the FESR's for $\gamma K\to(\eta,\eta')K$. The prediction, normalized to the $\rho\eta\gamma$ above, is $\Gamma_{\rm total}(\eta')=480\pm120$ keV. This also implies that $\Gamma(A_2\to\pi\eta')$ must be close to its present upper bound of 1 MeV.

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must conclude that some of the F system parameters may have to be adjusted. A similar check in [14] and [14′] compared with (66) reveals no discrepancy for the D system.

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