Scale-violating quark model for large-p_T processes. Two-hadron inclusive reactions and correlations

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A quark-parton model with logarithmically decreasing moments and a number of features dictated by asymptotically free field theories was shown to successfully account for most of the data on single-hadron inclusive distributions in hadron collisions. This model is now used to study two-hadron inclusive reactions. Integrated correlations at large transverse momenta (p_T) are calculated. Comparison is made with experimental data on $pp \to h_1 h_2 + X$ with the two hadrons h_1 and h_2 at 90° in the c.m. of the intial protons either on opposite sides or on the same side. The model accounts fairly well for the p_T and s dependences as well as for the absolute magnitude of the correlation data. Rapidity and transverse-momentum sharing-variable (x_e) distributions are also calculated for trigger and secondary on opposite sides. The model predicts reasonably well their absolute magnitude, as well as a p_T dependence of the x_e distribution in the direction indicated by experiment, although somewhat weaker. The shape of the rapidity distribution is also discussed.

I. INTRODUCTION

There has recently been much interest in a theory of elementary processes based on a fundamental quark-gluon interaction. Naturally this has resulted in renewed interest in the parton model of Berman, Bjorken, and Kogut (BBK),[1] in which the underlying mechanism of hadron reactions at large transverse momenta (p_T) is quark scattering via the exchange of a single gluon. In addition, CERN ISR correlation data[2-5] have now established a two-jet structure of the large-p_T events, which is one of the basic features of the BBK model.

Very recently the idea has been advanced[6] that some of the well-known defects of the original BBK model can be removed without modifying the basic dynamical mechanism by taking into account the violations of Bjorken scaling; such violations have been observed experimentally,[7-9] and predicted theoretically on the basis of asymptotically free field theories (AFFT) as well as of conventional field theories (CFT). In particular, in a very recent work[10] (hereafter referred to as I), a model of quark-parton distributions was presented which satisfied a number of requirements of AFFT and led to logarithmically decreasing moments and it was shown that this model accounts fairly well for the recent deep-inelastic lepton-nucleon data[8] and for all the essential large-p_T data on single-hadron production in proton-proton collisions.[10]

The purpose of the present work is to show that the model of I also accounts for most of the essential features of the two-hadron inclusive production and correlation data at large p_T.

In I the probability distributions for the quarks u, d, s, $\bar{u}$, $\bar{d}$, and $\bar{s}$ in a proton were taken as follows:

$$u = 2v_u(x, Q^2) + t(x, Q^2), \quad d = v_d(x, Q^2) + t(x, Q^2) , \tag{1.1}$$

$$\bar{u} = \bar{d} = s = \bar{s} = t(x, Q^2) , \tag{1.2}$$

where x is the usual scaling variable and Q is the four-momentum of the probe. The valence distributions $v_i(x, Q^2)$, $i = u, d$ and the sea distribution $t(x, Q^2)$ were determined on the basis of the following requirements:

(i) The valence distributions satisfy for all Q^2 the conditions[11]

$$\int_0^1 v_i(x, Q^2) dx = 1, \quad i = u, d$$

imposed by conservation of charge and of third component of isospin.

(ii) As Q^2 increases, at small $x (\lesssim 0.2)$, $v_i(x, Q^2)$ and $t(x, Q^2)$ increase, but at large x they decrease. This is known to be a very important property of scale-violating parton models[12] and leads to nucleon structure functions $\nu W_2(x, Q^2)$ with the scale-violating pattern of AFFT and CFT.

(iii) At some low Q^2 value, say $Q^2 = Q_0^2$, all the quark distributions reduce to certain specific forms, which have been determined by fitting the data on the structure functions νW_2 for $e^- p$, $e^- n$, νp, and $\bar{\nu} p$, inelastic scattering (these data are mainly at low Q^2). In particular, our quark distributions reduce to those of a modified Kuti-Weisskopf model, as specified in Refs. 13 and 14, in order to account for more recent data on νW_2.

(iv) The nth moment, $M_n(Q^2)$ of the structure function $\nu W_2^p(x, Q^2)$ for $e+p\to e+X$ and $\mu+p\to\mu+X$ is defined by

$$M_n(Q^2)=\int_0^1 x^n \nu W_2^p(x,Q^2)dx\ . \tag{1.4}$$

In the model of I we require that as $Q^2\to\infty$ all the moments for $n\geq 1$ decrease as inverse powers of $\ln Q^2$. This requirement is dictated by AFFT.

(v) At large Q^2 the lowest moment ($n=0$) is required to have the form

$$M_0(Q^2)\simeq A_0+A_1/\ln Q^2\ , \tag{1.5}$$

where A_0, A_1 are constants and $A_1>0$. AFFT predict a similar form.[15,16] In particular, for a Lagrangian containing n_ψ fermion fields (quarks) of charge e_j and n_v vector fields (gluons) AFFT predict that

$$A_0=\sum_j \frac{e_j^{\,2}}{2n_v+n_\psi}\ ; \tag{1.6}$$

this is required as well.

(vi) The leading term A_0 in (1.5) originates from the contribution of the sea $t(x,Q^2)$, while the ratio[10] $M_n^v(Q^2)/M_n^t(Q^2)\to 0$ as $Q^2\to\infty$. This has been shown in detailed studies of the contributions to νW_2 of nonsinglet as well as singlet Wilson operators.[17]

The final form of the distributions $v_i(x,Q^2)$ and $t(x,Q^2)$ obtained in I on the basis of these requirements is given below in Eqs. (2.3)–(2.6). We use precisely the same distributions to study some of the most important features of the large-p_T correlation data.[18]

During the last year there have been several studies of two-hadron correlations within the quark-parton model.[19–26] The differences between these studies and the present work can be summarized as follows:

In Refs. 19 and 20 the underlying dynamical mechanism is not specified at all. In Refs. 19–22 the question of scale violations is not considered. In particular, Ref. 21 assumes a quark subprocess of the BBK type, but their fragmentation functions are taken to scale (Q^2-independent), so that in order to obtain a single-hadron inclusive cross-section behaving like $\sim p_T^{-8}$ they are obliged to use purely *ad hoc* forms for the cross section $d\sigma/d\hat{t}$ of the quark subprocess. Moreover, Q^2 independence of the fragmentation functions amounts to structure functions νW_2 independent of Q^2, in contradiction with deep-inelastic muon-nucleon[8] and neutrino-nucleon data.[9]

References 24–26 consider scale-breaking effects; however, their moments $M_n(Q^2)$ behave asymptotically like inverse powers of Q^2. Probably such a pattern of scale violation, although not excluded by present data, is somewhat too strong; certainly it contradicts AFFT. As stated, in the present work (and in I) the scale violations are of logarithmic nature and are taken to satisfy the above requirements (i)–(iv), many of which are dictated by AFFT.

Other important differences between the present work and that of other authors are discussed in Secs. V and VI.

In Sec. II we present the essential results on the forms of the hadron and quark fragmentation functions obtained in I. In Sec. III we give the general expressions of the inclusive cross sections for two hadrons produced in opposite sides or in the same side. In Sec. IV we present and discuss the predictions of the model concerning the magnitude, momentum dependence, and energy dependence of the two-hadron correlation functions. Section V contains the predictions for the transverse-momentum sharing-variable (x_e) distribution. Section VI presents and discusses our results on the rapidity distributions of secondaries opposite a large-p_T trigger, and compares them with the results of other models. Finally, in the Appendix we discuss in some detail the form of the fragmentation function for a quark producing two (same-side) hadrons.

II. FRAGMENTATION FUNCTIONS

The fragmentation function $F_{a/A}(x,Q^2)$ is related to the differential probability dP that a hadron A is seen by a probe with four-momentum Q to contain a quark a with a fraction x of the hadron longitudinal momentum by the relation

$$dP=F_{a/A}(x,Q^2)dx/x\ . \tag{2.1}$$

When $A=$proton, depending on the type of quark a the function $F_{a/A}(x,Q^2)$ is related to the quark distributions (1.1), (1.2) as follows:

$$x^{-1}F_{a/A}(x,Q^2)=\begin{cases}2v_u(x,Q^2)+t(x,Q^2), & a=u\ ,\\ v_d(x,Q^2)+t(x,Q^2), & a=d\ ,\\ t(x,Q^2), & a=\bar{u},\bar{d},\bar{s},s\ .\end{cases} \tag{2.2}$$

On the basis of the requirements (i)–(vi) (Sec. I), the valence distributions $v_i(x,Q^2)$ were determined in I as follows:

$$v_i(x,Q^2)\ =\beta_i(Q^2)\left(\frac{Q^2}{Q_0^{\,2}}\right)^{-bx}x^{-1/2}q_i(x)\quad (i=u,d)\ , \tag{2.3}$$

with the constants $Q_0^{\,2}=1.5\ \text{GeV}^2$ and $b=1.2$, and with

$$q_u(x) = 0.895(1-x)^3(1+2.3x)\,,$$
$$q_d(x) = 1.107(1-x)^{3.1}\,; \tag{2.4}$$

$\beta_i(Q^2)$ is given by

$$\beta_i^{-1}(Q^2) = \int_0^1 \left(\frac{Q^2}{Q_0^2}\right)^{-bx} x^{-1/2} q_i(x)dx\,, \tag{2.5}$$

so that $v_i(x, Q^2)$ satisfy the conditions (1.3). Also, the sea distribution was determined as follows[26a]:

$$t(x,Q^2) = \left(0.2 + \tfrac{5}{56} b \ln\frac{Q^2}{Q_0^2}\right)\left(\frac{Q^2}{Q_0^2}\right)^{-bx} \times x^{-1}(1-x)^{11/2}\,. \tag{2.6}$$

The invariant inclusive cross section for $A+B \to C+X$ with the hadron C produced with transverse momentum p_T at an angle θ in the center of momentum (c.m.) of the hadrons A and B, via the quark-quark subprocess $a+b\to c+d$, is given by

$$E\frac{d\sigma}{d^3p}(p_T,\theta,s) = \frac{4}{\pi x_T^2}\sum_{a,b,c}\int\int dx_a dx_b F_{a/A}(x_a,Q^2)F_{b/B}(x_b,Q^2)\frac{d\sigma}{d\hat{t}}G_{C/c}(y,Q^2)\frac{\eta}{(1+\eta)^2} + (\theta \leftrightarrow \pi-\theta)\,. \tag{2.7}$$

Here $G_{C/c}(y, Q^2)$ is the quark fragmentation function, to be specified below and

$$\eta = \frac{x_a}{x_b}\tan^2(\tfrac{1}{2}\theta),\quad y = \frac{x_T(1+\eta)}{2x_a\tan\frac{1}{2}\theta} \tag{2.8}$$

$$Q^2 = -\hat{t}$$
$$= x_a s\tan\tfrac{1}{2}\theta\left(\frac{\tan\frac{1}{2}\theta}{x_b} + \frac{\cot\frac{1}{2}\theta}{x_a}\right)^{-1}, \tag{2.9}$$

where $x_T = 2p_T/\sqrt{s}$. The limits of the integration in (2.7) are determined from

$$0 \le x_a, x_b, y \le 1\,. \tag{2.10}$$

For scattering of spin-$\frac{1}{2}$ quarks via exchange of a massless vector gluon the differential cross section for $a+b\to c+d$ is

$$\frac{d\sigma}{d\hat{t}} = \frac{\pi\alpha_{\rm eff}^2}{\hat{s}^2}\Sigma(\eta)\,, \tag{2.11}$$

$$\Sigma(\eta) = 2\left[\frac{1}{\eta^2} + \left(1+\frac{1}{\eta}\right)^2\right], \tag{2.12}$$

where $\alpha_{\rm eff}$ is the fine-structure constant for the interaction between quarks and gluons.

We now turn to the quark fragmentation functions $G_{C/c}(y, Q^2)$. These are related to the differential probability dP that a quark c is seen by a probe of four-momentum Q to produce a hadron C carrying a fraction y of the quark longitudinal momentum by the relation

$$dP = G_{C/c}(y,Q^2)dy/y\,. \tag{2.13}$$

In I a scale breaking similar to that of Eq. (2.3) [and of the hadron fragmentation functions $F_{a/A}(x, Q^2)$] was assumed for $G_{C/c}(y, Q^2)$, namely the form

$$G_{C/c}(y,Q^2) = A(C,c)\beta_{C/c}(Q^2)\left(\frac{Q^2}{Q_0^2}\right)^{-by} \times (1-y)^{m(C,c)}\,, \tag{2.14}$$

where b and Q_0 are the same constants as before and the functions $\beta_{C/c}(Q^2)$ are determined from

$$\beta_{C/c}^{-1}(Q^2) = \int_0^1\left(\frac{Q^2}{Q_0^2}\right)^{-by}(1-y)^{m(C,c)}dy \tag{2.15}$$

[similar to (2.5)]. The exponents $m(C,c)$ and the coefficients $A(C,c)$ were determined in I on the basis of the following considerations:

When c is a valence quark of hadron C, $m(C,c)$ is determined from well known counting rules.[27] In the subsequent calculations we are mainly interested in the case C is a pion, and in I we took

$$m(\pi^\pm, c) = 1\,. \tag{2.16}$$

Also, on account of charge and isospin symmetry we took the coefficients $A(\pi, c)$ as follows:

$$A(\pi^+,u) = A(\pi^+,\bar{d}) = A(\pi^-,\bar{u}) = A(\pi^-,d)\,. \tag{2.17}$$

When c is a nonvalence quark of C the contributions $G_{C/c}$ are, in general, relatively small. In I, for $C=\pi^\pm$ we have chosen forms consistent with the results of the analysis of deep-inelastic neutrino-nucleon data.[28] Thus denoting by c' the quarks $\bar{u}, d, s, \bar{s}$ we choose the following:

$$m(\pi^+,c') = m(\pi^-,\bar{c}') = \tfrac{3}{2} \tag{2.18}$$

and

$$A(\pi^+,c') = A(\pi^-,\bar{c}') = \tfrac{1}{2}A(\pi^+,v)\,. \tag{2.19}$$

Finally, for neutral pions we took in I (as usual)

$$G_{\pi^0/c}(y,Q^2) = \tfrac{1}{2}[G_{\pi^+/c}(y,Q^2) + G_{\pi^-/c}(y,Q^2)] \tag{2.20}$$

for every kind of quark c.

We note that the quark fragmentation functions $G_{C/c}(y, Q^2)$ satisfy the sum rule

$$\sum_{\text{all } C}\int_0^1 G_{C/c}(y,Q^2)dy = 1\,, \tag{2.21}$$

and in view of (2.14)

$$\sum_{\text{all } C} A(C,c) = 1\,. \tag{2.22}$$

This sum rule was used in I to fix the absolute magnitude of the coefficients $A(C,c)$.

III. TWO-HADRON INCLUSIVE CROSS SECTIONS.

We begin by considering the reaction $A+B\to h_1+h_2+X$ when the two final hadrons h_1, h_2 are produced with large transverse momenta p_{T1}, p_{T2} in opposite directions in the c.m. system of the initial hadrons A, B (Fig. 1). Assuming that again the reaction takes place via the same quark-quark subprocess $a+b\to c+d$ the invariant inclusive cross section is[25]

$$E_1E_2\frac{d^2\sigma^{(-)}}{d^3p_1d^3p_2}=\frac{4}{\pi s x_{T1}^2x_{T2}^2}\sum_{a,b,c,d}\int\frac{dx_a}{x_a}z^2F_{a/A}(x_a,Q^2)F_{b/B}(x_b,Q^2)\frac{d\sigma}{d\hat{t}}G_{h_1/c}\left(\frac{x_{T1}}{z},Q^2\right)G_{h_2/d}\left(\frac{x_{T2}}{z},Q^2\right)D(\phi_1-\phi_2+\pi)$$
$$+(\theta_1\leftrightarrow\pi-\theta_1,\ \theta_2\leftrightarrow\pi-\theta_2, A\leftrightarrow B)\,. \tag{3.1}$$

Here $d\sigma/d\hat{t}$ is given by (2.11), (2.12), and

$$x_b=x_a\tan\tfrac{1}{2}\theta_1\tan\tfrac{1}{2}\theta_2,\quad \eta=\tan\tfrac{1}{2}\theta_1\cot\tfrac{1}{2}\theta_2\,, \tag{3.2}$$

and

$$z=\frac{2x_a\tan\frac{1}{2}\theta_1\tan\frac{1}{2}\theta_2}{\tan\frac{1}{2}\theta_1+\tan\frac{1}{2}\theta_2}\,,\quad Q^2=\frac{x_a^2s\tan^2(\frac{1}{2}\theta_1)}{1+\eta}\,. \tag{3.3}$$

In (3.1) the integration region is determined from the conditions

$$0\le x_a, x_b, \frac{x_{T1}}{z}, \frac{x_{T2}}{z}\le 1\,. \tag{3.4}$$

The function $D(\phi_1-\phi_2+\pi)$ peaks at $\phi_2=\phi_1+\pi$ and will be specified in detail in Sec. IV.

We turn next to the case $A+B\to h_1+h_2+X$ with the hadrons h_1, h_2 produced with large transverse momenta p_{T1}, p_{T2} on the same side ($\theta_1\approx\theta_2$, $\phi_1\approx\phi_2$). Now we introduce the differential probability d^2P for a quark c to produce a hadron h_1 of momentum fraction y_1 and a hadron h_2 of fraction y_2,

$$d^2P=G_{h_1h_2/c}(y_1,y_2,Q^2)dy_1dy_2\,. \tag{3.5}$$

Then, in terms of the fragmentation function $G_{h_1h_2/c}$ in two hadrons, the invariant inclusive cross section is[25]

$$E_1E_2\frac{d^2\sigma^{(+)}}{d^3p_1d^3p_2}=\frac{8}{\pi s x_{T1}x_{T2}}\sum_{a,b,c,d}\int\int\frac{dx_a}{x_a}\frac{dx_b}{x_b}F_{a/A}(x_a,Q^2)F_{b/B}(x_b,Q^2)\frac{d\sigma}{d\hat{t}}G_{h_1h_2/c}(y_1,y_2,Q^2)$$
$$\times\Delta(\theta_1-\theta_2)D(\phi_1-\phi_2)\,. \tag{3.6}$$

For same-side hadrons h_1, h_2 we are interested in the case $\theta_1\approx\theta_2\approx 90°$ (and $\phi_1\approx\phi_2$) in the c.m. of the colliding hadrons. Then

$$y_j=\tfrac{1}{2}x_{Tj}(x_a^{-1}+x_b^{-1}),\quad j=1,2\,. \tag{3.7}$$

In (3.6) the integration region is determined from

$$0\le x_a, x_b, y_1+y_2\le 1\,. \tag{3.8}$$

The functions $\Delta(\theta_1-\theta_2)$, $D(\phi_1-\phi_2)$ peak at $\theta_1=\theta_2$, $\phi_1=\phi_2$, respectively, and will be specified in Sec. IV.

The fragmentation function $G_{h_1h_2/c}(y_1,y_2,Q^2)$ in two hadrons is related to the fragmentation function $G_{h/c}(y,Q^2)$ in one hadron via the sum rule[19,25]

$$\sum_{h_2}\int_0^{1-y_2}dy_2\,y_2G_{h_1h_2/c}(y_1,y_2,Q^2)=\frac{1-y_1}{y_1}G_{h_1/c}(y_1,Q^2)\,. \tag{3.9}$$

Apart from this restriction, $G_{h_1h_2/c}$ is unknown, in general. To proceed in our calculation of the correlation function for two same-side hadrons (Sec. IV) we have used the form

$$G_{h_1h_2/c}(y_1,y_2,Q^2)=G_{h_1/c}(y_1,Q^2)\times G_{h_2/c'}\left(\frac{y_2}{1-y_1},Q^2\right)\frac{1}{y_1y_2}\,, \tag{3.10}$$

where c' is a quark of type $\bar{h}_1c$. The details of our calculation, the satisfaction of the sum rule (3.9), and the physical picture corresponding to the form (3.10) are discussed in the Appendix.

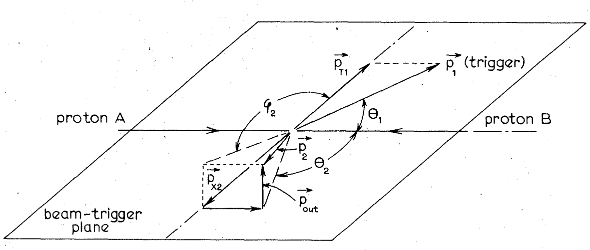

FIG. 1. Kinematics of the large-p_T event.

IV. MOMENTUM AND ENERGY DEPENDENCE OF CORRELATION FUNCTIONS

There is much experimental information on inclusive production $p+p\to\pi^0+h^\pm+X$ (Refs. 4 and 2), where $h^\pm$ is a charged hadron and the two final particles $\pi^0, h^\pm$ are produced at 90° in the c.m. of the colliding protons, either in opposite sides (Fig. 2) or in the same side (Fig. 3). The data are presented in terms of the correlation functions

$$F_\mp(p_{T2}, s) = \frac{\int_{\Delta\Omega} d\eta_1 d\phi_1 \int_{p_0}^\infty dp_{T1} p_{T1} p_{T2}(E_1E_2 d^2\sigma^{(\mp)}/d^3p_1 d^3p_2)}{\int_{\Delta\Omega} d\eta_1 d\phi_1 \int_{p_0}^\infty dp_{T1} p_{T1}(E_1 d\sigma/d^3p_1)}, \tag{4.1}$$

corresponding to π^0-$h^\pm$ in opposite sides (F_-) or in the same side (F_+). Here ϕ_1, p_{T1} are the azimuthal angle and transverse momentum of π^0, and η_1 its (pseudo)rapidity, defined by

$$\eta_1 = -\ln\tan\tfrac{1}{2}\theta_1 , \tag{4.2}$$

and p_{T2} is the transverse momentum of the charged hadron $h^\pm$. In the data the lower limit of the p_{T1} integration is $p_0 = 3$ GeV and the acceptance $\Delta\Omega$ corresponds to $\Delta\theta = \pm 20°$ and $\Delta\phi = \pm 7°$.[4]

To compare the predictions of our model with these data we first specify the function $D(\phi_1 - \phi_2 + \pi)$ in Eq. (3.1) and the functions $\Delta(\theta_1 - \theta_2)$, $D(\phi_1 - \phi_2)$ in Eq. (3.6). For this we use the experimental data regarding the distribution of large-p_T hadrons perpendicular to the beam-trigger plane.[2,29] All data suggest that $D(\phi)$ decreases rapidly with ϕ, and we choose the following form (symmetric in p_{T1} and p_{T2}):

$$D(\phi) = \frac{p_{T1}p_{T2}}{\pi d(p_{T1}^2 + p_{T2}^2)^{1/2}} \times \exp\left(-\frac{p_{T1}^2 p_{T2}^2}{p_{T1}^2 + p_{T2}^2}\frac{\phi^2}{\pi d^2}\right). \tag{4.3}$$

We have carried calculations with $d = 0.3$ GeV (dash-dotted lines in Figs. 2 and 3) corresponding to an average hadron's momentum perpendicular to the beam-trigger plane $= 0.3$ GeV. However, more recent information and related considerations[30] suggest that the average momentum is, probably, somewhat larger. Thus we have also carried calculations with $d = 0.6$ GeV (solid lines in Figs. 2 and 3). Finally, in (3.6) we have taken $\Delta(\theta_1 - \theta_2) = D(\theta_1 - \theta_2)$.

To fix the absolute magnitude of the correlation functions $F_\mp$ we need the absolute magnitude of the coefficients $A(C, c)$ of the fragmentation functions $G_{C/c}(y, Q^2)$. In view of the relations (2.17), (2.19), and (2.20) these coefficients are determined once the magnitude of one of them, e.g., $A(\pi^+, u)$, is fixed. To fix $A(\pi^+, u)$ we have used in I the sum rule (2.22) together with (2.17)–(2.20) and similar relations for C = kaon or baryon. However, as we discussed in detail in I, the experimental data on $p + p \to p + X$ require for the ratio $A(p,u)/A(\pi^+,u)$ a value as large as ~10. Such a large value is also found to be necessary in other quark-parton models[31,32]; it certainly contradicts SPEAR data, where mesons dominate the quark-jet fragmentation, and so far it has not been understood within the class of the present quark model. Now if we use $A(p,u)/A(\pi^+,u) \sim 10$ the sum rule (2.22) for $c = u$ is dominated by the contribution of $A(p,u)$, and this much affects the determination of the other coefficients $A(\pi^+,u)$, $A(\pi^0,u)$, etc. In particular, with $A(p,u) = 10A(\pi^+,u)$ the sum rule (2.22) gives $A(\pi^+,u) \sim \frac{1}{10}$, whereas if we neglect the contribution of baryons $[A(p,u) = 0]$ we obtain

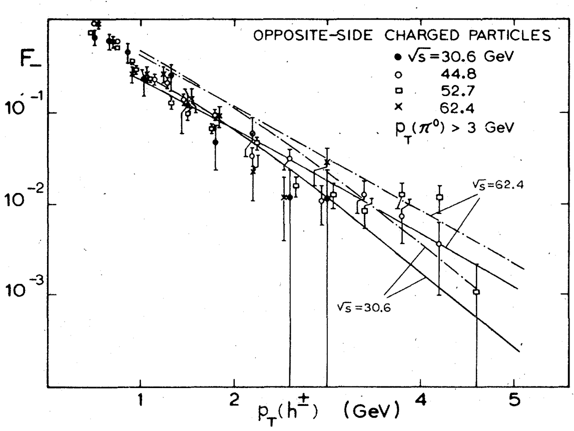

FIG. 2. The correlation function F_- for $p + p \to \pi^0 + h^\pm + X$ with the π^0 and the charged hadron $h^\pm$ in opposite sides, at 90° in the c.m. of the colliding protons. Dash-dotted lines: predictions of the model with $d = 0.3$ GeV [Eq. (4.3)]; solid lines: predictions with $d = 0.6$. Data are from Ref. 4.

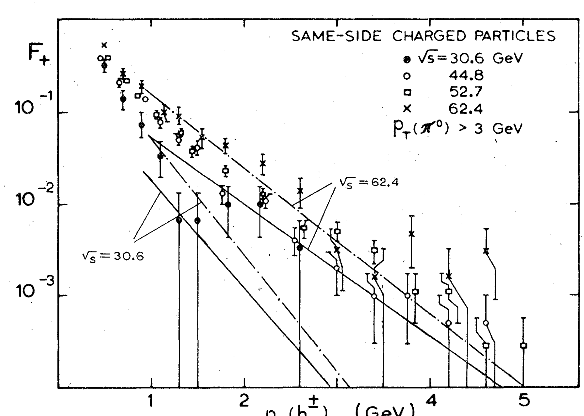

FIG. 3. The correlation function F_+ for $p + p \to \pi^0 = h^\pm + X$ with π^0, $h^\pm$ in the same side, at 90° in the c.m. of the colliding protons. Notation as in Fig. 2. Data are from Ref. 4.

$A(\pi^+,u)=\frac{1}{3}$.

Because of this problem, in all the subsequent calculations (including Secs. V and VI) we have proceeded as follows: First we have fixed $A(\pi^+,u)=\frac{1}{4}$. Notice that the fits to the single-hadron inclusive cross sections (given in detail in I) for $A(\pi^+,u)=\frac{1}{4}$ correspond to $\alpha_{\rm eff}=1.15$ effective quark-gluon coupling and such a value is, probably, somewhat higher than $\alpha_{\rm eff}$ inferred from deep-inelastic lepton-nucleon scattering, but of the same order of magnitude. Next, whenever the charged hadron $h^\pm$ is not specified by experiment, we calculated the inclusive cross sections (3.1) and (3.6) for $h^\pm=\pi^\pm$ and multiplied the final result by a factor 1.66. Such a factor is suggested by rough considerations.[2,19,25]

The results of the calculations for $\sqrt{s}=30.6$ and 62.4 GeV are given in Figs. 2 and 3. Within the above procedure we see that the model predicts roughly the correct magnitude of F_-, but F_+ somewhat lower than the data. The p_T dependence is also correctly predicted for F_-, at lower $\sqrt{s}$ is somewhat too strong for F_+.

It is of interest that with increasing $\sqrt{s}$ the model predicts a strong energy dependence for F_+ (in particular at larger p_T), but very weak energy dependence for F_-. This feature can be (very roughly) understood as follows: At some fixed x_a and x_b, in view of the form (3.10), the integrand of (3.6) contains a factor $G_{h_2/c'}(y_2/(1-y_1),Q^2)$. As $\sqrt{s}$ increases, both y_2 and y_1 decrease and the argument $y_2/(1-y_1)$ decreases rapidly. In particular, it can be seen that most of the contribution to the integrand of (3.6) comes from relatively large y_1; then the argument $y_2/(1-y_1)$ is very sensitive to changes in $\sqrt{s}$ and produces a rapid increase of $G_{h_2/c'}(y_2/(1-y_1),Q^2)$. This results in a rapid increase of F_+. On the other hand, the integrand of (3.1) contains instead the factor $G_{h_2/d}(x_{T2}/z,Q^2)$, the argument of which increases less rapidly, and this results in weak variation of F_-.

The faster increase of F_+ with $\sqrt{s}$ seems to be supported by the data and has been emphasized in Refs. 2–4. Qualitatively similar results were also obtained in the scale-violating models of Ref. 25 (for both F_+ and F_-) and Ref. 26 (this studies only F_-); as mentioned in Sec. I, the scale violation in Refs. 24–26 is powerlike in Q^2 (instead of the logarithmic of the present work).

V. TRANSVERSE-MOMENTUM SHARING-VARIABLE (x_e) DISTRIBUTIONS

In this section (and the next) we consider the predictions of the model concerning certain characteristic distributions of the secondaries produced opposite-side a large-p_T trigger. In terms of Fig. 1 we define the transverse-momentum sharing variable as follows:

$$x_e=p_{x2}/p_{T1}\,. \tag{5.1}$$

The importance of the distributions with respect to this variable has already been emphasized[29,30]; in particular, simple considerations based on exact scaling suggest that at sufficiently large p_{T1} the x_e distributions, properly normalized, should be independent of the trigger momentum p_{T1}.

The normalized x_e distributions are defined by

$$\frac{1}{N}\frac{dN}{dx_e}=\int_{\Delta p_{\rm out}}dp_{\rm out}\int_{\Delta\eta_2}d\eta_2\,\frac{d\sigma^{(-)}}{dx_e d\eta_2 dp_{\rm out}d^3p_1}\Big/\frac{d\sigma}{d^3p_1}\,, \tag{5.2}$$

where η_2 is the (pseudo)rapidity of the secondary and $p_{\rm out}$ is the component of the secondary's momentum $\vec{p}_2$ perpendicular to the beam-trigger plane (Fig. 1). The ranges $\Delta\eta_2$ and $\Delta p_{\rm out}$ are specified in the experiment. It can be shown that

$$d\eta\, dp_x dp_{\rm out}=d^3p/E\,, \tag{5.3}$$

so that

$$\frac{1}{N}\frac{dN}{dx_e}=p_{T1}\int_{\Delta p_{\rm out}}dp_{\rm out}\int_{\Delta\eta_2}d\eta_2\,E_1E_2 \times\frac{d^2\sigma^{(-)}}{d^3p_1d^3p_2}\Big/E_1\frac{d\sigma}{d^3p_1}\,. \tag{5.4}$$

We have calculated this distribution for several

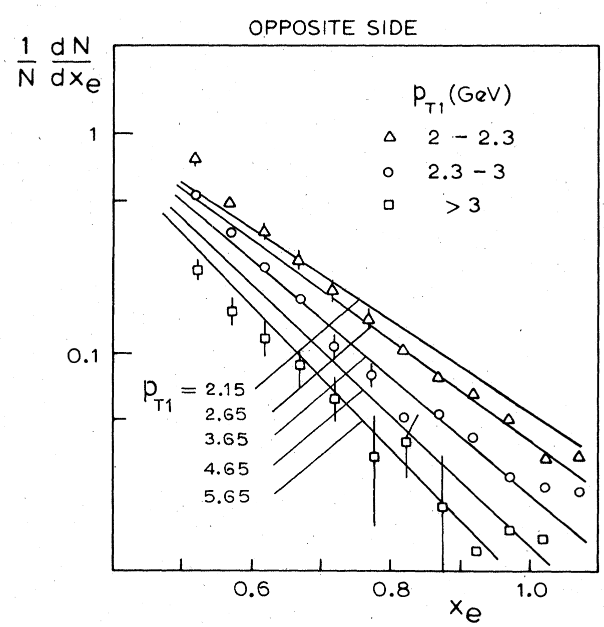

FIG. 4. Transverse-momentum sharing-variable (x_e) distributions for a pion trigger at 45° and $\sqrt{s}=52.5$ GeV. Data are from Ref. 38 corresponding to $|p_{\rm out}|\leq 0.6$ GeV.

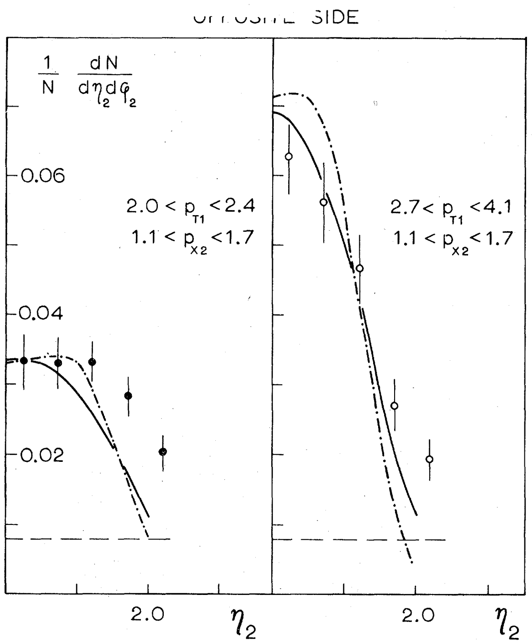

FIG. 5. Rapidity distributions at $\sqrt{3}=53$ GeV of charged secondaries opposite side a π^0 trigger at $\theta_1=90°$. Dash-dotted lines: predictions of the model without θ_2-smearing [Eq. (6.3)]; solid lines: predictions with θ_2-smearing [Eq. (6.4)]; dashed lines: mean charged particle densities for minimum bias triggers. Data from Ref. 28.

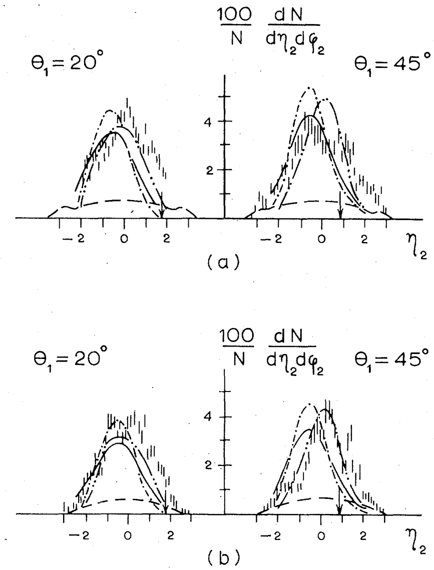

FIG. 6. Rapidity distributions at $\sqrt{s}\simeq 53$ GeV of charged secondaries with $p_{T2}>1$ GeV opposite-side a charged trigger with $\theta_1=20°$ and 45° and $\langle p_{T1}\rangle = 2.5$ GeV. Dash-dotted lines: predictions of the model (vector gluon exchange) without θ_2-smearing; solid lines: predictions with θ_2-smearing; dash-dot-dotted lines: predictions corresponding to vector + scalar gluon exchange [Eq. (6.10)] and without θ_2-smearing; dashed lines: distributions for minimum bias triggers. Data from Ref. 5. (a) positive trigger, positive secondaries (b) positive trigger, negative secondaries.

p_{T1} of a trigger at $\theta_1=45°$ and for $|\Delta\eta_2|=2$ and $|\Delta p_{\rm out}|=0.6$ GeV; we compare with relevant data in Fig. 4. Note that insofar as $|\Delta\eta_2|\geq 2$ our results are insensitive to the exact value, for our η_2 distributions are very small when $|\eta_2|>2$ (see Figs. 5 and 6). The x_e distributions of Fig. 4 have been calculated with $d=0.6$ GeV in Eq. (4.3) and with $A(\pi^+,u)=\frac{1}{4}$ as in Sec. IV.

We see that with this normalization the predictions are somewhat higher than the data, but on the whole are of the correct order of magnitude. Also, the x_e dependence is, probably, somewhat weaker than the data.

It is remarkable that at fixed x_e the model predicts $N^{-1}dN/dx_e$ decreasing with the p_{T1} of the trigger, in accordance with the data. Such a feature can be traced to the scale-breaking character of the model and has also been shown in other models incorporating scale violations.[25] However, the predicted p_{T1} dependence is somewhat weaker than the data, at least in the range $2.15\lesssim p_{T1}\lesssim 3.65$ GeV (Ref. 33). In Fig. 4 we have also included predictions at higher p_{T1}.

It is known that simple quark-parton models neglecting the effects of scale violations predict $N^{-1}dN/dx_e$ completely independent of the trigger p_{T1}.[30,31]

VI. RAPIDITY DISTRIBUTIONS

Another important quantity that strongly reflects the detailed dynamics of the quark-quark scattering subprocess is the distribution in rapidity of the secondaries produced opposite a large-p_T trigger. It is known that the related predictions of various quark models vary considerably.[21,22,25,26]

Let N be the number of events with a trigger at a fixed transverse momentum p_{T1} and rapidity η_1. With η_2 and ϕ_2 the rapidity and azimuthal angle of an opposite-side secondary, the rapidity distribution to be considered is

$$\frac{1}{N}\frac{dN}{d\eta_2 d\phi_2} = \frac{1}{\Delta\phi_2}\int_{\Delta p_{T2}} dp_{T2}\int_{\Delta\phi_2} d\phi_2 \frac{d\sigma^{(-)}}{dp_{T2}d\phi_2 d\eta_2 d^3p_1} \Big/ \frac{d\sigma}{d^3p_1}, \tag{6.1}$$

the ranges Δp_{T2} and $\Delta\phi_2$ are specified by experiment. It is

$$\frac{d^3p}{E} = p_T dp_T d\eta d\phi, \tag{6.2}$$

so that

$$\frac{1}{N}\frac{dN}{d\eta_2 d\phi_2}=\frac{1}{\Delta\phi_2}\int_{\Delta p_{T2}} dp_{T2}\int_{\Delta\phi_2} d\phi_2 p_{T2}E_1E_2\frac{d\sigma^{(-)}}{d^3p_1 d^3p_2}\Big/ E_1\frac{d\sigma}{d^3p_1}\,. \tag{6.3}$$

There are two particularly interesting aspects concerning these rapidity distributions and reflecting into the dynamics of the quark-quark subprocess[21-26]:

(a) The existence of a dip at $\eta_2=0$, in particular when the trigger is at $\theta_1=90°$. The data of the ACHM collaboration[29] show no such dip; certain data, however, could be used as supporting a dip (or a double-hump structure).[5]

(b) The symmetry of $dN/d\eta_2 d\phi_2$ with respect to $\eta_2\simeq 0$ even when the trigger is at $\eta_1\neq 0$ (away from 90°). The data of the CCHK collaboration do support, on the whole, such a symmetry. However, data of the ACHM collaboration[34] peak at $\eta_2\approx-\eta_1$ (indicating a back-to-back effect).

Before we present our predictions we remark that in the expression (3.1) for the inclusive cross section of two opposite-side hadrons, no attempt was made to account for the fact that the momentum of the final hadron can have a component transverse to the momentum of the producing quark not only perpendicular to the beam-trigger plane but also parallel to this plane (θ- or η-smearing). Accounting for such an effect perpendicular to this plane [via the distribution $D(\phi_1-\phi_2+\pi)$] was necessary in the calculations of Secs. IV and V because of the coplanarity of the quark-quark subprocess. In many cases θ-smearing effects are not particularly important; however, rapidity distributions are an exception. Such effects can be taken into account by using

$$\frac{1}{N}\frac{dN}{d\eta_2 d\phi_2}=\int d\eta'\,\mathfrak{D}(\eta'-\eta_2)\frac{1}{N}\frac{dN}{d\eta' d\phi_2}\bigg|_{\text{unsm}}, \tag{6.4}$$

where

$$\frac{1}{N}\frac{dN}{d\eta' d\phi_2}\bigg|_{\text{unsm}}$$

is given by (6.3).

Concerning the function $\mathfrak{D}(\eta)$ we have considered two forms:

(i) A Gaussian form, on the basis of the form (4.3) for $D(\phi)$, namely

$$\mathfrak{D}(\eta)=\frac{p_{T2}}{\pi d}\exp\left(\frac{-p_{T2}{}^2\eta^2}{\pi d^2}\right); \tag{6.5}$$

(ii) An exponential form, suggested from the form of the inclusive pion distributions at low p_T (< 1 GeV), namely

$$\mathfrak{D}(\eta)=\frac{ap_{T2}}{2}\exp(-ap_{T2}|\eta|)\,, \tag{6.6}$$

with $a=2\ \text{GeV}^{-1}$. In both cases we smear only with respect to the secondary hadron because the unsmeared distributions (6.3) depend very little on the trigger rapidity. The resulting effect is very similar for forms (i) and (ii), and we subsequently present calculations only for the former with $d=0.6$ GeV.

We compare first our predictions with data of the ACHM collaboration[29] for a π^0 trigger at 90° (Fig. 5). Calculations and data correspond to $|180°-\phi_2|<35°$. First, with the normalization specified in Sec. IV $[A(\pi^+,u)=\frac{1}{4}]$ we see that the model predicts correctly the overall magnitude; however, the predicted distributions are somewhat narrower than the data. Now, regarding the shape around $\eta_2\approx 0$, the unsmeared distributions (dash-dotted lines) show a tiny dipping at $\eta_2=0$. However, when smearing is introduced (solid line) the dipping completely disappears. A stronger dip at $\eta_2=0$ (for essentially unsmeared distributions) has been observed in other quark models[21,24,26] and typically arises when the c.m. of the quark-quark subprocess differs significantly from the c.m. of the colliding hadrons. On the other hand, smearing due to distributions transverse to the jet axis (θ-smearing) is expected to fill in and eliminate the dip.[30]

Next we compare with data of the CCHK collaboration[5] for a trigger at $\theta_1=20°$ ($\eta_1=1.74$) and $\theta_1=45°$ ($\eta_1=0.88$) (Fig. 6). Again, the overall magnitude is reasonably well predicted. Now, both without smearing (dash-dotted lines) and with smearing (solid lines) the model predicts a peak at $\eta_2\approx-0.5$; clearly, at least when the trigger is at $\theta_1=20°$ the predicted peak is at $\eta_2>-\eta_1$, i.e., somewhere between the back-to-back and the symmetric situation. As stated, on the whole the CCHK data can be said to favor symmetry with respect to $\eta_2=0$; however, certain ACHM data[34] show a back-to-back effect.

The prediction of back-to-back or back-antiback effects in hard-scattering models is known to be related to the "peripherality" of the cross section $d\sigma/d\hat{t}$ of the quark-quark subprocess, i.e., on its dependence on the ratio $\hat{s}/\hat{t}$.[22] From the kinematic relations (2.8), (2.9) and with the subprocess c.m. energy $\sqrt{\hat{s}}=(x_a x_b s)^{1/2}$ it follows easily that

$$-\hat{s}/\hat{t}=1+1/\eta\,. \tag{6.7}$$

Therefore, for vector-gluon exchange between the scattered quarks, which so far has been assumed throughout this work, the function Σ of Eq. (2.12) becomes

$$\Sigma\left(\frac{\hat{s}}{\hat{t}}\right)=2\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\,. \tag{6.8}$$

For small $\hat{t}/\hat{s}$, $\Sigma \sim (\hat{s}/t)^2$, and this is a rather peripheral form that tends to produce back-to-back effects.

A less peripheral form is obtained if, instead of vector, we assume a scalar gluon exchange.[35] Then (2.11) holds again with[36]

$$\Sigma = 2 . \tag{6.9}$$

Such a form produces back-antiback effects (at moderate p_{T1}).

We have considered the combined effect of vector plus scalar exchange, i.e., a combination of (6.8) and (6.9). The combination was adjusted so that for the trigger at $\theta_1 = 20°$ and at $\eta_2 = 0$,

$$\frac{dN}{d\eta_2 d\phi_2}\,(\text{scalar}) = \frac{1}{3}\,\frac{dN}{d\eta_2 d\phi_2}\,(\text{vector}) . \tag{6.10}$$

The results are shown in Fig. 6 with dash-dot-dotted lines (no smearing). Now the rapidity distributions peak at $\eta_2 \approx 0$.

VII. CONCLUSIONS

In Ref. 10 (referred to as I) a scale-violating quark model for large processes was proposed. The essential idea was to examine whether the basic BBK mechanism, namely quark-quark scattering via single-gluon exchange, supplemented by scale breaking, as suggested by lepton-nucleon deep-inelastic data and AFFT and CFT, can account for the dominant features of large-p_T hadron production. In view of the recent interest in fundamental quark-gluon interactions, such an approach, if successful, might be a step towards a unified description of phenomena at short distances. In I single-hadron distributions were studied, and in the present work two-hadron distributions and correlations were studied.

We have encountered certain difficulties, the most important of which we would like to summarize as follows:

(A) The magnitude and energy dependence of $pp \to pX$. This has been much discussed in I and mentioned in Sec. IV of this paper; also, the magnitude of $pp \to \bar{p}X$.

(B) The magnitude of α_{eff}. As discussed in Sec. IV and in I we need a quark-gluon coupling $\alpha_{\text{eff}} \simeq 1$. Certain estimates based on AFFT, in particular the approach to scaling of the total cross section $\sigma(e^+e^- \to \text{hadrons})$, do not contradict such a value; however, other applications lead to smaller α_{eff}, perhaps $\alpha_{\text{eff}} \simeq 0.3-0.6$ (see, though, below).

(C) Perhaps the magnitude of the dependence on the trigger momentum p_{T1} of the x_e distribution of opposite-side secondaries (Sec. V).

(D) For single-vector-gluon exchange the exact form of the rapidity distribution (peaking at $\eta_2 \approx 0$) of opposite-side secondaries (weak back-to-back).

An effect receiving much recent attention is the transverse motion of the quarks with respect to the initial hadrons and of the final hadrons with respect to the fragmenting quarks.[37-40] Properly speaking, the former effect has not been introduced in the present work. Our fragmentation functions $F_{a/A}$, $F_{b/B}$ have been taken independent of the quark transverse momentum. Introduction of such an effect is easily seen to increase the predicted single-hadron inclusive cross sections, and detailed estimates give a factor of ~2 at ISR energies and higher factors at lower energies.[32,39,40] Finally, addition of quark-gluon and gluon-gluon scattering[40] to the quark-quark subprocess further increases the predicted single-particle cross section and reduces α_{eff} to the usually accepted value

$$\alpha_{\text{eff}} = 12\pi/[25 \ln(Q^2/\Lambda^2)] , \quad \Lambda = 0.3-0.5 \text{ GeV}.$$

Thus, on the whole, it can be said that the scale-violating approach describes large-p_T phenomena reasonably well. To us this seems to be particularly true in comparison with other quark-quark scattering models which completely neglect scale breaking and use arbitrary forms for the subprocess cross section $d\sigma/d\hat{t}$.[31,32,21] With respect to difficulty (A), those models fail even worse because, in addition, they predict the wrong p_T dependence of $p + p \to p + X$; the scale-breaking approach predicts, at least, correct p_T dependence.[10,26] With respect to (B), since in those models $d\sigma/d\hat{t}$ is arbitrary, there is no prediction of the magnitude of the inclusive distributions. With respect to (C), they predict complete p_{T1} independence. With respect to (D), the form of $d\sigma/d\hat{t}$ of Ref. 32 gives back-to-back effects at least as strong as of our basic model[32] (solid lines of Fig. 6).

Anyway, some of the issues, in particular (C) and (D), should require better experimental data before they are definitely settled. We may anticipate this in the near future.

Note added in proof. A recent work[40] studies large-p_T hadron production with quark and gluon distributions satisfying exact AFFT (quantum chromodynamics) requirements and partons carrying transverse momentum. With $\alpha_{\text{eff}} = 12\pi/[25 \ln(Q^2/\Lambda^2)]$ the predicted cross sections agree with the data reasonably well, in particular at ISR energies. Momentum correlations and opposite-side rapidity distributions are similar to those of Secs. IV and VI of the present work; and the x_e distributions are reasonable, if spectator partons are taken into account.

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APPENDIX

We consider the fragmentation function $G_{h_1h_2/c}(y_1, y_2, Q^2)$ for a quark producing two hadrons h_1, h_2 (Sec. III). First, it is easy to show that the form

$$G_{h_1h_2/c}(y_1, y_2, Q^2) = G_{h_1/c}(y_1, Q^2) \times G_{h_2/c'}\left(\frac{y_2}{1-y_1}, Q^2\right) \qquad \text{(A1)}$$

satisfies the sum rule (3.9)

$$\sum_{h_2} \int_0^{1-y_1} dy_2\, y_2 G_{h_1h_2/c}(y_1, y_2, Q^2) = \frac{1-y_1}{y_1} G_{h_1/c}(y_1, Q^2)\,. \qquad \text{(A2)}$$

Indeed (neglecting Q^2 in all the arguments)

$$\sum_{h_2} \int_0^{1-y_1} dy_2\, y_2 G_{h_1h_2/c}(y_1, y_2) = G_{h_1/c}(y_1)\frac{1}{y_1} \sum_{h_2} \int_0^{1-y_1} dy_2\, G_{h_2/c'}\left(\frac{y_2}{1-y_1}\right), \qquad \text{(A3)}$$

with a change of variable $y_2/(1-y_1) = y$ and taking into account the sum rule (2.21)

$$\sum_{h_2} \int_0^{1-y_1} dy_2\, G_{h_2/c'}\left(\frac{y_2}{1-y_1}\right) = (1-y_1)\sum_{h_2} \int_0^1 dy\, G_{h_2/c'} = 1-y_1\,. \qquad \text{(A4)}$$

To specify the quark c' in (A1) and to proceed in our calculations we assume the following physical picture (Fig. 7): The quark c produces first one of the hadrons, say h_1, then an intermediate state containing one or more quarks appears and finally the second hadron h_2 is produced together with one or more quarks. For simplicity we are restricted to h_1, h_2 = mesons, which is the case of our calculations. In general, there are two possibilities:

(a) The quark c is a valence quark of h_1 [Fig. 7(a1) and (a2)]; then c' is uniquely specified ($c' = ch_1$).

(b) The c is a nonvalence quark of h_1 [Fig. 7(b)]. Then, in general, the second hadron h_2 can be produced either by c or by one of the quarks of h_1 [denoted by $\bar{c}'$ and c'' in Fig. 7(b)].

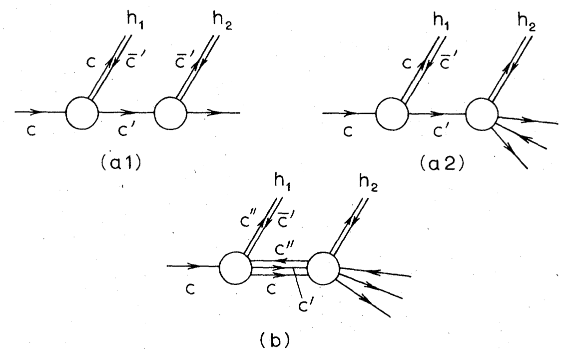

FIG. 7. Quark processes involved in the specification of the fragmentation function $G_{h_1h_2/c}$.

Although there is no particular difficulty in treating both possibilities, (a) and (b), in our calculations we have ignored the latter. The justification is that (b) leads to a minimum of three residual quarks and considerably reduces the probability of producing a second high-momentum hadron. Anyway, inclusion of (b) only complicates our procedure without qualitatively affecting any of the results of Sec. IV.

Assuming (a) to be the dominant process there are clearly two possibilities, in general:

(a1) The quark c' is a valence quark of h_2 [Fig. 7(a1)]. We denote the corresponding quark fragmentation function by $G^{(1)}(y, Q^2)$.

(a2) The c' is not a valence quark of h_2 [Fig. 7(a2)], and the corresponding fragmentation function is $G^{(2)}(y, Q^2)$.

We have then

$$G_{h_2/c'}(y, Q^2) = \lambda(h_2, c')G^{(1)}(y, Q^2) + G^{(2)}(y, Q^2)\,. \qquad \text{(A5)}$$

For the cases of interest in our calculations ($h_2 = \pi^\pm, \pi^0$, see Sec. IV),

$$\lambda(\pi^+, u) = \lambda(\pi^+, \bar{d}) = \lambda(\pi^-, d) = \lambda(\pi^-, \bar{u}) = 1\,, \qquad \text{(A6)}$$

$$\lambda(\pi^0, u) = \lambda(\pi^0, d) = \lambda(\pi^0, \bar{u}) = \lambda(\pi^0, \bar{d}) = \tfrac{1}{2}\,, \qquad \text{(A7)}$$

and all other $\lambda(h_2, c')$ are zero [e.g., $\lambda(\pi^+, d) = \lambda(\pi^+, s) = \cdots = \lambda(\pi^-, u) = \lambda(\pi^-, s) = \cdots = \lambda(\pi^0, s) = \lambda(\pi^0, \bar{s}) = 0$]. Moreover, it is easy to see that

$$G^{(2)}(y, Q^2) = G_{\pi^+/d}(y, Q^2)\,, \qquad \text{(A8)}$$

$$G^{(1)}(y, Q^2) = G_{\pi^+/u}(y, Q^2) - G_{\pi^+/d}(y, Q^2)\,, \qquad \text{(A9)}$$

where $G_{\pi^+/u}$ and $G_{\pi^+/d}$ are specified in Sec. II (and in Ref. 10).

Finally we symmetrize our expressions with respect to $h_1 \leftrightarrow h_2$. In this way $G_{h_1h_2/c}$ is completely specified for all the cases of our interest.

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