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Electroproduction of low hadronic masses

M. G. Olsson and E. T. Osypowski*
Physics Department, University of Wisconsin, Madison, Wisconsin 53706

Evelyn H. Monsay
Argonne National Laboratory, Argonne, Illinois 60439

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A model satisfying analytic, unitary, and soft-meson requirements is found to account for all electroproduction data in the Δ(3,3) region even at very high photon momentum transfers. The Δ, pion, and nucleon axial-vector form factors are discussed within the framework of this simple yet accurate model.

I. INTRODUCTION

Although much theoretical work has been done[1,2] on electroproduction in the Δ(3,3) region there are several reasons for continued effort.

(1) A number of recent experiments shed light on the pion, axial-vector nucleon, and Δ form factors. Since the extraction of these form factors is model dependent a dependable simple model is required.

(2) There is no simple accurate model available at present. Dispersive calculations have become quite complicated[1] and, furthermore, do not ensure soft-pion results. Isobar-type models[2] generally contain many parameters, are not unitary, and also are often inconsistent with soft-pion requirements.

The model which we propose combines features of several previous approaches. We use a Lagrangian framework which incorporates the soft-meson theorems and treats the Δ on the same footing as the nucleon. In those multipoles where the Δ pole appears we unitarize in a manner consistent with analyticity. The observables can thus be directly calculated and compared with experiment. In previous work we have shown[3,4] that this type of model accounts for all of the observed photoproduction multipoles in the Δ region. The extension to electroproduction involves the introduction of form-factor variation of the photon coupling constants. Except for these form factors the coupling-constant values are the same as obtained from photoproduction.

Because there is not yet a multipole analysis for electroproduction, the unitarization step is crucial. In previous "isobar" analyses[2] a width factor was inserted into the denominator of the Δ-pole term. This is an incorrect procedure since the total resonant multipole including the Born term will no longer have the phase $\delta_{3,3}$ required by unitarity. In our present model we impose unitarity without destroying the analytic behavior. Our unitarization method is local in the sense that the Lagrangian amplitude modified only in the vicinity of the resonance.

There are three independent couplings of the on-shell Δ to the nucleon and photon. The three gauge-invariant Lagrangians first proposed by Gourdin and Salin[5] correspond in a simple way to the three Pauli-type form factors of Mathews.[6] We will discuss a class of sum rules which can sort out these couplings from the data before a detailed comparison is made. In this paper we evaluate two of these coupling-constant sum rules using photoproduction data. The third coupling vanishes for real photons and we are not able to use our sum rules since the required data, separated into longitudinal and transverse parts at fixed photon momentum transfer, are not available at present. There is, however, a very stringent upper limit for this third coupling provided by the inclusive longitudinal cross section in the Δ region. The result of our analyses is that only the nonderivative $\Delta N\gamma$ coupling $C_3(k^2)$ is allowed. The $\Delta N\gamma$ form factor falls off more rapidly in photon four-momentum transfer $(k^2)^{1/2}$ than the nucleon magnetic factor. To good accuracy we find the $\Delta N\gamma$ form factor $C_3(k^2)$ varies as

$$C_3(k^2) = C_3(0)(1+k^2/M_v^2)^{-2}(1+k^2/4M_v^2)^{-1}, \qquad (1)$$

where $C_3(0) \simeq 0.315$ and $M_v \simeq 0.84$ GeV. This behavior is similar to the nucleon isovector form factor $F_2^V(k^2)$.

The nucleon axial-vector form factor $F_A(k^2)$ appears in the electroproduction amplitude due to partial conservation of axial-vector current (PCAC). By comparing our model with π^+ coincidence experimental data near threshold ($k^2 < 0.9$ GeV^2) we find that $F_A(k^2)$ is consistent with dipole behavior with a mass

$$M_A = 1.15 \pm 0.10 \text{ GeV}. \qquad (2)$$

The pion form factor is evaluated using forward π^+ data. Our result is consistent with a monopole falloff with masses of 0.69 or 0.77 GeV or with a dipole mass of 0.84 GeV.

We show that serious discrepancies with the data exist for some dispersive models and that these difficulties are largely resolved with our model. The dispersive models are found to badly disagree with the data at large k^2 for single-arm experiments and at large-pion angles both above and below the Δ mass in π^0 electroproduction.

In Sec. II we discuss the Lagrangian model. The construction and evaluation of Δ coupling sum rules is found in Sec. III. Section IV discusses the unitarization problem. We compare our model calculations with experimental data in Sec. V and summarize and conclude in Sec. VI. Section VI also contains a discussion of work by previous authors.

II. THE MODEL

Before describing the details of our model we shall discuss briefly the kinematics of electroproduction and our notation. We treat the pion electroproduction process (see Fig. 1)

$$e(k_1)+N(p_1)\to e'(k_2)+N(p_2)+\pi^a(q)$$

in the one-photon-exchange approximation which allows us to separate the scattering amplitude T^a into leptonic and hadronic parts,

$$T^a=\epsilon_\mu T^a_\mu ,$$

where

$$J^a_\mu = {}_{\text{out}}\langle p_2; q, a\,|j^{\text{em}}|p_1\rangle_{\text{in}}, \qquad \epsilon_\mu=\frac{e}{k^2}\,\bar u(k_2)\gamma_\mu u(k_1), \quad k=k_1-k_2. \tag{2.1}$$

We choose the following decomposition of the scattering amplitude in terms of Lorentz and isospin matrices:

$$T^a=\bar u(p_2)\sum_{i=1}^{6}\{V_i^{(0)}\tau^a+V_i^{(+)}\delta^{a3}+V_i^{(-)}\tfrac{1}{2}[\tau^a,\tau^3]\}u(p_1)O_i , \tag{2.2}$$

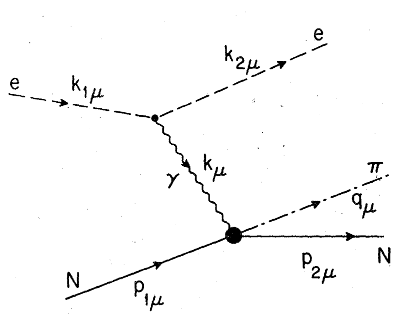

FIG. 1. Pion electroproduction.

where

$$\begin{aligned}
&O_1=\tfrac{1}{2}i\gamma_5\gamma_\mu\gamma_\nu f_{\mu\nu},\\
&O_2=2i\gamma_5P_\mu q_\nu f_{\mu\nu},\\
&O_3=\gamma_5\gamma_\mu q_\nu f_{\mu\nu},\\
&O_4=2\gamma_5\gamma_\mu P_\nu f_{\mu\nu}-2MO_1,\\
&O_5=i\gamma_5k_\mu q_\nu f_{\mu\nu},\\
&O_6=\gamma_5k_\mu\gamma_\nu f_{\mu\nu},\\
&f_{\mu\nu}=\epsilon_\mu k_\nu-\epsilon_\nu k_\mu,\\
&P=\tfrac{1}{2}(p_1+p_2)\\
&M=\text{nucleon mass.}
\end{aligned} \tag{2.3}$$

The invariant amplitudes are functions of k^2, the square of the photon momentum, and the usual Mandelstam variables

$$\begin{aligned}
&s=-(p_1+k)^2=-(q+p_2)^2,\\
&t=-(t-q)^2=-(p_1-p_2)^2,\\
&u=-(k-p_2)^2=-(q-p_1)^2,\\
&s+t+u=2M^2+\mu^2-k^2,\\
&\mu=\text{pion mass.}
\end{aligned} \tag{2.4}$$

Two other useful invariants are

$$\nu=-\frac{P\cdot k}{M}=\frac{s-u}{4M}, \quad \nu_B=\frac{q\cdot k}{2M}=\frac{t-\mu^2+k^2}{4M}. \tag{2.5}$$

The crossing symmetry properties of the invariant amplitudes are specified by

$$V_i^{(\pm0)}(s,t,u,k^2)=(\pm+)\eta_iV_i^{(\pm0)}(u,t,s,k^2), \tag{2.6}$$

where

$$\eta_1=\eta_2=\eta_4=+1 \;\text{ and }\; \eta_3=\eta_5=\eta_6=-1.$$

Our model for the electroproduction amplitude in the (3, 3) resonance region consists of Born approximation terms obtained from the axial-vector (A) theory of pion-nucleon coupling and contributions from $\Delta(3,3)$-exchange diagrams calculated using the most general Δ propagator. In addition, the resonant multipoles are projected out of the Born-Δ background and unitarized by means of a technique described in Sec. IV.

The motivation for inclusion of the axial-vector rather than the pseudoscalar Born terms is based on the observation of Dombey and Read[7] that axial-vector theory encompasses all of the low-energy theorems obtained from PCAC and current algebra, *provided* that a choice of form factors is made according to current-algebra prescriptions. Recently Adler[8] extended an older model[1] for weak pion production in the (3,3) resonance region so as to incorporate the low-energy constraints. The key modification in the vector part of the amplitude was the addition of "consistency-condition" terms to the pseudoscalar Born approximation which, in fact, leads to precisely the axial-vector Born approximation advocated by Dombey and Read.[7]

The axial-vector Born approximation is given by

$$T_A^a=\left(\frac{fe}{\mu}\right)\bar{u}(p_2)\Big\{\gamma_\mu\gamma_5F_A(k^2)\tfrac{1}{2}[\tau^a,\tau^3] + (\not{k}-\not{q})\gamma_5\tau^c\frac{(2q-k)_\mu}{\mu^2-t}F_\pi(k^2)i\epsilon^{a3c} - \not{q}\gamma_5\tau^a\frac{1}{M+i(\not{p}_1+\not{k})}\Gamma_\mu(k) - \Gamma_\mu(k)\frac{1}{M+i(\not{p}_2-\not{k})}\not{q}\gamma_5\tau^a\Big\}u(p_1)\epsilon_\mu. \quad (2.7)$$

In the above, f is the axial-vector πN coupling constant for which we use $f^2/4\pi=0.079\pm0.001$; $F_\pi(k^2)$ is the pion electromagnetic form factor with normalization $F_\pi(0)=1$; $\Gamma_\mu(k)$ is the γNN vertex

$$\Gamma_\mu(k)=i\left[\gamma_\mu\left(\frac{F_1^S(k^2)+F_1^V(k^2)\tau^3}{1}\right) - \frac{1}{M}\left(\frac{F_2^S(k^2)+F_2^V(k^2)\tau^3}{2}\right)\sigma_{\mu\nu}k_\nu\right] \quad (2.8)$$

expressed in terms of the nucleon isoscalar and isovector form factors with normalization $F_1^V(0)=F_1^S(0)=1$, $F_2^S(0)=\kappa^S\simeq-0.06$, $F_2^V(0)=\kappa^V\simeq1.85$, and finally $F_A(k^2)=G_A(k^2)/G_A(0)$, where $G_A(k^2)$ is the usual axial-vector nucleon form factor with normalization $G_A(0)=1.34$ given by the Goldberger-Treiman relation.

Expressed in terms of the invariant-amplitude V_i the axial-vector Born approximation is

$$\begin{aligned}
V_1^{(0,+)} &= \frac{ef}{2\mu}\left[-2MF_1^{S,V}(S+U)+\frac{2}{M}F_2^{S,V}\right],\\
V_2^{(0,+)} &= \frac{ef}{2\mu}\left[\frac{4MF_1^{S,V}}{2q\cdot k}(S+U)\right],\\
V_3^{(0,+)} &= \frac{ef}{2\mu}[2F_2^{S,V}(S-U)],\\
V_4^{(0,+)} &= \frac{ef}{2\mu}[2F_2^{S,V}(S+U)],\\
V_5^{(0,+)} &= V_6^{(0,+)}=0,\\
V_1^{(-)} &= \frac{ef}{2\mu}[-2MF_1^V(S-U)],\\
V_2^{(-)} &= \frac{ef}{2\mu}\left[\frac{4MF_1^V}{2q\cdot k}(S-U)\right],\\
V_3^{(-)} &= \frac{ef}{2\mu}[2F_2^V(S+U)],\\
V_4^{(-)} &= \frac{ef}{2\mu}[2F_2^V(S-U)],\\
V_5^{(-)} &= \frac{ef}{2\mu}\left[\frac{4M}{k^2}\left(\frac{2F_\pi}{t-\mu^2}-\frac{F_1^V}{q\cdot k}\right)\right],\\
V_6^{(-)} &= \frac{ef}{2\mu}\left[\frac{2}{k^2}(F_A-F_1^V)\right],\\
S &\equiv \frac{1}{M^2-s},\quad U\equiv\frac{1}{M^2-u}.
\end{aligned} \quad (2.9)$$

Our treatment of the Δ-exchange contributions generalizes that employed is previous analyses of photoproduction.[3] The most general Δ propagator is

$$P_{\mu\nu}(p)=\frac{1}{M_\Delta+i\gamma\cdot p}\left[\delta_{\mu\nu}-\tfrac{1}{3}\gamma_\mu\gamma_\nu+\frac{i}{3M_\Delta}(\gamma_\mu p_\nu-\gamma_\nu p_\mu)+\frac{2}{3M_\Delta^2}p_\mu p_\nu\right] + \frac{i}{6M_\Delta^2}\left[\frac{2(A+1)}{2A+1}(\gamma_\mu p_\nu+\gamma_\nu p_\mu)-\left(\frac{A+1}{2A+1}\right)^2\gamma_\mu(\gamma\cdot p)\gamma_\nu - iM_\Delta\frac{2A(A+1)}{(2A+1)^2}\gamma_\mu\gamma_\nu\right], \quad (2.10)$$

where M_Δ is the $\Delta(3,3)$ mass and A is a parameter reflecting an ambiguity in the choice of the off-mass-shell terms. The $\Delta N\pi$ interaction

$$\mathcal{L}_{\Delta N\pi}=g_\Delta\bar{\Delta}_\mu^a\Theta_{\mu\nu}N\partial_\nu\pi^a+\text{H.c.},\qquad \Theta_{\mu\nu}=\delta_{\mu\nu}+[\tfrac{1}{2}(1+4Z)A+Z]\gamma_\mu\gamma_\nu, \quad (2.11)$$

introduces another off-mass-shell parameter Z. However, the form of $\Theta_{\mu\nu}$ is chosen so that Δ contributions to the S matrix depend on Z but not A.

In order to choose the $\Delta N\gamma$ vertex we investigated the most general coupling introduced by Gourdin and Salin[5] for their original isobar model for photoproduction:

$$\begin{aligned}
\mathcal{L}_{\Delta N\gamma} &= \mathcal{L}_3+\mathcal{L}_4+\mathcal{L}_5,\\
\mathcal{L}_3 &= \frac{e}{\mu}C_3\bar{\Delta}_\nu^3\gamma_\mu\gamma_5NF_{\mu\nu}+\text{H.c.},\\
\mathcal{L}_4 &= \frac{e}{\mu^2}C_4\partial_\mu\bar{\Delta}_\nu^3\gamma_5NF_{\mu\nu}+\text{H.c.},\\
\mathcal{L}_5 &= -\frac{e}{\mu^2}C_5\bar{\Delta}_\nu^3\gamma_5\partial_\mu NF_{\mu\nu}+\text{H.c.},\\
F_{\mu\nu} &= \partial_\mu A_\nu-\partial_\nu A_\mu,
\end{aligned} \quad (2.12)$$

For electroproduction the coupling constants C_3, C_4, and C_5 are regarded as form factors depending on k^2 and can be identified in terms of Mathews Pauli-type form factors[6]

$$G_1(k^2) = C_3(k^2),$$
$$G_2(k^2) = \tfrac{1}{2}[C_4(k^2) + C_5(k^2)], \qquad (2.13)$$
$$G_3(k^2) = \tfrac{1}{2}[C_4(k^2) - C_5(k^2)].$$

Various considerations including a study of photoproduction sum rules discussed in Sec. III lead us to use only the coupling $\mathfrak{L}_3$ in our calculation of the electrproduction amplitude.

More precisely, consistent with our treatment of the $N\pi\Delta$ interaction, we add to $\mathfrak{L}_3$ an off-mass-shell term giving

$$\mathfrak{L}_{\Delta N\gamma} = \frac{e}{\mu} C_3 \overline{\Delta}{}_\mu^3 \, \Phi_{\mu\nu\lambda} N F_{\nu\,\lambda} + \text{H.c.},$$
$$\Phi_{\mu\nu\lambda} = \{\delta_{\mu\nu} + [Y + \tfrac{1}{2}(1+4Y)A]\gamma_\mu\gamma_\nu\}\gamma_\lambda\gamma_5, \qquad (2.14)$$

where Y is a new parameter. We anticipate that Y along with Z will have the same values as in the photoproduction case.

The couplings $\mathfrak{L}_{\Delta\pi N}$ and $\mathfrak{L}_{\Delta N\gamma}$ along with the Δ propagator $P_{\mu\nu}$ lead to the following Δ-exchange contributions to the invariant amplitudes:

$$V_i^{(\pm)} = \frac{e}{\mu} g_\Delta C_3 \binom{2/3}{-1/3} \times \left[\left(\frac{1}{M_\Delta^2 - s} \pm \eta_i \frac{1}{M_\Delta^2 - \mu}\right)\Delta_i(t, k^2) + R_i^{(\pm)}(s, t, k^2)\right], \qquad (2.15)$$

$$\Delta_1 = t/2 - \frac{M}{3}(E_\Delta + M) + \frac{M}{2}(M + M_\Delta)\xi + (1-\xi)\frac{k^2}{2},$$
$$\Delta_2 = -1 + \xi\frac{k^2}{2q\cdot k},$$
$$\Delta_3 = -\tfrac{1}{2}(M + M_\Delta) + \frac{M}{2}\xi,$$
$$\Delta_4 = \tfrac{1}{2}(M + M_\Delta) + \frac{M}{2}\xi, \qquad (2.16)$$
$$\Delta_5 = \xi\frac{(M_\Delta^2 - M^2)}{2q\cdot k},$$
$$\Delta_6 = -\tfrac{1}{2}(M + M_\Delta)\xi + \tfrac{1}{3}(E_\Delta + M),$$
$$(E_\Delta + M) = \frac{1}{2M_\Delta}[(M + M_\Delta)^2 - \mu^2],$$
$$\xi = \frac{1}{3M_\Delta^2}(2M_\Delta^2 + MM_\Delta - M^2 + \mu^2),$$

$$R_1^{(+)} = \frac{1}{3M_\Delta^2}\left[-M(M + M_\Delta) + \tfrac{1}{2}(1-\alpha)k^2 + \frac{\alpha\beta}{4}(t - \mu^2 + k^2) + \frac{\beta}{4}(t - 5\mu^2 + k^2)\right],$$
$$R_2^{(+)} = \frac{1}{3M_\Delta^2}(\alpha - 1)\frac{k^2}{2q\cdot k}, \qquad (2.17)$$
$$R_4^{(+)} = \frac{1}{3M_\Delta^2}\left[\alpha\beta M_\Delta + \frac{M}{2}(\alpha + \beta + \alpha\beta - 1)\right],$$
$$R_5^{(+)} = \frac{1}{3M_\Delta^2}\left[\tfrac{1}{2}(\alpha - 1)(s-u)\frac{1}{2q\cdot k}\right],$$
$$R_3^{(+)} = R_6^{(+)} = 0,$$
$$\alpha = 1 + 4Z, \quad \beta = 1 + 4\gamma,$$

$$R_1^{(-)} = \frac{1}{3M_\Delta^2}\frac{\beta}{4}(1-\alpha)(s-u),$$
$$R_3^{(-)} = \frac{1}{3M_\Delta^2}[\alpha\beta M_\Delta + \frac{M}{2}(\alpha + \beta + \alpha\beta - 1)],$$
$$R_5^{(-)} = \frac{1}{3M_\Delta^2}\tfrac{1}{2}(1-\alpha), \qquad (2.18)$$
$$R_6^{(-)} = \frac{1}{3M_\Delta^2}(M + M_\Delta),$$
$$R_2^{(-)} = R_4^{(-)} = 0.$$

Our invariant amplitudes V_i, Pauli amplitudes $\mathfrak{F}_i$, and electric and magnetic multipoles reduce to those of Chew-Goldberger-Low-Nambu (CGLN) for $k^2 = 0$. To obtain the corresponding quantities with Adler's normalization[1],

$$V_i^{\text{Adler}} = \frac{2}{e}V_i,$$
$$\mathfrak{F}_i^{(\pm,0)\text{Adler}} = \left(\frac{4\pi W}{Me}\right)\mathfrak{F}_i^{(\pm,0)},$$
$$\mathfrak{M}^{(\pm,0)\text{Adler}} = \left(\frac{4\pi W}{Me}\right)\mathfrak{M}^{(\pm,0)},$$

where $\mathfrak{M}$ is a magnetic or electric multipole amplitude. The four electroproduction reactions $\gamma_v p \to \pi^0 p$, $\gamma_v n \to \pi^0 n$, $\gamma_v p \to \pi^+ n$, and $\gamma_v n \to \pi^- p$ are described by the linear combinations $V_i^{(0)} + V_i^{(+)}$, $V_i^{(0)} - V_i^{(+)}$, $\sqrt{2}(V_i^{(0)} + V_i^{(-)})$, and $\sqrt{2}(V_i^{(0)} - V_i^{(-)})$. Finally, the isovector transition amplitude $V_i^{(\pm)}$ can be expressed in terms of the amplitude $V_i^{(2I)}$ corresponding to isospin $\frac{1}{2}$ and $\frac{3}{2}$ in the final-state

$$V_i^{(1)} = V_i^{(+)} + 2V_i^{(-)}, \quad V_i^{(3)} = V_i^{(+)} - V_i^{(-)}.$$

For the relations between the invariant, Pauli, helicity, and multipole amplitudes we refer to the relevant Appendices of Adler's[1] work. The unitarization of the resonant multipoles described in Sec. IV is incorporated into the model by subtracting out the resonant Lagrangian multipoles and adding back the unitarized ones. The calculation of the observable quantities is discussed in Sec. V.

III. RESONANT SUM RULES

The Δ-resonance coupling constants appearing in the Lagrangian of the preceding section cannot be directly determined from the resonant multipoles alone. The reason for this, of course, is the singularity at the Δ mass. Although this singularity does not appear in the nonresonant multipoles it implies a serious weakness of the model since the resonant multipoles are often dominant features in the observables. There are several ways of dealing with this problem:

(1) Compare the model only with nonresonant multipoles and resonant multipoles far from resonance thus ignoring the problem completely. This method is useful in the case of photoproduction where accurate energy independent multipole analyses exist. The photoproduction results are very encouraging,[3] but if no multipole analysis exists, as with electroproduction, we must do better.

(2) Subthreshold behavior: Dispersion relations can be used to calculate the invariant amplitudes below threshold. Model comparisons then can be made without serious difficulties with unitarity. This has been done successfully with πN elastic scattering,[9] and it could also be done in the photoproduction case where detailed knowledge of the multipoles is available.[10]

(3) Resonant sum rules: These sum rules provide a method of directly extracting the Lagrangian-model coupling constants from the observables in a model independent way. Using the couplings so determined we can predict the nonresonant and subthreshold amplitudes. This is the main subject of the present section.

(4) Unitarization of the resonant amplitudes: Using unitarity and analyticity we construct a "physical" amplitude which reduces to the Lagrangian model result above and below resonance. The observables can then be directly computed and compared with experiment. The method of derivation will be discussed in Sec. IV.

The evaluation of sum rules for the resonant coupling constants has a distinct advantage. Before a detailed data comparison is made, the various types of resonant coupling (for example, the C_3, G_2, and G_3 types in electroproduction) can be examined and the coupling constants fixed. We discuss the construction of sum rules for Δ couplings in elastic, electroproduction, and Compton scattering. Subsequently, we evaluate the photoproduction sum rules for C_3 and G_2.

An invariant amplitude $V^{(\pm)}(\nu, t)$ satisfies the fixed-t dispersion relation

$$\mathrm{Re}V^{(\pm)}(\nu,t) = \mathrm{Born} + \frac{1}{\pi}\,\mathrm{P}\int_{\nu_1}^{\infty} \mathrm{Im}V^{(\pm)}(\nu',t) \times\left(\frac{1}{\nu'-\nu}+\frac{\eta}{\nu'+\nu}\right)d\nu', \tag{3.1}$$

where $\eta=\pm1$ depending on the crossing property. If we separate out the direct channel Δ-excitation term and project the isospin-$\frac{3}{2}$ part we obtain

$$\mathrm{Re}V^{3/2}(\nu,t) = (\mathrm{Background})^{I=3/2} + \frac{1}{\pi}\,\mathrm{P}\int_{\nu_1}^{\nu_c} \frac{\mathrm{Im}V^{3/2}(\nu',t)d\nu'}{\nu'-\nu}, \tag{3.2}$$

where the "background" consists of Born, resonance exchanges, and nonresonant terms. The Lagrangian model can directly predict the first term on the right side. By comparing the dispersive and Lagrangian-model results for direct Δ excitation we obtain sum rules for the Δ coupling constants. The Lagrangian model expression for direct Δ production is

$$V_L^{3/2}(\nu,t) = \frac{1}{\nu_\Delta-\nu}\,(\Sigma_i\, g_i\, a_i), \tag{3.3}$$

where g_i are coupling constants and a_i are known kinematic terms. In the integral term of Eq. (3.2) we assume the following:

(1) The Δ-excitation partial waves are the only ones which contribute to the imaginary part of V.

(2) $\mathrm{Im}V^{(3/2)}$ is sharply peaked in the vicinity of $\nu=\nu_\Delta$. On either side of the Δ resonance the term $\nu'-\nu$ can be replaced by $\nu_\Delta-\nu$ and we obtain

$$\Sigma_i\, g_i\, a_i = \frac{1}{\pi}\int_{\nu_1}^{\nu_c} \mathrm{Im}V^{3/2}(\nu,t)d\nu\,. \tag{3.4}$$

Using the convenient integration variable

$$\omega = \frac{s-M^2-\mu^2}{2M} \tag{3.5}$$

which is just the incident laboratory pion energy for elastic πN scattering and differs from ν by a constant at fixed t, we find sum rules of the form

$$\Sigma_i\, g_i\, a_i = \frac{1}{\pi}\int_{1}^{\omega_c} \mathrm{Im}V^{3/2}(\omega)d\omega\,. \tag{3.6}$$

The integrand may be evaluated either by a partial-wave analysis or in some cases by an optical theorem.

Elastic scattering

The sum rule for πN elastic scattering was evaluated by Höhler *et al.*[11] and we only include it here for completeness. Here there is only one coupling constant g_Δ so V can be taken to be either of the πN invariant amplitudes. Using the partial-wave expansion we have

$$\mathrm{Im}A^{3/2}(\omega) = \frac{4\pi}{3}\left(\frac{\alpha_1+\alpha_2 t}{E+M}\right)\frac{\mathrm{Im}f_{33}}{|\vec{q}|^2}, \qquad (3.7)$$

where the notation is standard.[11] The Lagrangian-model expression for the left side of Eq. (3.6) is

$$g_\Delta a = \frac{g_\Delta{}^2}{18M}(\alpha_1+\alpha_2 t), \qquad (3.8)$$

so the sum rule is then

$$\frac{g_\Delta{}^2}{4\pi} = \frac{3}{\pi}\int_1^{\omega_c}\frac{2M}{E+M}\,\frac{\mathrm{Im}f_{33}}{|\vec{q}|^2}\,d\omega, \qquad (3.9)$$

where we have divided out the slowly varying factor $\alpha_1+\alpha_2 t$. This last step is necessary for self-consistency since the same sum rule must be obtained from the $B(\nu,t)$ amplitude.

When the latest data is used the sum rule yields the value[12]

$$\frac{g_\Delta{}^2}{4\pi} = (0.28 \pm 0.015)\mu^{-2}, \qquad g_\Delta = (1.88 \pm 0.05)\mu^{-1}. \qquad (3.10)$$

This result is quite insensitive to the cutoff energy ω_c here taken to be about 1500 MeV. The coupling constant g_Δ obtained in this way agrees well with an analysis of nonresonant πN scattering.[9]

Photoproduction

In electroproduction there are six independent gauge-invariant dispersive amplitudes and three coupling constants each with form factor variation in k^2. Unfortunately, a multipole analysis has only been done at $k^2=0$ (photoproduction). In this limit there are only two couplings $C_3(0)$ and $G_2(0)$ (the G_3 amplitudes vanish at $k^2=0$), and four invariant amplitudes. Using the same method as in the elastic case (but with more algebra) we obtain two sum rules,

$$eg_\Delta C_3(0) = 48M\mu\int_1^{\omega_c}\frac{d\omega}{|\vec{q}||\vec{k}|}\,\frac{W}{(W+M)^2}\,\frac{D_1}{D_2}\times(\mathrm{Im}M_{1+}+\mathrm{Im}E_{1+}), \qquad (3.11)$$

$$eg_\Delta G_2(0) = 24M\mu^2\int_1^{\omega_c}\frac{d\omega}{|\vec{q}||\vec{k}|}\,\frac{1}{(W+M)^2}\,\frac{D_1}{D_2}\times\left[\mathrm{Im}M_{1+}+\left(\frac{3W+M}{W-M}\right)\mathrm{Im}E_{1+}\right], \qquad (3.12)$$

where $D_i=(E_i+M)^{1/2}$; $E_1=(M^2+|\vec{k}|^2)^{1/2}$, $E_2=(M^2+|\vec{q}|^2)^{1/2}$, and M_{1+} and E_{1+} are two resonant photoproduction multipoles (in electroproduction we have in addition the longitudinal resonant multipole L_{1+}).

The evaluation of these sum rules proceeds straightforwardly by the use of multipole analyses in the Δ region and above.[10] Since M_{1+} is roughly an order of magnitude larger and of the opposite sign as E_{1+} we see that there is a large cancellation between the terms in the G_2 sum rule. When evaluated in detail we obtain

$$g_\Delta C_3(0) = (0.62 \pm 0.04)\mu^{-1}, \qquad g_\Delta G_2(0) = (-0.018 \pm 0.020)\mu^{-1}. \qquad (3.13)$$

Thus G_2 is consistent with zero and is certainly much smaller than C_3. The result for $C_3(0)$ is quite consistent with our previous analysis value[3,4]:

$$g_\Delta C_3(0) \simeq 0.61\mu^{-1}. \qquad (3.14)$$

Unfortunately, there is no way to determine G_3 by use of photoproduction data and there is no reliable multipole analysis for $k^2 \neq 0$. There is, however, a method of obtaining electroproduction sum rules by considering the Compton amplitude.

Compton scattering

For Compton scattering with massive photons, there are two spin averaged optical theorems[13] in terms of σ_T and σ_L, the transverse and longitudinal electroproduction total cross sections. Thus if σ_T and σ_L were known as a function of W(for a fixed k^2) two of the coupling constants could be found for this value of k^2. Since σ_L and σ_T have not been extracted as a function of W (at fixed k^2) we shall not proceed further. As we will see in Sec. V there are excellent separated data taken at fixed electron angle which will essentially rule out the G_3 coupling type.

IV. UNITARIZATION

As stressed earlier a method must be worked out to modify the Lagrangian-model result for resonant multipoles if observables are to be predicted. The method presented in this section is based on three principles:

(1) The unitarized multipole must reduce to the pole-model result away from the resonance.

(2) Unitarity must be preserved: In the case of electroproduction multipoles in the Δ region this means that the complex phase at all energies must be the same as the elastic (3,3) phase shift. This implies that the simple prescription of inserting a width factor in the resonance denominator cannot

be used since the phase of the total multipole including the Born term will no longer be the (3, 3) phase shift.

(3) Analyticity: The resonant multipole will be assumed to have the normal simple analytic structure consisting of a branch point at hadronic threshold, a Δ resonance pole on the second sheet, and other singularities due to the Born terms.

For a resonant multipole $M(\omega)$ the Lagrangian-model expression is of the form

$$M_L(\omega) = B(\omega) + \frac{C_3 g_\Delta R(\omega)}{\omega_\Delta - \omega}, \tag{4.1}$$

where $R(\omega)$ is a known kinematic factor characteristic of the specific multipole and $B(\omega)$ is the background, mostly Born, projected into this multipole. For the (3, 3) partial wave the elastic πN amplitude Lagrangian result is

$$f_L(\omega) = f_B(\omega) + \frac{g_\Delta{}^2 R_e(\omega)}{\omega_\Delta - \omega}, \tag{4.2}$$

where

$$R_e(\omega) = \frac{MM_\Delta(E_2+M)(W+M)|\vec{q}|^2}{12\pi s},$$

and f_B is the background (again mostly Born) projected into the (3, 3) partial wave.

Below the double-pion-production threshold unitarity requires that both $f(\omega)$ and $M(\omega)$ have the same complex phase and that the (3, 3) partial-wave amplitude be of the form

$$f_{1+}^{3/2}(\omega) = e^{i\delta} \sin\delta / |\vec{q}| . \tag{4.3}$$

We propose that the resonant multipole be given by

$$M(\omega) = \frac{M_L(\omega)}{f_L(\omega)} f_{1+}^{3/2}(\omega). \tag{4.4}$$

First observe that if the elastic amplitude $f(\omega)$ approaches the Lagrangian value $f_L(\omega)$ away from resonance the multipole does likewise. This is certainly the case near threshold, where it has been verified[14] that $f_{1+}^{3/2} \simeq f_L$. Secondly, since M_L

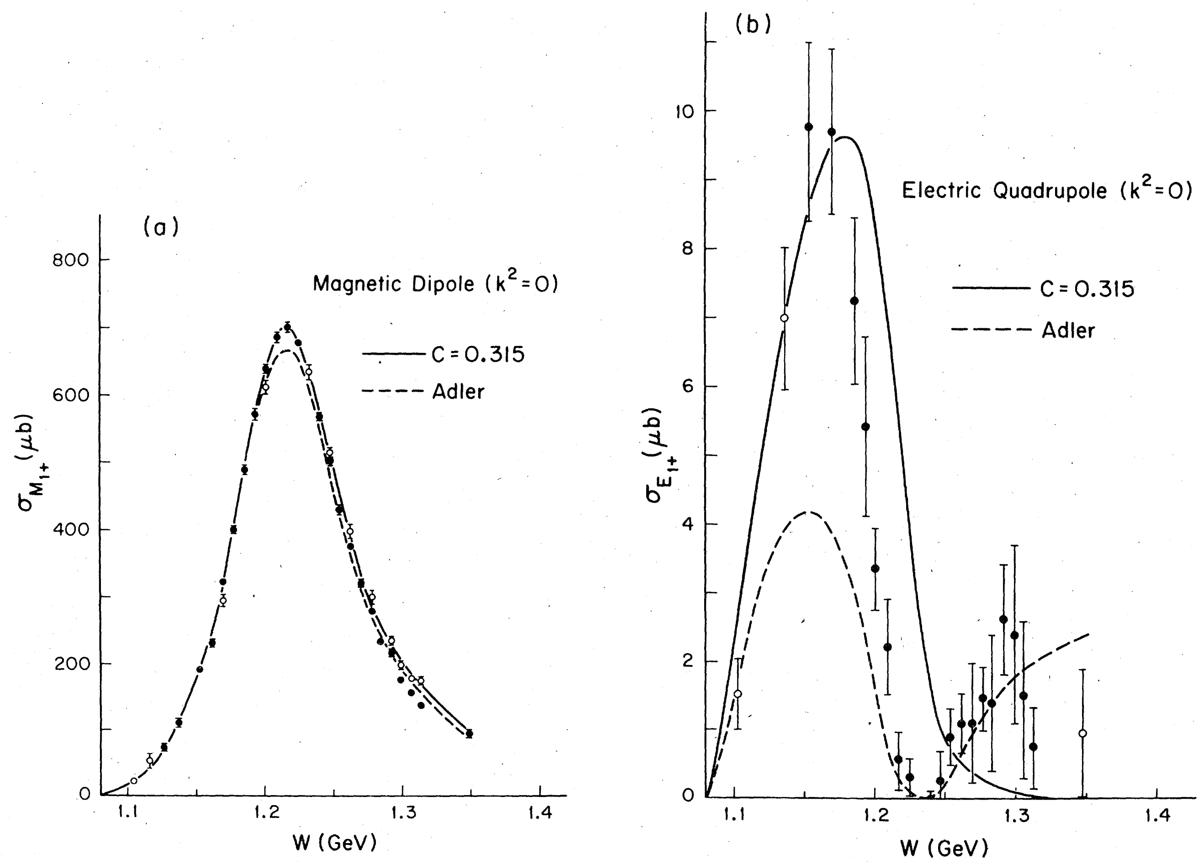

FIG. 2. (a) Magnetic-dipole cross section at $k^2 = 0$. The solid curve is our model calculation with $C_3(0) = 0.315$. The dashed curve is the prediction of Adler's model (Ref. 1). The energy-independent photoproduction analyses (Ref. 10) of Berends and Donnachie (solid dots) and Pfeil and Schwela (open dots) have been used to provide the experimental data. (b) Electric-quadrupole cross section. The curves and data are the same as in Fig. 2(a).

and f_L are real, M_L automatically has the same phase as $f_{1+}^{3/2}(\omega)$ satisfying the unitarity requirement.

Finally, we see that $M(\omega)$ has the required analyticity properties. Near threshold $M(\omega)\simeq M_L(\omega)$ so that $M(\omega)$ has the correct branch point. The pole at $\omega=\omega_\Delta$ occurring in the Lagrangian amplitudes cancels in the unitarized multipole expression (4.4) and the second sheet pole in $M(\omega)$ comes from $f_{1+}^{3/2}(\omega)$.

Combining Eqs. (4.3) and (4.4) we obtain

$$M(\omega)=\frac{M_L(\omega)}{|\vec{q}|f_L(\omega)}\,e^{i\delta}\sin\delta\,. \qquad (4.5)$$

For photoproduction we show the comparison for the M_{1+} and E_{1+} cross sections in Fig. 2. The M_{1+} cross section given by

$$\sigma_M=\frac{8\pi|\vec{q}|}{|\vec{k}|}|M_{1+}|^2 \qquad (4.6)$$

fits well if we choose

$$g_\Delta C_3(0)=(0.58\pm0.01)\mu^{-1}$$

in agreement with the resonant sum rule value (3.13). The E_{1+} cross section

$$\sigma_E=\frac{24\pi|\vec{q}|}{|\vec{k}|}|E_{1+}|^2$$

prediction is sensitive to the Δ mass and we find a value of about 1.210 GeV is optimum.

In principle the nonresonant multipoles should also have unitarity corrections using the prescription given in Eq. (4.5). Since the nonresonant πN phase shifts are small in the Δ region we do not expect these effects to be large. In Ref. 4 nonresonant unitarity corrections were calculated in the photoproduction case and found to be quite negligible so we feel justified in their neglect in our case also.

V. DIFFERENTIAL CROSS SECTIONS AND COMPARISONS WITH DATA

The electroproduction differential cross section can be written in the gneral form[15]

$$\frac{d^3\sigma}{d\Omega_{e'}^L dE'd\Omega_\pi}=\frac{M(e^2)^2}{4(2\pi)^5(k^2)^2}\frac{E'}{E}\frac{|\vec{q}|}{W}\times(T_0+T_1\cos\phi_\pi+T_2\cos2\phi_\pi)\,, \qquad (5.1)$$

where $E(E')$ is the initial (final) electron laboratory energy, k^2 is the lepton four-momentum transfer, $\vec{q}$ is the pion three-momentum in the isobaric (pion-nucleon center of mass) frame, W is the invariant πN mass, and ϕ_π is the pion azimuthal angle in the isobaric frame as illustrated

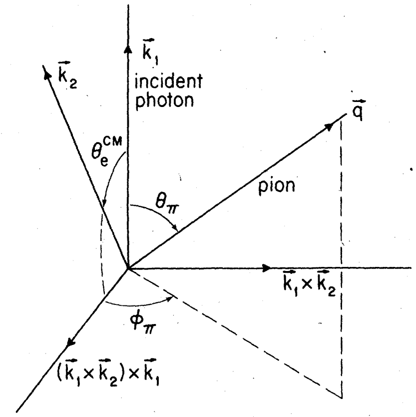

FIG. 3. Isobaric-frame electroproduction kinematics.

in Fig. 3. The coefficients T_i can be expanded in terms of the pion polar angle in the isobaric frame θ_π, as

$$\begin{aligned}T_0&=t_0+t_1\cos\theta_\pi+t_2\cos^2\theta_\pi+\cdots\,,\\ T_1&=\sin\theta_\pi(t_0+t_1\cos\theta_\pi+t_2\cos^2\theta_\pi+\cdots)\,,\\ T_2&=\sin^2\theta_\pi(t_0+t_1\cos\theta_\pi+t_2\cos^2\theta_\pi+\cdots)\,.\end{aligned} \qquad (5.2)$$

The triple differential cross section can also be written as a product of two factors[16]: the flux factor Γ_t, describing the electron- photon vertex, and a second factor describing the virtual-photon–hadron interaction. In this case, we write

$$\begin{aligned}\frac{d^3\sigma}{d\Omega_{e'}^L dE'd\Omega_\pi}&=\Gamma_t\frac{d\sigma}{d\Omega_\pi}\,,\\ \Gamma_t&=\frac{e^2}{4\pi}\frac{K}{4\pi^2}\frac{E'}{E}\left(\frac{2}{k_2}+\frac{\cot^2(\frac{1}{2}\theta_e)}{|\vec{k}|^2}\right)\,,\end{aligned} \qquad (5.3)$$

where $K=(s-M^2)/2M$, θ_e is the isobaric-frame angle between incoming and outgoing lepton momenta, $\vec{k}$ is the isobaric photon momentum, and

$$\frac{d\sigma}{d\Omega_\pi}=\frac{d\sigma_T}{d\Omega_\pi}+\epsilon\frac{d\sigma_L}{d\Omega_\pi}+\epsilon\sin^2\theta_\pi U(\theta_\pi)\cos2\phi_\pi+[\tfrac{1}{2}\epsilon(\epsilon+1)]^{1/2}S(\theta_\pi)\sin\theta_\pi\cos\phi_\pi\,, \qquad (5.4)$$

with electron polarization

$$\epsilon=\left[1+2\frac{|\vec{k}|^2}{k^2}\tan^2\left(\frac{\theta_e}{2}\right)\right]^{-1}.$$

σ_T and σ_L are called the transverse and longitudinal cross sections, referring to the two types of polarizations possible for virtual photons.

Various aspects of the electroproduction process, each capable of limiting the behavior of certain parameters of our model, can be studied by fixing the electron or pion angles and/or integrating over some of the kinematic variables. In this section, we will compare our model with data

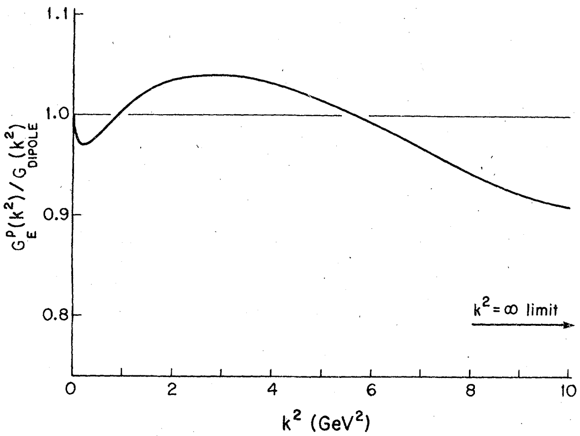

FIG. 4. Experimental proton form factor $G_E^p(k^2)$ plotted to show deviations from the usual dipole expression $G_{\rm dipole}(k^2) = (1 + k^2/0.71\ \text{GeV}^2)^{-2}$.

relevant to the fixing of the form factors used in the model, in particular, we will investigate the behavior of the form factors $C_3(k^2)$ [$C_3(0)$ is taken to be the value required by photoproduction data], $G_3(k^2) = \frac{1}{2}[C_4(k^2) - C_5(k^2)]$, $F_A(k^2)$, and $F_\pi(k^2)$. The behavior of the proton and neutron electric and magnetic form factors $G_E^{p,n}$ and $G_M^{p,n}$, which is seen to deviate substantially from a simple dipole form, was fixed by investigation of the data of Stein *et al.*[17] and Atwood,[18] assuming the form factor scaling relation $G_M^{p,n} = \mu^{p,n} G_E^p$ ($G_E^n = 0$). We found that the functional form for $G_M^{p,n}$ (or G_E^p) at high k^2 proposed by Atwood was not capable of reproducing the behavior at the low k^2 of G_E^p graphed by Stein *et al.* We use the following form for $G_E^p(k^2)$, which is consistent with both low-k^2 and high-k^2 data (see Fig. 4):

$$G_E^p = \begin{cases} \lambda(k^2) G_{\rm dipole}(k^2), & k^2 < 5.8\ \text{GeV}^2 \\ 0.4/k^4, & k^2 > 5.8\ \text{GeV}^2 \end{cases} \tag{5.5}$$

$$\lambda^2(k^2) = 1 - (0.053 + 0.017k) \sin\left(\frac{4.00k}{1 + 0.22k}\right)$$

$$k = (k^2)^{1/2},$$

where $G_{\rm dipole}(k^2) = (1 + k^2/0.71\ \text{GeV}^2)^{-2}$.

In this analysis we fix the Δ coupling constants at $k^2 = 0$ to those values obtained by photoproduction. In Sec. III we found the sum rules yield

$$C_3(0) = \frac{g_\Delta C_3(0)}{g_\Delta} = \frac{0.62 \pm 0.04}{1.88 \pm 0.05} = 0.33 \pm 0.03, \tag{5.5}$$

$$G_2(0) \simeq 0.$$

When the unitarized form of the multipole amplitudes was compared with the resonant photoproduction multipoles in Sec. IV we found

$$C_3(0)) \frac{g_\Delta C_3(0)}{g_\Delta} = 0.315 \pm 0.01. \tag{5.6}$$

For the remainder of this paper we shall assume that

$$\begin{aligned} C_3(0) &= 0.315, \\ G_2(0) &= 0, \\ Z &= 0, \\ Y &= 0.25. \end{aligned} \tag{5.7}$$

The above values Z and Y are obtained in a previous photoproduction analysis.[3] Slightly different values of the off-shell coupling constants were found[4] when vector-meson exchanges were considered. For simplicity we have neglected the generally small vector-mesonic effects and hence the earlier[3] values for Z and Y have been used. The neglect of vector-meson exchanges is most evident in the isoscalar multipoles $E_{0+}^{(0)}$ and $M_{1-}^{(0)}$.

Axial-vector nucleon form factor $F_A(k^2)$

The behavior of the triple differential cross section at threshold is particularly sensitive to the axial-vector form factor $F_A(k^2)$. If we expand the coefficients T_i of Eq. (5.2) in terms of $|\vec{q}|$ and θ_π, then at threshold, $|\vec{q}| \to 0$, the triple differential cross section behaves as

$$\begin{aligned} \frac{1}{\Gamma_t} 4\pi \frac{K}{|\vec{q}|} \frac{d^3\sigma}{d\Omega_e^L dE' d\Omega_\pi} &= A_1 + A_2|\vec{q}|\cos\theta_\pi + A_3|\vec{q}|^2\cos^2\theta_\pi \\ &+ A_4|\vec{q}|\sin\theta_\pi\cos\phi_\pi \\ &+ A_5|\vec{q}|^2\sin^2\theta_\pi\cos 2\phi_\pi + A_6|\vec{q}|^2 \\ &+ A_7|\vec{q}|^2\sin\theta_\pi\cos\theta_\pi\cos\phi_\pi. \end{aligned}$$

Good data[19] exist for the coefficients A_1 and A_4.

The A_i can be expressed as sums of process multipoles. If we assume that only s- and p-wave multipoles need be considered in the cross section, then we can write the coefficients A_1 and A_4 as[15]

$$A_1 = \lim_{|\vec{q}| \to 0} \frac{4\pi W}{M}\left(|E_{0+}|^2 + \epsilon \frac{k^2}{k_0^2}|L_{0+}|^2\right),$$

$$A_4 = \left[\frac{\epsilon(\epsilon+1)}{2}\right]^{1/2} \frac{k}{k_0}$$

$$\times \lim_{|\vec{q}| \to 0} \frac{2}{|\vec{q}|} \frac{4\pi W}{M}\{-\text{Re}[E_{0+}(L_{1-} - 2L_{1+})^*]$$

$$+ \text{Re}[(-M_{1-} + M_{1+} - 3E_{1+})L_{0+}^*]\},$$

where k_0 is the photon isobaric energy. Data from several experimental investigations of π^+ electroproduction[19] for k^2 up to 0.9 GeV^2 is plotted in

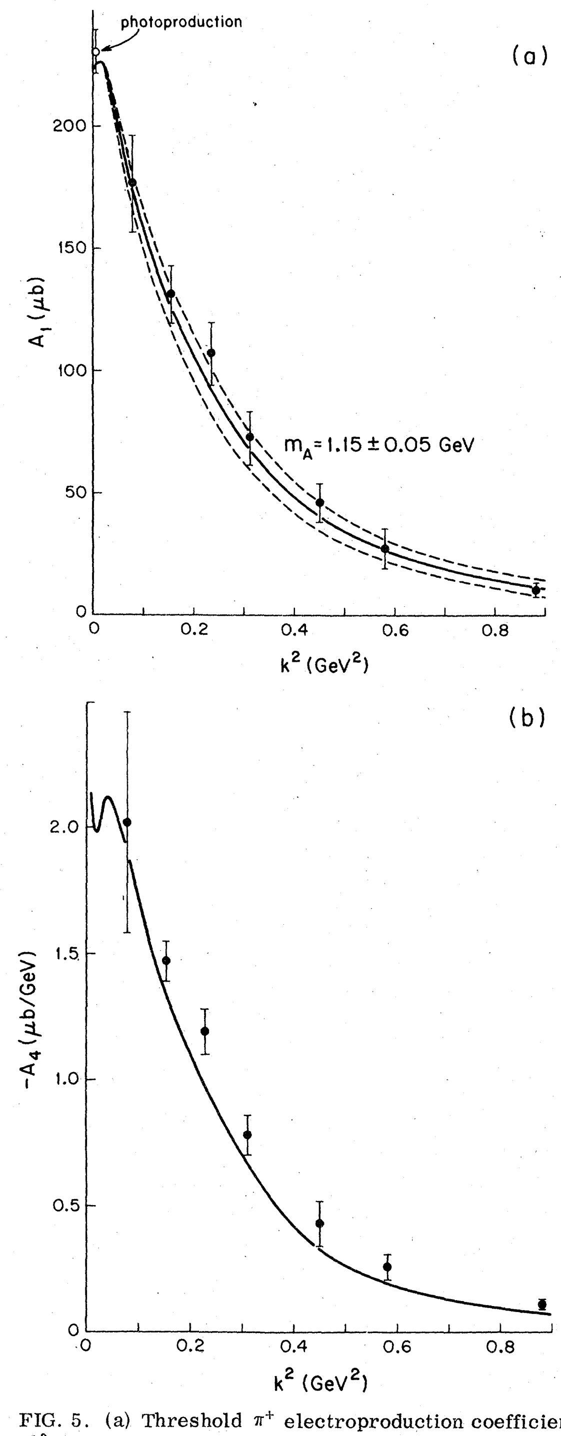

FIG. 5. (a) Threshold π^+ electroproduction coefficient $A_1(k^2)$ from the experiment of Del Guerra *et al.* (Ref. 19) with photoproduction point (Ref. 19). The theoretical curves show the sensitivity of $A_1(k^2)$ to the nucleon dipole axial-vector form-factor mass. The three curves correspond to $M_A(1.15 \pm 0.05)$ Gev. (b) π^+ electroproduction coefficient $A_4(k^2)$. The experimental data is from Del Guerra *et al.* (Ref. 19) and the theoretical curve is calculated for $M_A = 1.15$ GeV. This coefficient is not as sensitive to the nucleon axial-vector form factor as is $A_1(k^2)$.

Fig. 5 for the coefficients A_1 and A_4. A_1 is particularly sensitive to $F_A(k^2)$, varying by as much as 75% at $k^2 \simeq 0.9$ GeV^2 for M_A between 1.00 and 1.50 GeV. Our model predictions are plotted for those values of the dipole axial-vector mass parameter M_A found most consistent with all of the available data on A_1, i.e., $M_A = 1.15 \pm 0.05$ GeV. A_4 data and our model prediction ($M_A = 1.15$ GeV) are also shown. We find, as noted by Bloom *et al.* and Nambu and Yoshimura,[20] that a larger value of M_A, $M_A \simeq 1.5$ GeV, is required for agreement with large-k^2 threshold data from the SLAC inelastic ep scattering experiments.[17,18]

We believe that the present π^+ coincidence data is certainly consistent with a value for the dipole nucleon axial-vector form factor of

$$m_A = 1.15 \pm 0.10 \text{ GeV}.$$

The larger value obtained using the SLAC data is bothersome, but we feel that the systematic errors such as radiative corrections are better understood in a coincidence experiment. A more serious problem lies in the comparison with neutrino quasielastic scattering where a recent analysis[21] gives

$$m_A^{(\nu)} = 0.94 \pm 0.05 \text{ GeV}. \tag{5.8}$$

If the neutrino and electroproduction data continue to provide different values of m_A the consequences could be serious for the current-algebra PCAC picture.[7] On the other hand, the discrepancy does not seem too serious at present and as the experimental data improves one would hope that a unified picture will evolve.

Pion form factor $F_\pi(k^2)$

Whereas the threshold region is particularly sensitive to the parametrization of $F_A(k^2)$, the W region immediately above threshold and yet still below the $\Delta(1232)$ peak is found to be sensitive to the choice of $\{Y, Z\}$ made and to the k^2 dependence of the pion electromagnetic form factor $F_\pi(k^2)$. Positive pion electroproduction data has been taken at the Saclay electron linac[22] with $W = 1.175$ GeV for $\theta_\pi = 0°$. From Eqs. (5.3) and (5.4) we see that for this choice of θ_π, the triple differential cross section reduces to a sum of two terms

$$\frac{1}{\Gamma_t} \frac{d^3\sigma}{d\Omega_e^L dE' d\Omega_\pi} = \frac{d\sigma_T}{d\Omega_\pi} + \epsilon \frac{d\sigma_L}{d\Omega_\pi}. \tag{5.9}$$

By varying ϵ, the transverse and longitudinal differential cross sections can be separated. Both the transverse and longitudinal differential cross sections are found to be very sensitive to the choice of $\{Y, Z\}$ made. The values of $\{Y, Z\} = \{\frac{1}{4}, 0\}$ suggested by the application of the our model to the problem of photoproduction[3] were reexamined here. Consistency with the Saclay electroproduction data, as well as with the photoproduction ($k^2=0$) point[23] for $d\sigma_T/d\Omega_\pi$, was best achieved with the choice $\{Y,Z\}=\{\frac{1}{4},0\}$, although, for very high k^2, $k^2\simeq 9$ GeV^2, the choice $\{Y,Z\}=\{\frac{1}{4},\frac{1}{2}\}$ also works well.

If we set $\{Y,Z\}=\{\frac{1}{4},0\}$, we might also be able to use the Saclay data to constrain the k^2 dependence of $F_\pi(k^2)$. Several forms of $F_\pi(k^2)$ were investigated, including (a) a single-pole form found by Bebek *et al.*[24] with a mass parameter $m=0.69$ GeV, (b) a single-pole form with $m=m_\rho\simeq 0.77$ GeV, and (c) a dipole form with $M=M_V=0.84$ GeV. Although our model is sensitive to the form of $F_\pi(k^2)$ used in the region of W and k^2 studied by the Saclay group, their data is not sufficiently accurate to rule out any of these forms of k^2 dependence completely. Furthermore, as can be seen in Fig. 6 the transverse-cross-section measurements are apparently not normalized correctly to the photoproduction data points, and we note in fact that the sum of the longitudinal and transverse cross sections agrees very well with the theoretical values.

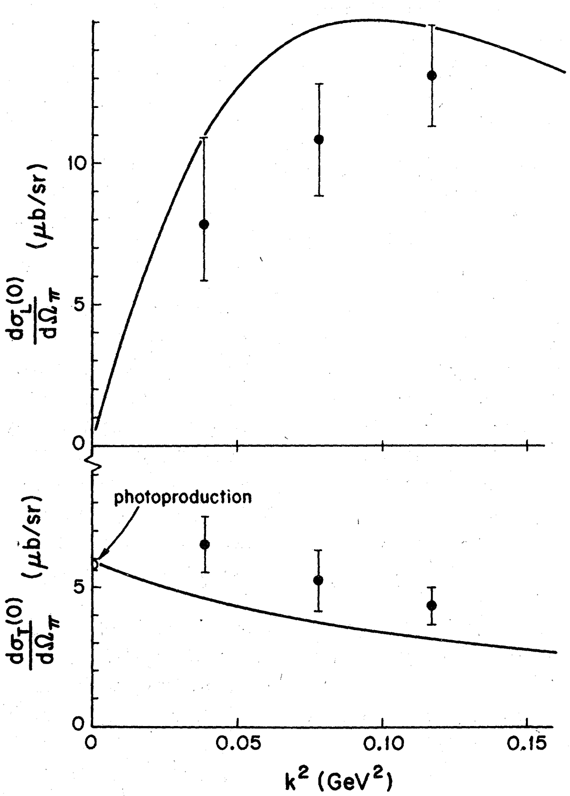

FIG. 6. Separated π^+ transverse and longitudinal cross sections for pions in the same direction as the virtual photon ($\theta_\pi=0$). The $k^2\neq 0$ data is from Bardin *et al.* (Ref. 22) and the photoproduction point is from Fischer *et al.* (Ref. 23).

The forward transverse cross section is particularly model dependent at $W=1175$ MeV because of the near cancellation of the Δ and Born terms. A better analysis could be done with data at both $\theta_\pi=0$ and 180 degrees and also including larger values of k^2. Although it is widely assumed that the forward longitudinal cross section is model independent[25,22] we note that it too is quite sensitive to the off-mass-shell Δ coupling constants Y and Z.

Finally, we might mention that even at low-W values, such as 1.175 GeV, the forward cross section cannot be accurately calculated with only s- and p-wave multipoles due to the pion term; the full amplitude must be used.

Δ Form factors $C_3(k^2)$ and $G_3(k^2)$

Much of the experimental data concerning electroproduction involves the double differential cross section obtained by integrating over the pion solid angle in Eqs. (5.3) and (5.4), i.e., one looks at

$$\frac{1}{\Gamma_t}\frac{d^2\sigma}{d\Omega_{e'}^L dE'}=\sigma_T(W,k^2)+\epsilon\sigma_L(W,k^2). \tag{5.10}$$

In this case, fixing θ_e^L and measuring E', for a given E, fixes the value of k^2 for a given invariant mass W. This form of the double differential cross section, as well as an alternative form

$$\frac{d^2\sigma}{d\Omega_{e'}^L dE'}=\frac{\alpha^2\cos^2(\theta_e^L/2)}{4E^2\sin^4(\theta_e^L/2)}\times\left[W_2(k^2,\nu_s)+2\tan^2\left(\frac{\theta_e^L}{2}\right)W_1(k^2,\nu_s)\right], \tag{5.11}$$

where $\nu_s=E-E'$, and α is the fine-structure constant, are often used to investigate scaling behavior in the hadronic system. Usually, the experimenters look at inclusive ep scattering; we will use their data in the region $1.08\lesssim W\lesssim 1.35$ GeV. The structure functions $W_1(k^2,\nu_s)$ and $W_2(k^2,\nu_s)$ are related to $\sigma_T(W,k^2)$ and $\sigma_L(W,k^2)$ by

$$W_1=\frac{K}{4\pi^2\alpha}\sigma_T\quad W_2=\frac{K}{4\pi^2\alpha}\frac{k^2}{|\vec{k}|^2}(\sigma_T+\sigma_L).$$

Data has been reported by Bartel *et al.*[26] for $0.3\lesssim k^2\lesssim 0.9$ GeV^2, by Stein *et al.*[17] for $0.1\lesssim k^2\lesssim 1.8$ GeV^2, by Cone *et al.*[27] for $1\lesssim k^2\lesssim 4$ GeV^2, by Breidenbach[28] for $0.5\lesssim k^2\lesssim 7.0$ GeV^2, and by Atwood[18] for $k^2\simeq 9$ GeV^2. The data of Bartel *et al.* has been separated into σ_T and σ_L components. The wide range of k^2 investigated in these experiments has allowed us to parametrize $C_3(k^2)$. This was done by examining the deviation of $C_3(k^2)$ from a simple dipole form, $(1+k^2/M_V^2)^{-2}$, where $M_V=0.84$

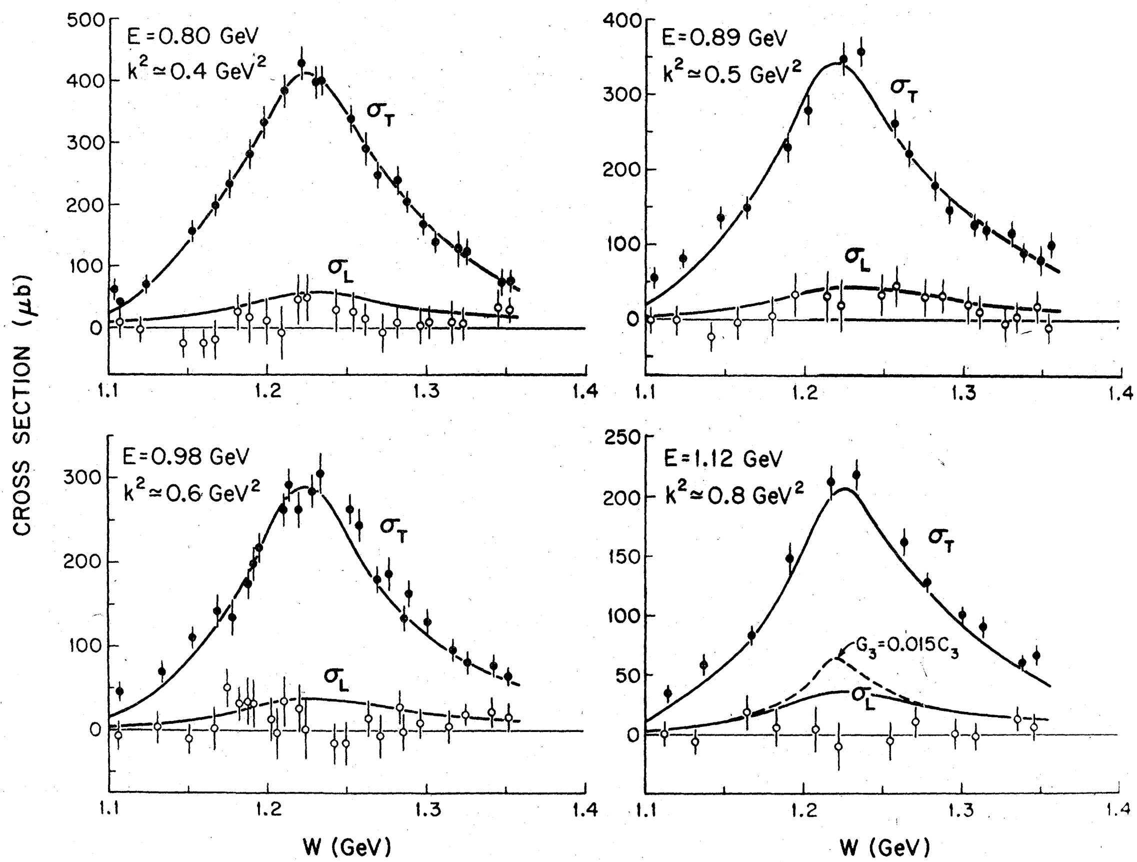

FIG. 7. Total inelastic electron scattering data due to Bartel *et al.* (Ref. 26) separated into transverse and longitudinal parts. The solid curves are our model values with $G_3(k^2)=0$ and $C_3(k^2)$ given by Eq. (5.12). The dashed curve on the $E=1.12$ GeV data results if $G_3(k^2)=0.015\,C_3(k^2)$. The data is consistent with assuming $G_3(k^2)=0$.

GeV. It has been noted previously,[29] that the Δ form factor falls more rapidly with k^2 than a simple dipole. Shaw[29] suggested that $C_3(k^2)$ might have the same k^2 dependence of $F_2(k^2)$, i.e., $C_3(k^2)\simeq(1+k^2/M_V^2)^{-2}(1+k^2/4M^2)^{-1}$. In this form, $C_3(k^2)$ still does not fall rapidly enough with k^2. The parametrization given by Dufner and Tsai[29] also does not reproduce the data. We find that a good fit to all the available data is obtained if we take

$$C_3(k^2)=\frac{0.315}{(1+k^2/M_V^2)^2(1+k^2/4M_V^2)}\,. \qquad (5.12)$$

The separated Bartel *et al.*[26] data are graphed in Fig. 7. The solid curves are our model expectations with $C_3(k^2)$ Δ-coupling alone. The Stein *et al.* data[17] along with our model predictions are shown in Fig. 8. We compare in Fig. 9 our model and that of Adler[15] with the Cone *et al.*[27] data at their largest photon masses and finally in Fig. 10 we show the data of Atwood[18] at extremely large values of k^2. The inelastic data protrayed varies in k^2 from near zero to about 9 GeV.^{2} The cross sections correspondingly fall over eight orders of magnitude to the picobarn range at $k^2\simeq 9$ GeV^2. As k^2 increases we see a relative rising trend for the cross section above the Δ. This rise is particularly evident in the Atwood data and apparently is due to the tails of higher resonances whose form factor dependence falls off less rapidly than that of the Δ. At the highest k^2 values the Δ peak is no longer prominent and, in fact, the cross-section variation is given by the deep-inelastic scaling curve.

The longitudinal cross section σ_L is particularly sensitive to contributions from $G_3(k^2)=\frac{1}{2}[C_4(k^2)-C_5(k^2)]$ and we can use this sensitivity to limit the value of this form factor. Using the separated data of Bartel *et al.*,[26] we find that a contribution with $G_3(0)$ of only 1.5% of $C_3(0)$, giving $G_3(k^2)$ the same k^2 dependence as $C_3(k^2)$, is decidedly ruled out, as can be seen in Fig. 7. Even a value for $G_3(0)$ of 0.5% of $C_3(0)$ proves troublesome at $k^2\simeq 9$ (GeV)^2, causing an uncomfortably highly peaked W distribution. The solid curves on Fig. 7 are

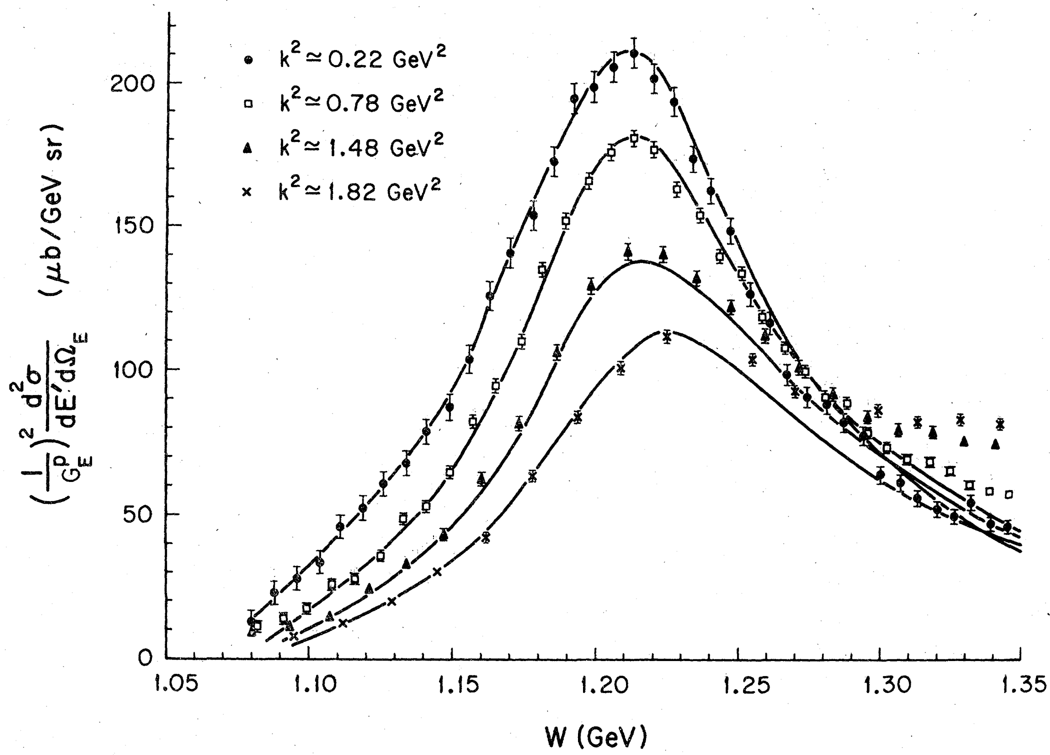

FIG. 8. Inelastic ep data of Stein *et al.* (Ref. 17) scaled by $[G_E^p(k^2)]^2$. The actual data at the Δ peak falls from about $200\,\mu$b at $k^2 \simeq 0.22$ GeV^2 to about $0.6\,\mu$b at $k^2 \simeq 1.8$ GeV^2. The onset of a background at large W and k^2 can be seen. This background will become the scaling curve at very large k^2.

calculated with $C_3(k^2)$ alone and are seen to account nicely for both σ_T and σ_L; hence we feel that $G_3(k^2)$ is consistent with zero.

π^0 electroproduction

Several experimental studies of π^0 electroproduction have been made by Mistretta *et al.*,[30] by

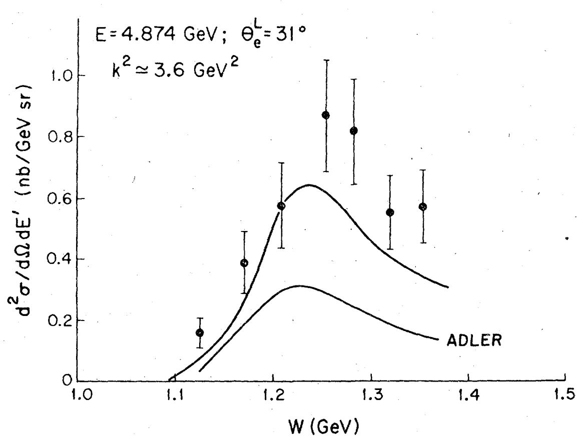

FIG. 9. Inelastic ep data of Cone *et al.* (Ref. 27) at $k^2 \simeq 3.6$ GeV. Shown also on this curve is the prediction of Adler (Ref. 15).

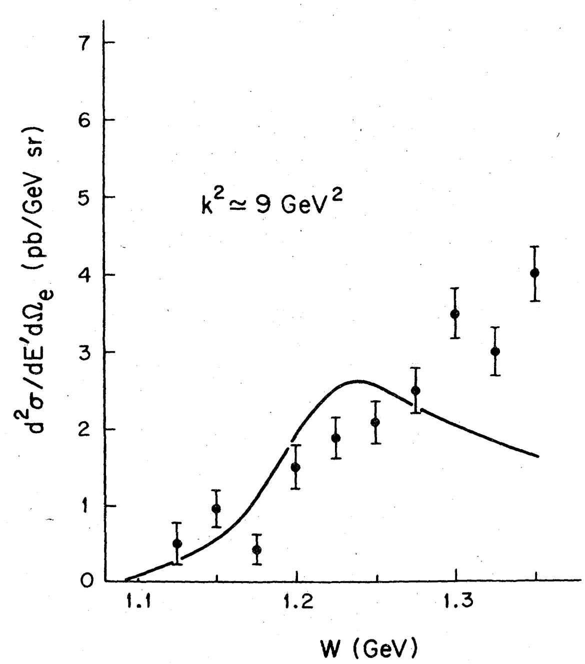

FIG. 10. Inelastic ep data of Atwood (Ref. 18) at very high k^2. The scaling background on the upper W range is clearly seen.

Albrecht *et al.*,[31,32] and others,[33] for W in the first resonance region and for k^2 from 0.25 to 1.0 GeV^2. These groups investigated the distributions in θ_π and ϕ_π of the virtual-photon differential cross section $d\sigma/d\Omega_\pi$ of Eqs. (5.3) and (5.4). $d\sigma/d\Omega_\pi$ may be rewritten as

$$\frac{d\sigma}{d\Omega_\pi} = A + B\cos\theta_\pi + C\cos^2\theta_\pi + (D + E\cos\theta_\pi)\sin\theta_\pi\cos\phi_\pi + F\sin^2\theta_\pi\cos2\phi_\pi . \tag{5.13}$$

The coefficients A through F may be expressed in terms of s- and p-wave

$$\begin{aligned}
A &= \frac{|\vec{q}|W}{MK}\{|E_{0+}|^2 + \tfrac{5}{2}|M_{1+}|^2 + \tfrac{9}{2}|E_{1+}|^2 + |M_{1-}|^2 + \mathrm{Re}[M_{1-}M_{1+}^* - 3E_{1+}(M_{1+} - M_{1-})^*] \\
&\quad + \epsilon\frac{k^2}{k_0^2}[|L_{1-}|^2 + 4|L_{1+}|^2 + |L_{0+}|^2 - 4\,\mathrm{Re}L_{1-}L_{1+}^*]\}, \\
B &= \frac{2|\vec{q}|W}{MK}\{\mathrm{Re}[E_{0+}(M_{1+} - M_{1-} + 3E_{1+})^*] + \epsilon\frac{k^2}{k_0^2}\mathrm{Re}[L_{0+}(4L_{1+} + L_{1-})^*]\}, \\
C &= \frac{3|\vec{q}|W}{MK}\left\{-\tfrac{1}{2}|M_{1+}|^2 + \tfrac{3}{2}|E_{1+}|^2 + \mathrm{Re}[M_{1+}(3E_{1+} - M_{1-})^* - M_{1-}E_{1+}^*] + \epsilon\frac{4k^2}{k_0^2}[|L_{1+}|^2 + \mathrm{Re}L_{1-}L_{1+}^*]\right\}, \\
D &= [\tfrac{1}{2}\epsilon(\epsilon+1)]^{1/2}(-2)\frac{|\vec{q}|W}{MK}\frac{k}{k_0}\{\mathrm{Re}[E_{0+}(L_{1-} - 2L_{1+})^* + L_{0+}(3E_{1+} + M_{1-} - M_{1+})]\}, \\
E &= [\tfrac{1}{2}\epsilon(\epsilon+1)]^{1/2}\frac{(-12)|\vec{q}|W}{MK}\frac{k}{k_0}\{\mathrm{Re}[E_{1+}(L_{1-} + L_{1+})^* + L_{1+}(M_{1-} - M_{1+})^*]\}, \\
F &= \frac{3\epsilon}{2}\frac{|\vec{q}|W}{MK}\{3|E_{1+}|^2 - |M_{1+}|^2 - 2\,\mathrm{Re}[E_{1+}M_{1+}^* + M_{1-}(M_{1+} - E_{1+})^*]\}.
\end{aligned} \tag{5.14}$$

The data and model predictions are graphed in Figs. 11–13. Generally, we find good agreement with the exception of the coefficient E, for which our model predictions are high, and the coefficient C, for which our model predictions are low, especially at high k^2. We also find, for the high-k^2 data, a crossing from negative to positive values for the coefficients B and D not seen in the data. In Fig. 14, our model predictions for the ϕ_π distributions at fixed θ_π and W are compared

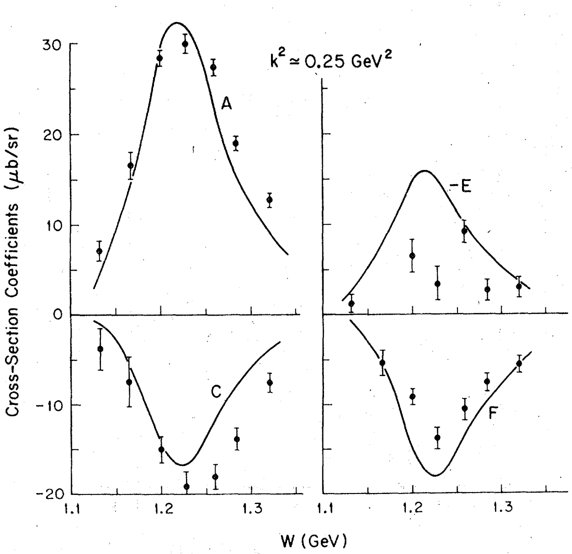

FIG. 11. Angular distribution coefficients as in Eq. (5.13) for the data of Mistretta *et al.* (Ref. 30) for π^0 electroproduction.

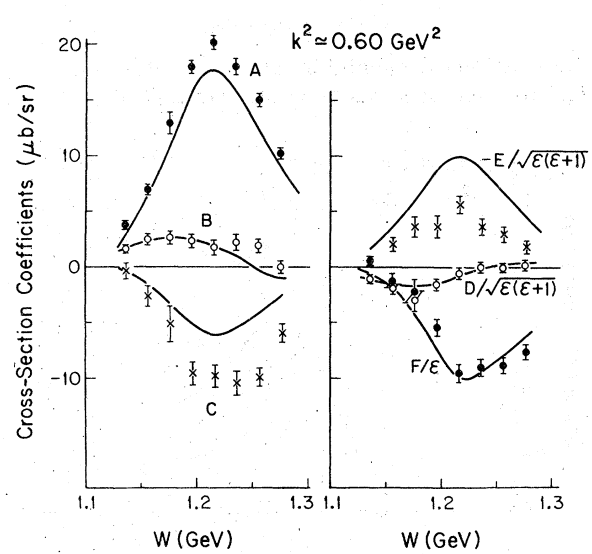

FIG. 12. Angular distribution coefficients for π^0 electroproduction as defined in Eq. (5.13). The data is from Albrecht *et al.* (Ref. 31).

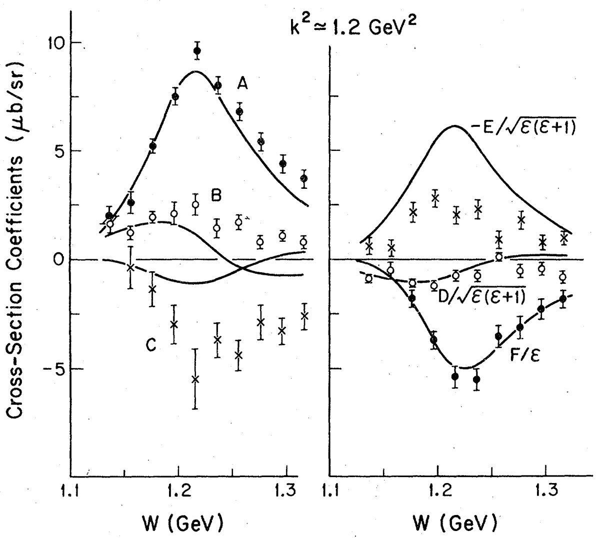

FIG. 13. π^0 electroproduction angular coefficients defined in Eq. (5.13). The experimental data is from Albrecht *et al.* (Ref. 32).

with the predictions of the dispersion-relation models of Adler[15] and Zagury[1] and with the data of Mistretta *et al.*[30] We find that the Lagrangian model tends to reproduce the data at least as well as, and often better than, these dispersion-relation models of electroproduction.

VI. DISCUSSION AND CONCLUSIONS

This paper has discussed electroproduction of a single pion in the Δ region. Our model is quite simple. It is based on Lagrangian couplings which ensure the content of current algebra and PCAC will be reflected in the scattering amplitude. The analytic structure of the resulting amplitude is correct in the tree approximation except for the three resonant multipoles. In these cases we modify the Lagrangian result such that the resonant multipoles have a pole on the second sheet and satisfy unitarity. We have considered the most general coupling of the Δ to $(N\gamma)$ and $(N\pi)$ and find only the nonderivative photon coupling $C_3(k^2)$ is required and that the off-shell couplings $\{Y, Z\}$ have the

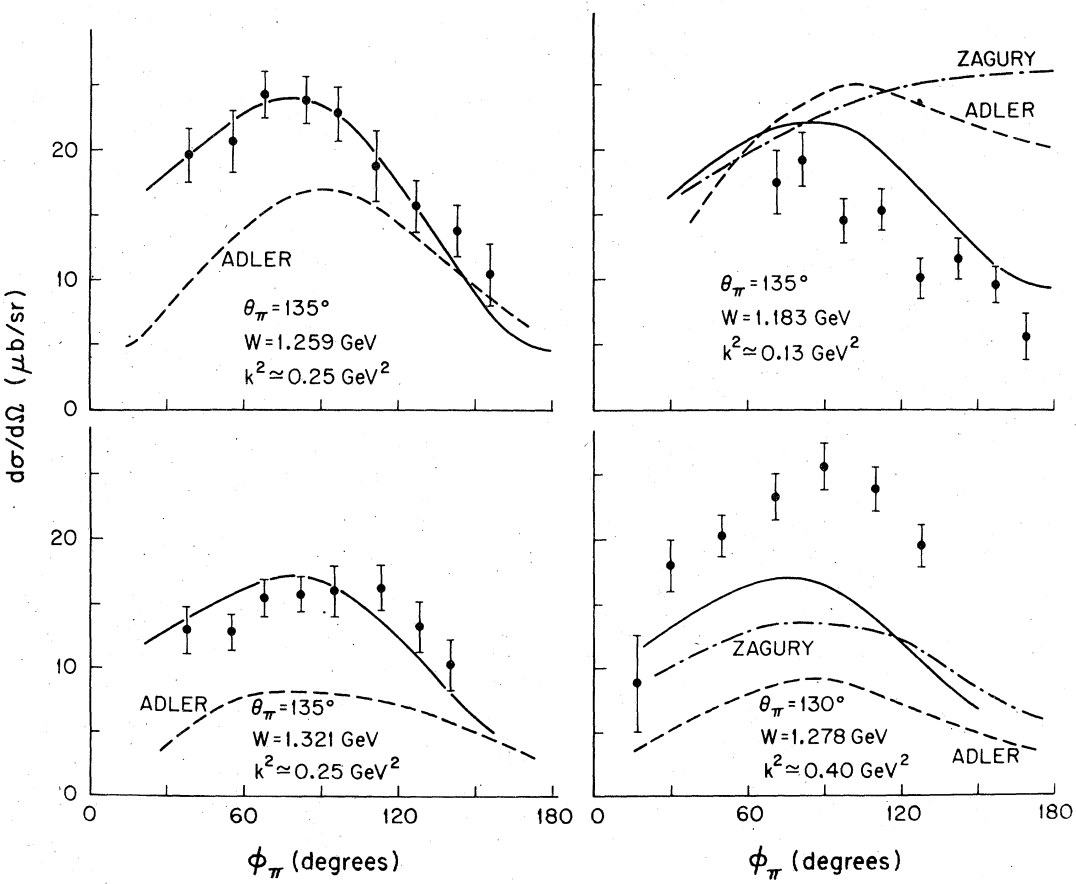

FIG. 14. ϕ_π angular distributions at fixed θ_π and W for π^0 electroproduction. The data is from Mistretta *et al.* (Ref. 30). The solid curves are our results. The dashed curves are from the models of Adler (Ref. 15) and Zagury (Ref. 1).

same values as in an earlier analysis of photoproduction.

The form factors, depending on the photon mass, are free in this model and by comparison with the data we have found that the $\Delta N\gamma$ transition form factor $C_3(k^2)$ falls off faster than the usual dipole expression. The additional factor $(1+k^2/4m_v^2)$ with $m_v^2=0.71$ GeV^2 accounts for the $\Delta N\gamma$ coupling strength out to $k^2 \simeq 9$ GeV^2. Our results for the nucleon axial-vector form factor and the pion form factor are consistent with earlier work.

We are able to account for a wide range of electroproduction and photoproduction data within a simple model with few parameters. There are, however, two refinements which might be made. First, we noted in the preceding section that since the higher-nucleon-resonance transition form factors do not fall off as rapidly as the Δ's the tails of these higher resonances could be important at large k^2 in the Δ mass range. At the highest available k^2 values of about 9 GeV^2, the individual-resonance effects seem to disappear and a smooth background remains which is consistent with the deep-inelastic scaling result. This background effect is of increasing importance at high k^2 in the Δ region as evident in Figs. 8 and 10.

A second improvement in our model concerns the inclusion of vector-mesnonic effects. As in the photoproduction case we expect ω and ρ exchange to manifest themselves primarily in the E_{0+} and M_{1-} multipoles of the isovector (+) and isoscalar amplitudes. These multipoles interfere with the dominant M_{1+} multipole in several of the π^0 angular coefficients given by Eq. (5.14).

Because of the substantial efforts already devoted to the problem of electroproduction we would like to discuss briefly the different approaches and compare them with ours. By far the most popular type of calculation[1] is an extension of the CGLN method[34] to electroproduction. In this calculation fixed-momentum-transfer dispersion relations are projected into an infinite set of coupled integral equations for the electric and magnetic multipoles in the "elastic" region where an additional pion cannot be produced. This set of equations breaks down when inelasticities are important and also contains little information about t-channel exchanges. The coupling of a given multipole to the others is described by a set of kernels in a coupling intergral. These kernels are algebraicly complex and their calculation is difficult. The solution of the integral equations is carried out in some approximation often in effect neglecting some of the kernel terms. Usually an ansatz solution is proposed for the dominant M_{1+} multipole and then one or more iterations of the integral equations are considered.

The main strength of the dispersive method is its "fundamental" nature since the Δ coupling and form factor are fixed within the model. The difficulties are mainly due to practical restriction to "elastic" amplitudes where Watson's theorem fixes the multipole phase. Since the integrals range over all energies some cutoff procedure must be devised. The low-energy behavior of the small multipoles (e.g., $E_0^{(+)}$) involve a cancellation between the large pseudoscalar Born terms and the rescattering contributions. The later is sensitive to high-energy behavior and thus the low-energy amplitude can be quite model dependent. In particular the low-energy theorems of current algebra and PCAC are not assured. The dispersive approach cannot say much about possible t-channel exchanges. Since in the usual model the k^2 dependence of the Born terms is specified the introduction of vector-meson exchanges may involve serious double counting. Finally, the numerical work involved in finding an accurate solution is great and there are difficulting associated with the numerical stability of a set of coupled singular integral equations.

An alternative procedure known as the "isobar" or "propagator" model[2] was introduced by Gourdin and Salin.[5] Here one evaluates Feynman diagrams for the various particle and resonance exchanges. The virtue is that now the calculation is mechanically very simple as there are no integral equations to solve. The equivalence of a propagator model to the dispersive calculation was demonstrated by Amati and Fubini.[35] Again, one encounters several problems. First, the soft-pion results do not automatically follow and secondly, without modification the propagator model will certainly violate unitarity since the resonances have been treated as stable particles.

Our model reformulates the propagator model to avoid the above defects. The effective Lagrangian method with axial-vector coupling ensures the soft-pion theorems and our unitarization method shifts the resonance pole from the real axis to its proper position in a way which maintains the unitarity of the entire multipole as well as its analytic properties. In addition we find that the "off-mass-shell" Δ coupling is quite important.[3,4]

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*Permanent address: University of Wisconsin, Marathon County Campus, Wausau, Wisconsin 54401.

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