Geometrical scaling for rising photon-nucleon cross sections in dispersion-relation analyses

Van Chang, N. W. Dean, and K. E. Lassila
Ames Laboratory-DOE and Department of Physics, Iowa State University, Ames, Iowa 50011
(Received 5 August 1977; revised manuscript received 3 January 1978)

Geometrical-scaling principles are used to deduce the asymptotic photoabsorption cross section. Insertion of such a logarithmically rising total photon-nucleon cross section into dispersion relations for the Compton amplitude suggests that previous conclusions about the real part must be modified.

Dispersion relations for the photon-nucleon amplitude were first evaluated in a classic paper by Damashek and Gilman[1] in 1970. Using the data available at that time, they calculated the real part of the spin-averaged forward amplitude $f(\nu)$ from the equation

$$\mathrm{Re} f(\nu) = -\frac{\alpha}{M_p} + \frac{\nu^2}{2\pi^2} \mathrm{P}\int_{\nu_0}^{\infty} \frac{d\nu'}{\nu'^2-\nu^2}\,\sigma_T(\nu')\,, \quad (1)$$

where α is the fine-structure constant, M_p is the proton mass, ν is the photon energy, ν_0 is the pion-photoproduction threshold, and $\sigma_T(\nu)$ is the total γN cross section. Comparison of this calculated, asymptotic real part with the real part predicted by standard Regge theory showed a constant difference in good agreement with the Thomson limit $-\alpha/M_p \sim -3.0$ μbGeV. (In Regge language this limit would correspond to a fixed pole at $J=0$.)

For the high-energy total cross section, however, they used a parametrization $\sigma_T(\nu)=\sigma_\infty + b\nu^{-1/2}$. More recently, predictions[2,3] have been made that the photon-nucleon total cross section would rise analogously to other total cross sections. (While this paper was in review, these predictions were fulfilled by new data in the energy range between 40 and 180 GeV.)

Clearly, a change in the high-energy behavior of $\sigma_T(\nu)$ will cause a change in the calculated value of Re $f(\nu)$ via (1). By the same token, a different asymptotic form of $f(\nu)$ will have a different Regge phase. Consequently, the question of whether the constant term in (1) is indeed consistent at high energy with the Regge phase must be re-evaluated. We report in this paper such an investigation, with the conclusion that the asymptotic form of the amplitude retains at most a small vestige of the fixed pole when geometrical scaling determines the rise.

The parametrization of $\sigma_T(\nu)$ which we use is based on the hypothesis of geometrical scaling,[4,5,6] which has produced numerous useful correlations among the high-energy properties of hadron elastic and total cross sections. We extend this hadron-projectile work to the photon case. The amplitude $T(s,b)$ as a function of the c.m. energy squared (s) and impact parameter b can be written as $T(b/R(s))$, where $R(s)$ is a radial scale parameter containing all the s dependence. As first[6] considered in proton-proton collisions, shrinkage suggests the proton is expanding so, e.g., for an inelastic collision, the matter density does not change with energy when plotted as a function of the scaled radius. Geometrical scaling thus accounts for the growth of total cross sections, constant cross-section ($\sigma_{\mathrm{elastic}}/\sigma_{\mathrm{total}} = \sigma_{\mathrm{el}}/\sigma_t$) ratios, constant slope- (B) to-cross-section ratios (B/σ_t), and shrinkage of the forward elastic peak. It was found that R was given by[4]

$$R^2 = R_0^{\,2} + R_1^{\,2}\ln s\,, \quad (2)$$

where $R_1/R_0 = 0.262$ and $\sigma_{\mathrm{el}} \propto R^2$, $\sigma_t \propto R^2$, $\sigma_{\mathrm{inel}} \propto R^2$, $B \propto R^2$. Such relations were shown to be valid for numerous other hadron-hadron collisions.[5]

From the Compton sum rule, it has been deduced that $\sigma_T(\nu)$ and the elastic γp cross section would show rising behavior above $\nu \sim 50$ GeV.[2] Because of the well documented hadronlike character of high-energy photon interactions, it seems natural to generalize the hadronic geometrical-scaling considerations by writing

$$\sigma_t(\gamma p) = N[1 + (0.262)^2 \ln s]\,. \quad (3)$$

The normalization N can be determined from the empirical observation[2,7] that the photon in hadron reactions is very hadronlike and a constant factor [7] $(217)^{-1}$ times πN data produces the corresponding γN result. We therefore know the asymptotic $\sigma_t(\gamma p)$ behavior from geometrical scaling and

$$\sigma_t(\gamma p) = (217)^{-1}(\tfrac{1}{2})[\sigma_t(\pi^+ p) + \sigma_t(\pi^- p)]\,,$$

so that

$$N = \tfrac{1}{2}(217)^{-1}(28.2\ \mathrm{mb})[\sigma_t(\pi^+ p)/\sigma_t(pp) + \sigma_t(\pi^- p)/\sigma_t(pp)]_{p_{\mathrm{lab}}=360\,\mathrm{GeV}/c}$$
$$= 81.7 \pm 1.5\ \mu\mathrm{b}\,, \quad (4)$$

where the ratio of cross sections in Eq. (4) is evaluated at the highest available pion momentum,[8]

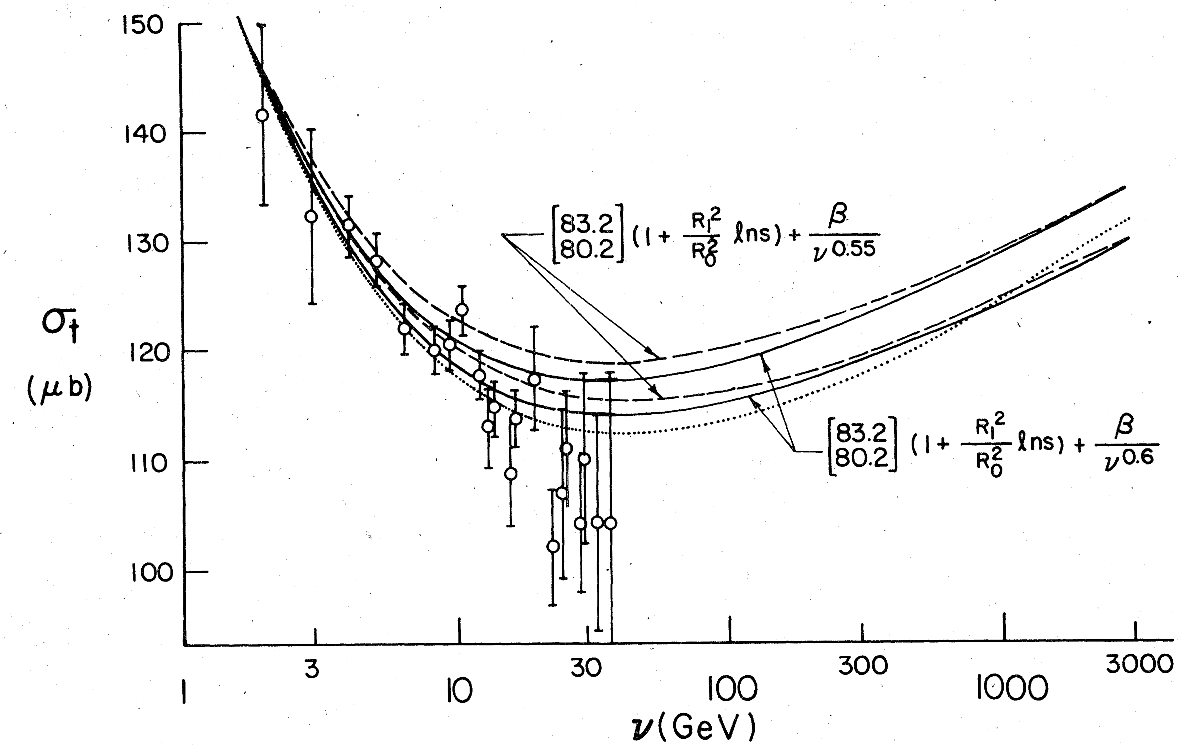

FIG. 1. Geometrical-scaling predictions for the photon-energy dependence of the photoabsorption cross section. The solid and dashed lines represent the bands within which $\sigma_t(\gamma p)$ should lie for Regge (see text) intercepts $\alpha(0)=0.40$ and 0.45, respectively. The dots represent a fit to the Daresbury, SLAC, and Serpuhkov actual data, plus theoretically simulated data reproducing the results of geometrical scaling up to 3000 GeV of the form $\sigma_t = C_1 + C_2\nu^{-1/2} + C_3 \ln\nu$; $C_1 = 67.83$ μb, $C_2 = 102.3$ μb, and $C_3 = 7.737$ μb, when ν is in GeV. All curves match the end of the resonance region at 1.68 GeV.

and $N = 28.2$ mb for the pp case.

To evaluate the Compton-amplitude dispersion integrals, we need $\sigma_t(\gamma p)$ as a function of photon laboratory energy E_γ (or ν) for all energies. The low-energy resonance region up to $E_\gamma = 1.68$ GeV is well parametrized for us by the work of Damashek and Gilman. The high-energy dependence required by geometrical scaling is displayed in Fig. 1 for the two approaches taken. First, it was required that the strict functional form of Eq. (3) hold at large energies, with standard Regge trajectories (P' and A_2) with average intercepts $\alpha(0) = 0.45$ or 0.40 to account for the falling cross section in the intermediate-energy region. The bands in Fig. 1 therefore are the regions bounded by

$$\sigma_t = (80.2 \text{ and } 83.2\ \mu\text{b})(1 + 0.07 \ln s) + \beta\nu^{\alpha(0)-1}, \tag{5}$$

with the solid curves given by $\alpha(0) = 0.40$ and the dashed curves by $\alpha(0) = 0.45$, with β constrained to match the end of the resonance region at $\nu = 1.68$ GeV. The curves describe the data[9] obtained at SLAC quite well but are high relative to the Serpukhov results[10] in the 13.4–36.9 GeV range. The 89 data points between 2 and 4.2 GeV measured at Daresbury[11] are very well described; two representative points at 2 and 3 GeV are on the plot. The other approach used the SLAC, Daresbury, and Serpukhov data[9-11] to find the best fit of the form

$$\sigma_t(\gamma p) = C_1 + C_2\nu^{-1/2} + C_3 \ln\nu . \tag{6}$$

However, these data were unable to determine C_3, so we added (theoretical) data calculated from geometrical scaling Eq. (4) in the range $\nu = 300$–3000 GeV. The resulting fit is indicated by the dots in Fig. 1. Figure 2 shows the comparison between the real part of f as calculated using these parametrizations and the classic earlier result found by Damashek and Gilman, labeled DG on the plot. The notation for the other curves correspond to that used in Fig. 1. Clearly, the Ref curves following from geometrical scaling reach a minimum near $\nu = 15$ GeV, and turn toward zero above 15 GeV; in contrast, the DG curve smoothly decreases as $\sim\sqrt{\nu}$. When the DG curve was compared[1] with the real part of the simple-Regge-pole contribution

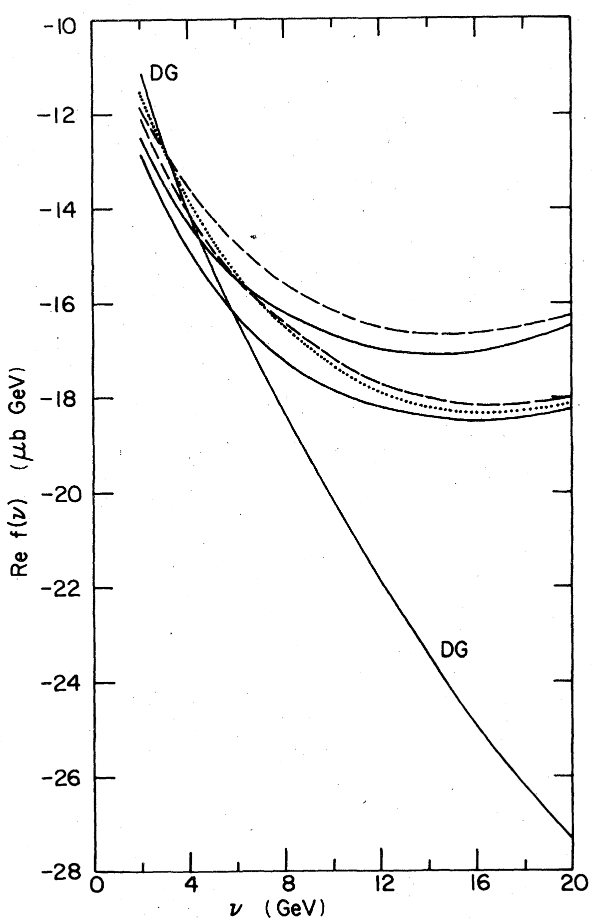

FIG. 2. Comparison of the real part of the Compton amplitude predicted by geometrical scaling with that deduced in the classic analysis of Ref. 8 labeled DG. The solid, dashed, and dotted curves given by geometrical scaling behavior follow the labeling of Fig. 1.

a constant difference of $-\alpha/M_p = -3$ μb GeV was found as required by a fixed pole satisfying the Thompson limit. (An alternative evaluation, using derivative analyticity relations,[12] agreed with the DG result, apparently without any indication of rising total cross sections.) This same comparison of a simple Regge-pole term with the real parts suggested by geometrical scaling indicates no such constant difference to characterize a fixed pole. But such a comparison is naive, as this real part should now be compared with the real part of the amplitude leading to (3). It is known[13] than an amplitude with correct analyticity and crossing symmetry can be obtained from the total cross section by replacing ν appropriately by ν/i , since a real analytic function of ν/i automatically satisfies those requirements. When the comparison is made with the real part obtained from this prescription, after the method of DG for the Regge comparison, a difference much less than -3.0 μb GeV results. Using the fits shown in Fig. 1, we find differences more of the order of -1 μb GeV.

A logarithmic rise in cross section thus implies, through dispersion relations, a considerably different real part to a Compton amplitude from that given by a monotonically decreasing cross section. Geometrical scaling, resulting from an increase in size of the photon-proton interaction in similar fashion to hadron-hadron interactions, requires that the cross section $\sigma_t(\gamma p)$ rise with a particular functional form. The resulting ratio of real to imaginary parts appears to behave similarly to that for e.g., pp scattering. We expect that the ideas of increasing photon-hadron interactions deduced from geometrical scaling for the diffraction

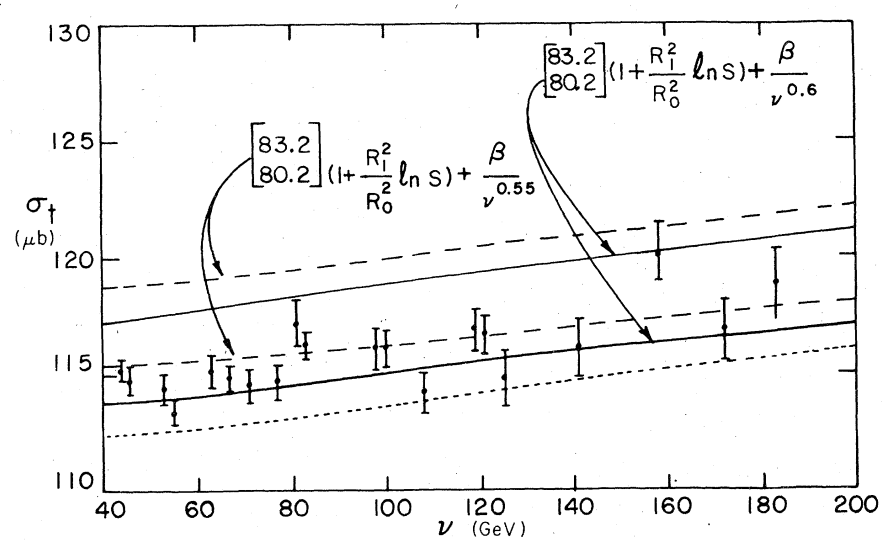

FIG. 3. Theoretical curves of Fig. 1 for the photon energy 40 GeV < ν < 200 GeV compared with data presented at the Hamburg Conference, Ref. 14.

region will soon be put to stringent tests.

A test has become possible since the initial submission of this paper. Figure 3 shows data submitted to the Hamburg Conference by Caldwell *et al.*[14] compared with the curves in Fig. 1; agreement is excellent. Correspondingly, since then another approach has been given by Margolis,[15] who suggests that vector-meson states in the $J/\psi, \psi', \ldots$ sequence explain the data shown in Fig. 3. Vector-meson dominance was used to deduce the contribution of such hidden charm states to the photoabsorption cross section. Likewise, contributions from the photoproduction of many particles, one of which has a c quark and another particle a $\bar{c}$ quark, can be deduced from recent reports.[16] However, these are not really relevant[17] to the present work. To use the dispersion integral reliably we require a reasonable representation of the photon-proton total cross section, $\sigma_T(\nu)$ in Eq. (1). Our curves in Fig. 3 essentially "blanket" the data shown in this figure and connect smoothly to the earlier low-energy data shown in Fig. 1. Our representation is above the measurements in the Serpuhkov energy range, as are the new data, and the presentations[14] of these new data do not include comparisons with Ref. 10 but simply the smooth transition into the SLAC range.[9] In these data, no sudden increase in $\sigma_T(\nu)$ is evident anywhere which might be taken as an explicit threshold effect. However, we[18] have parametrized the charm-production cross section in the manner of Barger[5] which describes the threshold behavior of photoproduction extremely well. The effect on the real part of the spin-averaged amplitude is very small. Therefore we feel our evaluation of the Compton dispersion integral within the framework of the geometrical-scaling picture is quite reliable.

This work was supported by the U.S. Department of Energy, Division of Basic Sciences. One of the authors (K.E.L.), would like to thank Dr. B. L. Young for conversations during their early work, Ref. 2, and all thank Professor F. Halzen for useful discussions on Ref. 16.

[1]M. Damashek and F. J. Gilman, Phys. Rev. D 1, 1319 (1970).

[2]B. L. Young and K. E. Lassila, Phys. Rev. D 12, 3424 (1975). (We note that the second entry in the first column in Table I labeled a should be 23.12 mb rather than 28.12 mb.)

[3]V. Chang, N. Dean, and K. Lassila, Iowa State University Report No. ISU 4232, submitted to the International Conference on Photon and Lepton Interactions, Hamburg, 1977 (unpublished). Also, W. Weise, Phys. Rep. 13C, 53 (1974), discusses the possibility that the photon-proton cross section follows the asymptotic form of H. Cheng and T. Wu [Phys. Rev. Lett. 24, 1456 (1970)] with $\sigma_T(\nu) = A + B \log^2(s/s_0)$. The values $s_0 = 125$ GeV^2, $A = 105$ μb, and $B = 24$ μb are suggested as possible parameters on the basis of data below 20 GeV photon energy. The author concedes that this "consideration is purely hypothetical," and this prediction is in poor agreement with the new measurements given in Fig. 3. However, Weise does calculate the real part of $f(\nu)$ with the result that Re$f(\nu)$ will change sign at $\nu \sim 200$ GeV.

[4]V. Barger, in *Proceedings of the XVII International Conference on High Energy Physics at London, 1974*, edited by J. R. Smith (Science Research Council, Rutherford Laboratory, Chilton, Didcot, Berkshire, England, 1974), p. I-193.

[5]V. Barger and R. J. N. Phillips, Nucl. Phys. B97, 452 (1975); V. Barger, J. Luthe, and R. J. N. Phillips, *ibid.* 88, 237 (1975); V. Barger, Lecture given at the McGill Institute of Particle Physics International Summer School, 1975 (unpublished).

[6]J. Dias de Deus, Nucl. Phys. B59, 231 (1973); A. J. Buras and J. Dias de Deus, *ibid.* 71, 481 (1974).

[7]J. J. Sakurai, in *Laws of Hadronic Matter*, proceedings of the 1973 International Summer School "Ettore Majorana," Erice, Italy, edited by A. Zichichi (Academic, New York, 1975), p. 252.

[8]A. Firestone *et al.*, Phys. Rev. D 14, 2092 (1976), provide the $\pi^- p$ measurement at 360 GeV/c while the $\pi^+ p$ cross section is calculated from the fit given in Refs. 2 and 5.

[9]D. O. Caldwell *et al.*, Phys. Rev. D 7, 1362 (1973); Phys. Rev. Lett. 25, 613 (1970).

[10]A. S. Belousov *et al.*, Yad. Fiz. 21, 556 (1975) [Sov. J. Nucl. Phys. 21, 289 (1975)].

[11]T. A. Armstrong *et al.*, Phys. Rev. D 5, 1640 (1972).

[12]J. B. Bronzan, G. L. Kane, and U. P. Sukhatme, Phys. Lett. 49B, 272 (1974).

[13]See, e.g., N. W. Dean, *Introduction to the Strong Interactions* (Gordon and Breach, New York, 1976), pp. 262–264.

[14]D. O. Caldwell *et al.*, work presented at the International Conference on Photon and Lepton Interactions, Hamburg, 1977 (unpublished). These data were presented by Dr. W. Y. Lee in his rapporteur's talk. See also T. Nash, Report No. Fermilab-Conf-77/61-Exp (unpublished).

[15]B. Margolis, Phys. Rev. D 17, 1310 (1978).

[16]F. Halzen and D. M. Scott, Phys. Lett. 72B, 404 (1978); L. Jones and H. W. Wyld, Jr., Phys. Rev. D 17, 2332 (1978).

[17]We would like to thank F. Halzen for this observation

[18]Van Chang and K. E. Lassila, Iowa State University Report No. ISU 4327, 1977 (unpublished).