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# Hadronic polarization in neutrino scattering

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Using a parton-model framework we show how the polarization of unstable hadrons produced in neutrino scattering may be used to study the helicity structure of the weak neutral current. Consistency tests of the framework are also suggested.

Deep-inelastic electron and neutrino scattering has been extremely important in recent progress in the understanding of the fundamental forces. Experiment has supported a parton picture of hadrons which in turn has suggested that the strong interactions are described by an asymptotically free gauge theory. Neutrino scattering has deepened our understanding of the weak interaction, suggesting a unified weak and electromagnetic gauge theory. Most of the earlier experiments have focused on the energy dependence of the cross section. Recently, however, there has been considerable theoretical and experimental interest in the hadronic final state produced in deep-inelastic scattering.[1] In the parton language, the struck quark fragments into the observed hadrons, and these fragments contain information about the original quark. For example, this has been used by Sehgal to unravel the isospin sturcture of the weak neutral current.[2] The work presented here describes a method whereby hadronic polarization in the final state may be used to determine the quark helicity structure of the weak currents. Another way to obtain the same information from the spatial distribution of fast pions has been suggested by Nachtmann.[3]

To present the method, consider first the process $\nu+p\rightarrow\mu+\rho^0+X$ at value of $x$ where the quark sea may be neglected. More general cases will be considered later. At the quark level, the reactions of interest are

$$\begin{aligned}&\nu+d\rightarrow\mu^-+u\,,\\ &\overline{\nu}+u\rightarrow\mu^++d\,,\\ &\nu(\overline{\nu})+u\rightarrow\nu(\overline{\nu})+u\,,\\ &\nu(\overline{\nu})+d\rightarrow\nu(\overline{\nu})+d\,.\end{aligned}\tag{1}$$

We employ the parton framework that allows us to separate the quark production and quark fragmentation processes. SU(2) invariance tells us that the fragmentation distributions of $u$ and $d$ quarks into $\rho^0$ are identical. In the charged current reactions the $u$ and $d$ quarks are produced lefthandedly (i.e., with helicity $-\frac{1}{2}$). It is possible that this helicity may result in a polarization of the $\rho^0$. To study this, define the fragmentation function including helicity indices,

$${}^{\lambda\lambda'}\rho_{\mu\mu'}\,,$$

which is a double helicity density matrix describing the transfer of polarization from a quark system described by the indices $\lambda,\lambda'$ to a $\rho^0$ with indices $\mu,\mu'$. That is, the density matrix for the $\rho$ is given by

$$\rho_{\mu\mu'}=\sum_{\lambda\lambda'}F_{\lambda\lambda'}\,{}^{\lambda\lambda'}\rho_{\mu\mu'}\,,\tag{2}$$

where $F_{\lambda\lambda'}$ is the helicity density matrix for the quarks. We will use the notation of

$${}^{(-1/2)(-1/2)}\rho_{\mu\mu'}={}^{L}\rho_{\mu\mu'}\,,$$

$${}^{(1/2)(1/2)}\rho_{\mu\mu'}={}^{R}\rho_{\mu\mu'}\,.$$

This matrix is a function of $z=E_{\rho^0}/E_{\text{quark}}$.

Hermiticity implies

$${}^{\lambda\lambda'}\rho_{\mu\mu'}=({}^{\lambda'\lambda}\rho_{\mu'\mu})^*\tag{3}$$

and parity conservation of the fragmentation process implies

$${}^{(-\lambda)(-\lambda')}\rho_{-\mu-\mu'}=(-1)^{\mu-\mu'+\lambda-\lambda'}\,{}^{\lambda\lambda'}\rho_{\mu\mu'}\,.\tag{4}$$

The most convenient normalization is

$$1=\sum_{\mu}{}^{L}\rho_{\mu\mu}=\sum_{\mu}{}^{R}\rho_{\mu\mu}\,.\tag{5}$$

In charged-current (cc) processes, both for neutrinos and antineutrinos, the density matrix for the $\rho^0$ is

$$\rho_{\mu\mu'}={}^{L}\rho_{\mu\mu'}\,.\tag{6}$$

We study the density matrix by the decay of the $\rho^0$, $\rho^0\rightarrow\pi^+\pi^-$, in the $\rho^0$ rest frame. The $z$ axis is the $\rho^0$ direction of motion, and we choose the $x$ axis to be in the plane of the total charged-hadron momentum. A better choice for the axis would be in the plane of the actual quark direction

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$q$, but this is generally not experimentally known. The fluctuations of the charged-hadron direction about the quark direction will result in a weakening of the desired signal. Note, however, that the distributions that we will be mainly interested in have a node on the $x$ axis and hence are least sensitive to such fluctuations. The decay distribution for the $\pi^+$ traveling in direction $\theta, \phi$ is

$$W(\theta,\phi)=\frac{1}{4\pi}\Big[1+\tfrac{1}{2}(1-3\cos^2\theta)(\rho_{11}+\rho_{-1-1}-2\rho_{00})-3\sqrt{2}\,\sin\theta\cos\theta\cos\phi\,\mathrm{Re}(\rho_{10}-\rho_{-10})$$
$$-3\sin^2\theta\cos2\phi\,\mathrm{Re}\rho_{1-1}+3\sqrt{2}\,\sin\theta\cos\theta\sin\phi\,\mathrm{Im}(\rho_{10}+\rho_{-10})+3\sin^2\theta\sin2\phi\,\mathrm{Im}\rho_{1-1}\Big]. \tag{7}$$

The last two terms in the angular distribution would be absent if parity were conserved in production process and hence are directly sensitive to the quark helicity. Experimentally these may be observed by the following asymmetries:

$$A_1=(1/N_{\mathrm{tot}})[N(0\le\theta\le\tfrac{1}{2}\pi,\ 0\le\phi\le\pi)+N(\tfrac{1}{2}\pi\le\theta\le\pi,\ \pi\le\phi\le2\pi)$$
$$-N(0\le\theta\le\tfrac{1}{2}\pi,\ \pi\le\phi\le2\pi)-N(\tfrac{1}{2}\pi\le\theta\le\pi,\ 0\le\phi\le\pi)]$$
$$=(2\sqrt{2}/\pi)\,\mathrm{Im}(\rho_{10}+\rho_{-10}) \tag{8}$$

and

$$A_2=(1/N_{\mathrm{tot}})[N(0\le\phi\le\tfrac{1}{2}\pi)+N(\pi\le\phi\le\tfrac{3}{2}\pi)-N(\tfrac{1}{2}\pi\le\phi\le\pi)-N(\tfrac{3}{2}\pi\le\phi\le2\pi)]$$
$$=(4/\pi)\,\mathrm{Im}\rho_{1-1}. \tag{9}$$

If $A_i^{\mathrm{cc}}$ is measured to result from the fragmentation of left-handed quark, parity conservation of the fragmentation process tells us that $-A_i^{\mathrm{cc}}$ would result if the quark were right-handed, i.e., these asymmetries change sign under $L\to R$. These terms will be called "helicity sensitive." The other terms in the angular distribution are even under $L\to R$, and will be referred to as "helicity even."

These asymmetries are not readily calculable. It is ultimately an experimental question as to whether or not they are nonzero. Their observations would be interesting in themselves. The fact that neutrino and antineutrino charged-current scattering produce the same polarization when sea quarks are unimportant is a strong test of the model. In addition, since these asymmetries are a property of the fragmentation process, once they are measured in the charged currents they may be used to obtain information about the neutral current, as we now demonstrate.

The effective neutral-current (nc) interaction may be written as

$$H^{\mathrm{nc}}=K\bar{\nu}\gamma_\mu(1+\gamma_5)\nu[g_L^u\bar{u}\gamma^\mu(1+\gamma_5)u+g_R^u\bar{u}\gamma^\mu(1-\gamma_5)u+g_L^d\bar{d}\gamma^\mu(1+\gamma_5)d+g_R^d\bar{d}\gamma^\mu(1-\gamma_5)d+\ldots]. \tag{10}$$

The overall scale $K$ is unimportant in a polarization measurement. The quarks are produced with a helicity density matrix (not yet normalized) in neutrino scattering,

$$F_{\lambda\lambda'}(x,y)=\begin{bmatrix}(1-y)^2[u(x)(g_R^u)^2+d(x)(g_R^d)^2] & 0\\ 0 & [u(x)(g_L^u)^2+d(x)(g_L^d)^2]\end{bmatrix}. \tag{11}$$

For antineutrino scattering the $(1-y)^2$ factor multiplies the left-handed coupling constants instead of the right-handed ones. The density matrix for $\rho^0$ production is then

$$\rho_{\mu\mu'}=\frac{F_{\lambda\lambda'}(x,y)^{\lambda\lambda'}\rho_{\mu\mu'}(z)}{\mathrm{Tr}F}. \tag{12}$$

This can be expressed entirely in terms of the density matrix measured in charged current reactions. Those elements of the angular distribution that are even under $L\to R$ [i.e., $(\rho_{11}+\rho_{-1-1}-2\rho_{00})$, $\mathrm{Re}(\rho_{10}-\rho_{-10})$, and $\mathrm{Re}\rho_{1-1}$] are equal to their charged-current values, independent of $x$ and $y$. However, the helicity-sensitive asymmetries $A_1$ and $A_2$ are changed.

If we denote by $\eta_i$ the integral over the experimental region in $x$ of the distribution function for quark $i$, e.g.,

$$\eta_u=\int u(x)\,dx\,, \tag{13}$$

and average over all $y$, the asymmetries are of the form[4]

$$A_i^{\nu}=\frac{[\eta_u(g_L^u)^2+\eta d(g_L^d)^2]-\frac{1}{3}[\eta_u(g_R^u)^2+\eta_d(g_R^d)^2]}{[\eta_u(g_L^u)^2+\eta_d(g_L^d)^2]+\frac{1}{3}[\eta_u(g_R^u)^2+\eta_d(g_R^d)^2]}A_i^{cc} \tag{14}$$

for neutrino scattering, and

$$A_i^{\bar\nu}=\frac{\frac{1}{3}[\eta_u(g_L^u)^2+\eta_d(g_L^d)^2]-[\eta_u(g_R^u)^2+\eta_d(g_R^d)^2]}{\frac{1}{3}[\eta_u(g_L^u)^2+\eta_d(g_L^d)^2]+[\eta_u(g_R^u)^2+\eta_d(g_R^d)^2]}A_i^{cc} \tag{15}$$

for antineutrino scattering. Note that both asymmetries $A_1$ and $A_2$ are reduced by the same factor. As an example, let us consider $\nu p$ scattering in the Weinberg-Salam model using $\eta_u/\eta_d=2$. Then with $S\equiv\sin^2\theta_W$,

$$A_i^{\nu}=\frac{9-20S+8S^2}{9-20S+16S^2}A_i^{cc}, \tag{16}$$

$$A_i^{\bar\nu}=\frac{9-20S-24S^2}{9-20S+48S^2}A_i^{cc}. \tag{17}$$

These are plotted as a function of $\sin^2\theta_W$ in Figs. 1 and 2. Note how sensitive $A_i^{\bar\nu}$ is to values of $\sin^2\theta_W$ in the region of interest.

For $\sin^2\theta_W=0.3$ we have

$$A_i^{\nu}=0.84\,A_i^{cc}, \qquad A_i^{\bar\nu}=0.11\,A_i^{cc}. \tag{18}$$

Measurement of these asymmetries in neutral-current reactions yields information on the helicity structure of the neutral current. This information is also available from the traditional analysis of inclusive neutral-current scattering. The above procedure provides an independent method that is directly sensitive to helicity of the quarks.

Another test of the model would be in electroproduction. There parity invariance forbids either of the asymmetries, but the three helicity-even combinations of density matrix elements are predicted to be the same as in neutrino reactions.

The major experimental problem in the study of $\rho^0$ production is the background of $\pi^+\pi^-$ not associated with a $\rho^0$. The backgrounds can be studied outside the $\rho$ peak. The simplest expectation is that the background will not exhibit the asymmetries that we are interested in. They are manifestations of a tensor polarization and it seems unlikely that the uncorrelated production of two pseudoscalars would mimic this polarization. If this is the case, then a study of the asymmetries of the "$\rho^0$ plus background" system will yield the desired signal. However, the asymmetries must be normalized by the total number

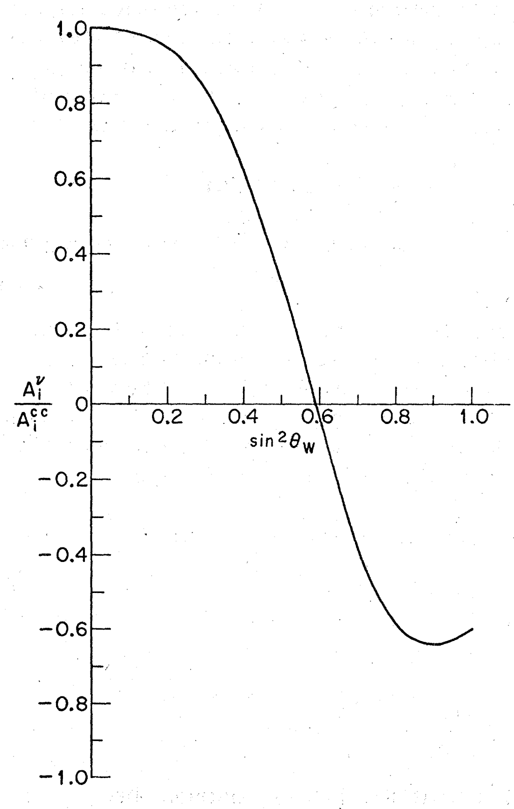


FIG. 1. The ratio of neutral-current to charged-current asymmetries for neutrinos.

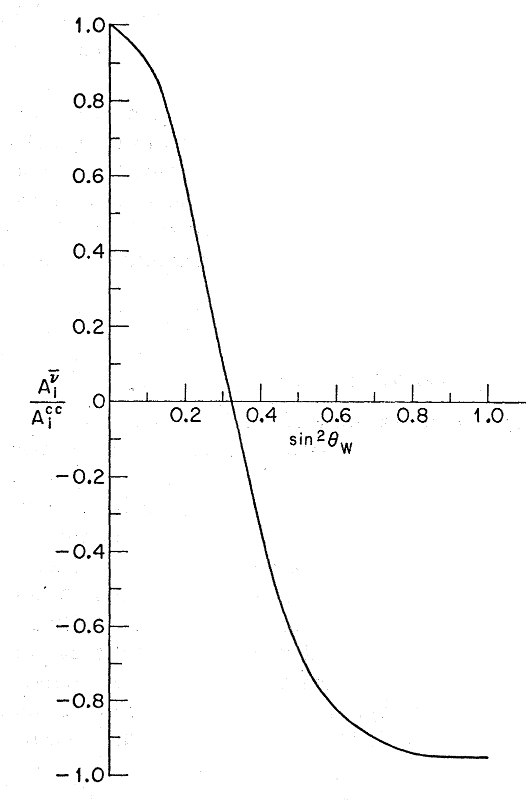


FIG. 2. The ratio of neutral-current to charged-current asymmetries for antineutrinos.

of $\rho^0$'s, not the total number of $\pi^+\pi^-$ pairs in the $\rho^0$-plus-background system. This is because the parton model suggests that the number of $\rho^0$'s scales with the multiplicity of pions, while the number of false $\pi^+\pi^-$ pairs scales roughly with the square of the multiplicity. If the background does exhibit the asymmetries in question they would also be helicity sensitive and contain the same information as the $\rho^0$ signal. The study of the background would then be as enlightening as a study of the $\rho^0$ decay.

To include the sea quarks in the charged current define

$$r(x)=\frac{\bar{u}(x)}{d(x)}\,,\qquad r'(x)=\frac{\bar{d}(x)}{u(x)}\,, \tag{19}$$

where both $r(x)$ and $r'(x)$ are expected to be small except at low $x$. The density matrix for a left-handed antiquark to produce a $\rho^0$ is the same as a right-handed quark. If ${}^L\rho_{\mu\mu'}$ is the density matrix measured above small $x$, the density matrix at all $x$ is

$$\begin{aligned}{}^{\nu cc}\rho_{\mu\mu'}&=\frac{{}^L\rho_{\mu\mu'}+r(x)(1-y)^2(-1)^{\mu-\mu'}\,{}^L\rho_{-\mu-\mu'}}{\langle 1+r(x)(1-y)^2\rangle}\,,\\ {}^{\bar\nu cc}\rho_{\mu\mu'}&=\frac{(1-y)^2\,{}^L\rho_{\mu\mu'}+r(x)(-1)^{\mu-\mu'}\,{}^L\rho_{-\mu-\mu'}}{\langle(1-y)^2+r'(x)\rangle}\,,\end{aligned} \tag{20}$$

Again, the helicity-even elements of the angular distribution are unchanged and remain independent of $x$ and $y$. The antineutrino distribution is most sensitive to the sea content. For example, if we average over all $x$, $y$, and find $\langle r\rangle=\langle r'\rangle=0.1$, the helicity-sensitive asymmetries are reduced by

$$\begin{aligned}A_i^{\nu\,cc}&=0.94\,A_i^{cc}\,,\\ A_i^{\bar\nu\,cc}&=0.54\,A_i^{cc}\,.\end{aligned} \tag{21}$$

Thus the $x$ dependence of the asymmetries in charged-current processes provides another test of the model. In neutral current reactions the production of strange quarks and antiquarks from the sea brings in fragmentation functions not measured in charged current processes. This implies an uncertainty in the low-$x$ region of the analysis of neutral-current $\rho^0$ production. This uncertainty is expected to be small, but there is no general way to make this expectation precise. Neglecting strange quarks however, it is a simple exercise to put in the sea quarks as was done in Eq. (20).

This method may be applied to other semi-inclusive reactions also. In these other processes however, it is necessary to neglect the sea quarks entirely unless experiments become precise enough in the future to sort out their effects by the $x$ and $y$ distributions.

In the process $\nu+T\rightarrow K^{*\pm}+X$, the procedure is to measure the left-handed up quark fragmentation matrix to $K^{*\pm}$ in the neutrino charged-current interaction, and that of the left-handed down quark in antineutrino interactions. The neutral current is then formed by

$${}^{\nu}\rho_{\mu\mu'}=\frac{F^u_{\lambda\lambda'}{}^{\lambda\lambda'}\rho^u_{\mu\mu'}+\gamma F^d_{\lambda\lambda'}{}^{\lambda\lambda'}\rho_{\mu\mu'}}{\mathrm{Tr}(F^u+\gamma F^d)}\,, \tag{22}$$

where $F^u$ and $F^d$ are the portions of Eq. (11) referring to $u$ and $d$ quarks respectively and $\gamma$ is the measured total ratio of quarks fragmenting to $K^{*\pm}$ relative to $u$ quarks,

$$\gamma=\frac{D_d^{K^*}(z)}{D_u^{K^*}(z)}\,. \tag{23}$$

The decay distributions and asymmetries are the same as for the $\rho^0$, but are sensitive to different combinations of $u$ and $d$ quarks. In the neutral current the correct formulas may be obtained from Eqs. (14) and (15) by the replacement $\eta_d\rightarrow\gamma\eta_d$.

If fast hyperons are observed in the quark fragments they will provide other interesting quantities as their weak decay yields information on their polarization in the production plane. Again $u$ and $d$ fragmentation matrices must be measured separately in neutrino and antineutrino charged current reactions. The angular distribution of the final-state baryon in hyperon decay is

$$\begin{aligned}W(\theta,\phi)=\frac{1}{4\pi}\,[&1-2\,\mathrm{Im}\rho_{+-}\sin\theta\sin\phi\\ &+(\rho_{++}-\rho_{--})\alpha\cos\theta+2\,\mathrm{Re}\rho_{+-}\,\alpha\sin\theta\cos\phi]\,.\end{aligned} \tag{24}$$

The first term is helicity even, and will be the same in all processes, while the last two are helicity sensitive, changing sign if the fragmenting quark changes from $L\rightarrow R$. The quantity $\alpha$ is the usual $S$-$P$ interference term. There are two asymmetries of interest here,

$$\begin{aligned}A_3&=\frac{1}{N_{\rm tot}}\,[N(\theta\leq\tfrac12\pi)-N(\theta\geq\tfrac12\pi)]\\ &=\tfrac12\,\alpha(\rho_{++}-\rho_{--})\end{aligned} \tag{25}$$

and

$$\begin{aligned}A_4&=\frac{1}{N_{\rm tot}}\,[N(-\tfrac12\pi\leq\phi\leq\tfrac12\pi)-N(\tfrac12\pi\leq\phi\leq\tfrac32\pi)]\\ &=\alpha\,\mathrm{Re}\rho_{+-}\,.\end{aligned} \tag{26}$$

If $A_i^u$ and $A_i^d$ are the asymmetries observed in neutrino and antineutrino charged currents, the neutral-current asymmetry will be

$$A_i^{\nu} = \frac{\eta_u[(g_L^u)^2 - \frac{1}{3}(g_R^u)^2]A_i^u + \gamma\,\eta_d[g_L^{d^2} - \frac{1}{3}(g_R^d)^2]A_i^d}{\eta_u[(g_L^u)^2 + \frac{1}{3}(g_R^u)^2] + \gamma\,\eta_d[(g_L^d)^2 + \frac{1}{3}(g_R^d)^2]}, \tag{27}$$

with an obvious change for $A_i^{\bar{\nu}}$.

The measurements suggested in this paper are not easy and are only now becoming experimentally possible. If they are observed, they will provide us with a better understanding of neutrino scattering, and may help us to unravel further the physics involved in this important process.

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[1]This interest dates back to R. P. Feynman, *Photon Hadron Interactions* (Benjamin, New York, 1972). For a review of recent work see L. M. Sehgal, Aachen Report No. PITHA-(1977), NR-81 (unpublished).

[2]L. M. Sehgal, Phys. Lett. 71B, 99 (1977).

[3]O. Nachtman, Nucl. Phys. B127, 314 (1977).

[4]Other averages over $y$ are easy to do, and they weight the helicities in a different way.