Unconfined quarks and gluons*

A. De Rújula
The Physics Laboratories, Harvard University, Cambridge, Massachusetts 02138

R. C. Giles and R. L. Jaffe
Center for Theoretical Physics, Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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We explore the possibility that fractionally charged colored quarks and electrically neutral colored gluons may exist as real particles. Our framework is a renormalizable, spontaneously broken version of color dynamics [quantum chromodynamics (QCD)], in which the gluons are given a common mass μ in the Lagrangian. Color SU(3) remains an exact global symmetry, and for small μ our understanding of color-singlet hadrons in QCD is unaltered. For small μ the masses of physical quarks and gluons are large: $O((2\pi\alpha'\mu)^{-1})$ with α' the slope of Regge trajectories. There is at least one stable hadron for every representation of color SU(3). In the unbroken QCD limit ($\mu\rightarrow 0$) the conventional picture of color confinement is recovered. To implement quasiconfinement in detail we use the MIT bag model. For $\mu = 0$ this model describes confinement. For $\mu \neq 0$, it describes quasiconfinement with no further modification. Production cross sections for quarks and gluons turn out to be very small. Quark-nucleon and gluon-nucleon cross sections, on the other hand, are very large. Quarks and gluons have a large nuclear appetite: Sequential absorption of nucleons by a free quark is exothermic up to large values of total baryon number. If quarks are very heavy, primordial quarks left over from the big bang will be found in superheavy quark-nucleon complexes with large nonintegral charge and baryon number. Primordial gluons will have the appearance of integrally charged superheavy nucleon complexes.

I. INTRODUCTION

Candidates for fractionally charged matter have been reported on several occasions, the first one by Millikan himself.[1] Two new very intriguing candidates recently have been reported.[2] This gives us an excuse to speculate on the theoretically unfashionable but most interesting possibility that unconfined fractionally charged quarks and unconfined electrically neutral colored gluons may exist.

Over the past few years the point of view that fractionally charged quarks should be confined in colorless hadrons has gained general acceptance. Quantum chromodynamics (QCD), a gauge theory in which the interactions between colored quarks are mediated by massless vector colored gluons, has emerged as the best candidate for a theory of the strong interactions. In QCD gluons are massless and conventional (u, d, s) quarks are light. QCD is formally analogous to QED in many respects, the major exception being the non-Abelian nature of color SU(3). If the gauge group of QCD is an exact local symmetry the color-triplet quarks and color-octet gluons of the theory are not expected to exist as real particles. But suppose that the evidence for unconfined quarks became uncontroversial. We would then have to face the following questions: Is the existence of unconfined quarks and gluons compatible with our successful understanding of colorless hadrons in a QCD context? Does Archimedes's principle[3] apply to quarks and gluons, i.e., could they behave as light particles inside colorless hadrons and yet be heavy when unconfined? We will answer both questions affirmatively. At a more phenomenological level we would like to develop a theoretical framework to guide further searches for unconfined quarks and gluons. At this level we study the masses of unconfined quarks and gluons, and make rough estimates of production cross sections in different reactions. We investigate the binding of quarks with nucleons relevant to the searches for quarks in stable matter.

It is conjectured that if the local gauge symmetry of QCD remains *unbroken*, only color-singlet hadrons exist. A proof of color and quark confinement in QCD does not exist. For practical purposes we must rely on models such as the MIT bag model,[4,5] where the hadron is an extended structure *ab initio* and confinement is automatic in QCD if the gauge symmetry is unbroken. Such models, despite their shortcomings, are rather successful at describing the properties of color-singlet hadrons.[6,7]

In the bag model, local energy-momentum conservation imposes boundary conditions on quark and gluon fields at the bag's surface. Together with the Gauss law for exact QCD, these force all states to be color singlets. Our strategy to allow for the existence of unconfined quarks and gluons is to make the minimum alteration in this scheme. We implement a spontaneous breakdown of the local

SU(3) color gauge symmetry but retain the *global* SU(3) color symmetry and the calculational framework of the bag model. By breaking the local gauge symmetry spontaneously we generate gluon mass terms in the Lagrangian while maintaining the sacrosanct renormalizability of the theory. We use a specific Higgs mechanism to generate a common Lagrangian mass for all eight gluons A_α^a, $a=1\cdots8$:

$$\mathcal{L}_\mu = \tfrac{1}{2}\,\mu^2 A_\alpha^a A^{a\alpha}\,. \tag{1.1}$$

The free parameter μ measures the breakdown of the local gauge symmetry and is zero in the standard QCD limit. When μ is no longer zero the color field of a single quark of gluon is of a Yukawa type. Gauss's law no longer holds; the bag boundary conditions can be satisfied, and colored objects are allowed to exist. As $\mu\to 0$ the boundary conditions clash with the emerging Gauss law. Colored objects become infinitely large and infinitely heavy and finally leave the spectrum altogether when μ is zero. Because color remains an exact global symmetry, familiar hadrons remain color singlets. Our understanding of their spectrum remains intact. Exact QCD confines color exactly, approximate QCD approximately.

Our model predicts that the masses of free quarks and gluons are

$$M_Q = \frac{1}{2\pi\alpha'\mu} + O(\mu^{1/3})\,, \tag{1.2a}$$

$$M_G = \tfrac{3}{2} M_Q + O(\mu^{1/3})\,, \tag{1.2b}$$

where $\alpha' = 0.88\ \text{GeV}^{-2}$ is the slope of Regge trajectories. Notice that as $\mu\to 0$ the masses of free quarks and gluons diverge as μ^{-1}, and the conventional picture of color confinement is recovered. We consider this inverse relation between gluon masses in the Lagrangian and the masses of free quarks and gluons to be a very satisfactory property of our model. For small μ, quarks and gluons are light "inside" color-singlet hadrons and heavy "outside".[3] The successful phenomenology of confinement requires any realistic model of quark quasiconfinement to satisfy this principle in one way or another.[8]

In Sec. II we discuss the problem of breaking of the local SU(3) invariance of QCD via the Higgs mechanism while retaining color SU(3) as an exact global symmetry. We are forced to introduce color triplets of Higgs mesons whose masses and couplings must be adjusted to avoid the formation of low-lying fractionally charged bound states. Whether these problems are serious depends on whether Higgs mesons are real or just an artifact of our poor understanding of spontaneous symmetry breakdown. Finally, we discuss the approximate asymptotic freedom of broken QCD.

In Sec. III we compute the masses of colored objects in the bag model. We find that the leading $O(1/\mu)$ results of Eq. (1.2) are independent of the details of the model. The mass of a colored object turns out to be linear in its color charge, defined as the square root of the quadratic Casimir operator of the color-SU(3) representation to which the colored object belongs. It is a corollary that there is at least one stable colored object per SU(3) representation We discuss these questions and the spectrum of colored states in Sec. IV.

In Sec. V we study the binding of nucleons to quarks. This question is most relevant to the searches for "natural" quarks in stable matter. The free quark is a large lump of hadronic matter with a radius $R\sim O(\mu^{-1/3})$. If a nucleon "falls" into a quark, the nucleon's constituent quarks can "relax" into the large quark lump with considerable reduction in their kinetic energy. A quark may exothermically absorb nucleons until the exclusion principle makes it unfavorable to absorb more nucleons into the quark cavity. We are confronted with the somewhat unusual problem of calculating the number of nucleons in a quark. We call the expected number of nucleons which combine with a single quark its "appetite," which we find to be of the order of magnitude $A_{\max}\sim M_Q/m_p$. If unconfined quarks are very massive, this may be a reason to expect to find quarks in association with heavy nuclei. In our model, free, massive, electrically neutral colored gluons must also exist. Gluons are also expected to absorb nucleons. Such states will not have the telltale fractional charge of quark nuclei, but will have unusually large masses.

In Sec. VI we discuss the search for quarks and gluons. The feature of our model most relevant to accelerator and cosmic-ray searches is the radius of the colored object, which is large by colorless-hadron standards. We argue that a large radius strongly suppresses production cross sections. With this suppression taken into account, the present lower limits on quark masses from accelerator experiments are only of the order of a few GeV. Further, a large radius implies quark-proton and gluon-proton cross sections that are larger than typical strong-interaction cross sections. We estimate, for instance, that

$$\sigma_{\text{tot}}(Qp) = \frac{1}{4}\left[1+\left(\frac{2M_Q}{m_p}\right)^{1/3}\right]^2. \tag{1.3}$$

The capture cross sections for the quark or gluon to absorb part or the totality of a nuclear target may be a large fraction of σ_{tot}.

We end the paper with a discussion of the most difficult problem: the cosmic history of primordial quarks. In the event that quarks and gluons are not permanently confined, nature may have seen fit to leave some about at the beginning of time. We are unable to estimate this initial density. However, we can conclude rather safely that any primordial quarks would have satiated their nuclear appetite during the early minutes. Such primordial quarks would not be found in the baryon-number-$\frac{1}{3}$ form but rather as anomalous heavy nuclei with fractional charge. Quarks created more recently, e.g., in collisions of high-energy cosmic rays, are more likely to be in their pristine $B=\frac{1}{3}$ state since collisions are very rare in the interstellar vacuum. Primordial gluons, likewise, may be found as anomalously heavy nuclei.

II. A CONSERVATIVE BREAKDOWN OF COLOR GAUGE SYMMETRY

Here we show that it is possible to break the color SU(3) gauge symmetry of QCD in such a way that we have the following:

(1) All eight gluons acquire the same mass μ.

(2) The theory remains renormalizable.

(3) Global color SU(3) remains an exact symmetry. Modifications of the spectrum and other properties of low-lying color-singlet states are small for small μ.

(4) The theory remains able to describe Bjorken scaling in regions explored to date, though deviations are possible at very large momentum transfers.

We present our arguments qualitatively in the first part of the section; all technicalities are postponed to the second part.

In order to preserve renormalizability, we introduce a spontaneous breaking of the symmetry in the manner of Higgs. We shall see below that the introduction of three elementary color-triplet, flavor-singlet Higgs scalars is sufficient for our purposes. We choose a Higgs potential whose symmetry ensures the survival of global SU(3) as an exact symmetry. It is a corollary that all eight gluons acquire a common mass μ, and the spectrum and quantum numbers of conventional color singlets are not affected.

The effects of elementary color-triplet scalars on the spectrum and asymptotic properties of the theory are potentially drastic. The color-singlet sector will contain many states in addition to the usual bound states of quarks. These include, for example, flavor-singlet bound states of scalars, and fractionally charged states consisting of a quark bound to a scalar.[9] If the scalars were light these would appear to be light fractionally charged hadrons.

We will not explore the spectrum of color-singlet states including Higgs scalars for two reasons. First, their masses may be made so heavy that Higgs-meson-containing states are of no phenomenological importance. Second (and more important), we suspect that the Higgs mesons may be only artifacts of our rather clumsy way of breaking local gauge symmetries and that nature, being more clever, can do it without introducing these new degrees of freedom. In any case we regard hypothetical color-singlet matter made of scalars and quarks as an even more speculative subject than unconfined quarks and will not pursue it here.

Elementary scalars destroy the asymptotic freedom of QCD. Asymptotic freedom is the standard explanation of the observed Bjorken scaling. To maintain Bjorken scaling in the explored domain of momentum transfers, we must again make the mass of the Higgs particles relatively large and/or their self-couplings relatively small. This ensures that the theory is "temporarily" free and that deviations from scaling only occur at very large momentum transfers.

We now turn to a brief technical description of a model embodying the ideas outlined above. The technique to break the local SU(3) gauge symmetry in such a way that color remains exact as a global symmetry has been invented by Bardacki and Halpern in a different context and has recently been discussed by Mohapatra, Pati, and Salam.[10] Let there be three color triplets of Higgs scalars ϕ_1, ϕ_2, ϕ_3, that we arrange into a 3×3 matrix,

$$\Phi = (\phi_1, \phi_2, \phi_3) = \begin{bmatrix} \phi_1^R & \phi_2^R & \phi_3^R \\ \phi_1^B & \phi_2^B & \phi_3^B \\ \phi_1^G & \phi_2^G & \phi_3^G \end{bmatrix} . \tag{2.1}$$

Color SU(3) acts on columns and the SU(3) to be left over as a global symmetry acts on rows; Φ is a $(3,\bar{3})$ under $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_G$. The most general renormalizable $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_G$-invariant Higgs-meson self-couplings correspond to the potential

$$V(\Phi) = M^2\,\mathrm{Tr}A + m(a\,\mathrm{Tr}\Phi^\dagger\Phi - 2\det\Phi) + \lambda_1\,\mathrm{Tr}(A^2) + \lambda_2(\mathrm{Tr}A)^2 + \mathrm{H.c.}\,, \tag{2.2}$$

where

$$A \equiv \Phi^\dagger\Phi - b^2\underline{1} \tag{2.3}$$

and a, b, m, and M are parameters with dimensions of mass.

We may use an $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_G$ transformation to write Φ in the diagonal form

$$\Phi = e^{i\delta}\begin{bmatrix}\alpha & & \\ & \beta & \\ & & \gamma\end{bmatrix} \tag{2.4}$$

with α, β, γ real. For $M^2 < 0$ the usual Higgs mechanism takes place and $V(\Phi)$ has an absolute minimum at

$$\alpha^2 = \beta^2 = \gamma^2 = a^2 = b^2 + \frac{|M|^2}{2(\lambda_1 + 3\lambda_2)}\,, \tag{2.5a}$$

$$\delta = 0\,. \tag{2.5b}$$

The Higgs fields have acquired nonzero vacuum expectation values (VEV's)

$$\langle\phi_1\rangle = \begin{bmatrix}a\\0\\0\end{bmatrix},\quad \langle\phi_2\rangle = \begin{bmatrix}0\\a\\0\end{bmatrix},\quad \langle\phi_3\rangle = \begin{bmatrix}0\\0\\a\end{bmatrix}. \tag{2.6}$$

The kinetic terms $D_\nu \Phi_i^\dagger D^\nu \Phi_i$ in the Higgs Lagrangian now generate a global-SU(3)-invariant common mass μ for all eight gluons. The values of μ and of the Higgs mass m_Φ are

$$\mu = |ga|\,, \tag{2.7a}$$

$$m_\Phi = |\lambda^{1/2} a|\,, \tag{2.7b}$$

where g is the QCD gauge coupling constant and λ is a combination of the quartic coupling constants λ_1 and λ_2 of Eq. (2.2). The above equations are to be interpreted[11] as masses renormalized at Euclidean momenta of the order of a. If the VEV a is small the coupling constants may be large and Eqs. (2.7) need not be numerically correct. They simply indicate that μ and m_Φ are independent and nonvanishing.

As we have already mentioned, the asymptotic freedom[12,13] of QCD does not in general survive spontaneous symmetry breakdown. The ultraviolet behavior of the theory we have constructed (and of many other theories) has been studied by Politzer and Ross.[14] We will present their results very schematically. The renormalization-group β functions of the gauge coupling constant g and the quartic coupling constants λ have the following form, to leading orders in g and λ:

$$\beta_g = -a_1 g^3 + b_1 g^3 \lambda\,, \tag{2.8a}$$

$$\beta_\lambda = a_2 g^4 + a_3 \lambda^2 - a_4 g^2 \lambda\,. \tag{2.8b}$$

In the above equations, and the remainder of the discussion, we refer to just one quartic coupling constant λ, to present the gist of our argument in a simplified context. The quantities a_i in Eqs. (2.8) are positive numbers.[14] In broken QCD the origin ($g = \lambda = 0$) is not an ultraviolet-stable point. But for initial values of g and λ that are not very large the theory is "temporarily" free. It mimics a truly asymptotically free theory: Coupling constants decrease with increasing momenta until enormous values of the relevant momenta.[14] The observed electroproduction scaling remains unaffected. For large values of the Higgs mass m_Φ our theory is even closer to conventional QCD than the theories explored by Politzer and Ross, who studied the limit where all masses are negligible relative to the relevant momenta. Let the momentum transfer in electroproduction be Q^2 and let all quantities be renormalized at Euclidean momenta M_0 with $M_0^2 \sim Q^2$. In the regime $Q^2 \ll m_\Phi^2$, the quantity b_1 in Eq. (2.8a) is now replaced by a power $(M_0/m_\Phi)^8 \ll 1$. The exponent in the suppression factor can be found by counting powers in diagrams such as the one shown in Fig. 1, which are responsible for the $g^3\lambda$ term of Eq. (2.8a). The suppression also occurs to higher orders in perturbation theory. This power suppression is a manifestation of the Appelquist-Carrazone[15] theorem, which (in brief) states that heavy particles do not affect renormalizable theories at momenta much smaller than their masses. The effects of the Higgs particles on β_g and on the analysis of electroproduction are negligible, provided the Higgs particles are sufficiently heavy. Yet, if unconfined quarks exist, the model we have outlined would predict the electroproduction scaling will not last forever.[16]

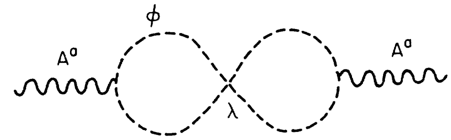

FIG. 1. A diagram responsible for the leading Higgs meson contribution to the momentum evolution of the running gauge coupling constant of broken QCD. The wavy lines are gluons. The dashed lines are Higgs mesons.

III. QUASICONFINEMENT IN THE BAG MODEL

We reformulate the bag model of QCD with massive gluons following the prescription of Sec. II. Inside the bag the quarks, gluons, and Higgs scalar obey the usual equations of motion of (spontaneously broken) QCD. At the bag's boundary each field obeys a "linear" boundary condition:

$$\text{quarks: } i\not{n}\, q_\alpha = q_\alpha\,, \tag{3.1a}$$

$$\text{gluons: } n_\mu F_a^{\mu\nu} = 0\,, \tag{3.1b}$$

where $\alpha(1,2,3)$ and $a(1,2\cdots 8)$ are color indices. (The Higgs scalars are irrelevant to this discussion.) The spacelike four-vector n_μ is the covariant outward normal to the bag's surface. There is an additional boundary condition corresponding to the physical requirement that the field pressure equal the universal bag pressure B ($B^{1/4}=146$ MeV) on the boundary

$$B=\sum_a \tfrac{1}{2}(\vec{E}_a^{\,2}-\vec{B}_a^{\,2}+\mu^2 A_a^{02}-\mu^2\vec{A}_a^{\,2})-\tfrac{1}{2}n\cdot\partial\bar{q}q\,, \tag{3.2}$$

where the sum on quark color indices is implicit. We must emphasize that Eqs. (3.1) and (3.2) are not only boundary conditions on the fields but also equations of motion for the surface of the bag. Static surfaces of various shapes are not, in general, consistent with Eqs. (3.1) and (3.2). Nevertheless the time-averaged structure of an unexcited bag state is thought to be adequately represented by approximating the bag as a static cavity.[17] For a static bag Eq. (3.2) requires that the total energy (field energy plus bag energy) be stationary with respect to local variations of the bag surface. In looking for low-energy configurations this requirement may be approximated by minimizing the total energy with respect to the bag's volume and a few parameters which describe its shape.

If the gluon mass were zero Eq. (3.1b) would guarantee color confinement. In the local rest frame of a given point on the surface it reads

$$\vec{n}\cdot\vec{E}_a=0\,, \tag{3.3a}$$

$$\vec{n}\times\vec{B}_a=0\,. \tag{3.3b}$$

When μ is zero Gauss's law combines with Eq. (3.3a) to force all hadrons to be color singlets. When μ is not zero, Eqs. (3.3a) and (3.3b) still hold, but Gauss's law does not. Colored states are possible. We proceed to calculate their masses to leading order in $1/\mu$.

Suppose a static cavity of arbitrary shape (volume V) contains a time-independent, classical color charge density $\rho_a(x)$ due to quarks and/or gluons. The equation for the color (scalar) potential is

$$(\nabla^2-\mu^2)A_a^0(x)=-g\rho_a(x)\,. \tag{3.4}$$

Integrating this over the bag we obtain

$$\oint_S \hat{n}\cdot\vec{\nabla}A_a^0\,d^2s-\mu^2\int_V A_a^0\,d^3x=-gQ_a\,, \tag{3.5}$$

where Q_a is the total color charge matrix. $\int_S d^2s$ spans the surface of the bag. In ordinary ($\mu=0$) QCD Eq. (3.5) reduces to the Gauss law. Combined with the bag boundary condition Eq. (3.3a) one obtains $Q_a=0$: confinement. In fact, the first term in Eq. (3.5) always vanishes on account of Eq. (3.3a), leaving

$$\int_V d^3x\,A_a^0=-gQ_a/\mu^2\,. \tag{3.6}$$

This may be satisfied in two ways. Either $Q_a=0$ and $\int_V d^3x A_a^0=0$, or $Q_a\neq 0$ and A_a^0 must contain a term which diverges as $\mu\to 0$. States of the first kind are ordinary, color-singlet hadrons. States with $Q_a\neq 0$ are unconfined quarks and gluons. The appearance of a divergence in $\int d^3x\,A_a^0$ is a manifestation of the well-known inconsistency of the Neumann boundary conditions for a classical electrodynamical system with nonzero total charge. We may isolate the divergent term in $A_a^0(x)$:

$$A_a^0(x)\equiv\frac{gQ_a}{\mu^2 V}+\bar{\Phi}_a(x)\,. \tag{3.7}$$

Upon substitution in Eq. (3.1) and (3.4) we find

$$(\nabla^2-\mu^2)\bar{\Phi}_a(x)=-g[\rho_a(x)-Q_a/V]\,, \tag{3.8a}$$

$$\hat{n}\cdot\vec{\nabla}\bar{\Phi}_a=0\,, \tag{3.8b}$$

$$\int\bar{\Phi}_a(x)\,d^3x=0\,. \tag{3.8c}$$

It is easy to show that $\bar{\Phi}_a(x)$ is well behaved as $\mu\to 0$:

$$\lim_{\mu\to 0}\bar{\Phi}_a(\mu^2)=\bar{\Phi}_a+O(\mu^2)\,. \tag{3.9}$$

For small gluon mass the leading contribution to the field energy within the bag comes from the $O(1/\mu^2)$ term in A_a^0:

$$E_f=\frac{2\pi\alpha_c C^2}{\mu^2 V}+O(1)\,, \tag{3.10}$$

where we have set $g^2=4\pi\alpha_c$ and $\sum_a Q_a^{\,2}\equiv C^2$, the quadratic Casimir operator of color SU(3) (our normalization is such that $C^2=\frac{16}{3}$ for a triplet representation). We argue that all other terms in E_f remain finite in the limit $\mu\to 0$. Quark and gluon kinetic energies are controlled by their masses or by $1/R$ (R is a typical cavity dimension), whichever is larger. The vector potential $\vec{A}_a$ does not contribute a term of order $1/\mu^2$ to E_f. The field equation for $\vec{A}_a$ is similar to Eq. (3.4), but its source, $\vec{j}_a$, is a static, divergenceless current with no monopole moment. Quantum fluctuations respect the global conservation of color charge and therefore will not require new shifts of order $1/\mu^2$ in A_a^0. Quantum corrections to our semiclassical calculations will therefore be of the same order as those in color-singlet bags. Thus we believe the result in Eq. (3.10) to be rather general.

The field pressure due to the leading term in A_a^0 is uniform. Therefore, to leading order in μ^{-2}, the pressure balance, Eq. (3.2), may be satisfied in a cavity of arbitrary shape. In this case Eq. (3.2) is equivalent to minimizing the total energy with respect to the volume V. The total energy of a colored bag combines the bag energy, BV, with the field energy E_f of Eq. (3.10). To leading order we have

$$E=\frac{2\pi\alpha_c C^2}{\mu^2 V}+BV, \tag{3.11}$$

whence

$$V=\frac{1}{4\pi\alpha'\mu B}\left(\frac{3C^2}{16}\right)^{1/2} \tag{3.12}$$

and

$$E=2BV=\frac{1}{2\pi\alpha'\mu}\left(\frac{3C^2}{16}\right)^{1/2}. \tag{3.13}$$

For convenience we have introduced the bag expression for the Regge slope[18]:

$$\alpha'=\frac{1}{16\pi}\ (3/2\pi\alpha_c B)^{1/2}. \tag{3.14}$$

Except for quark mass terms, which are small, the corrections to Eq. (3.11) are scaled by a typical linear dimension of the bag ($1/R$). Typically $R \sim V^{1/3} \sim \mu^{-1/3}$, so the corrections to Eq. (3.12) are $O(\mu^{1/3})$ and are indeed small for small μ.

The shape of the free-quark bag is determined by the next-order terms in the expansion of the field pressure in μ^2. These include pressures due to the residual color potential $\bar{\Phi}_a$ and the vector potential $\vec{A}_a$, and the pressure of confined quarks and gluons. The same terms appear in the study of color-singlet bag states. Thus we expect the lowest-lying colored states, like ordinary S-wave hadrons, to be spherical. The quark kinetic energy, for example, is minimized in a spherical shape (for fixed V) because the minimum quark momentum is determined by the smallest linear dimension.

A particular alternative to a spherical configuration, namely a stringlike shape, should be discussed in detail. It has been conjectured[8] that in lattice versions of QCD a free quark is a string (with a quark on one end) which peters out over a distance of order $1/\mu$. Since the string tension equals $(2\pi\alpha')^{-1}$, the mass of this stringlike quark would be of order $(2\pi\alpha'\mu)^{-1}$—in agreement with our shape-independent result [Eq. (3.13)]. In the bag model, however, the stringlike solution is unstable against collapse to a roughly spherical shape. We have checked this explicitly by estimating the energy of a stringlike bag of length L with a quark on one end. We find, minimizing E with respect to cross section and taking $L \sim 1/\mu$,

$$E(L)=\frac{1}{2\pi\alpha'\mu}\ (y \coth y)^{1/2}, \tag{3.15}$$

where $y \equiv \mu L$. Application of the pressure balance, Eq. (3.2), requires $E(L)$ to be minimized with respect to L. But $E(L)$ is a monotonically increasing function of L with a minimum $L=0$: $E(0) = (2\pi\alpha'\mu)^{-1}$. Thus the stringlike solution is energetically unstable against contraction.

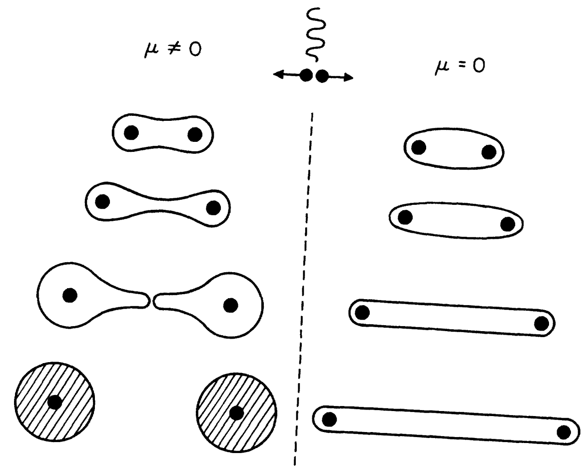

FIG. 2. Evolution of a $Q\bar{Q}$ pair produced in e^+e^- annihilation. The dominant process, the breakdown of the interquark color field into $Q\bar{Q}$ pairs, is not shown. For $\mu \neq 0$ the quarks may separate far enough for the hadron to fission, resulting in the liberation of quarks. For $\mu=0$ the color field strength does not weaken as the interquark separation increases, and quarks remained confined.

For μ identically zero, the bag state with a quark and antiquark separated by a large distance L is indeed stringlike.[18] This might seem to suggest that for $\mu \approx 0$ single quark states might remain stringlike. As we have discussed this is not the case: The limits $\mu \to 0$ and $L \to \infty$ do not commute. This is illustrated in Fig. 2. For $\mu=0$ isolated quarks do not exist. As L increases the stringlike bag grows longer and larger at a cost in energy of order $L/2\pi\alpha'$. For $\mu \neq 0$ and $\mu L \ll 1$ the state is "cigarlike" in shape, as in the $\mu=0$ case. As L becomes comparable to $1/\mu$ the bag becomes dumbbell-shaped and finally breaks into two roughly spherical bags of radius $\sim 1/\mu.^{1/3}$ With this picture of unconfined quark states we proceed to discuss their physical properties.

IV. THE SPECTRUM OF COLOR

The parameter μ measures the breakdown of the color gauge symmetry and μ^{-1} measures the overall mass scale of colored hadrons. Unfortunately, we can offer no theoretical predilection on the actual value of μ. The mass of a liberated quark can be read off Eq. (3.13) to be

$$M_Q=\frac{1}{2\pi\alpha'\mu}=0.18\ \text{GeV}^2\mu^{-1}. \tag{4.1}$$

A quark mass of 10 GeV, for example, corresponds to a value of μ of 18 MeV. This is considerably smaller than the inverse radius of colorless

TABLE I. The spectrum of color.

Color representation	Examples	Mass/M_Q
3	quark, diquark ($\bar{3}$)	1
8	octet, quark-antiquark	1.5
6	diquark, gluon+quarks	1.58
15	gluon+quark	2
10	triquark	2.12
15′	tetraquark	2.65

hadrons. Thus our understanding of them as systems of quarks bound by massless gluons will not be significantly affected.

The mass of a gluon (a color octet) can also be read off Eq. (3.13). The result is $m_G = 1.5 m_Q$. Thus we expect a gluon (or a quark-antiquark with octet color) to be stable under decay into a quark-antiquark pair. The masses of colored objects in different representations (normalized to the mass of an unconfined quark) are summarized in Table I. Recall that in our theory color is an exact global symmetry. Inspection of Table I then leads to the conclusion that the lowest mass state of a given color charge is stable. This result is general, and applies to all color charges, not only the ones in Table I. If a colored hadron in the $\underline{R}$ representation is to decay into two colored states in the $\underline{r}_1$ and $\underline{r}_2$ representations, respectively, global color conservation requires $\underline{R}$ to be contained in $\underline{r}_1 \otimes \underline{r}_2$. But this condition is sufficient to prove the following relation between Casimir operators (color charges):

$$C(\underline{R}) \leq C(\underline{r}_1) + C(\underline{r}_2), \tag{4.2}$$

where the equal sign holds only if $\underline{r}_1$ or $\underline{r}_2$ is a color singlet. According to Eq. (3.13) the mass of a colored state is proportional to the corresponding C. Thus Eq. (4.2) implies that the hadron $\underline{R}$ is below threshold to decay into $\underline{r}_1$ and $\underline{r}_2$. If color can be unconfined there is a very rich spectrum of stable colored hadrons—at least one for every distinct color-SU(3) representation.

Colored hadrons, like conventional hadrons, will have excited states. These can be radial excitations, angular excitations, or correspond to the addition of more quarks or gluons to the original colored hadron. In all cases, the scale of the splittings between states of the same color is set by R^{-1}, the inverse radius of the state. The radius of the state can be read off Eq. (3.12). The result is that the mass splittings between states of the same color C have an order of magnitude

$$\Delta(C) \sim \left(\frac{8B\pi}{3M_C}\right)^{1/3} \sim O(\mu^{1/3}), \tag{4.3}$$

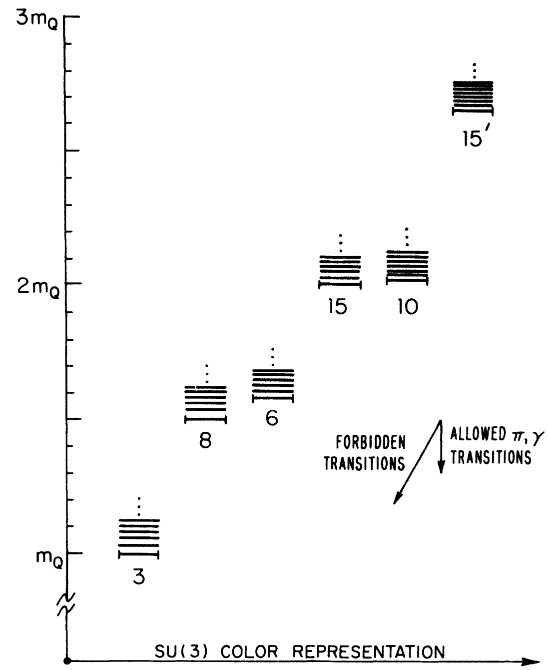

FIG. 3. The spectrum of color.

where M_C is the mass of the state. These mass splittings are small by conventional hadron standards. For a 10-GeV quark, for instance, they are of the order of 75 MeV. Figure 3 is a representation of the general features of the spectrum of color in our model.

We expect unconfined strange and charmed quarks to be heavier than unconfined up or down quarks by amounts comparable to the mass differences between the corresponding colorless hadrons. Unconfined s (c) quarks will decay into unconfined u (s) quarks with typical weak lifetimes. We expect the unconfined down quark to be a few MeV heavier than the unconfined up quark and to decay into it with a lifetime comparable to the neutron lifetime.

V. THE BINDING OF QUARKS AND NUCLEONS

Unconfined quarks and gluons are larger than ordinary hadrons. We predict that a 10-GeV quark, for example, has radius ~2.8 F. As μ goes to zero the quark radius diverges like $\mu^{-1/3}$. As mentioned in the Introduction, the large radius of unconfined quarks or gluons suggests that these particles will absorb baryons from ordinary matter on contact.[19] The dynamics of this process will be essential in understanding where free quarks or gluons reside in the real world and how they behave in the laboratory. Typically we will have to understand the behavior of an unconfined quark (or gluon) in the vicinity of a supply of color-singlet baryons. Examples of this situation are a primordial quark in the early universe, a cosmic-ray quark incident on the earth's atmosphere, or an accelerator produced quark traveling through an experiment's apparatus. Initially it will be energetically favorable for the quark to absorb a baryon:

$$Q+N\rightarrow(QN)+\pi's\,. \tag{5.1}$$

As more color-singlet quark triplets are added to the interior of the unconfined quark, the long wavelength modes in it fill up. Quarks from newly absorbed baryons are forced by the exclusion principle to more energetic modes. A Fermi degeneracy pressure builds up. Eventually the Fermi sea of quarks reaches such a level that it is no longer energetically favorable to absorb another baryon. At this point the chemical potential of the quarks equals one-third the mass of the nucleon.

We call the baryon number of the resulting object the "appetite," $A_{\max}$, of the original quark. We refer to an unconfined quark that has absorbed one or more nucleons as a "quark-nucleon complex" (or QNC). This is to emphasize that the binding of quarks and nucleons is, in our model, entirely different from the binding between nucleons. A quark-nucleon complex is, to lowest order, a single bag of many noninteracting quarks, *not* a collection of nucleons with an added quark.

To be more quantitative, consider a QNC consisting of $3N+1$ quarks and having the total color charge of a single quark. Ignoring for the moment electromagnetic effects, we identify three contributions to the QNC's energy: the color field energy, the kinetic and interaction energy of the quarks, and the bag energy.

The static color field energy is

$$E_1=2\pi\alpha_c C^2/\mu^2 V\,, \tag{5.2}$$

where V is the volume. As argued in Sec. III, E_1 depends only on the hadron's total color (to leading order in $1/\mu^2$) so the presence of a color-singlet quark sea is irrelevant. The quark energy is

$$E_2=\frac{3V}{2\pi^2}k_F{}^4 f(\alpha_c)\,, \tag{5.3}$$

where k_F is the Fermi momentum and $f(\alpha_c)$ is an interaction correction to be discussed soon. The Fermi momentum is determined by the relation

$$3N+1=\frac{2}{\pi^2}Vk_F{}^3\cong 3A\,. \tag{5.4}$$

In Eqs. (5.3) and (5.4) we have taken a degeneracy factor of 12 (3 colors, 2 spins, and 2 flavors). We ignore the possible appearance of strange quarks by leakage from the highly filled u and d quark Fermi seas.[20] The quantity $f(\alpha_c)$ measures the change in quark energies due to interquark interactions. $f(\alpha_c)$ depends only on α_c for dimensional reasons so long as the u and d quarks are massless and the strange (and charmed) quarks are ignored. The function $f(\alpha_c)$ has been evaluated perturbatively in studies of quark stars[21]:

$$f(\alpha_c)=1+\frac{8\alpha_c}{3\pi}+O(\alpha_c{}^2,\alpha_c{}^2\ln\alpha_c)\,. \tag{5.5}$$

Here α_c is the renormalization-group-improved QCD fine-structure constant, and is a function of k_F.[22] The values of k_F relevant to this calculation are small, $O(m_p/3)$. At small momenta, the value of α_c is large and cannot be reliably inferred from what is known about α_c from studies of deep-inelastic scattering.[13] Fortunately, the value of α_c at the relevant momenta can be inferred from the study of conventional colorless hadrons. The "hyperfine" splittings between colorless hadrons of different spins belonging to the same representation of SU(6) [spin SU(2)×flavor SU(3)] have been interpreted by the authors of Ref. 6 and Ref. 7 as being due to one-gluon exchange between constituent quarks. Comparison with experiment then yields $\alpha_c=0.55$ (Ref. 6) and $\alpha_c=0.4$ (Ref. 7). There is yet another independent determination. In the bag model Eq. (3.14) can be interpreted as a prediction for α_c in terms of the slope of Regge trajectories. This gives again the result $\alpha_c=0.55$.[18] For the purpose of illustration we will use $\alpha_c=0.55$ in Eq. (5.5). We cannot take the result too seriously. The reason is twofold. First, with so large a value of α_c higher orders in perturbation theory are bound to be significant. Second, and more to the point, our results are rather sensitive to $f(\alpha_c)$ so even small corrections due to higher orders in α_c will be important. Thus, we will also give predictions with $f(\alpha_c)$ treated as an independent unknown parameter.

The third contribution to the energy of a QNC comes from the bag pressure:

$$E_3=BV\,. \tag{5.6}$$

The final ingredient is the pressure balance at the bag's boundary. The quark pressure is $E_2/3V$; the static gluon contribution is identical to that of Sec. III:

$$B=E_2/3V+2\pi\alpha_c/\mu^2V^2\,. \tag{5.7}$$

We now have the necessary formulas to calculate the appetite of an unconfined quark and other relevant binding properties. In so doing, it is convenient to regard the physical parameters of the system as functions of the quark pressure measured in units of B: $x\equiv E_2/3BV$, with $x\leq 1$ required by Eq. (5.7). In terms of x we find for the total energy

$$E=\frac{1}{2\pi\alpha'\mu}\frac{1+x}{(1-x)^{1/2}}, \tag{5.8}$$

for the baryon number

$$A=\frac{1}{2\pi\alpha'\mu B^{1/4}}\left(\frac{1}{3\sqrt{\pi}}\right)\frac{[2x/f(\alpha_c)]^{3/4}}{(1-x)^{1/2}}, \tag{5.9}$$

and for the volume

$$V=\frac{1}{4\pi\alpha'\mu B}\frac{1}{(1-x)^{1/2}}. \tag{5.10}$$

To calculate the appetite it is necessary to know the rate of change of the mass of the system with baryon number

$$\frac{\partial E}{\partial A}=3(2\pi^2 Bx)^{1/4}f(\alpha_c)^{3/4}. \tag{5.11}$$

Equations (5.8)–(5.11) are to be understood as equations for E and V as functions of A.

Absorption of an additional baryon will be energetically favorable so long as $\partial E/\partial A$ is less than a proton's mass. The quark's appetite, $A_{\max}$, is that value of A for which $\partial E/\partial A$ first exceeds m_p. In principle it might be that $\partial E/\partial A$ never exceeds m_p regardless of how large A becomes. But for large enough A even the presence of a liberated quark (or gluon) becomes unimportant, so if $\lim_{A\to\infty}\partial E/\partial A < m_p$ it would appear that ordinary nuclei would prefer to collapse into single bags. There is ample evidence that this is not the case in nature. The parameters of our model must be such that $\lim_{A\to\infty}\partial E/\partial A \geq m_p$, and unconfined quarks have finite appetite. According to Eq. (5.9) $A\to\infty$ implies $x\to 1$. From Eq. (5.11) we obtain the condition

$$3(2\pi^2 B)^{1/4}f(\alpha_c)^{3/4}\geq m_p. \tag{5.12}$$

We use the universal bag constant $B^{1/4}=0.146$ MeV to obtain the inequality $f(\alpha)\geq 1.013$. If $f(\alpha)$ is only slightly greater than unity ordinary nuclei do not collapse into single bags and unconfined quarks have less than infinite appetite. Before we present our results for the quark appetite, we must make a final aside. In nuclear physics it is important to consider the effects of Coulomb energy: Heavy nuclei contain more neutrons than protons. The same will be true for quarks which have absorbed nucleons. For a given baryon number A there will be a "valley of stability" with its minimum at some nonzero T_3 (third component of isospin). We denote this value by $T_3(A)$. To include this effect, we have modeled a QNC as a uniformly charged sphere of radius R. The corresponding Coulomb energy is

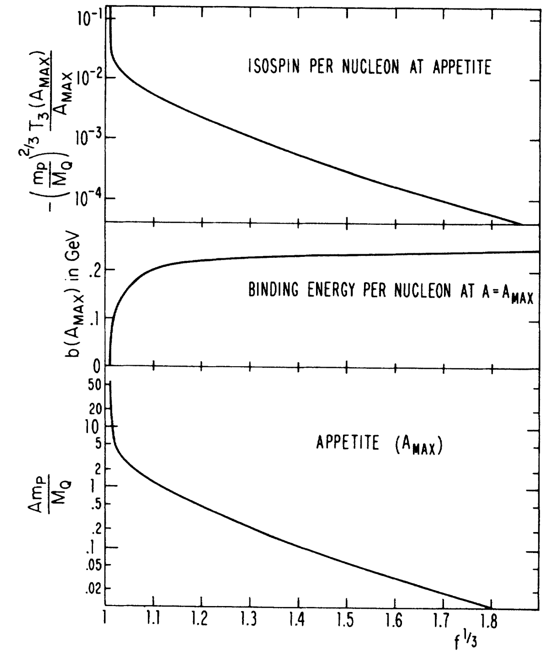

FIG. 4. The dependence of satiated quark parameters on $f(\alpha)$. The abscissa has been chosen to be $f(\alpha)^{1/3}$ for convenience. Powers of (M_Q/m_p) have been scaled out of the appetite and fractional isospin to give universal curves. The figure shows $(T_3/A)(m_p/M_Q)^{2/3}$, $b(A_{\max})$ in GeV, and $A_{\max}m_p/M_Q$.

$$E_c=\tfrac{3}{5}\alpha\frac{(A/2+T_3)^2}{R}. \tag{5.13}$$

We have modified our analysis to include this effect to order α and find

$$T_3(A)=-\tfrac{27}{20}\alpha\frac{A^2}{R}[2\pi^2 Bf^3(\alpha_c)x]^{-1/4}, \tag{5.14}$$

where x and R are the functions of A determined by Eqs. (5.9) and (5.10). This expression for the isospin of the most stable QNC of baryon number A has all the properties that one may have expected. It decreases as the radius increases and the Coulomb interactions vanish, and it is quadratic in A, reflecting the fact that the Coulomb energy is proportional to the number of *pairs* of charged constituents.

Our results are summarized in Figs. 4 and 5. Our model has two parameters: μ, which sets the scale and therefore can be incorporated in the way we label our axes, and $f(\alpha)$, which cannot. Therefore in Fig. 4 we have plotted everything as a function of $f(\alpha)$. The figure shows the appetite, $A_{\max}$, of an unconfined quark, the binding energy per nucleon,

$$b(A)\equiv m_p-(E-M_Q)/A \tag{5.15}$$

at $A=A_{\max}$, and the isospin of the QNC at the ap-

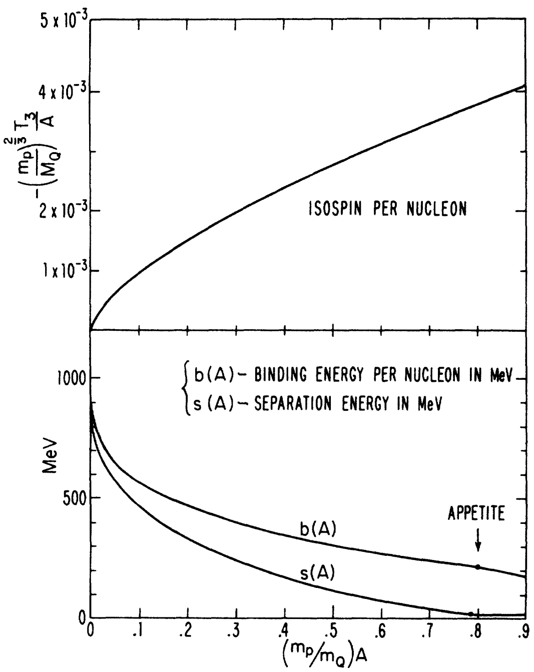

FIG. 5. Dependence of separation energy $S(A)$, binding energy $b(A)$, and fractional isospin T_3/A on A with $f(\alpha)=1.47$. The appetite and fractional isospin have been rescaled as in Fig. 4. The appetite is $A_{max}m_p/M_Q=0.83$.

petite. In presenting these figures we have ignored the effect of the Coulomb energy on the appetite and on $b(A)$. The effect is small (unless A is very large). Although the appetite measures the maximum number of nucleons which will coalesce with a quark to form a single hadron, this does not mean that further nucleons cannot bind to this system by conventional nuclear interactions. At A_{max}, however, the dynamics goes over from novel, color-dependent forces to conventional nuclear physics.

The reader will note from Fig. 4 that the value of A_{max}, and hence the scale of the entire QNC problem, is rather sensitive to $f(\alpha_c)$ especially as it approaches its minimum. To understand QNC physics quantitatively it is necessary to know both the quark mass and $f(\alpha_c)$. For the purposes of illustration, suppose the second-order estimate of $f(\alpha_c)$ [Eq. (5.5)] were adequate for $\alpha_c=0.55$ $[f(\alpha_c)=1.47]$. The resulting QNC physics is summarized in Fig. 5. There we have plotted quantities of interest from the nuclear physics point of view: the binding energy per nucleon, defined in Eq. (5.15), and the separation energy of the last nucleon,

$$S(A)\equiv m_p-\frac{\partial E}{\partial A} \tag{5.16}$$

and the isospin, $T_3(A)$, of the most stable QNC for each A. Both $S(A)$ and $b(A)$ break at the appetite, A_{max}, where we assume additional nucleons may be added to the system with typical nuclear binding energies.

For a specific choice of $f(\alpha_c)$, the appetite and isospin at the appetite are simply related to the quark mass. For $f(\alpha_c)=1.47$,

$$A_{max}=0.83\,M_Q/m_p\,, \tag{5.17a}$$

$$T_3(A_{max})=-3.2\times10^{-3}(M_Q/m_p)^{5/3}\,. \tag{5.17b}$$

The considerations of this section apply to gluons as well as quarks. Unconfined gluons will tend to absorb nuclear matter up to an appetite $\frac{3}{2}$ as large as that of the quarks. The formulas derived in this section all apply to a gluon-nucleon complex with the substitution of the quark mass by the gluon mass ($M_G=\frac{3}{2}M_Q$).

VI. THE SEARCH FOR UNCONFINED QUARKS AND GLUONS

In this section we discuss the implications of our model for quark hunts in accelerator and cosmic-ray experiments, and for searches for "natural" quarks bound in stable matter. Our considerations are very approximate. Many properties of hadrons are merely reflections of their size. We have seen that unconfined quarks, if heavy, are large compared to colorless hadrons. We suspect this to be a general feature of colored hadrons in the context of broken QCD and not merely a peculiarity of our particular model of quasiconfinement. We may draw two conclusions which form the basis for further phenomenological considerations:

(i) Production cross sections for unconfined quarks and gluons are probably much smaller than production cross sections for pointlike particles of the same mass.

(ii) Quark-matter or gluon-matter cross sections are probably much larger than proton-matter cross sections.

Quark-matter and gluon-matter cross sections

Quark hunters often assume in the analysis of their experiments[1] that quark-matter cross sections are relatively small. A primary or secondary cosmic quark, for instance, is sometimes assumed to have a large probability of descending unhampered from the upper atmosphere. Define the cross-section ratio

$$r_Q=\frac{\sigma_{tot}(Qp)}{\sigma_{tot}(pp)}\,. \tag{6.1}$$

It is sometimes argued that this ratio may be $\frac{1}{3}$. The argument is based on "quark additivity," a successful recipe in predicting the ratio of meson-nucleon to nucleon-nucleon high-energy cross sections. The same recipe would predict that gluon-matter cross sections vanish. We believe these arguments to be wrong.

In our broken-QCD model, a liberated quark or gluon is a blob of hadronic matter larger in size than a conventional colorless hadron, but with a typical hadronic density. The bulk of a total hadron-hadron cross section is due to soft peripheral processes, not to hard, short-distance quark-quark scattering. Total hadron-hadron cross sections are of the order of magnitude as black-disk geometrical cross sections. We estimate the ratio of quark-proton (or gluon-proton) cross sections to proton-proton cross sections to be the ratios of the corresponding geometrical cross sections. Combining the virial theorems $m_c = 2BV_c$ and $m_p = 4BV_p$ for a colored particle and proton, respectively, we find

$$r_c \equiv \frac{\sigma_{\text{tot}}(Cp)}{\sigma_{\text{tot}}(pp)} \sim \frac{1}{4}\left[1+\left(\frac{2M_C}{m_p}\right)^{1/3}\right]^2 . \quad (6.2)$$

For a 10-GeV quark or gluon, for instance, our estimate of $\sigma_{\text{tot}}(Cp)$ is ~145 mb.

We have argued in the preceding section that the capture of nucleons by heavy unconfined quarks or gluons is energetically a very favorable reaction. Thus, we expect a sizable fraction of the total quark-matter or gluon-matter cross section to result in nucleon capture. The signatures of a free quark or gluon in an accelerator search are unconventional and striking. An unconfined quark traveling through a detector is likely to increase in mass and positive charge by successive interactions. A free gluon may appear as a neutral particle that increases in mass and positive charge as it interacts.

Quark separation in electron-positron annihilation and other current-induced reactions

Define the cross-section ratios

$$R_Q(\text{exclusive}) = \frac{\sigma(e^+e^- \to Q\overline{Q})}{\sigma(e^+e^- \to \mu^+\mu^-)} , \quad (6.3a)$$

$$R_Q(\text{inclusive}) = \frac{\sigma(e^+e^- \to Q\overline{Q} + \text{anything})}{\sigma(e^+e^- \to \mu^+\mu^-)} , \quad (6.3b)$$

where Q ($\overline{Q}$) denotes an unconfined quark (antiquark). The ratio R_Q(exclusive) is, by definition, the square of the quark charge times a combination of its timelike electric and magnetic form factors. The calculation of a form factor is a task beyond our power, even for a colorless hadron. To estimate R_Q(exclusive) we would have to know how often the color flux lines developing between two separating electroproduced quarks manage *not* to break into quark-antiquark pairs. In conventional unbroken QCD with $\mu = 0$, the probability for this to happen is thought to vanish. But for $\mu \neq 0$ we expect the colored flux "tube" to become thinner for quark separations of $O(1/\mu)$ and to have a nonzero amplitude to break into liberated quarks, as in the first column of Fig. 2. We will refer to the probability amplitude for this to happen as the quark "separation factor." We proceed to develop a rough educated guess of the actual value of this quantity.

At small *spacelike* momentum transfers the form factors of known hadrons have the form

$$F(q^2) \sim \frac{1}{(1 - q^2\langle R^2\rangle/6)^p} , \quad (6.4)$$

where $p\langle R^2\rangle$ is the mean square radius of the charge distribution. The same form with roughly the same $\langle R^2\rangle$ also fits form factors at large spacelike $-q^2$. p is found to be 1 (monopole) or 2 (dipole). The *timelike* form factors of colorless hadrons, away from prominent resonances or averaged over sufficiently large intervals, Δq^2, are expected on analyticity grounds to be described by the same function, Eq. (6.4).

Qualitatively, the hadron radius enters the form factor because it characterizes the scale over which phase coherence must be maintained in order to preserve the identity of the struck object. Similarly, we expect the form factor of a quark to have a large negative slope at the origin, whose scale is determined by the quark radius R_c and which varies as $(-R_c^2 q^2)^{-p}$ for some power p. There exists an argument[23] that the power p can be predicted by counting the minimum number of gluons that must be exchanged to "inform" all quarks in a multiquark system that they must travel in the same direction as the electrically struck quark. For an unconfined quark this argument would predict $p = 0$. We do not expect this argument to apply to unconfined quarks. In the case of an unconfined quark, it is clearly necessary to "inform" the static color field of the struck quark to go along if the process is to be elastic.

Thus, we do not expect the form factors of a quark at large spacelike q^2 to be very different in functional form from the form factors of colorless hadrons. We analytically continue this expectation to the timelike domain and estimate a quark separation factor as in Eq. (6.4), with $\langle R^2\rangle$ computed for a uniform charge distribution within the radius R_c. Thus we have

$$R_Q(\text{exclusive}) \propto Q^2(1 + q^2 R_c^2/10)^{-2p} \times (1 - 4M_Q^2/q^2)^{1/2} , \quad (6.5)$$

where Q is the quark charge.

For the inclusive production of quarks we guess

$$R_Q(\text{inclusive}) \propto Q^2(1 + 4M_Q^2 R_c^2/10)^{-2p} \times (1 - 4M_Q^2/q^2)^{1/2} . \quad (6.6)$$

The appearance of $4M_Q^2$ in Eq. (6.6) instead of the q^2 of Eq. (6.5) reflects our belief that, above the threshold for quark production, any extra energy can be spent in pion production with no further suppression.

In Fig. 6 we have plotted R_Q(exclusive) and in Fig. 7 we show R_Q(inclusive) for charge $\frac{2}{3}$ quarks of different masses, and for $p = 1$ and 2. It is clear from the figures that, if our rough arguments are correct, it may be very difficult to detect quarks in e^+e^- annihilation, even if they are relatively light.

All current-induced reactions should be comparably inefficient at producing unconfined quarks. We expect the ratio of unconfined-quark production to the total cross section in electroproduction or neutrino scattering to be of the same order of magnitude as R_Q(inclusive), with the q^2 variable replaced by the invariant mass squared of the final hadronic system.

We have no argument to make a numerical estimate of gluon production in e^+e^- annihilation, but we expect it to be smaller than quark production. An unconfined gluon pair in the final state must have a large invariant mass ($>4m_G^2$). Feynman diagrams in which the original quarks transfer large momenta to the would-be unconfined gluons have large suppression factors in quark propagators. Thus we expect gluon production in current-induced reactions to be orders of magnitude below quark production.

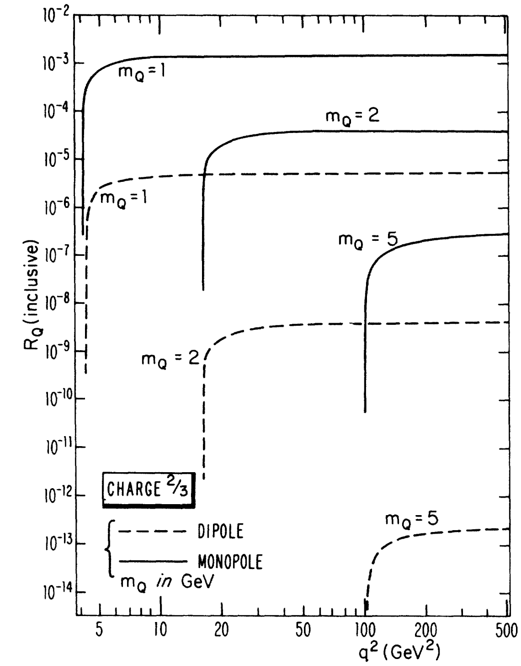

FIG. 7. Theoretical estimates of the ratio R_Q (inclusive). The labeling is as in Fig. 6.

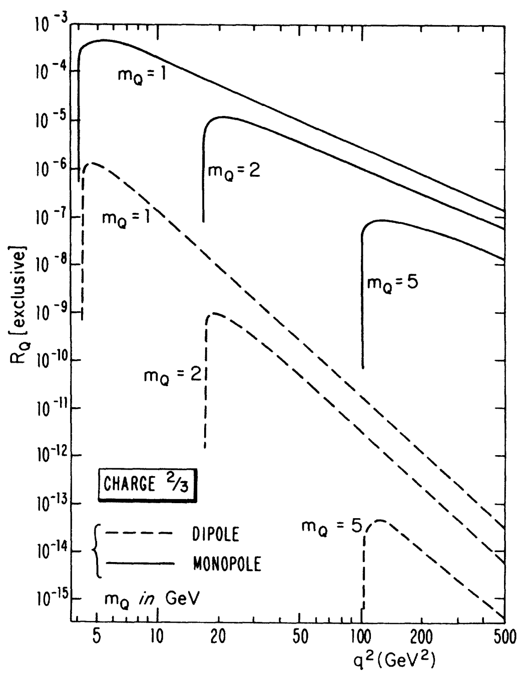

FIG. 6. Theoretical estimates of the ratio R_Q (exclusive) as a function of quark mass. The labels dipole and monopole correspond to two choices of the quark separation factor.

Quark separation in hadronic collisions

In the context of broken QCD, we expect cross sections for unconfined quark production in hadronic processes to be suppressed by separation factors similar to those in current-induced processes. We estimate the order of magnitude of quark production using the parton model.[24] The inclusive production cross section is an incoherent sum over parton distributions of cross sections for scattering of light pointlike partons into heavy unconfined quarks. We concentrate on the production of unconfined quark-antiquark pairs because we expect processes with minimum numbers of fractional baryon number particles in the final state to dominate. We also neglect contributions due to annihilation of constituent gluons of whose distributions we are ignorant. On general grounds, we expect that the gluon annihilation contribution is not significantly larger than that of quark annihilation. Our estimates are sufficiently crude that a factor of 2 is irrelevant. Further, we find that annihilation graphs are suppressed relative to scattering graphs by kinematics and combinator-

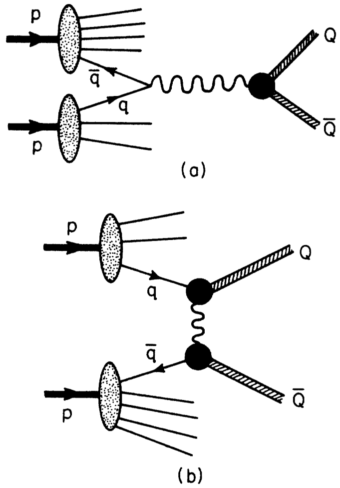

FIG. 8. Quark annihilation and scattering diagrams used in estimating liberated quark production in pp collisions.

ics, so in fact the gluon-annihilation contribution is expected to be relatively unimportant.

The dominant parton-model diagrams contributing to the production of a given unconfined quark-antiquark pair ($Q\bar{Q}$) in pp scattering are shown in Fig. 8. In this figure, the wavy lines are gluons and $q, \bar{q}$ are light pointlike quark partons. The inclusive pp production cross section at center-of-mass energy $\sqrt{s}$ is

$$\sigma_{pp\to Q\bar{Q}}(s) = \sum_{q,\bar{q}} \int_{4M_Q^2/s}^{1} y^2\, dy^2 \int_{y}^{1/y} dr/r\, f_q(yr) f_{\bar{q}}(y/r) \times \sigma_{q\bar{q}\to Q\bar{Q}}(y^2 s)\,, \tag{6.7}$$

where f_q and $f_{\bar{q}}$ are the appropriate parton distribution functions and $\sigma_{q\bar{q}\to Q\bar{Q}}$ is the cross section for light quarks scattering into heavy quarks at center-of-mass energy $\sqrt{Q^2} = (y^2 s)^{1/2}$. We estimate $\sigma_{q\bar{q}\to Q\bar{Q}}$ by computing the corresponding pointlike cross section and multiplying by the inclusive quark separation factors defined above, evaluated at $4M_Q^{\,2}$. In the case of the annihilation graph, it seems clear that the separation factor should not depend significantly on whether the quark pair has been produced by electrons or by quarks. In the case of the t-channel exchange graph, the argument is an order of magnitude one. We guess that the square of the color transition form factor from pointlike parton to unconfined quark is of the same order as a single separation factor. The diagrams in Fig. 8 would seem to imply that the hadronic debris, as well as the quarks themselves, may carry color. We think this not to be true and do not introduce an additional suppression factor to take it into account. Much as in the original parton model of electroproduction, the ultimate final-state debris need not be colored. Final-state interactions occurring over a longer time scale have nearly unit probability to neutralize the color of the debris by transmitting it to one of the quarks.

The $q\bar{q}$ cross section is dominated by the t-channel graph. The annihilation amplitude is proportional to $1/Q^2$ while the t-channel amplitude is proportional to $1/t$. Q^2 is always greater than $4M^2$ while $|t|_{\min}$ is always less than M^2 and decreases as Q^2 increases above threshold. Further, since gluons are flavor singlets, the annihilation graph contributes only to production of pairs in the flavor-singlet channel. For comparison to the strictest experimental limits on production in accelerators, we must compute the cross section for inclusive production of fractional charge, to which flavor-nonsinglet $Q\bar{Q}$ states of course contribute. Thus the t-channel graphs are relatively enhanced by combinatorics. We are therefore justified, at this level of estimation, in ignoring the annihilation amplitude of Fig. 8(a) and the gluon-annihilation process.

The t-channel cross section is

$$\sigma_{q\bar{q}\to Q\bar{Q}}(Q^2) = \frac{64\pi\bar{\alpha}_c^{\,2}}{9Q^2} F(4M_Q^{\,2})^2 \times \int_{\tau_-}^{\tau_+} \frac{d|\tau|}{Q^2}\left[1 - \frac{2(Q^2 - M_Q^{\,2})}{|\tau|} + \frac{2(Q^2 - M_Q^{\,2})^2 - M_Q^{\,4}}{|\tau|^2}\right], \tag{6.8}$$

where

$$\tau_\pm \equiv \tfrac{1}{2}Q^2 - M_Q^{\,2} \pm \tfrac{1}{2}Q^2(1 - 4M_Q^{\,2}/Q^2)^{1/2} \tag{6.9}$$

and $\bar{\alpha}_c$ is the running fine-structure constant of QCD[13] evaluated at $q^2 = 4M_Q^{\,2}$,[22]

$$\bar{\alpha}_c = \frac{\pi}{9\ln(4M_Q^{\,2}/\Lambda^2)} \tag{6.10}$$

with

$\Lambda \sim 500$ MeV.

We plot our predictions for $\bar{u}$-quark production at $s = 562\ \text{GeV}^2$ and $\bar{d}$-quark production at $s = 375\ \text{GeV}^2$ in Figs. 9 and 10, respectively. The production and subsequent decay of charmed and strange quarks is relatively damped by their higher masses and has been neglected. The predictions are given as functions of quark mass, for monopole and dipole separation factors. The predictions are compared in the Figs. 9 and 10 with upper limits on cross sections as reported by one of the most restrictive accelerator quark searches.[25] The model

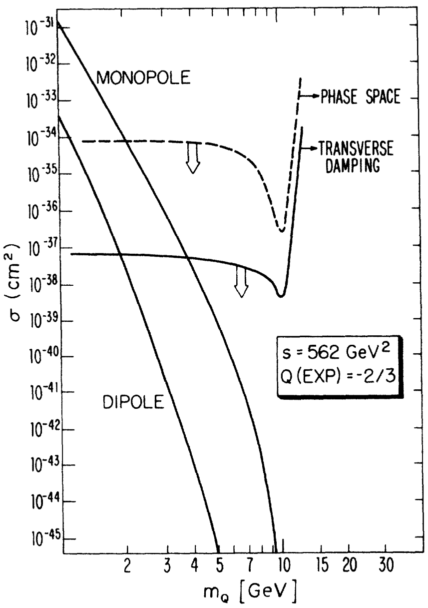

FIG. 9. Comparison of rough theoretical estimates and experimental upper limits on the production of quarks of charge $-\frac{2}{3}$ in pp collisions. The curves labeled monopole and dipole correspond to two choices of quark separation factors. The experimental upper limits labeled "phase space" and "transverse damping" correspond to two ways of analyzing the data (see text and Ref. 25 for details).

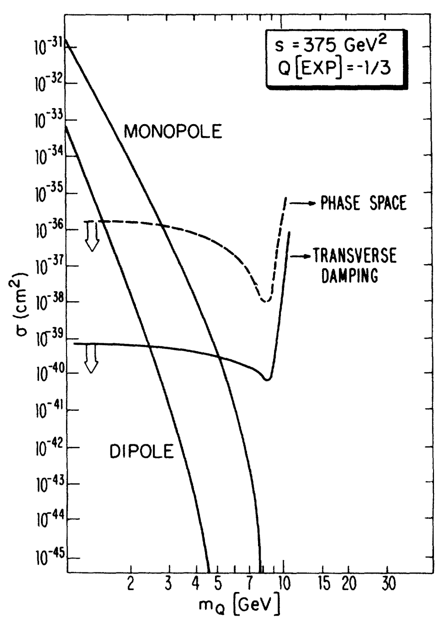

FIG. 10. The analog of Fig. 9 for quarks with charge $-\frac{1}{3}$.

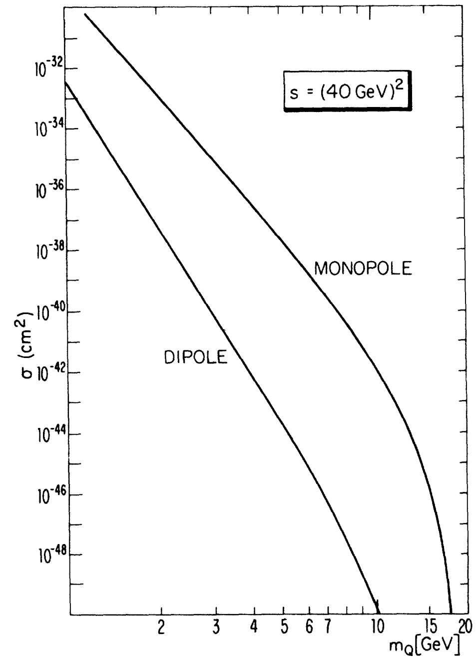

FIG. 11. Estimate of up or down quark or antiquark production cross section in pp collisions at a typical present colliding-beam energy.

used in the experimental analysis is based on the reaction $pp \to ppQ\overline{Q}$. The "phase-space" assumption is that the matrix element is a constant. The "transverse-damping" assumption is that the quark transverse-momentum distribution falls off as $\exp(-6p_T/1\text{ GeV})$. In our model the p_T falloff is in between these two, at least for $p_T \lesssim M_Q$. The experimental limits are given with the assumption that quarks do not interact in the detector. The limits should be raised by a factor of 3 to 10 for the very strongly interacting quarks of our model.[25] The conclusion of the comparison of experiment and theory is that, if our very rough estimates are not very wrong, unconfined quarks heavier than a few GeV would not have been found in past accelerator searches.

Figures 11 and 12 show similar cross-section estimates at present and future storage-ring energies: $s = 1600\text{ GeV}^2$ and $s = 640\,000\text{ GeV}^2$. The predominant increase in the cross sections is due to the decrease of the average $|t|_{\min}$.

We expect gluon pair production in proton-proton collisions to be dominated by the parton-model diagrams such as the ones shown in Fig. 13. The intermediate wavy lines in the diagrams are again gluons. The objects labeled G are the outgoing unconfined gluons. Again, the diagrams are not supposed to imply that the hadronic debris other than

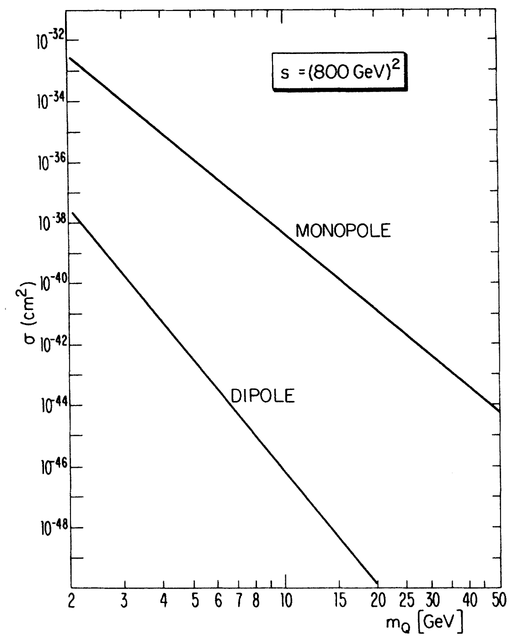

FIG. 12. Estimate of up or down quark or antiquark production cross section in pp collisions at a future colliding-beam energy.

the gluons themselves are colored. We expect the gluon separation factors to be similar to quark separation factors. In the absence of experimental upper limits on gluon production, we have not endeavored to estimate the production cross section. Up to the explicit color factors of order 1, they should be of the same order of magnitude as quark production cross sections at the same s and colored particle mass. In our model unconfined gluons are 50% heavier than unconfined quarks and should be correspondingly harder to produce, since at a given s the production cross section decreases very fast with mass (see Figs. 9–12). But the search for gluons should not be neglected; their signatures, as we have discussed, are rather striking.

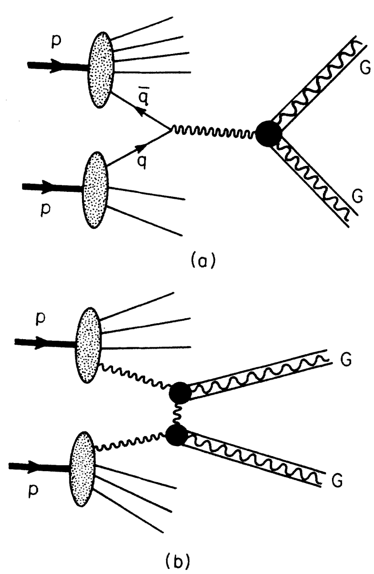

FIG. 13. Quark-annihilation and gluon-annihilation diagrams used in the discussion of gluon liberation in pp collisions.

Primordial and cosmic quarks

If quarks and gluons can exist as free particles, we expect that they may have been produced by natural processes at some times during the long history of the universe. For example, a few unconfined quarks may have remained after the majority had combined into colorless hadrons during the first few moments of time. We call these "primordial" quarks to distinguish them from "cosmic" quarks which may have been created much later in stellar or galactic processes. Perhaps cosmic quarks are produced in the neighborhood of black holes, in supernovas, quasars, or in other catastrophic celestial events.

We cannot say much with assurance about the nature of these extremely energetic processes. We know little about the dynamics of unconfined quarks and perhaps even less about very-high-energy astrophysics. We do not attempt to estimate the number of quarks created either in the big bang or later. Instead, we focus on the practical question: Suppose quarks had been created, where would they be now? Would they be in nucleon complexes or in isolation?

Our conclusions depend on the environment in which the quarks were created. Primordial quarks, we argue, would absorb many neutrons during the first ten or twenty minutes of the universe. They would therefore all be found as QNC's with baryon number nearly equal to A_{max}. Cosmic quarks, on the other hand, are likely to be found with $A < A_{max}$ and might even be found in their pristine baryon-number-$\frac{1}{3}$ form in interstellar space. Quarks with $A \ll A_{max}$ entering the earth's atmosphere are likely to quickly pick up several units of baryon number and charge before settling into the earth's crust. The remainder of this section is devoted to discussing these results in more detail.

The density of unconfined quarks may have been high in the very early universe. The reaction in which quarks combine color-singlet baryons would have occurred rapidly until unconfined quarks were comparatively rare. As the temperature of the universe dropped, such quarks would begin to accumulate baryon number. So long as the unconfined quarks remained in thermal equilibrium they would absorb up to roughly the baryon number $A(T)$ at which the separation energy $S(A)$ of a baryon from the growing QNC equaled the ambient temperature $S(A(T)) \cong kT$. They would be termally distributed about this A with width $\sim[kT/S'(A)]^{1/2}$. Heavy conventional nuclei are thought not to have been made in the big bang because of the barriers presented by the $A=5$ and $A=8$ systems. In contrast, baryon absorption on a growing QNC is very exothermic all the way up to $A=A_{max}$, so we expect no similar barrier to its growth. Eventually the Coulomb barrier about a QNC puts an end to its absorption of protons. According to the standard cosmology,[26] however, neutrons were almost as plentiful as protons in the early universe—at least until times of the order $t_0 = 918$ sec (the neutron lifetime). A growing QNC would continue to absorb upon thermal neutrons until their density become too low to maintain thermal equilibrium. We have estimated this and found that the neutron flux is sufficient to maintain equilibrium until times ranging from $\frac{1}{3}$ to $3t_0$ depending on the absorption cross section[27] and the primordial baryon density. At those times the temperature of the universe was very low on a hadronic scale—roughly 10 to 100 keV. Since this temperature is so low one might expect to find all primordial quarks in the form of QNC's with baryon number equal to the appetite, A_{max}. This is, however, unrealistic. At energies as high as several MeV, the details of nuclear physics become important. Just as among heavy nuclei, there may be some QNC's with anomalously large or small neutron absorption cross sections, or QNC's which are anomalously stable. Perhaps a QNC at the appetite still has an affinity to bind nucleons by more conventional nuclear forces. We cannot hope to calculate these effects with our present knowledge. We note, however, that because the separation energy increases rapidly for decreasing A, we expect very few QNC's with $A<A_{max}$.

QNC's formed in the early universe are unlikely to be altered by subsequent events. They are stable and have small cross sections for absorbing further baryons. The implications for quark searches in ordinary matter may be rather far reaching. Quarks may be very specific. A strong limit on the occurrence of quarks in (say) oxygen would be misleading if primordial quarks occur as QNC's with (say) approximately the nuclear charge of copper, the mass of zinc, and (since they are always ionized) with a chemistry unlike anything currently known.

What cosmic processes may have produced unconfined quarks after the first hours of the life of the universe, we do not know. But several types of very violent celestial catastrophes are known or conjectured to occur. A quark freed in one of these processes may have been subject to high nuclear densities and may have reached the earth with its appetite saturated, much like a primordial quark. A more interesting possibility is that some unconfined quarks may have escaped this fate and may have traveled through interstellar space as cosmic quarks in their pristine baryon-number-$\frac{1}{3}$ form. Interstellar space has a low average nuclear density: A quark traversing our whole galaxy through its center would on the average only suffer ~ 0.04 nuclear collisions per 100 mb of quark-proton cross section. A quark not going through the center of a galaxy may easily reach the upper atmosphere unimpaired. But our atmosphere is dense. A cosmic-ray quark-reaching sea level would suffer an average of ~ 60 atmospheric collisions per 100 mb of quark-proton cross section. The chances that it appears at sea level as an object with charge $\pm\frac{1}{3}$ or $\pm\frac{2}{3}$ are very small. The cosmic quark, if massive, will have accumulated a large positive charge Q in its descent. Thus, our model favors mountain-top or sattelite cosmic quark searches. Sea-level searches require very good measurement of Q or Q/M.

VII. CONCLUSIONS AND OUTLOOK

Years ago the absence of free quarks was considered to be evidence against the quark model. As the successes of the quark model accumulated and quarks still failed to present themselves the notion of permanent confinement began to emerge. In QCD, which is the best candidate for a theory of hadrons at this time, none of the fundamental quanta neither quarks nor gluons, are supposed to exist as free particles. Now, rather than an embarrassment, confinement is regarded as a desirable and aesthetically attractive feature of the theory.

We have reconsidered the possibility that fractionally charged color states may exist, and have demonstrated a smooth connection between exact QCD, in which such states (presumably) do not exist, and slightly broken QCD, in which unconfined quarks and gluons are very heavy and the successful phenomenology of color-singlet hadrons is altered only slightly.

Since our theory of unconfined quarks and gluons is smoothly connected to phenomenological models of permanent color confinement, we have been able to predict some of the properties of the hypothetical colored particles. Unconfined quarks and gluons have large masses of $O(\mu^{-1})$, large radii and scattering cross sections, large nuclear appetite, and small production cross sections. A quark bound in a color-singlet hadron acts as a light particle while an unconfined quark has an extensive color field that makes it heavy. The quark "parton" within an unconfined quark would also be light when studied with a short-distance probe: It is only its infrared color field that makes the free quark state heavy.

We have argued that if quarks are very heavy primordial quarks left over from the big bang will have absorbed large numbers of nucleons. They will naturally occur in the disguise of fractionally charged nuclei, with large electrical charges and large mass. Their total baryon number will be of the order of M_Q/m_p and their mass of order $2M_Q$. Primordial gluons will similarly be disguised as integrally charged anomalously heavy nuclei, with masses of order $M_G = 3/2M_Q$. About the geological history of an atom with a heavy quark nucleus we can only speculate. The atom will be permanently "fractionally" ionized and its chemistry may be bizarre. Even for atoms with conventional chemistry and nuclear masses, the selection, fractionization, and purification processes in the earth's crust are amazingly selective. Rare materials, such as gold, are enormously concentrated by nature, and can be found in gold mines. Perhaps quarks are to be found in quark mines.

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