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Multibaryon states in the bag model

A. Th. M. Aerts, P. J. G. Mulders, and J. J. de Swart
Institute for Theoretical Physics, Nijmegen, The Netherlands
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In the MIT bag model we calculate the masses of all colorless N-quark states, for $N = 3, 6, 9, 12, 15$, and 18, with the quarks in $s_{1/2}$ states of the bag. For fixed N these states belong to only one SU(6) irreducible representation. We give SU(6) mass formulas for these states. Several candidates for the lightest $N = 6$ states are already available. Especially in the ΛN channel the lowest three predicted states seem to show up in the data.

I. INTRODUCTION

In the naive quark model[1] one was able to explain many features of the hadrons. Relations between mass differences within SU(3) multiplets could be derived. However, it was impossible to say anything sensible about the masses of the individual hadrons. In particular any indication about the masses of the exotic mesons ($q^2\bar{q}^2$), the exotic baryons ($q^4\bar{q}$), and the dibaryons (q^6) was lacking. One of the reasons was our ignorance about the interactions between the quarks.

In the last few years this situation has changed. It is now generally thought that the quarks are colored and that the interaction between the quarks is mediated by colored gluons.[2,3] A particular version of this gluon-quark model is the MIT bag model,[4,5] in which colored quarks are confined to a certain region of space, called a bag. In the simplest version, in which the bag is taken to be a sphere, one was able to reproduce rather well the masses of the colorless s-wave $q\bar{q}$ mesons and of the q^3 baryons, using in the case of three flavors only five physically interpretable parameters. Without introducing new parameters one can calculate in this bag model also the masses of exotic states, like the s-wave $q^2\bar{q}^2$ mesons[6] and the s-wave six quark dibaryons.[7]

The results of these exotic-meson calculations were quite surprising and pleasing. The lowest states formed a nonet of scalar mesons, the lightest one being an isoscalar at ± 650 MeV, which is obviously the long known, but still quite puzzling ϵ meson. The isovector and second isoscalar mesons are degenerate and have a mass of about 1.1 GeV. These states are probably the δ and S^* enhancements around the $K\bar{K}$ threshold.

The results of the dibaryon calculations were also quite interesting. It was shown that one must expect some six-quark states with relatively low mass. These states must show up as resonance in NN, ΛN, and ΣN scattering, and in the $\Lambda\Lambda$, ΞN, $\Sigma\Lambda$, and $\Sigma\Sigma$ channels. Especially significant are the predictions of a $\Lambda\Lambda$ bound state with a binding energy of about 50 MeV and of possible NN resonances.

Experimental verification of these predictions is quite important, because the existence or nonexistence of these states will be quite an important test of the applicability of the present form of the MIT bag model to exotic states.

Although these six-quark states and in general the colorless N-quark states ($N=3, 6, 9, \ldots$) occur as resonances in scattering processes like pd, $p\,{}^3\mathrm{H}$, or Λd, they are different from nuclear states like ${}^3\mathrm{He}$, ${}^4\mathrm{He}$, or hypernuclear states like ${}^3_\Lambda\mathrm{H}$, because they are *single* hadron states. They are unaccounted for by the spectrum of resonances and bound states arising in standard potential model or shell-model calculations.[8]

In this paper we will consider all colorless N-quark states, where the quarks are in the $s_{1/2}$ states of a spherical bag. These hadrons have thus all positive parity. Since all particles should be color singlets and since the color symmetry is unbroken, the old mass formulas[9-11] obtained from specific assumptions about the breaking of flavor-spin symmetry are not affected. The difficulty in applying these mass formulas was that one had to determine the coefficients for each flavor-spin multiplet separately from the experimentally known masses of the hadrons. The MIT bag model offers a way to calculate these coefficients for the colorless N-quark states, which will belong for fixed N to only one flavor-spin SU(6) irreducible representation. Because the allowed states must be totally antisymmetric with respect to flavor, spin, and color, the color-spin tensor operators occurring in the spherical-bag mass operator can be expressed in simple flavor-spin tensor operators. We then can identify the contributions of the different SU(6)-breaking tensor operators, the coefficients being known functions of the bag radius.

In order to satisfy one of the boundary conditions in the bag model, the mass of a particular state is found by minimizing the expectation value of the mass operator with respect to the bag radius R. As this radius does not vary too much between different members of the same SU(6) multiplet, it is possible to choose only one R value for an entire multiplet without introducing significant numerical inaccuracies. This way we obtain for fixed N mass formulas, which suffice to obtain the masses of all the s-wave $N=6$, 9, 12, 15, and 18 quark states, without introducing any new parameters.

At this point we would like to stress the importance of the prediction of several NN resonances, because these predictions could at present be checked experimentally, if their width (about which we cannot say anything sensible) is not too large. We expect NN resonances in the ${}^3S_1$ channel at $T_{\rm lab} \simeq 0.61$ GeV, in the ${}^1S_0$ channel at $T_{\rm lab} \simeq 0.79$ GeV and two (almost?) degenerate resonances in the ${}^1D_2$ and ${}^3D_3$ channels at $T_{\rm lab} \simeq 1.04$ GeV.

In the hyperon-nucleon (ΛN and ΣN) channel many resonances are expected. In the experimental data (only available for the lower energies) several enhancements can be seen next to the resonance H, seen[12] at 2127 MeV. The resonance H is certainly not a six-quarks-in-*one*-bag state, because it can quite naturally be explained in the ordinary potential picture.[13]

In the $Y=0$ channels ($\Lambda\Lambda$, ΞN, $\Lambda\Sigma$, and $\Sigma\Sigma$) we expect an $I=0$ ${}^1S_0$ bound state about 30 MeV below the $\Lambda\Lambda$ threshold. The first resonances in the $I=0$ and $I=1$ ${}^3S_1$ channels are found at $E \simeq 2.35$ GeV and $E \simeq 2.39$ GeV. The Pauli principle forbids the $I=0$ resonance at $E \simeq 2.35$ GeV to decay in the $\Lambda\Lambda$ channel, it can only decay in ΞN.

II. CLASSIFICATION OF THE N-QUARK STATES

In the MIT bag model[4, 5] we will consider multibaryon states with baryon number B, described by a wave function of $N=3B$ quarks, all in $s_{1/2}$ states of the bag. These states have an SU(2, J) classification for the space-spin part, an SU(3, F) classification for the flavor part (assuming only 3 flavors), and an SU(3, C) classification for the color part. Because of generalized Fermi statistics the N-quark states must be totally antisymmetric. We therefore can place up to 18 colored quarks in these states of the bag. Because

$$\mathrm{SU}(18) \supset \mathrm{SU}(3,F) \otimes \mathrm{SU}(2,J) \otimes \mathrm{SU}(3,C)\,,$$

a classification of the states with the help of the group SU(18) is quite useful. This does not mean that we consider SU(18) a good symmetry group of the Hamiltonian. The permutation symmetry of the states we will describe with the help of Young diagrams.[14, 15] The states contain N quarks, so the corresponding Young diagrams contain N boxes. Because of Fermi statistics the N-quark states must belong to the totally antisymmetric irreducible representation (irrep) of SU(18), described by a Young diagram of only one column and N rows.

To get some of the important quantum numbers of these states we consider the decomposition of SU(18) in the old-fashioned (flavor-spin) SU(6, FJ) and SU(3, C)

$$\mathrm{SU}(18) \supset \mathrm{SU}(6,FJ) \otimes \mathrm{SU}(3,C).$$

The physical states must be SU(3, C) singlets. The corresponding Young diagram for the SU(3, C) part of the state, therefore, is rectangular and contains 3 rows and $B=N/3$ columns. Because the state must be totally antisymmetric, the permutation symmetry of the SU(6, FJ) part of the state is described by the associate diagram of the diagram describing the permutation symmetry of the color part of the state. This associate diagram thus has three columns and B rows. This uniquely determines the SU(6, FJ) irrep $[\mu]$ to which the colorless states belong. They are given in Table I. At this point we should note that in SU(n) the irrep described by the rectangular Young diagram with x columns and y rows is the complex conjugate irrep of the irrep described by the Young diagram with x columns and $(n-y)$ rows. We see this property clearly reflected in Table I. Next we consider the decomposition

$$\mathrm{SU}(6,FJ) \supset \mathrm{SU}(3,F) \otimes \mathrm{SU}(2,J).$$

For the relevant SU(6, FJ) irreps $[\mu]$ the decomposition

$$[\mu] = \sum_{\oplus} (\underline{n}, J)$$

in the different SU(3, F) irreps $\underline{n}$ together with their spins J is given in Table II. For the content of SU(3, F) irreps we refer to Ref. 16.

TABLE I. The SU(6, FJ) irreps $[\mu]$ of the colorless N-quark states.

N	$[\mu]$
3	[56]
6	[490]
9	[980]
12	[490*]
15	[56*]
18	[1]

TABLE II. The decomposition of SU(6,FJ) irreps in flavor and spin. The states are ordered according to increasing mass. It will turn out that the states $(\underline{27},2)$ and $(\underline{10}^*,3)$ in [490], $(\underline{27},\frac{1}{2})$, $(\underline{8},\frac{7}{2})$, and $(\underline{1},\frac{9}{2})$ in [980], and $(\underline{27},2)$ and $(\underline{10},3)$ in [490*] are degenerate as long as there is no mixing (see Sec. VI).

$B=1$	$[56]=(\underline{8},\frac{1}{2})\oplus(\underline{10},\frac{3}{2})$
$B=2$	$[490]=(\underline{1},0)\oplus(\underline{8},1)\oplus(\underline{8},2)\oplus(\underline{10},1)\oplus(\underline{10}^*,1)\oplus(\underline{27},0)$
	$\oplus(\underline{27},2)\oplus(\underline{10}^*,3)\oplus(\underline{35},1)\oplus(\underline{28},0)$
$B=3$	$[980]=(\underline{1},\frac{3}{2})\oplus(\underline{1},\frac{5}{2})\oplus(\underline{8},\frac{1}{2})\oplus(\underline{8},\frac{3}{2})\oplus(\underline{8},\frac{5}{2})\oplus(\underline{10},\frac{3}{2})\oplus(\underline{10}^*,\frac{3}{2})$
	$\oplus(\underline{27},\frac{1}{2})\oplus(\underline{8},\frac{7}{2})\oplus(\underline{1},\frac{9}{2})\oplus(\underline{27},\frac{3}{2})\oplus(\underline{27},\frac{5}{2})\oplus(\underline{35},\frac{1}{2})$
	$\oplus(\underline{35}^*,\frac{1}{2})\oplus(\underline{64},\frac{3}{2})$
$B=4$	$[490^*]=(\underline{1},0)\oplus(\underline{8},1)\oplus(\underline{8},2)\oplus(\underline{10},1)\oplus(\underline{10}^*,1)\oplus(\underline{27},0)$
	$\oplus(\underline{27},2)\oplus(\underline{10},3)\oplus(\underline{35}^*,1)\oplus(\underline{28}^*,0)$
$B=5$	$[56^*]=(\underline{8},\frac{1}{2})\oplus(\underline{10}^*,\frac{3}{2})$

Another useful decomposition[11] is determined by

$$\mathrm{SU}(6,FJ)\supset \mathrm{U}(1,Y)\otimes\mathrm{SU}(4,IJ_n)\otimes\mathrm{SU}(2,J_s)\ ,$$

where J_n (J_s) is the total spin of the nonstrange (strange) quarks, I is the isospin, and Y the hypercharge. The decomposition

$$[\mu]=\sum_{\oplus}(Y,(\nu),J_s)$$

is given in Table III. Here (ν) denotes the SU(4, IJ_n) irrep. They are given by their dimension and if necessary an extra index. This decomposition is necessary, because when we calculate the SU(6, FJ) breaking we shall consider the nonstrange and strange quarks contained in a state separately. The decomposition of the group SU(4, IJ_n) in isospin SU(2, I) and nonstrange spin SU(2, J_n)

$$\mathrm{SU}(4,IJ_n)\supset \mathrm{SU}(2,I)\otimes\mathrm{SU}(2,J_n)$$

is the nonstrange analog of the flavor-spin decomposition in flavor and spin. We get

$$(\nu)=\sum_{\oplus}(I,J_n)\ .$$

These decompositions are given in Table IV.

TABLE III. The hypercharge, SU(4,IJ_n), and strange spin content of the SU(6,FJ) irreps. For [490*] and [56*] the decompositions are the same as for [490] and [56], except that the Y eigenvalue changes sign and (ν) becomes (ν^*).

[56]	$=(1,(20_s),0)\oplus(0,(10),\frac{1}{2})\oplus(-1,(4),1)\oplus(-2,(1),\frac{3}{2})$
[490]	$=(2,(50),0)\oplus(1,(60),\frac{1}{2})\oplus(0,(45),1)\oplus(0,(20_2),0)$
	$\oplus(-1,(20_s),\frac{3}{2})\oplus(-1,(20_1),\frac{1}{2})\oplus(-2,(10),1)$
	$\oplus(-2,(6),0)\quad\oplus(-3,(4),\frac{1}{2})\oplus(-4,(1),0)$
[980]	$=(3,(20_s),0)\oplus(2,(45),\frac{1}{2})\oplus(1,(60),1)\oplus(1,(36),0)$
	$\oplus(0,(64),\frac{1}{2})\oplus(0,(50),\frac{3}{2})\oplus(-1,(60),1)\oplus(-1,(36),0)$
	$\oplus(-2,(45),\frac{1}{2})\oplus(-3,(20_s),0)$

III. THE HAMILTONIAN IN THE BAG MODEL[5]

The MIT bag model provides us with a method to calculate the masses of the various N-quark states. In this model the hadron is an extended object (bag) to which the quarks are confined. The bag is taken to be a sphere of radius R and the quarks are placed in $s_{1/2}$ states. Inside this bag the quarks can move freely, except for a weak one-gluon-exchange interaction between the color charges ($\sim g\lambda_\alpha^c/2$) and between the color magnetic

TABLE IV. The isospin and nonstrange spin content of the SU(4,IJ_n) irreps (ν). In this specific decomposition the contents of (ν^*) are identical to the contents of (ν).

(1)	$=(0,0)$
(4)	$=(\frac{1}{2},\frac{1}{2})$
(6)	$=(1,0)\oplus(0,1)$
(10)	$=(1,1)\oplus(0,0)$
(20_s)	$=(\frac{3}{2},\frac{3}{2})\oplus(\frac{1}{2},\frac{1}{2})$
(20_1)	$=(\frac{3}{2},\frac{1}{2})\oplus(\frac{1}{2},\frac{3}{2})\oplus(\frac{1}{2},\frac{1}{2})$
(20_2)	$=(2,0)\oplus(1,1)\oplus(0,2)\oplus(0,0)$
(36)	$=(\frac{3}{2},\frac{3}{2})\oplus(\frac{3}{2},\frac{1}{2})\oplus(\frac{1}{2},\frac{3}{2})\oplus(\frac{1}{2},\frac{1}{2})$
(45)	$=(2,1)\oplus(1,2)\oplus(1,1)\oplus(1,0)\oplus(0,1)$
(50)	$=(3,0)\oplus(2,1)\oplus(1,2)\oplus(1,0)\oplus(0,3)\oplus(0,1)$
(60)	$=(\frac{5}{2},\frac{1}{2})\oplus(\frac{3}{2},\frac{3}{2})\oplus(\frac{3}{2},\frac{1}{2})\oplus(\frac{1}{2},\frac{5}{2})\oplus(\frac{1}{2},\frac{3}{2})\oplus(\frac{1}{2},\frac{1}{2})$
(64)	$=(2,1)\oplus(2,0)\oplus(1,2)\oplus2(1,1)\oplus(1,0)\oplus(0,2)\oplus(0,1)$

moments ($\sim g\lambda_\alpha^c\sigma_k/2$). The eight generators of SU(3, C) in the irrep $\underline{3}$ we denote by $\lambda_\alpha^c/2$ with $\alpha = 1$ to 8. They are normalized such that $\mathrm{Tr}\lambda_\alpha^2 = 2$. The three generators of SU(2, J) in the $J = \frac{1}{2}$ irrep are $\sigma_k/2$ with $k = 1$ to 3 and $\mathrm{Tr}\sigma_k{}^2 = 2$. The mass operator of an N-quark system is given by

$$M = E_B + E_Q + E_M + E_E \ . \tag{1}$$

The energy E_B associated with the bag is

$$E_B = \frac{4\pi}{3} BR^3 - \frac{Z_0}{R} \ , \tag{2}$$

where B is the bag pressure and Z_0 is associated with the zeropoint energy.[5] The rest energy and kinetic energy E_Q of the quarks in the bag is

$$E_Q = \sum_i N_i \frac{\alpha_i(R)}{R} = N_n \frac{\alpha_n(R)}{R} + N_s \frac{\alpha_s(R)}{R} \ . \tag{3}$$

We use i and j for the particle indices of the quarks. The indices n and s more specifically refer to the nonstrange and strange quarks. N_i is the number operator for the quarks i. The energy eigenvalue of a quark in a spherical bag is (see Table V)

$$\alpha_i(R)/R = \alpha(m_i R)/R \ ,$$

where m_i is the mass of the ith quark. The energy E_M due to the color-magnetic interaction between the quarks is

$$E_M = -\frac{\alpha_c}{R} \sum_{i>j} M_{ij}(R) \left(\frac{\lambda^c\sigma}{2}\right)_i \left(\frac{\lambda^c\sigma}{2}\right)_j \ . \tag{4}$$

The energy E_E due to the color-electric interaction between the quarks is

$$E_E = \frac{\alpha_c}{R}\left(\frac{2}{3}\sum_i E_{ii}(R) + \sum_{i>j} E_{ij}(R)\frac{\lambda_i^c}{2}\cdot\frac{\lambda_j^c}{2}\right) \ . \tag{5}$$

Here

$$\lambda_i \cdot \lambda_j = \sum_\alpha (\lambda_\alpha)_i (\lambda_\alpha)_j$$

and

$$\sigma_i \cdot \sigma_j = \sum_k (\sigma_k)_i (\sigma_k)_j \ .$$

The gluon coupling constant g appears in $\alpha_c = g^2/4\pi$. The functions $M_{ij}(R) = M(m_i R, m_j R)$ and $E_{ij}(R) = E(m_i R, m_j R)$ are functions of the products of R and the quark masses m_i and m_j (see Table V).

The bag radius R is determined according to one of the boundary conditions in the model by minimizing M with respect to R. This should be done for each hadron separately. In order to have a useful mass formula expressed in flavor-spin tensor operators, we take an average R_{av} for each entire SU(6, FJ) multiplet. For a particular state we have

$$M(R) = \frac{4\pi}{3} BR^3 - \frac{Z_0}{R} + \sum_i N_i \frac{\alpha_i(R)}{R} + \frac{f(R)}{R} ,$$

where $f(R)$ contains the R dependence of E_M and E_E coming from the functions M_{ij} and E_{ij}. Minimalization gives

$$R_{\min} = (4\pi B)^{-1/4}\left[\sum_i N_i\left(\alpha_i - R\frac{\partial \alpha_i}{\partial R}\right) - Z_0 + \left(f - R\frac{\partial f}{\partial R}\right)\right]^{1/4}_{R=R_{\min}} .$$

As long as the functions α_i, M_{ij}, and E_{ij} are about linear we may approximate:

$$\alpha_i(R) - R\frac{\partial \alpha_i}{\partial R} \simeq \alpha_i(0) = \alpha(0) \ .$$

In the bag model the nonstrange-quark mass m_n is chosen to be zero.[5] So we have $\alpha(0) = \alpha(m_n R)$

TABLE V. Average radii for multibaryon multiplets and values of functions α_s, M_{ij}, E_{ij}, $\bar{M} = M_{nn} + M_{ss} - 2M_{ns}$, and $\bar{E} = E_{nn} + E_{ss} - 2E_{ns}$.

B	1	2	3	4	5	6
$\langle(\lambda^C\sigma)^2\rangle$	0	12	24	60	96	144
R_{av} (GeV^{-1})	5.22	6.70	7.57	8.39	9.02	9.60
α_n	2.043	2.043	2.043	2.043	2.043	2.043
α_s	2.914 9	3.211 3	3.393 5	3.570 0	3.708 5	3.838 1
M_{nn}	0.177	0.177	0.177	0.177	0.177	0.177
M_{ss}	0.112 7	0.098 1	0.090 5	0.083 9	0.079 1	0.075 1
M_{ns}	0.140 6	0.131 0	0.125 6	0.120 8	0.117 3	0.114 1
E_{nn}	0.278 4	0.278 4	0.278 4	0.278 4	0.278 4	0.278 4
E_{ss}	0.409 1	0.441 5	0.459 2	0.474 8	0.486 3	0.496 3
E_{ns}	0.334 8	0.346 6	0.352 8	0.358 2	0.362 0	0.365 3
$\bar{M}$	0.008 43	0.013 18	0.016 25	0.019 27	0.021 66	0.023 88
$\bar{E}$	0.017 99	0.026 69	0.031 96	0.036 94	0.040 73	0.044 17

$=\alpha_n(R)=\alpha_n$. The same we have for $f(R)$. So a reasonable average value for the radius of a whole multiplet is found by minimizing the bag mass for a system of nonstrange quarks, taking an average value for the color-magnetic interaction term:

$$R_{\rm av}=(4\pi B)^{-1/4}[N\alpha_n - Z_0 + \tfrac{1}{4}\alpha_c M_{nn}\langle(\lambda^c\sigma)^2\rangle]^{1/2},$$

where $\langle(\lambda^c\sigma)^2\rangle$ is an average value of $\sum(\lambda^c\sigma)_i\cdot(\lambda^c\sigma)_j$ in an SU(6, FJ) multiplet. In this case the color-electric part does not contribute. Since we work in the neighborhood of a minimum, the R dependence of $M(R)$ is not too strong. The masses of the hadrons obtained with $R_{\rm av}$ are only slightly larger than the masses obtained with the radius coming from the minimalization procedure. The differences can easily be estimated and are $\lesssim 20$ MeV. Of the five parameters, m_n is fixed to be zero and B, Z_0, α_c, and m_s are made to fit the light hadron mass spectrum, under the condition that we have one radius for the baryons involved. The parameters are: $B^{1/4}=0.146$ GeV, $Z_0=1.89$, $\alpha_c=2.12$, $m_n=0$, and $m_s=0.285$ GeV. As was already noted above $\alpha_n(R)$, $M_{nn}(R)$, and $E_{nn}(R)$ are independent of R since $m_n=0$. The values of $R_{\rm av}$ and the values of the functions α_i, M_{ij}, E_{ij}, at $R=R_{\rm av}$ are given in Table V.

Using the total-quark-number operator $N=N_n+N_s$ and hypercharge operator $Y=(N_n-2N_s)/3$, we can rewrite (3) as

$$E_Q = N\frac{2\alpha_n+\alpha_s}{3R} - \frac{(\alpha_s-\alpha_n)}{R}Y\,. \qquad (7)$$

It also useful to separate the summations in (4) and (5) into three parts: a summation over all quarks, a summation over only the nonstrange quarks, and a summation over only the strange quarks. Then we may write

$$E_M = -\frac{\alpha_c}{4R}\left(M_{ns}\sum_{i>j}(\lambda^c\sigma)_i\cdot(\lambda^c\sigma)_j + (M_{nn}-M_{ns})\sum_{n_1>n_2}(\lambda^c\sigma)_1\cdot(\lambda^c\sigma)_2 + (M_{ss}-M_{ns})\sum_{s_1>s_2}(\lambda^c\sigma)_1\cdot(\lambda^c\sigma)_2\right) \qquad (8)$$

and

$$E_E=\frac{\alpha_c}{4R}\left[E_{ns}\left(\tfrac{8}{3}N+\sum_{i>j}\lambda_i^c\cdot\lambda_j^c\right)+(E_{nn}-E_{ns})\left(\tfrac{8}{3}N_n+\sum_{n_1>n_2}\lambda_1^c\cdot\lambda_2^c\right)+(E_{ss}-E_{ns})\left(\tfrac{8}{3}N_s+\sum_{s_1>s_2}\lambda_1^c\cdot\lambda_2^c\right)\right]. \qquad (9)$$

IV. EVALUATION OF THE COLOR-MAGNETIC AND COLOR-ELECTRIC TERMS

We will make use of the permutation symmetry of the states to replace the sums of the color and color-spin tensor operators in (8) and (9) by more useful sums of flavor, spin, and flavor-spin tensor operators. We introduce the following permutation operators:

for SU(2), $P_{ij}=\frac{1}{2}(1+\sigma_i\cdot\sigma_j)$,

for SU(3), $P_{ij}=\frac{1}{2}(\frac{2}{3}+\lambda_i\cdot\lambda_j)$.

The three summation ranges in (8) and (9) we will consider separately.

A. The sum over all quarks

The states can be labeled by quantum numbers belonging to the groups

$$SU(3,F)\otimes SU(2,J)\otimes SU(3,C).$$

The wave function is antisymmetric with respect to flavor, spin, and color, so

$$P_{ij}^F P_{ij}^J P_{ij}^C = -1\,.$$

This gives

$$\lambda_i^C\cdot\lambda_j^C = -1-\tfrac{1}{3}\sigma_i\cdot\sigma_j - \tfrac{1}{2}\lambda_i^F\cdot\lambda_j^F - \tfrac{1}{2}(\lambda^F\sigma)_i\cdot(\lambda^F\sigma)_j\,,$$

$$-(\lambda^C\sigma)_i\cdot(\lambda^C\sigma)_j = 1+\tfrac{1}{3}\sigma_i\cdot\sigma_j+\tfrac{3}{2}\lambda_i^F\cdot\lambda_j^F - \tfrac{1}{2}(\lambda^F\sigma)_i\cdot(\lambda^F\sigma)_j\,.$$

Before proceeding we introduce the 35 SU(6, FJ) generators A_a with $a=1$ to 35. For the irrep [6] these can be found as the direct product of the generators and the unity operators in the irrep $\underline{3}$ of SU(3, F) and the irrep $J=\frac{1}{2}$ of SU(2, J). These 35 generators, normalized to $\mathrm{Tr}A_a^2=1$, are $\frac{1}{2}(\lambda_\alpha^F\otimes\underline{1})$, $(1/\sqrt{6})\,(\underline{1}\otimes\sigma_k)$, and $\frac{1}{2}(\lambda_\alpha^F\otimes\sigma_k)$ with $\alpha=1$ to 8 and $k=1$ to 3. The quadratic Casimir operator C_6 for SU(6, FJ) has in the irrep $[\mu]$ the eigenvalue $C_6(\mu)$ and is given by

$$C_6=\sum_{i,j}A_i\cdot A_j\,,$$

where

$$A_i\cdot A_j=\sum_a (A_a)_i(A_a)_j\,.$$

This implies that

$$2\sum_{i>j}A_i\cdot A_j = C_6 - NC_6(6) = C_6-\tfrac{35}{6}N\,.$$

Using the explicit expression for A_a we find

$$2\sum_{i>j} A_i \cdot A_j = \sum_{i>j} \tfrac{1}{3}\sigma_i \cdot \sigma_j + \tfrac{1}{2}\lambda_i^F \cdot \lambda_j^F + \tfrac{1}{2}(\lambda^F\sigma)_i \cdot (\lambda^F\sigma)_j \,.$$

This implies that we may write

$$\sum_{i>j} \lambda_i^C \cdot \lambda_j^C = -\sum_{i>j} (1 + 2A_i \cdot A_j),$$

$$-\sum_{i>j} (\lambda^C\sigma)_i \cdot (\lambda^C\sigma)_j = \sum_{i>j} (1 - 2A_i \cdot A_j + \tfrac{2}{3}\sigma_i \cdot \sigma_j + 2\lambda_i^F \cdot \lambda_j^F) \,.$$

Next we note the relations

$$\sum_{i>j} \sigma_i \cdot \sigma_j = 2\vec{J}^2 - \tfrac{3}{2}N \,,$$
$$\sum_{i>j} \lambda_i^F \cdot \lambda_j^F = 2C_3 - 2NC_3(3) = 2C_3 - \tfrac{8}{3}N \,, \tag{10}$$

where

$$C_3 = F^2 = \tfrac{1}{4}\sum_{i,j} \lambda_i \cdot \lambda_j$$

is the quadratic Casimir operator[16] in $SU(3, F)$, which has the value[17]

$$C_3(n) = f^2 = \tfrac{1}{3}(p^2 + pq + q^2) + p + q$$

in the irrep $\underline{n} = D(p, q)$. The equivalent expression for Eq. (10) for color operators, applied to states which are color singlets, gives

$$\sum_{i>j} \lambda_i^C \cdot \lambda_j^C = -\tfrac{8}{3}N \,. \tag{11}$$

This implies then that

$$C_6 = \tfrac{1}{2}N(18 - N) \tag{12}$$

and, therefore,

$$-\sum_{i>j} (\lambda^C\sigma)_i \cdot (\lambda^C\sigma)_j = N(N-10) + \tfrac{4}{3}\vec{J}^2 + 4C_3 \,. \tag{13}$$

B. The sum ranges over all nonstrange quarks

The states can be labeled by quantum numbers belonging to the groups

$$U(1, Y) \otimes SU(4, IJ_n) \otimes SU(2, J_s) \otimes SU(3, C) \,.$$

The wave function for the N_n nonstrange quarks is antisymmetric with respect to nonstrange spin, isospin, and color; therefore,

$$P_{12}^{J_n} P_{12}^{I} P_{12}^{C} = -1 \,.$$

This gives

$$\lambda_1^C \cdot \lambda_2^C = -\tfrac{7}{6} - \tfrac{1}{2}\sigma_1 \cdot \sigma_2 - \tfrac{1}{2}\tau_1 \cdot \tau_2 - \tfrac{1}{2}(\tau\sigma)_1 \cdot (\tau\sigma)_2 \,,$$

$$-(\lambda^C\sigma)_1 \cdot (\lambda^C\sigma)_2 = \tfrac{3}{2} + \tfrac{1}{6}\sigma_1 \cdot \sigma_2 + \tfrac{3}{2}\tau_1 \cdot \tau_2 - \tfrac{1}{2}(\tau\sigma)_1(\tau\sigma)_2 \,.$$

Here $\tfrac{1}{2}\tau_k$ with $k = 1$ to 3, normalized such that $\mathrm{Tr}\tau_k^2 = 2$, are the three $SU(2, I)$ generators in the $SU(2, I)$ irrep with $I = \tfrac{1}{2}$. We next introduce the 15 $SU(4, IJ_n)$ generators B_b with $b = 1$ to 15. For the irrep (4) of SU(4) they are normalized such that $\mathrm{Tr}B_b^2 = 1$ and are given by $\tfrac{1}{2}(\tau_k \otimes \underline{1})$, $\tfrac{1}{2}(1 \otimes \sigma_l)$, and $\tfrac{1}{2}(\tau_k \otimes \sigma_l)$ with $k, l = 1, 2, 3$. The quadratic Casimir operator for $SU(4, IJ_n)$ has in the irrep (ν) the eigenvalue $C_4(\nu)$ and is given by

$$C_4 = \sum_{n_1, n_2} B_1 \cdot B_2 \,.$$

Therefore,

$$2\sum_{n_1 > n_2} B_1 \cdot B_2 = C_4 - NC_4(4) = C_4 - \tfrac{15}{4}N \,.$$

Using the explicit expression for B_α we find

$$2\sum_{n_1 > n_2} B_1 \cdot B_2 = \tfrac{1}{2}\sum_{n_1 > n_2} [\sigma_1 \cdot \sigma_2 + \tau_1 \cdot \tau_2 + (\tau\sigma)_1 \cdot (\tau\sigma)_2] \,.$$

This implies that we may write

$$\sum_{n_1 > n_2} \lambda_1^C \cdot \lambda_2^C = -\sum_{n_1 > n_2} (\tfrac{7}{6} + 2B_1 \cdot B_2) = -\tfrac{7}{12}N_n^2 + \tfrac{13}{3}N_n - C_4 \,, \tag{14}$$

$$-\sum_{n_1 > n_2} (\lambda^C\sigma)_1 \cdot (\lambda^C\sigma)_2 = \sum_{n_1 > n_2} (\tfrac{3}{2} - 2B_1 \cdot B_2 + \tfrac{2}{3}\sigma_1 \cdot \sigma_2 + 2\tau_1 \cdot \tau_2) = \tfrac{3}{4}N_n^2 - N_n - C_4 + \tfrac{4}{3}\vec{J}_n^2 + 4\vec{I}^2 \,. \tag{15}$$

C. The sum ranges over all strange quarks

The states again can be labeled by the quantum numbers belonging to the groups

$$U(1, Y) \otimes SU(4, IJ_n) \otimes SU(2, J_s) \otimes SU(3, C) \,.$$

The wave function of the N_s strange quarks is antisymmetric with respect to strange spin and color. Therefore,

$$P_{12}^{J_s} P_{12}^{C} = -1 \,.$$

This gives

$$\lambda_1^C \cdot \lambda_2^C = -\tfrac{5}{3} - \sigma_1 \cdot \sigma_2 \,,$$

$$-(\lambda^C\sigma)_1 \cdot (\lambda^C\sigma)_2 = 3 - \tfrac{1}{3}\sigma_1 \cdot \sigma_2 \,.$$

Therefore, we get

$$\sum_{s_1 > s_2} \lambda_1^C \cdot \lambda_2^C = -\tfrac{5}{6}N_s^2 + \tfrac{7}{3}N_s - 2\vec{J}_s^2 \,, \tag{16}$$

$$-\sum_{s_1 > s_2} (\lambda^C\sigma)_1 \cdot (\lambda^C\sigma)_2 = \tfrac{3}{2}N_s^2 - N_s - \tfrac{2}{3}\vec{J}_s^2 \,. \tag{17}$$

In the same way we obtained Eq. (11) we get

$$\sum_{n_1>n_2} \lambda_1^C \cdot \lambda_2^C = 2C_3(C,n) - \tfrac{8}{3} N_n\,,$$

$$\sum_{s_1>s_2} \lambda_1^C \cdot \lambda_2^C = 2C_3(C,s) - \tfrac{8}{3} N_s\,,$$

where $C_3(C,n)$ and $C_3(C,s)$ are the quadratic SU(3, C) Casimir eigenvalues for the nonstrange and strange quarks. These do not vanish, but the color irreps of the nonstrange and strange quarks must be the complex conjugate of each other, so $C_3(C,n)=C_3(C,s)$. Using Eqs. (14) and (16), $N_n = \frac{2}{3}N+Y$ and $N_s=\frac{1}{3}N-Y$. We then find

$$\vec{J}_s^{\,2} - \tfrac{1}{2}C_4 + \tfrac{1}{8}Y^2 = \tfrac{1}{12}N(N-18) + \tfrac{2}{3}(N-9)Y\,. \qquad (18)$$

This equation and Eq. (12) result from the fact that we consider decompositions of totally antisymmetric states in SU(18), which contain an SU(3, C) color singlet. They enable us to calculate the quadratic Casimir eigenvalues for SU(6, FJ) irreps and SU(4, IJ_n) irreps occurring in the decomposition of SU(18).

V. THE MASS OPERATOR AND SU(6,FJ) TENSOR OPERATORS

Because of conservation of spin, isospin, and hypercharge the mass operator M must transform as a spin and isospin singlet with $Y=0$. It therefore can be expressed in irreducible SU(6, FJ) tensor operators $M(\mu, n)$ transforming as the $I=Y=0$ member of the flavor multiplet $\underline{n}$ with $J=0$ contained in the flavor-spin multiplet $[\mu]$. Thus,

$$M=\sum_{\mu,n} M(\mu,n)\,.$$

In this version of the bag model these operators $M(\mu,n)$ are quadratic operators constructed from the SU(6, FJ) tensor operators A_a which transform as members of the SU(6, FJ) irrep [35]. The mass operator therefore has parts transforming according to

$$[35]\otimes[35]=[1]\oplus[35]_s\oplus[35]_a\oplus[189]\oplus[405]\oplus[280]\oplus[280^*]\,,$$

where s and a mean the symmetric and antisymmetric combinations. From the tensor operators A_a we can make quadratic combinations $\Omega(\mu,n)$ transforming as the $I=Y=0$ member of the flavor multiplet $\underline{n}$ with $J=0$ contained in the flavor-spin multiplet $[\mu]$. They are[11]

$$\Omega(1,1)=1,\quad \Omega(189,1)=[C_3-\vec{J}^2]-\tfrac{1}{10}C_6,$$

$$\Omega(405,1)=[C_3+\vec{J}^2]-\tfrac{5}{14}C_6\,,$$

$$\Omega(35_a,8)=Y,\quad \Omega(35_s,8)=\vec{J}_s^{\,2}-\tfrac{1}{2}C_4+\tfrac{1}{8}Y^2+\tfrac{1}{6}C_6,$$

$$\Omega(189,8)=3[\vec{I}^2-\tfrac{1}{4}Y^2-\vec{J}_n^{\,2}+\vec{J}_s^{\,2}]-[C_3-\vec{J}^2]-\tfrac{3}{4}[\vec{J}_s^{\,2}-\tfrac{1}{2}C_4+\tfrac{1}{8}Y^2+\tfrac{1}{6}C_6]\,,$$

$$\Omega(405,8)=3[\vec{I}^2-\tfrac{1}{4}Y^2+\vec{J}_n^{\,2}-\vec{J}_s^{\,2}]-[C_3+\vec{J}^2]+\tfrac{21}{8}[\vec{J}_s^{\,2}-\tfrac{1}{2}C_4+\tfrac{1}{8}Y^2+\tfrac{1}{6}C_6]\,,$$

$$\Omega(189,27)=\tfrac{4}{3}[\vec{I}^2-\tfrac{1}{4}Y^2-\vec{J}_n^{\,2}+\vec{J}_s^{\,2}]-[C_3-\vec{J}^2]+\tfrac{4}{3}[\vec{J}_s^{\,2}-\tfrac{1}{2}C_4+\tfrac{1}{8}Y^2+\tfrac{1}{6}C_6]-\tfrac{20}{3}[\vec{J}_s^{\,2}-\tfrac{1}{4}Y^2]+\tfrac{1}{2}C_6,$$

$$\Omega(405,27)=\tfrac{4}{3}[\vec{I}^2-\tfrac{1}{4}Y^2+\vec{J}_n^{\,2}-\vec{J}_s^{\,2}]-[C_3+\vec{J}^2]-\tfrac{4}{3}[\vec{J}_s^{\,2}-\tfrac{1}{2}C_4+\tfrac{1}{8}Y^2+\tfrac{1}{6}C_6]+\tfrac{20}{3}[\vec{J}_s^{\,2}+\tfrac{3}{4}Y^2]-\tfrac{5}{18}C_6\,,$$

$$\Omega(280,8)=\Omega(280^*,8)=0.$$

The mass operator for the N-quark states in the bag model, therefore, can be rewritten as

$$M=\sum_{\mu,n} m(\mu,n)\Omega(\mu,n)\,,$$

where $m(\mu,n)$ are constants calculable in the model. Using for convenience the specific operator combinations, occurring in the Ω's, we can write

$$M=m_0+m_1[C_3-\vec{J}^2]+m_2[C_3+\vec{J}^2]+m_3Y+m_4[\vec{J}_s^{\,2}-\tfrac{1}{2}C_4+\tfrac{1}{8}Y^2]+m_5[\vec{I}^2-\tfrac{1}{4}Y^2]+m_6[\vec{J}_n^{\,2}-\vec{J}_s^{\,2}]+m_7\vec{J}_s^{\,2}+m_8Y^2.$$

Different from the mass operator containing only the contributions $M(\mu,1)$ and $M(\mu,8)$[11] are the contributions $\sim\vec{J}_s^{\,2}$ and $\sim Y^2$. These tensors come in with the $M(\mu,27)$.

To see which tensors contribute in a particular SU(6, FJ) irrep, we have to consider the Clebsch-Gordan series

$$[56]\otimes[56^*]=[1]\otimes[35]\otimes[405]\otimes[2695]\,,$$

$$[490]\otimes[490^*]=[1]\otimes[35]\otimes[189]\otimes[405]\otimes[2695]\otimes\cdots\,,$$

$$[980]\otimes[980]=[1]\otimes[35]\otimes[175]\otimes[189]\otimes[405]\otimes\cdots\,.$$

The irrep [35] appears only once in all these products. Therefore, the matrix elements of the operators $\Omega(35_a,8)$ and $\Omega(35_s,8)$ must be proportional as can be seen in Eq. (18). In principle, we then are left with a mass operator with 8 nonzero coefficients. However, there is some symmetry left, due to the simple form of the bag Hamiltonian. The operators C_3, $\vec{J}^2$, $\vec{I}^2$, and $\vec{J}_n^{\,2}$ coming only from E_M appear in the bag mass operator in the specific combinations $C_3+\frac{1}{3}\vec{J}^2$ and $\vec{I}^2+\frac{1}{3}\vec{J}_n^{\,2}$ [see Eqs. (13) and (15)]. The resulting mass operator then has the following structure:

$$M=a_0+a_1[C_3+\tfrac{1}{3}\vec{J}^2]+a_2Y+a_3[(\vec{I}^2-\tfrac{1}{4}Y^2)+\tfrac{1}{3}(\vec{J}_n^{\,2}-\vec{J}_s^{\,2})]+a_4\vec{J}_s^{\,2}+a_5Y^2. \qquad (19)$$

TABLE VI. The coefficients for the general mass formula [see Eq. (19)].

B	a_0	a_1	a_2	a_3	a_4	a_5
1	0.9337	0.0571	−0.1896	0.0148	−0.004 22	−0.000 24
2	2.2091	0.0414	−0.1613	0.0146	−0.004 92	−0.000 19
3	3.4789	0.0352	−0.1297	0.0144	−0.005 23	−0.000 16
4	4.8003	0.0305	−0.0992	0.0142	−0.005 48	−0.000 12
5	6.1666	0.0276	−0.0680	0.0140	−0.005 64	−0.000 09
6	7.5766	···	···	···	···	···

Moreover, in the product $[56]\otimes[56^*]$ the irrep $[189]$ does not occur. Therefore, the matrix elements of $\Omega(189,n)$ disappear between states belonging to the irreps $[56]$ or $[56^*]$ for $\underline{n}=\underline{1},\ \underline{8},\ \underline{27}$. This gives

$$\langle 56|C_3-\vec{J}^2|56\rangle=\tfrac{9}{4},$$

$$\langle 56|\vec{J}_n^{\,2}-\vec{J}_s^{\,2}|56\rangle=\langle 56|\vec{I}^2-\tfrac{1}{4}Y^2+Y-\tfrac{3}{4}|56\rangle,$$

$$\langle 56|\vec{J}_s^{\,2}|56\rangle=\langle 56|\tfrac{1}{4}Y^2-Y+\tfrac{3}{4}|56\rangle.$$

In the relations for the irrep $[56^*]$ Y has to be replaced by $-Y$. The mass operator for the $B=1$ and $B=5$ states, therefore can be simplified to

$$M=b_0+b_1\vec{J}^2+b_2Y+b_3(\vec{I}^2-\tfrac{1}{4}Y^2)+b_4Y^2. \qquad (20)$$

Up to the term $\sim Y^2$ coming from the $M(405,27)$ contribution in this specific case, this is the familiar SU(6) mass operator.[10]

Having performed the summations for the color-magnetic and color-electric interaction terms in Sec. IV we may collect all terms to yield the following mass operator:

$$M=\frac{4\pi}{3}BR^3-\frac{Z_0}{R}+N\frac{2\alpha_n+\alpha_s}{3R}-\frac{\alpha_s-\alpha_n}{R}\,Y+\frac{\alpha_c}{4R}M_{ns}[N(N-10)+4(C_3+\tfrac{1}{3}\vec{J}^2)]$$
$$+\frac{\alpha_c}{4R}(M_{nn}-M_{ns})[\tfrac{3}{4}N_n^2-N_n-C_4+4(\vec{I}^2+\tfrac{1}{3}\vec{J}_n^{\,2})]+\frac{\alpha_c}{4R}(M_{ss}-M_{ns})(\tfrac{3}{2}N_s^2-N_s-\tfrac{2}{3}\vec{J}_s^{\,2})$$
$$+\frac{\alpha_c}{4R}(E_{nn}-E_{ns})[\tfrac{7}{12}N_n(12-N_n)-C_4]+\frac{\alpha_c}{4R}(E_{ss}-E_{ns})[\tfrac{5}{6}N_s(6-N_s)-2\vec{J}_s^{\,2}].$$

M can be rewritten in combinations occurring in (19). The coefficients are

$$a_0=\frac{4\pi}{3}BR^3-\frac{Z_0}{R}+N\frac{2\alpha_n+\alpha_s}{3R}+\frac{\alpha_c}{4R}[N(N-10)(\tfrac{2}{3}M_{nn}+\tfrac{1}{3}M_{ss})+N(18-N)(\tfrac{1}{6}\bar{M}+\tfrac{5}{54}\bar{E})],$$
$$a_1=\frac{\alpha_c}{R}M_{ns},\quad a_2=-\frac{\alpha_s-\alpha_n}{R}+\frac{\alpha_c}{4R}[(\tfrac{5}{3}N-7)(M_{nn}-M_{ss})+(N-9)(\tfrac{2}{3}\bar{M}+\tfrac{5}{9}\bar{E})],$$
$$a_3=\frac{\alpha_c}{2R}(M_{nn}-M_{ns}),\quad a_4=-\frac{\alpha_c}{4R}(\tfrac{2}{3}\bar{M}+2\bar{E}),\quad a_5=\frac{\alpha_c}{4R}(\tfrac{3}{2}\bar{M}-\tfrac{5}{6}\bar{E}),$$

where $\bar{M}=M_{nn}+M_{ss}-2M_{ns}$ and $\bar{E}=E_{nn}+E_{ss}-2E_{ns}$. Using the values of the functions α_i, M_{ij}, and E_{ij} in Table V, we are able to calculate the coefficients a_0 to a_5 in Eq. (19) for $B=1$ to 6 and b_0 to b_4 in Eq. (20) for $B=1$ and 5. They are listed in Tables VI and VII.

VI. NUMERICAL ANALYSIS AND DISCUSSION

The bag parameters B, Z_0, α_c, and m_s were determined to give the best overall reproduction

TABLE VII. The coefficients for the mass formula for $B=1$ and 5 [see Eq. (20)].

B	b_0	b_1	b_2	b_3	b_4
1	1.054	0.0762	−0.1805	0.0197	−0.0013
5	6.221	0.0368	−0.0783	0.0187	−0.0015

TABLE VIII. Numerical results for $B=1$ (see text). All masses in GeV.

	A		B
b_0	1.054		1.062
b_1	0.762		0.717
b_2	−0.181		−0.192
b_3	0.020		0.035
b_4	−0.0013		−0.0020
	M_A	M_B	$M_{\exp}$
N	0.939	0.939	0.939
Λ	1.111	1.116	1.116
Σ	1.150	1.186	1.193
Ξ	1.300	1.323	1.318
Δ	1.227	1.260	1.232
Σ^*	1.379	1.401	1.385
Ξ^*	1.529	1.538	1.533
Ω	1.676	1.672	1.672

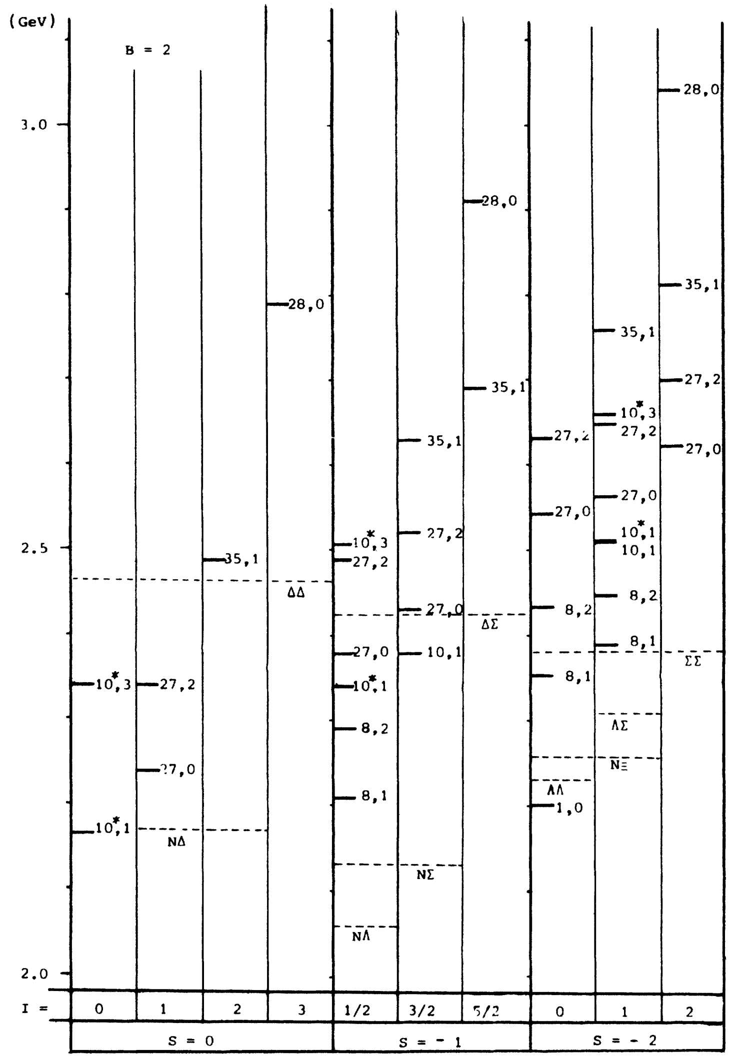

FIG. 1. The masses of the $B=2$-baryons, for $S=0$, -1, and -2. The states are characterized by the flavor representation that they dominantly belong to and their spin. Nearby thresholds are represented by dashed lines labeled with the name of the corresponding channel.

of the light ($B=1$) baryon masses, using one R value for the entire multiplet, as well as reasonable values for the K, K^*, ω, and ϕ meson masses. The model proves to be sensitive to variation in B, whereas the other dependencies do not seem to be very critical. Comparison with the values obtained by the MIT group[5] shows that α_c has become a little smaller and Z_0 a little larger. The second parameter shift causes the masses to be somewhat smaller correcting for the fact that, since we do not minimize for each state separately, our masses tend to be slightly above minimum values. The mass spectrum of the $B=2$ to $B=6$ baryons does not exhibit significant shifts, when changing from one set of parameters to the other.

In Table VIII the coefficients b_0 to b_4, following from our parameters, are listed (A), together with the values, which we found by treating these coefficients as independent parameters and determining them directly from the baryon spectrum (B). The resulting masses for both sets of coefficients are given together with the experimental values. Comparison gives an indication about the applicability of the mass formula and its MIT bag analog. The calculations were carried out under the assumption that the spherical-cavity approximation remains reasonable for higher B systems.

The computation of coefficient b_4, occurring in

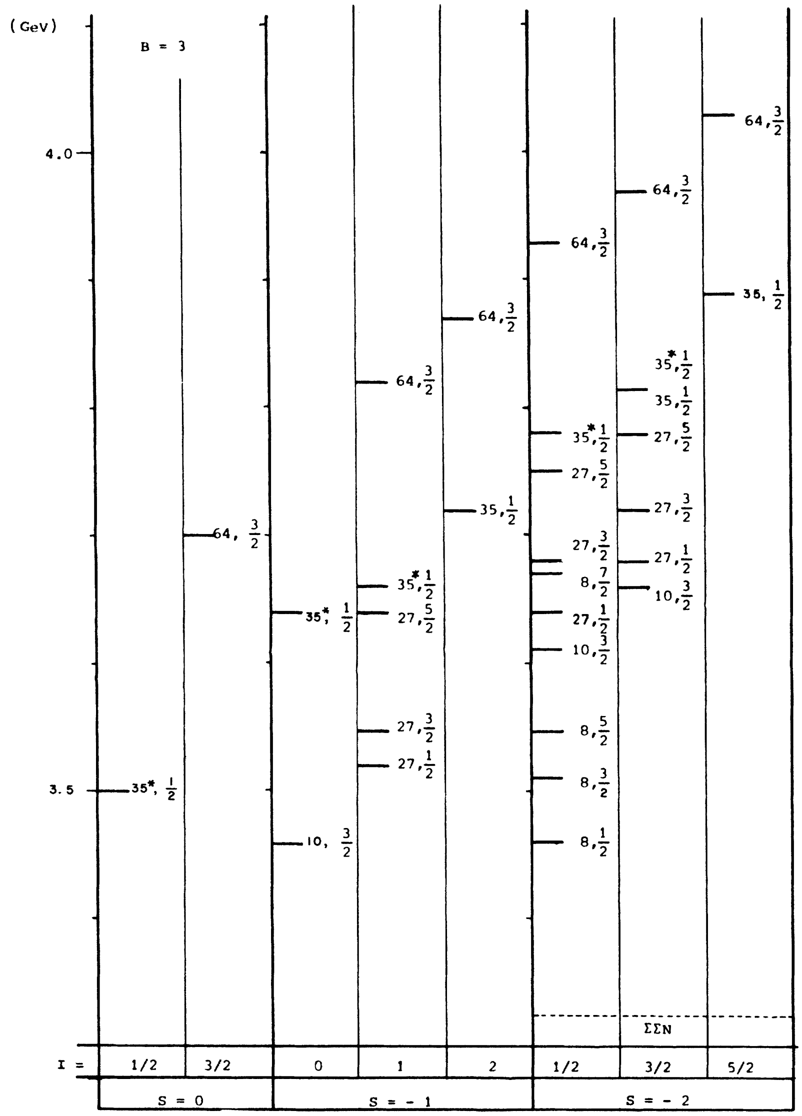

FIG. 2. The masses of the $B=3$ baryons for $S=0$, -1, and -2.

the term $b_4 Y^2$, which for instance breaks the equal spacing in the decuplet, gives the correct sign and order of magnitude as compared with the result in column B of Table VIII. The value of b_3 determining the Σ-Λ splitting, is too small, which seems to be inherent to the bag model.[5] The agreement with the experimental spectrum is fairly satisfactory.

For $B=2, 3, 4$ states the mass operator is diagonal with respect to J, Y, and I. Mixing occurs between different flavor multiplets with the same J, Y, and I, when a particular flavor state is a linear combination of some (J_n, J_s) states. Since the contribution of the SU(3, F) quadratic Casimir C_3 in the mass formula is much larger that the contribution of J_n, J_s $(a_1 > a_3, a_4)$, the mass operator is almost diagonal in flavor. In Figs. 1, 2, and 3 the masses of the multibaryon states with $B=2$, 3, and 4 and $S=0$, -1, -2, have been plotted together with the important thresholds. The states are denoted by their quantum numbers S, I, J, and the flavor multiplet they (mostly) belong to. In Tables IX to XII a complete list of the multibaryon masses has been given. The states that participate in mixing are supplied with a letter (a, b, c) that indicates the uncertainty induced by this mixing. Apart from these uncertainties, there are of course the ones due to the bag model. The almost

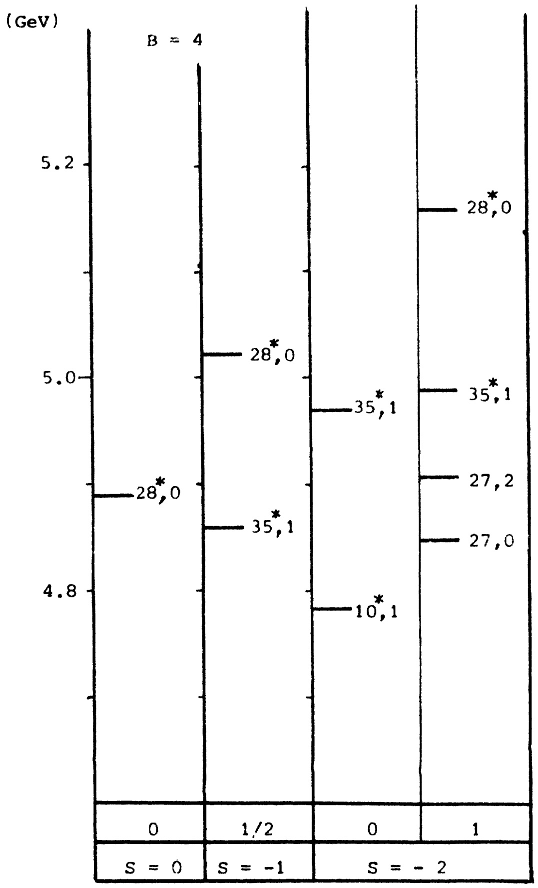

FIG. 3. The masses of the $B=4$ baryons for $S=0$, -1, and -2.

TABLE IX. Masses of the $B=2$ baryons in GeV. The uncertainties induced by the mixing are $a \leq 10$ MeV, $10 \leq b \leq 20$ MeV, and $20 \leq c \leq 30$ MeV.

Y	I	J	F	Mass (GeV)
2	3	0	28	2.79
	2	1	35	2.49
	1	2	27	2.34
		0	27	2.24
	0	3	10*	2.34
		1	10*	2.16
1	$\frac{5}{2}$	1	35	2.69
		0	28	2.91
	$\frac{3}{2}$	2	27	2.52
		1	35	2.63 a
			10	2.38 a
		0	27	2.42
	$\frac{1}{2}$	3	10*	2.51
		2	27	2.49 a
			8	2.29 a
		1	10*	2.34 a
			8	2.21 a
		0	27	2.38
0	2	2	27	2.70
		1	35	2.81
		0	28	3.04 a
			27	2.62 a
	1	3	10*	2.66
		2	27	2.65 a
			8	2.45 a
		1	35	2.76 b
			10	2.51 b
			10*	2.51 b
		1	8	2.39 b
		0	27	2.56
	0	2	27	2.63 b
			8	2.43 b
		1	8	2.35
		0	27	2.54 a
			1	2.20 a
-1	$\frac{3}{2}$	3	10*	2.82
		2	27	2.82
		1	35	2.94 a
			10*	2.69 a
		0	28	3.16 a
			27	2.74 a
	$\frac{1}{2}$	2	27	2.78 b
			8	2.57 b
		1	35	2.89 b
			10	2.64 b
			8	2.52 b
		0	27	2.71
-2	1	2	27	2.95
		1	35	3.06
		0	28	3.29 a
			27	2.87 a
	0	1	35	3.04 b
			10	2.79 b
-3	$\frac{1}{2}$	1	35	3.19
		0	28	3.41
-4	0	0	28	3.54

complete lack of data means that we can not say much about the absolute mass scales. This is mainly due to the fact that the hadron mass rather strongly depends on the volume term in E_B, which may be too simple a picture to maintain for higher-mass states. The relative positions seem to be more reliable as they depend on the color interaction.[2]

Next we will discuss some of the predictions.

NN system: We find one resonance at $E_{\text{c.m.}}=2.16$ GeV ($T_{\text{lab}}=610$ MeV) in the $^3S_1$-$^3D_1$ wave, one in the $^1S_0$ wave at $E_{\text{c.m.}}=2.24$ GeV ($T_{\text{lab}}=790$ MeV) and two (almost?) degenerate resonances: one in the $^1D_2$ and one in the $^3D_3$-$^3G_3$ waves at $E_{\text{c.m.}}=2.34$ GeV ($T_{\text{lab}}=1040$ MeV). There is some experimental evidence[18] for an enhancement at 2.38 GeV, which could be due to the degenerate $^1D_2$ and $^3D_3$ resonances. The bag model does not tell us anything about the width, but in the above experiment it is about 200 MeV.

YN system. In the ΛN channel we find an $(\underline{8}, 1^+)$ resonance at 2.21 GeV, an $(\underline{8}, 2^+)$ resonance at 2.29

TABLE X. Masses of the $B=3$ baryons. See caption of Table IX.

Y	I	J	F	Mass (GeV)
3	$\frac{3}{2}$	$\frac{3}{2}$	64	3.70
	$\frac{1}{2}$	$\frac{1}{2}$	35*	3.50
2	2	$\frac{3}{2}$	64	3.87
		$\frac{1}{2}$	35	3.72
	1	$\frac{5}{2}$	27	3.64
		$\frac{3}{2}$	64	3.82 *a*
			27	3.57 *a*
		$\frac{1}{2}$	35*	3.66 *a*
			27	3.52 *a*
	0	$\frac{3}{2}$	10*	3.46
		$\frac{1}{2}$	35*	3.63
1	$\frac{5}{2}$	$\frac{3}{2}$	64	4.03
		$\frac{1}{2}$	35	3.89
	$\frac{3}{2}$	$\frac{5}{2}$	27	3.78
		$\frac{3}{2}$	64	3.79 *b*
			27	3.73 *b*
			10	3.66 *b*
		$\frac{1}{2}$	35	3.82 *a*
			35*	3.82 *a*
			27	3.68 *a*
	$\frac{1}{2}$	$\frac{7}{2}$	8	3.67
		$\frac{5}{2}$	27	3.75 *a*
			8	3.57 *a*
		$\frac{3}{2}$	64	3.93 *b*
			27	3.68 *b*
			10*	3.61 *b*
			8	3.51 *b*
		$\frac{1}{2}$	35*	3.78 *a*
			27	3.64 *a*
			8	3.46 *a*
0	3	$\frac{3}{2}$	64	4.19
	2	$\frac{5}{2}$	27	3.92
		$\frac{3}{2}$	64	4.12 *b*
			27	3.87 *b*
		$\frac{1}{2}$	35	3.98 *b*
			35*	3.98 *b*
			27	3.84 *b*
0	1	$\frac{7}{2}$	8	3.79
		$\frac{5}{2}$	27	3.90 *b*
			8	3.72 *b*
		$\frac{3}{2}$	64	4.07 *c*
			27	3.83 *c*
			10	3.76 *c*
			10*	3.76 *c*
			8	3.65 *c*
		$\frac{1}{2}$	35	3.94 *a*
			35*	3.94 *a*
			27	3.80 *a*
			8	3.62 *a*
	0	$\frac{9}{2}$	1	3.79
		$\frac{7}{2}$	8	3.79
		$\frac{5}{2}$	27	3.86 *c*
			8	3.68 *c*
			1	3.58 *c*
		$\frac{3}{2}$	64	4.05 *c*
			27	3.80 *c*
			8	3.63 *c*
			1	3.52 *c*
		$\frac{1}{2}$	27	3.76 *b*
			8	3.58 *b*
−1	$\frac{5}{2}$	$\frac{3}{2}$	64	4.29
		$\frac{1}{2}$	35*	4.15
	$\frac{3}{2}$	$\frac{5}{2}$	27	4.04
		$\frac{3}{2}$	64	4.23 *b*
			27	3.99 *b*
			10*	3.92 *b*
		$\frac{1}{2}$	35	4.08 *a*
			35*	4.08 *a*
			27	3.94 *a*
	$\frac{1}{2}$	$\frac{7}{2}$	8	3.93
		$\frac{5}{2}$	27	4.01 *a*
			8	3.83 *a*
		$\frac{3}{2}$	64	4.19 *b*
			27	3.94 *b*
			10	3.87 *b*
			8	3.77 *b*

TABLE X. (*Continued*)

Y	I	J	F	Mass (GeV)
−1	$\frac{1}{2}$	$\frac{1}{2}$	35	4.04 *a*
			27	3.90 *a*
			8	3.72 *a*
−2	2	$\frac{3}{2}$	64	4.38
		$\frac{1}{2}$	35*	4.24
	1	$\frac{5}{2}$	27	4.16
		$\frac{3}{2}$	64	4.34 *a*
			27	4.09 *a*

Y	I	J	F	Mass (GeV)
−2	1	$\frac{1}{2}$	35	4.18 *a*
			27	4.04 *a*
	0	$\frac{3}{2}$	10	3.98
		$\frac{1}{2}$	35	4.15
−3	$\frac{3}{2}$	$\frac{3}{2}$	64	4.48
	$\frac{1}{2}$	$\frac{1}{2}$	35	4.28

TABLE XI. Masses of the $B=4$ baryons. See caption of Table IX.

Y	I	J	F	Mass (GeV)
4	0	0	28*	4.89
3	$\frac{1}{2}$	1	35*	4.86
		0	28*	5.03
2	1	2	27	4.91
		1	35*	4.99
		0	28*	5.16 *a*
			27	4.85 *a*
	0	1	35*	4.97
			10*	4.79
1	$\frac{3}{2}$	3	10	5.04
		2	27	5.04
		1	35*	5.13 *a*
			10	4.94 *a*
		0	28*	5.29 *a*
			27	4.98 *a*
	$\frac{1}{2}$	2	27	5.00 *b*
			8	4.85 *b*
		1	35*	5.08 *b*
			10*	4.90
			8	4.81 *b*
		0	27	4.95
0	2	2	27	5.18
		1	35*	5.26
		0	28*	5.43 *a*
			27	5.12 *a*
	1	3	10	5.14
		2	27	5.13 *a*
			8	4.98 *a*

Y	I	J	F	Mass (GeV)
0	1	1	35*	5.21 *b*
			10*	5.03 *b*
			10	5.03 *b*
			8	4.94 *b*
		0	27	5.06
	0	2	27	5.11 *b*
			8	4.96 *b*
		1	8	4.90
		0	27	5.04 *a*
			1	4.80 *a*
−1	$\frac{5}{2}$	1	35	5.40
		0	28*	5.57
	$\frac{3}{2}$	2	27	5.26
		1	35*	5.34 *a*
			10*	5.15 *a*
		0	27	5.19
	$\frac{1}{2}$	3	10	5.25
		2	27	5.23 *a*
			8	5.08 *a*
		1	10	5.11 *a*
			8	5.02 *a*
		0	27	5.15
−2	3	0	28*	5.70
	2	1	35*	5.47
	1	2	27	5.35
		0	27	5.27
	0	3	10	5.35
		1	10	5.20

TABLE XII. Masses of the $B=1$ and $B=5$ baryons.

Y	I	J	F	Mass (GeV)
		$B=1$		
1	$\frac{3}{2}$	$\frac{3}{2}$	10	1.227
	$\frac{1}{2}$	$\frac{1}{2}$	8	0.939
0	1	$\frac{3}{2}$	10	1.379
		$\frac{1}{2}$	8	1.150
	0	$\frac{1}{2}$	8	1.111
−1	$\frac{1}{2}$	$\frac{3}{2}$	10	1.529
		$\frac{1}{2}$	8	1.300
−2	0	$\frac{3}{2}$	10	1.676
		$B=5$		
2	0	$\frac{3}{2}$	10*	6.18
1	$\frac{1}{2}$	$\frac{3}{2}$	10*	6.29
		$\frac{1}{2}$	8	6.18
0	1	$\frac{3}{2}$	10*	6.40
		$\frac{1}{2}$	8	6.29
	0	$\frac{1}{2}$	8	6.25
−1	$\frac{3}{2}$	$\frac{3}{2}$	10*	6.50
	$\frac{1}{2}$	$\frac{1}{2}$	8	6.34

GeV, and a $(\underline{10}^*, 1^+)$ resonance at 2.34 GeV. Established[12, 19–22] is the Λp resonance *at* the $\Sigma^+ n$ threshold ($E = 2.13$ GeV). Because this state can very well be explained in potential theory[13] being the YN equivalent of the deuteron, it certainly is not one of the states mentioned above.

In the existing experiments three other energy regions show to be interesting. A region around $E = 2.14$ GeV just *above* the ΣN threshold, where there is weak evidence for another resonance.[12, 19–21] This could be the above-mentioned $(\underline{8}, 1^+)$ resonance. Another region around 2.25 GeV, where a resonance is found by Shahbazian[19] at 2.26 GeV. The Berkeley data[20] also show a peak at 2.24 GeV. This is probably the $(\underline{8}, 2^+)$ resonance. The third region is around 2.33 GeV, where both the Berkeley[20] and Dubna[19] data show peaks (statistically not significant). We would like to assign this effect to the $(\underline{10}^*, 1^+)$ resonance.

Of course additional information about J^P is needed to decide these questions.

YY and ΞN system. The most remarkable prediction is that of a bound $(\underline{1}, 0^+)$ state at 2.20 GeV. Furthermore, there are the $I=0$ states at $E = 2.35$ GeV $(\underline{8}, 1^+)$ and at 2.43 GeV $(\underline{8}, 2^+)$ and the $I = 1$ states at 2.39 GeV $(8, 1^+)$ and 2.45 GeV $(8, 2^+)$.

For $\Lambda\Lambda$ this means a bound state about 30 MeV below threshold in the $^1S_0$ wave. The $(8, 1^+)$ resonance should appear in the $^3S_1$- $^3D_1$ waves of ΞN and possibly $\Sigma\Sigma$. Because of the Pauli principle the isoscalar $J^P = 1^+$ state cannot decay in the $\Lambda\Lambda$ channel.

A possible candidate for the $(8, 2^+)$ $I = 0$ state is the $\Lambda\Lambda$ resonance at 2.37 GeV reported by Shahbazian *et al.*[19] and Beillère *et al.*[23]

If the above arguments are correct, we see that our lowest states are consistently 40–60 MeV high. This would mean that the $\Lambda\Lambda$ bound state may even be 90 MeV below threshold at 2.14–2.16 GeV.

Note added in proof. In the pp system a spin singlet state with mass 2.39 GeV (width 100 MeV) was reported in W. Grein and P. Kroll, Wuppertal Report No. WUB77-6 (unpublished), which coincides with the enhancement at 2.38 GeV in deuteron photodisintegration experiments.[18] These states could be the degenerate $^1D_2$ and $^3D_3$-$^3G_3$ resonances, predicted at 2.34 GeV.

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