PHYSICAL REVIEW D VOLUME 17, NUMBER 10 15 MAY 1978

Structure of asymptotic fields associated with permanently confined degrees of freedom in quantum field theory

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(Received 17 January 1978)

I give an interpretation of permanent color confinement which allows the existence of a single-particle state carrying any color representation, but forbids the existence of a scattering state with two or more separated particles each carrying a nonsinglet color representation. I show that the Haag in-field expansion of the Wightman field has a form consistent with this interpretation in all sectors of the second-quantized nonrelativistic harmonic oscillator. I replace the usual in-fields by the confined in-fields which, by construction, have nonvanishing matrix elements only between the vacuum and one-particle states. I propose that the occurrence of the confined in-field is the *sine qua non* of confinement in terms of asymptotic fields; and that the Haag expansion of the Wightman field in normal-ordered products of confined in-fields for fields carrying confined degrees of freedom, such as color, and ordinary in-fields for other fields, such as normal hadron fields, will be a useful tool to study confinement in relativistic theories, such as quantum chromodynamics.

I. INTRODUCTION

The apparent failure to detect fractionally charged quarks has led to the hypothesis that quarks and other color carrying objects are permanently confined in hadrons; in particular, the confinement hypothesis includes the notion that free fractionally charged quarks do not occur in nature despite the presence of fractionally charged quark fields in the theory of hadrons [for example, in quantum chromodynamics[1] (QCD)]. What is the physical interpretation of $q^\dagger(f)|0\rangle$, where q is a fractionally charged quark field and f is a test function, in such a theory? If this state has finite energy, which I will assume, then it represents a collection of objects with a total charge which is fractional. In addition, if the quark field carries a nonsinglet $SU(3)_{color}$ [or $SU(3)_c$, for short] representation, then the above state represents a collection of objects in a nonsinglet $SU(3)_c$ representation, and, further, a properly chosen polynomial in the color-carrying fields (the quark, antiquark, and gluon fields in QCD) applied to the vacuum will produce a state in any nonsinglet SU(3) representation. These observations seem to go against the possibility of permanent confinement of color; however, I will now give an interpretation of color confinement which is consistent with these observations.

I propose to interpret permanent confinement of color to mean that the macroscopic separation of color-carrying objects into freely moving particles, as in a scattering state, is absolutely forbidden, so that all color in a given state is localized in a single particle, but that any $SU(3)_c$ representation can occur in this way. Then a state such as $q^\dagger(f)|0\rangle$ consists of a superposition of systems, each of which contains a fractionally charged color-carrying single particle (a quark in this example) together with some collection of hadrons, each of which is an integrally charged color singlet.

The color-carrying particles might conceivably have anomalous kinematic properties (be some sort of jellylike object), but I will assume that they have normal single-particle kinematic properties, i.e., that they have a definite mass, intrinsic spin, etc. I emphasize that there must be single-particle states carrying every $SU(3)_c$ representation (without many-particle states of these color-carrying particles) as well as the usual single- and many-particle states of hadrons.

This interpretation of confinement is consistent with the failure to detect quarks or other color-carrying objects, because no quarks or other color-carrying particles could be produced in the laboratory from an initial color-singlet state, and, considering cosmic-ray experiments, since there is at most one color-carrying particle in any state of the universe, the flux of color-carrying particles incident on the earth's atmosphere is at most one per unit area per unit time, and most likely it is zero.

The presence of single-particle quark (and other color-carrying species) states would seem to imply the existence of quark in (and out) operators, and, via repeated application of the quark in-field to the vacuum, to imply the existence of many-quark states in direct contradiction to the interpretation of confinement just given. This objection seems so convincing that until recently I believed that the interpretation of confinement just given could not be valid.

I suggest that this objection can be answered by noticing that although repeated application of the quark in-field to the vacuum must lead to many-quark states, such states need not be created by repeated application of the quark Wightman field[2] to the vacuum. In particular, the asymptotic limits which allow construction of the quark in-field as a limit of the quark Wightman field in the Lehmann, Symanzik, and Zimmermann[3] (LSZ) or Haag and Ruelle[4] (HR) sense will not have their usual form because the vacuum expectation values will not have the required clustering properties,[5] so that it may be impossible for the quark in-field to be extracted from the quark Wightman field.

In Sec. II, I study a soluble nonrelativistic model with confinement in the lowest sector, and, incidentally, develop some properties of this formalism. In Sec. III, I introduce the confined in-field and give the form of the solution of the model of Sec. III in all sectors. In Sec. IV, I make conjectures about asymptotic operator structure in relativistic theories with confinement.

II. ASYMPTOTIC OPERATOR STRUCTURE OF A SOLUBLE MODEL WITH CONFINEMENT IN THE LOWEST SECTOR

To examine the possibility of having single-particle states without the associated many-particle scattering states, I consider a nonrelativistic harmonic oscillator in second-quantized form. The spectrum of this model has a unique discrete vacuum state, a continuum of single-particle states which can be labeled by the momentum, a countable infinity of continua of single-particle bound states of two of the quanta of the basic Wightman field which can be labeled by the discrete quantum numbers of the bound states of two quanta in the harmonic potential together with the total momentum of the bound state, and analogous sets of states which are single-particle bound states of three or more of the quanta of the basic field. There are no states with two or more freely moving separated particles in a scattering state in this model. All the states of this model correspond to the color-carrying single-particle states of QCD. Particles analogous to the usual hadrons are absent.

In terms of the charge (analogous to color, but Abelian here) carried by the Wightman field, there are single-particle states carrying charge one, two, three, etc., (analogous to the single-particle states carrying color) and, aside from the vacuum, no states with charge zero (analogous to the normal hadrons).

I will show that the Wightman field in this model has a Haag expansion[6] in the asymptotic field for the state with the quantum numbers of the Wightman field together with the asymptotic fields associated with each type of bound state[7] labeled by the number of quanta of the Wightman field contained in the bound state together with the internal quantum numbers of the bound state. I will exhibit the cancellations which remove the possible contradiction between the Haag expansion for the Wightman field and the absence of many-particle scattering states.

The Hamiltonian and charge are

$$H=(2m)^{-1}\int d^3x\, \vec{\nabla}\psi^\dagger(\vec{x},t)\cdot\vec{\nabla}\psi(\vec{x},t) + (k/2)\int d^3x d^3y \psi^\dagger(\vec{x},t)\psi^\dagger(\vec{y},t) \times(\vec{x}-\vec{y})^2\psi(\vec{y},t)\psi(\vec{x},t) \tag{2.1}$$

and

$$Q=\int d^3x\psi^\dagger(\vec{x},t)\psi(\vec{x},t)\,, \tag{2.2}$$

where $\psi(\vec{x},t)$ is a nonrelativistic Wightman field in three-dimensional space which satisfies Fermi equal-time anticommutation relations. The spectrum of this Hamiltonian has been described above. The equation of motion for ψ is

$$i\partial\psi(\vec{x},t)/\partial t=(2m)^{-1}\vec{\nabla}^2\psi(\vec{x},t) + k\int d^3y\psi^\dagger(\vec{y},t)(\vec{x}-\vec{y})^2\psi(\vec{y},t)\psi(\vec{x},t)\,. \tag{2.3}$$

The in-fields in the model are $\psi_{n\alpha,\mathrm{in}}(\vec{x},t)$, where n is the number of quanta of ψ in the bound state created by $\psi^\dagger_{n\alpha,\mathrm{in}}$ and α is the set of internal quantum numbers of this bound state. The Haag expansion of ψ in momentum space has the form

$$a(\vec{p},E)=a_{\mathrm{in}}(\vec{p})\,\delta(E-e(\vec{p})) + \int d^3q\, d^3r_1 d^3r_2\delta(\vec{p}-\vec{r}_1-\vec{r}_2+\vec{q})\delta(E-e(\vec{r}_1)-e(\vec{r}_2)+e(\vec{q}))f_{1;1^2}(\vec{q};\vec{r}_1,\vec{r}_2)\,a^\dagger_{\mathrm{in}}(\vec{q})a_{\mathrm{in}}(\vec{r}_1)a_{\mathrm{in}}(\vec{r}_2) + \int d^3q\, d^3r\delta(\vec{p}-\vec{r}+\vec{q})\,\delta(E-E_{2\alpha}(\vec{r})+e(\vec{q}))f_{1;2\alpha}(\vec{q};\vec{r})\,a^\dagger_{\mathrm{in}}(\vec{q})\,a_{2\alpha,\mathrm{in}}(\vec{r})+\cdots\,, \tag{2.4}$$

where

$$\psi(\vec{x},t)=\int d^3p\,dE\,a(\vec{p},E)\exp[-iEt+i\vec{p}\cdot\vec{x}], \tag{2.5}$$

$$\psi_{in}(\vec{x},t)=\int d^3p\,a_{\rm in}(\vec{p})\exp[-ie(\vec{p})t+i\vec{p}\cdot\vec{x}],\quad e(\vec{p})=\vec{p}^2/(2m),\quad \psi_{1\alpha,\,\rm in}\equiv\psi_{\rm in}, \tag{2.6}$$

$$\psi_{2\alpha,\,\rm in}(\vec{x},t)=\int d^3p\,a_{2\alpha,\,\rm in}(\vec{p})\exp[-iE_{2\alpha}(\vec{p})t+i\vec{p}\cdot\vec{x}], \tag{2.7}$$

and $\psi_{\rm in}$ and $\psi_{2\alpha,\,\rm in}$ are the asymptotic fields for particles with charge one and two, respectively, α labels the two-body states in the harmonic potential, and the f's are amplitudes to be determined from the operator equations of motion. The subscripts on the f's refer to the associated asymptotic fields; for example, $f_{1;1^2}$ has one $a^\dagger_{\rm in}$ and two $a_{\rm in}$'s, and $f_{1;2\alpha}$ has one $a^\dagger_{\rm in}$ and one $a_{2\alpha,\,\rm in}$. The a's have free (anti)commutation relations

$$[a_{\rm in}(\vec{p}),\,a^\dagger_{\rm in}(\vec{q})]_+=(2\pi)^{-3}\delta(\vec{p}-\vec{q}) \tag{2.8}$$

and

$$[a_{2\alpha,\,\rm in}(\vec{p}),\,a^\dagger_{2\beta,\,\rm in}(\vec{q})]_-=(2\pi)^{-3}\,\delta_{\alpha\beta}\,\delta(\vec{p}-\vec{q}); \tag{2.9}$$

where $[A,B]_\pm=AB\pm BA$. Other (anti)commutators vanish. The equation of motion for a is

$$[(\vec{p}^2/2m)-E]\,a(\vec{p},E)-(2\pi)^3k\int d^3q\,dF\,d^3r\,dG\,d^3s\,dH\,\delta(\vec{p}+\vec{q}-\vec{r}-\vec{s})\delta(E+F-G-H)$$
$$\times\, a^\dagger(\vec{q},F)[\vec{\nabla}_r^{\,2}\delta(\vec{r}-\vec{q})]a(\vec{r},G)\,a(\vec{s},H)=0\,. \tag{2.10}$$

The terms shown in (2.4) suffice to give the exact solution for the f's present in (2.4). Substitution of (2.4) into (2.10) followed by re-normal-ordering of the in-fields gives the following exact equations:

$$[(2m)^{-1}(\vec{r}_1+\vec{r}_2-\vec{q})^2-(2m)^{-1}(\vec{r}_1^{\,2}+\vec{r}_2^{\,2}-\vec{q}^{\,2})-k\vec{\nabla}_q^{\,2}][f_{1;1^2}(\vec{q};\vec{r}_1,\vec{r}_2)-f_{1;1^2}(\vec{q};\vec{r}_2,\vec{r}_1)]$$
$$-(2\pi)^3k[\vec{\nabla}_{r_1}^{\,2}\delta(\vec{r}_1-\vec{q})-\vec{\nabla}_{r_2}^{\,2}\delta(\vec{r}_2-\vec{q})]=0 \tag{2.11}$$

and

$$[(2m)^{-1}(\vec{q}-\vec{r})^2-E_{2\alpha}(\vec{r})+(2m)^{-1}\vec{q}^{\,2}-k\vec{\nabla}_q^{\,2}]f_{1;2\alpha}(\vec{q};\vec{r})=0\,. \tag{2.12}$$

Clearly $f_{1;1^2}$ must be antisymmetric in its last two arguments. No perturbation solution of (2.11) in powers of k exists, because of the singular nature of the inhomogeneous terms, whose singularities reflect the (confining) rising harmonic potential. The solution for $f_{1;1^2}$ is

$$f_{1;1^2}(\vec{q};\vec{r}_1,\vec{r}_2)=-\tfrac{1}{2}(2\pi)^3[\delta(\vec{q}-\vec{r}_1)-\delta(\vec{q}-\vec{r}_2)]\,. \tag{2.13}$$

This solution satisfies (2.11) for a general V.

Note that (2.13) has cluster decomposition violating singularities at $\vec{q}=\vec{r}_1$ and $\vec{q}=\vec{r}_2$. The x-space form of (2.13),

$$F(\vec{x};\vec{y}_1,\vec{y}_2)=-\tfrac{1}{2}[\delta(\vec{x}-\vec{y}_1)\delta(\vec{y}_2)-\delta(\vec{x}-\vec{y}_2)\delta(\vec{y}_1)], \tag{2.14}$$

where

$$F(\vec{x};\vec{y}_1,\vec{y}_2)=(2\pi)^{-9}\int d^3q\,d^3r_1\,d^3r_2\,f_{1;1^2}(\vec{q};\vec{r}_1,\vec{r}_2)\exp[-i\vec{q}\cdot\vec{x}+i\vec{r}_1\cdot\vec{y}_1+i\vec{r}_2\cdot\vec{y}_2] \tag{2.15}$$

is the Fourier transform of $f_{1;1^2}$, of course also violates the cluster-decomposition properties of theories without confinement, since F does not become small where its arguments are separated by a large distance.

To make clear the way in which models with and without confinement differ, I consider nonrelativistic models with an arbitrary $V(\vec{x})$ replacing the harmonic potential $k\vec{x}^2$. Then the equation for the partial Fourier transform

$$f(\vec{x};\vec{r}_1,\vec{r}_2)=\int d^3q\,f_{1;1^2}(\vec{q};\vec{r}_1,\vec{r}_2)\exp\{-i[\vec{q}-\tfrac{1}{2}(\vec{r}_1+\vec{r}_2)\cdot\vec{x}]\} \tag{2.16}$$

of $f_{1;1^2}$ is

$$[-m^{-1}\vec{\nabla}_x^{\,2}+V(\vec{x})][f(\vec{x};\vec{r}_1,\vec{r}_2)-f(\vec{x};\vec{r}_2,\vec{r}_1)]=(4m)^{-1}(\vec{r}_1-\vec{r}_2)^2[f(\vec{x};\vec{r}_1,\vec{r}_2)-f(\vec{x};\vec{r}_2,\vec{r}_1)]$$
$$+2(2\pi)^3i\,V(\vec{x})\sin[(\vec{r}_1-\vec{r}_2)\cdot\vec{x}]\,. \tag{2.17}$$

The solution for $f_{1;1^2}$ found above corresponds to

$$f(\vec{x};\vec{r}_1,\vec{r}_2)=(2\pi)^3 i\sin[\tfrac{1}{2}(\vec{r}_1-\vec{r}_2)\cdot\vec{x}]. \qquad (2.18)$$

For theories without confinement, the boundary conditions on the Haag expansion require the $t\to-\infty$ asymptotic limit of the Wightman field to be the in-field:

$$\lim_{t\to-\infty}\int d^3x'\mathfrak{D}(\vec{x}-\vec{x}',t-t')\psi(\vec{x}',t')=\psi_{\rm in}(\vec{x},t) \qquad (2.19)$$

where

$$\mathfrak{D}(\vec{x},t)=(2\pi)^{-3}\int d^3p\exp[-i\,e(\vec{p})\,t+i\,\vec{p}\cdot\vec{x}] \qquad (2.20)$$

is a solution of the free Schrödinger equation. Since the asymptotic limit (2.19) applied to the solution (2.13) for $f_{1;1^2}$ or (2.18) for f does not vanish, indeed the asymptotic limit applied to $-Q_1\psi_{\rm in}(\vec{x},t)$ just reproduces this term, the in-field boundary conditions rule out the solution by itself, for a theory without confinement. In such a theory, there is a continuum of scattering eigenfunctions of the homogeneous equations obtained from (2.11) or (2.17) by dropping the last term of the right-hand side, and a superposition of these added to (2.13) or (2.18) allows a vanishing contribution to the $(t\to-\infty)$ in-field limit and a nonvanishing contribution to the $(t\to\infty)$ out-field limit, thus giving scattering. By contrast, for the harmonic potential or any other confining potential these homogeneous equations do not have a continuum of scattering eigenfunctions, so that (2.13) or (2.18) is the only available solution.

The solutions for $f_{1;2\alpha}$ are the harmonic-oscillator eigenstates conveniently described in x space:

$$f_{1;2\alpha}(\vec{q};\vec{r})=f_{1;2\alpha}(\vec{q}-\tfrac{1}{2}\vec{r};0), \qquad (2.21)$$

$$F_{\vec{n}}(\vec{x})=(2\pi)^{-3}\int d^3q\,f_{1;2\alpha}(\vec{q};0)\exp(-i\vec{q}\cdot\vec{x}), \qquad (2.22)$$

$$(-m^{-1}\vec{\nabla}_x^2+k\vec{x}^2)F_{\vec{n}}(\vec{x})=\epsilon(\vec{n})F_{\vec{n}}(\vec{x}), \qquad (2.23)$$

$$\epsilon(\vec{n})=(n_1+n_2+n_3+\tfrac{3}{2})\epsilon,\quad \epsilon=2(k/m)^{1/2}, \qquad (2.24)$$

and

$$E_{2\alpha}(\vec{r})=\epsilon(\vec{n})+(4m)^{-1}\vec{r}^2, \qquad (2.25)$$

where $\alpha=\vec{n}=(n_1,n_2,n_3)$, n_i integral, labels the two-body states in the harmonic-oscillator potential.

The exact solution for ψ for terms of the form (2.4) is

$$\psi(\vec{x},t)=\psi_{\rm in}(\vec{x},t)-Q_1\psi_{\rm in}(\vec{x},t)+\sum_{\vec{n}}\int d^3x'F_{\vec{n}}(\vec{x}-\vec{x}')\psi^\dagger_{\rm in}(\vec{x}',t)\times\psi_{2\vec{n},\,{\rm in}}(\tfrac{1}{2}(\vec{x}+\vec{x}'),t), \qquad (2.26)$$

where $Q_1=\int d^3x\,\psi^\dagger_{\rm in}(\vec{x},t)\psi_{\rm in}(\vec{x},t)$ and $\psi_{2\vec{n},\,\rm in}$ was called $\psi_{2\alpha,\,\rm in}$ above. With this solution,

$$\psi^\dagger(\vec{y},t)|0\rangle=\psi^\dagger_{\rm in}(\vec{y},t)|0\rangle\neq 0, \qquad (2.27)$$

but

$$\psi^\dagger(\vec{x},t)\psi^\dagger(\vec{y},t)|0\rangle=\psi^\dagger_{\rm in}(\vec{x},t)(1-Q_1)\psi^\dagger_{\rm in}(\vec{y},t)|0\rangle+\sum_{\vec{n}}F^*_{\vec{n}}(\vec{x}-\vec{y})\psi^\dagger_{2\vec{n},\,\rm in}(\tfrac{1}{2}(\vec{x}+\vec{y}),t)|0\rangle. \qquad (2.28)$$

The first term on the right-hand side of (2.21) vanishes. Thus while the Wightman field $\psi^\dagger$ applied once to the vacuum creates a single-particle state with $Q=1$, $\psi^\dagger$ applied twice to the vacuum does not create a scattering state with two separated $Q=1$ particles, but rather creates a single-particle $Q=2$ state. This situation conforms exactly to the interpretation of confinement given in Sec. I. Equation (2.28) shows that $\psi^\dagger_{2\vec{n},\,\rm in}(\vec{x},t)$ creates a single-particle two-body state whose center of mass is at $\vec{x}$, and that $F^*_{\vec{n}}(\vec{x}-\vec{y})$ serves as the amplitude or wave function for the two constituent quanta to be at $\vec{x}$ and $\vec{y}$. The Pauli exclusion principle does not seem to be included: however the vanishing of the equal-time anticommutation relation (ETACR) for ψ (in particular, the coefficient of $\psi_{2\vec{n},\,\rm in}$) requires

$$F_{\vec{n}}(\vec{x})+F_{\vec{n}}(-\vec{x})=0, \qquad (2.29)$$

which gives the Pauli principle. Thus the $\Sigma_{\vec{n}}$ runs over the odd states only. The canonical ETACR for $[\psi,\psi]_+$ is satisfied to terms of the type A_2. The ETACR for $[\psi,\psi^\dagger]_+$ is satisfied to terms of the type $A_0+A_1^\dagger A_1$ provided

$$\sum_{\vec{n},\,\rm odd}F_{\vec{n}}(\vec{x})F^*_{\vec{n}}(\vec{y})=\delta(\vec{x}-\vec{y})-\delta(\vec{x}+\vec{y}), \qquad (2.30)$$

which holds because of the completeness relation for harmonic-oscillator wave functions,[8] using

$$F_{\vec{n},\,\rm odd}(\vec{x})=\tfrac{1}{2}[F_{\vec{n}}(\vec{x})-F_{\vec{n}}(-\vec{x})] \qquad (2.31)$$

and the fact that $\psi^\dagger_{2\vec{n},\,\rm in}$ creates a $Q=2$ state, so that $F_{\vec{n},\,\rm odd}$ is normalized to two

$$\int d^3x\,F^*_{\vec{n},\,\rm odd}(\vec{x})F_{\vec{n}',\,\rm odd}(\vec{x})=2\delta_{\vec{n},\vec{n}'}. \qquad (2.32)$$

The terms of the form[9] $A_1+A_1^\dagger A_2$ calculated above suffice to determine the terms of the form $A_1^\dagger A_1+A_2^\dagger A_2$ in the Hamiltonian and the charge operator. The result is

$$H=(2m)^{-1}\int d^3x\,\vec{\nabla}\psi^\dagger_{\rm in}(\vec{x},t)(1-Q_1)\cdot\vec{\nabla}\psi_{\rm in}(\vec{x},t)+\epsilon\sum_{\vec{n},\,\rm odd}\int d^3x\,\psi^\dagger_{2\vec{n},\,\rm in}(\vec{x},t)\psi_{2\vec{n},\,\rm in}(\vec{x},t)+(4m)^{-1}\int d^3x\,\vec{\nabla}\psi^\dagger_{2\vec{n},\,\rm in}(\vec{x},t)\cdot\vec{\nabla}\psi_{2\vec{n},\,\rm in}(\vec{x},t) \qquad (2.33)$$

and

$$Q=\int d^3x\,\psi^\dagger_{\rm in}(\vec{x},t)(1-Q_1)\,\psi_{\rm in}(\vec{x},t)$$

$$+2\sum_{\vec{n},\,{\rm odd}}\int d^3x\,\psi^\dagger_{2\vec{n},\,{\rm in}}(\vec{x},t)\psi_{2\vec{n},\,{\rm in}}(\vec{x},t)\,. \qquad (2.34)$$

Note that the first term, which I call H_1, in (2.33) has the properties that

$$H_1\,|0\rangle=0,\quad H_1|\vec{p}\,\rangle_{\rm in}=e(\vec{p})\,|\vec{p}\,\rangle_{\rm in},\quad H_1|\vec{p}_1,\vec{p}_2\,\rangle_{\rm in}=0\,, \qquad (2.35)$$

where

$$|\vec{p}\,\rangle_{\rm in}=a^\dagger_{\rm in}(\vec{p})\,|0\rangle,$$
$$|\vec{p}_1,\vec{p}_2\,\rangle_{\rm in}=a^\dagger_{\rm in}(\vec{p}_1)a^\dagger_{\rm in}(\vec{p}_2)\,|0\rangle\,,$$

which again agrees with the interpretation of confinement given in Sec. I.

In this model, there are no analogs of normal hadrons, and thus no scattering states of any kind, so that $\psi_{\vec{n}\alpha,\,{\rm in}}=\psi_{\vec{n}\alpha,\,{\rm out}}$.

The qualitative features of the harmonic oscillator hold for any particle-conserving model with confinement.

III. CONFINED ASYMPTOTIC OPERATORS AND SOLUTION IN ALL SECTORS OF PARTICLE-CONSERVING NONRELATIVISTIC MODELS WITH CONFINEMENT

In Sec. II I showed that for a model with confinement, the in operators $\psi_{\rm in}$ and $\psi^\dagger_{\rm in}$ are replaced by $(1-Q_1)\psi_{\rm in}$ and $\psi^\dagger_{\rm in}(1-Q_1)$ to order $A_1+A_1^\dagger A_2$ and $A_1^\dagger+A_2^\dagger A_1$, respectively. These replacements were derived from the operator equation of motion for the Wightman field, and guarantee the interpretation of confinement given in Sec. I. Now I introduce "confined in operators"

$$\psi_{n\alpha,\,{\rm con\text{-}in}}(\vec{x},t)=\Lambda_0\psi_{n\alpha,\,{\rm in}}(\vec{x},t)\,, \qquad (3.1)$$

and

$$\psi^\dagger_{n\alpha,\,{\rm con\text{-}in}}(\vec{x},t)=\psi^\dagger_{n\alpha,\,{\rm in}}(\vec{x},t)\Lambda_0\,, \qquad (3.2)$$

where

$$Q=\int d^3x\,\psi^\dagger(\vec{x},t)\psi(\vec{x},t)$$

$$=\sum_{n=1}^{\infty}n\sum_{\alpha}\int d^3x\,\psi^\dagger_{n\alpha,\,{\rm in}}(\vec{x},t)\psi_{n\alpha,\,{\rm in}}(\vec{x},t)\,, \qquad (3.3)$$

and

$$\Lambda_0=(\sin\pi Q)/(\pi Q) \qquad (3.4)$$

is the projection onto the vacuum. Note[10] that

$$\Lambda_0{}^2=\Lambda_0,\quad (\psi_{\rm con\text{-}in})^2=(\psi^\dagger_{\rm con\text{-}in})^2=0\,,$$
$$\psi_{\rm in}\Lambda_0=\Lambda_0\psi^\dagger_{\rm in}=0\,, \qquad (3.5)$$

and $\psi_{n\alpha,\,{\rm con\text{-}in}}$ and $\psi^\dagger_{n\alpha,\,{\rm con\text{-}in}}$ have nonvanishing matrix elements only between the vacuum and the state $\psi^\dagger_{n\alpha,\,{\rm in}}|0\rangle$. By construction, the confined in operators create at most one particle with confined degrees of freedom and thus guarantee the interpretation of confinement given in Sec. I. In models, such as the nonrelativistic harmonic oscillator discussed in Sec. II, where there are no scattering states at all, the confined in and out fields would be the same; however, in relativistic theories, such as QCD, it seems possible that a single quark or other particle with confined degrees of freedom (color, in QCD) could scatter off normal hadrons, and then the confined in- and out-fields would differ. Note that the confined in-operators for a given particle with a confined degree of freedom "know about" all the other confined in-operators for other particles with a confined degree of freedom via Λ_0, which contains all confined in-operators.

Now I show that the confined in-operators lead to the following simplified form of Haag expansion of the Wightman field:

$$a(\vec{p},E)=a_{\rm con\text{-}in}(\vec{p})\,\delta(E-e(\vec{p}))+\sum_{n=1}^{\infty}\sum_{\alpha,\beta}\int d^3q\,d^3r\,\delta(\vec{p}-\vec{r}+\vec{q})\,\delta(E-E_{(n+1)\beta}(\vec{r})+E_{n\alpha}(\vec{q}))f^{\rm con}_{n\alpha;\,(n+1)\beta}(\vec{q};\vec{r})$$
$$\times a^\dagger_{n\alpha,\,{\rm con\text{-}in}}(\vec{q})\,a_{(n+1)\beta,\,{\rm con\text{-}in}}(\vec{r}),\quad E_{1\alpha}(\vec{p})=e(\vec{p})\,, \qquad (3.6)$$

which provides a solution of the harmonic oscillator in all sectors. First, the term $a_{\rm con\text{-}in}\delta$ in (3.6) decouples from all other terms, and satisfies the same free equation of motion as $a_{\rm in}\delta$. Secondly, $f^{\rm con}_{1;2\beta}$ in (3.6) satisfies the same equation (2.12) as $f_{1;2\beta}$. The generic $f^{\rm con}$ satisfies

$$[(2m)^{-1}(\vec{r}-\vec{q})^2-E_{(n+1)\beta}(\vec{r})+E_{n\alpha}(\vec{q})]f^{\rm con}_{n\alpha,\,(n+1)\beta}(\vec{q};\vec{r})-k\sum_{\delta}M_{\alpha\delta}(\vec{q})f^{\rm con}_{n\delta,\,(n+1)\beta}(\vec{q};\vec{r})=0\,, \qquad (3.7)$$

where

$$M_{\alpha\delta}(\vec{q})=(2\pi)^{-3}\sum_{\gamma}\int d^3p\,[\vec{\nabla}_q{}^2 f^{{\rm con}*}_{(n-1)\gamma,\,n\alpha}(\vec{p};\vec{q})]f^{\rm con}_{(n-1)\gamma,\,n\delta}(\vec{p};\vec{q})\,. \qquad (3.8)$$

Starting with the solution for $f^{\mathrm{con}}_{1;2\beta}$ given in Sec. II, one can solve (3.7) recursively, since, for each n, (3.7) is a linear homogeneous equation for the f^{con} under consideration, and $M_{\alpha\delta}$ has already been determined by f^{con} for $n-1$ in (3.8).

I expect that the usual result, for models without confinement

$$H(\psi,\psi^\dagger)=\sum_{n,\alpha}H_0(\psi_{n\alpha,\,\mathrm{in}},\psi^\dagger_{n\alpha,\,\mathrm{in}}) \qquad (3.9)$$

will be replaced by

$$H(\psi,\psi^\dagger)=\sum\nolimits_{n,\alpha}H_{0,n\alpha}(\psi_{n\alpha,\mathrm{con\text{-}in}},\psi^\dagger_{n\alpha,\mathrm{con\text{-}in}}),$$

where $H_{0,n\alpha}$ is a free-field functional in the harmonic-oscillator model, and that similar replacements will hold for other relevant observables. For nonrelativistic models which have both confined particles which occur only in single-particle states and normal (unconfined) particles χ which have both single-particle states and many-particle scattering states, additional free-field functionals of the χ_{in}'s will occur in (3.10), and additional normal-ordered products of the χ_{in}'s will occur in (3.6). Note that the modified Haag expansion in terms of $\psi_{n\alpha,\mathrm{con\text{-}in}}$ is much simpler than the usual Haag expansion in terms of $\psi_{n\alpha,\,\mathrm{in}}$.

IV. SPECULATIONS ABOUT ASYMPTOTIC OPERATOR STRUCTURE IN RELATIVISTIC MODELS WITH CONFINEMENT

I speculate that the confined in-fields also play an important role in relativistic theories with confinement. For example, in a theory such as QCD, the usual free-quark in-field would be replaced by

$$q_{\mathrm{con\text{-}in}}(x)=\Lambda_1 q^{(+)}_{\mathrm{in}}(x)+q^{(-)}_{\mathrm{in}}(x)\,\Lambda_1\,, \qquad (4.1)$$

where $q^{(\pm)}_{\mathrm{in}}(x)$ are the positive-frequency (annihilation part) and negative-frequency (creation part) of the usual in-field, and

$$\Lambda_1=[\pi_{(f_1,f_2)\neq(0,0)}C_2^{(3)}(f_1,f_2)]^{-1}\times\{\pi_{(f_1,f_2)\neq(0,0)}[C_2^{(3)}(f_1,f_2)-C_2^{(3c)}]\}, \qquad (4.2)$$

is the projection onto $SU(3)_c$-singlet states. In (4.2),

$$C_2^{(3)}(f_1,f_2)=\tfrac{1}{3}(f_1^2-f_1f_2+f_2^2)+f_1$$

is the eigenvalue of the second-order Casimir operator for a representation of SU(3) with two rows of length $f_1\geq f_2\geq 0$, and $C_2^{(3c)}$ is the second-quantized form of this Casimir operator, which can be expressed in terms of either the Wightman quark and gluon fields or the infinite set[11] of in-fields carrying nonsinglet color, in analogy with (3.3) for the model of Sec. III. Note that

$$\Lambda_1^2=\Lambda_1,\quad (q^{(+)}_{\mathrm{con\text{-}in}})^2=(q^{(-)}_{\mathrm{con\text{-}in}})^2=0,$$
$$q^{(+)}_{\mathrm{in}}\Lambda_1=\Lambda_1 q^{(-)}_{\mathrm{in}}=0\,, \qquad (4.3)$$

in analogy with (3.5). The terms of the modified Haag expansion will be greatly simplified as was true for the models in Sec. III: the confined asymptotic fields which carry color will enter at most bilinearly, while the asymptotic fields for the color-singlet hadrons will enter in arbitrary degree as in the usual Haag expansion. The definition of normal ordering for the confined asymptotic fields differs in the way Λ_1 is treated from the usual normal ordering. The Hamiltonian and other relevant observables will have free-field form as a function of the confined asymptotic fields as in the models discussed in Sec. III.

ACKNOWLEDGMENTS

I thank Manoj Banerjee, Joseph Sucher, and Ching Hung Woo for helpful discussions. This work was supported in part by the National Science Foundation.

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[1]S. Weinberg, Phys. Rev. Lett. 31, 494 (1973); D. J. Gross and F. Wilczek, Phys. Rev. D 8, 3633 (1973); H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys. Lett. 47B, 365 (1973).

[2]A. S. Wightman, Phys. Rev. 101, 860 (1956); A. S. Wightman and L. Gårding, Ark. Fys. 28, 129 (1965); reviewed in R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics, and All That* (Benjamin, New York, 1964); R. Jost, *The General Theory of Quantized Fields* (American Mathematical Society, Providence, 1965); N. N. Bogoliubov, A. A. Logunov, and I. T. Todorov, *Introduction to Axiomatic Quantum Field Theory* (Benjamin, Reading, 1975).

[3]H. Lehmann, K. Symanzik, and W. Zimmermann, Nuovo Cimento 1, 205 (1955); 6, 319 (1957); O. W. Greenberg, Princeton Univ. thesis, 1956 (unpublished); and Ref. 2.

[4]R. Haag, Phys. Rev. 112, 669 (1958), and Nuovo Cimento Suppl. 14, 131 (1959). D. Ruelle, Helv. Phys. Acta 35, 147 (1962); and Ref. 2.

[5]See Ref. 2, and literature cited there.

[6]R. Haag, K. Dansk. Vidensk. Selsk. Mat.-Fys. Medd. 29, No. 12 (1955).

[7]O. W. Greenberg and R. J. Genolio, Phys. Rev. 150, 1070 (1966).

[8]A. Messiah, *Quantum Mechanics* (North-Holland, Amsterdam, 1964), Vol. I, p. 492.

[9]A_n stands for a product of in-field annihilation operators with total charge $Q=-n$ (formally, $[Q,A_n]_-=-nA_n$), and $A_n^\dagger$ stands for a product of in-field creation operators with total charge $Q=n$ (formally, $[Q,A_n^\dagger]_-=nA_n^\dagger$).

[10]Note also that $\Lambda_0=(\sin\pi Q)/(\pi Q)$ is equivalent to

$1-Q_1$ to order $1+A_1{}^\dagger A_1$. This equivalence follows to this order from normal ordering the in-fields in Λ_0.

[11]Just as there were infinitely many two-body, three-body, etc., in-fields and confined in-fields in the harmonic-oscillator model, there will also be infinitely many in-fields and confined in-fields for diquarks, digluons, and all composites with nonsinglet color in theories such as QCD.