

“Naturalness” of atomic parity conservation within left-right-symmetric unified theories

H. S. Mani

*International Centre for Theoretical Physics, Trieste, Italy
and Indian Institute of Technology, Kanpur 208016, India*

Jogesh C. Pati*

*Department of Physics, University of Maryland, College Park, Maryland 20742
and International Centre for Theoretical Physics, Trieste, Italy*

Abdus Salam

*International Centre for Theoretical Physics, Trieste, Italy
and Imperial College, London, England*

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The question of “naturalness” of atomic parity conservation for left-right-symmetric unified theories is examined. It is shown that the previously proposed patterns of spontaneous symmetry breaking do not offer a “natural” solution for such parity conservation. It may, however, be possible to secure this naturally if left-right-symmetry breaking in the neutral sector has a dynamically radiative origin.

Results of recent atomic parity experiments,¹ when compared with the present theoretical calculations,² appear to show that the strength of atomic parity violation in neutral-current interactions may perhaps be one to two orders of magnitude smaller than G_F (if not smaller still), in contrast to charged-current interactions where the magnitude is known to be of order G_F . Such a dichotomy between charged- and neutral-current interactions is not permissible within the simple “left-handed” $SU(2)_L \times U(1)$ theory.³ However, it can find a simple explanation (as can the entire body of currently known neutral-current data) within the left-right-symmetric theory⁴ $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$, proposed sometime ago with the primary motivation that nature must be *intrinsically* symmetric between left versus right.

The left-right-symmetric theory⁴ $SU(2)_L \times SU(2)_R \times U(1)$ {as well as *all* its quark-lepton-unifying extensions, e.g., the one based on⁴ $SU(2)_L \times SU(2)_R \times SU(4)_{L+R}$ or⁵ $[SU(4)]^4$ } have the distinguishing feature that for every left-handed ($V-A$) current coupled to the gauge particles (W_L^\pm, W_L^3), there must exist a *parallel* ($V+A$) current coupled to a *distinct* set of gauge particles (W_R^\pm, W_R^3) with equal strength ($g_L^{(0)} = g_R^{(0)}$). Parity violation at low energies arises in this class of theories due to spontaneously induced mass splittings between W_L 's and W_R 's.

The dichotomy between the degree of parity violation in the charged- versus neutral-current sectors can arise within this theory, if the spontaneously induced mass asymmetry between the charged gauge particles (W_L^\pm, W_R^\pm) is large, while at the same time the mass asymmetry between the neutral members ($W_{L,R}^3$) is small or “zero”.

To see how this may come about,⁶ consider Higgs fields $E_R = (1, 3, Y=0)$ and $E_L = (3, 1, Y=0)$ transforming as *vectors* under $SU(2)_{L,R}$. The appropriate vacuum expectation values contribute only to charged W' masses, but *not* to the masses of the neutral ones. Introduce also the scalar fields $B = (1, 2, Y=+1)$ and $C = (2, 1, Y=+1)$ transforming as *spinors* under $SU(2)_{L,R}$. These contribute [through their vacuum expectation value (VEV)] to the neutral as well as the charged W masses. Thus with

$$\langle E_R \rangle \gg \langle E_L \rangle,$$

but

$$\langle B \rangle \sim \langle C \rangle, \quad (1)$$

one would obtain a large mass asymmetry between the charged W 's, even though that between the neutral ones ($W_{L,R}^3$) may be small or “zero”.⁷ Correspondingly, parity violation in the charged sector would be large [$O(g_L^2/8m_{W_L^\pm}^2) \equiv O(G_F/\sqrt{2})$], while that in the neutral sector would be vanishingly small. In the limit $\langle B \rangle = \langle C \rangle$ and with $g_L = g_R$, neutral-current interactions would acquire the effective parity-conserving form ($VV+AA$). Allowing for *finite* $O(\alpha)$ radiative corrections^{4,8} to $(g_L - g_R)/g_L$ parity violation in neutral-current processes would arise (in this case) in order $G_F^{(N)}\alpha$ (where $G_F^{(N)}/\sqrt{2} \equiv g_L^2/8m_{N_1}^2$, and m_{N_1} is the mass of the lightest neutral weak-gauge particles). [Note, unlike standard $SU(2) \times U(1)$ vectorlike theories,⁹ an interaction possessing VV as well as AA pieces would distinguish between neutrinos (ν_L) and antineutrinos ($\bar{\nu}_R$), even though it conserves parity¹⁰ simply because the available neutrinos are left-handed, while the antineutrinos

are right-handed—produced as they are by dominantly $(V-A)$ charged-current interactions. Thus the theory would still predict $(\sigma_{\nu L p}^{NC} \neq \sigma_{\nu R p}^{NC})$, as observed experimentally.¹¹

Given the left-right-symmetric theory, and the picture of spontaneous symmetry breaking as outlined above, it is natural to ask: Is the solution of *vanishing* left-right mass asymmetry in the neutral sector ($\langle B \rangle - \langle C \rangle = 0$) "natural"? In other words, is this solution radiatively stable despite the mass asymmetry in the charged sector ($m_{w_R^+} \gg m_{w_L^+}$), in the sense that loop corrections induce at most finite and therefore calculable order- α corrections to the relevant asymmetry parameter $(\langle B \rangle^2 - \langle C \rangle^2) / \langle C \rangle^2$? The question is at the same level as the one which arises when we try to achieve a "natural" understanding of isospin conservation¹² [$m_n - m_p = O(\alpha)m_n$] within unified theories. The purpose of this paper is to examine the zeroth-order condition $\langle B \rangle = \langle C \rangle \neq 0$ and to remark that it is not natural in the above sense.

Now it appears that if one wishes to obtain $\langle B \rangle = \langle C \rangle \neq 0$ with $\langle E_R \rangle \neq \langle E_L \rangle$ in the zeroth order of spontaneous symmetry breaking, one has to impose the following restrictions on the relevant Higgs potential: (a) the mass parameters of B and C be equal in the bare Lagrangian ($\mu_b^{(0)2} = \mu_c^{(0)2}$); (b) their quartic couplings also be equal (this is required by natural $L \leftrightarrow R$ symmetry⁹); (c) the invariant quartic coupling (BB^*

$-CC^*)(E_R E_R^* - E_L E_L^*)$, though allowed by the gauge as well as $L \leftrightarrow R$ symmetry, be absent in the bare Lagrangian; and (d) the invariant term $(E_L^* E_L)(E_R^* E_R)$ be present. (This last term is essential to generate $\langle E_L \rangle \neq \langle E_R \rangle$ with $\mu_{E_R}^{(0)2} = \mu_{E_L}^{(0)2}$.) The point we wish to make is that at the least, condition (c) cannot be maintained when we consider the perturbative radiative corrections involving W_L and W_R loops. These reintroduce with infinite strength the omitted quartic coupling $(BB^* - CC^*)(E_R E_R^* - E_L E_L^*)$. The infinities may, of course, be absorbed at the expense, however, of introducing corresponding counter terms into the bare Lagrangian. This makes the renormalized value of the parameter $(\langle B \rangle^2 - \langle C \rangle^2) / \langle C \rangle^2$ in general non-vanishing and incalculable within the theoretical framework as currently available. The implications of this observation and a possible resolution are noted at the end of this paper.

To see the result stated above, we first write down the general Higgs potential involving B, C, E_R, E_L fields consistent with renormalizability and "natural" $L \leftrightarrow R$ symmetry. [Note that "natural" $L \leftrightarrow R$ symmetry, as defined in Ref. 8, requires that $L \leftrightarrow R$ discrete symmetry must be preserved everywhere, except possibly for scalar mass terms, so that radiative corrections to $(g_L - g_R)/g_L$ are finite and of order α .] The general potential subject to the discrete symmetry $E_{L,R} \leftrightarrow -E_{L,R}$ is given by

$$\begin{aligned} V(E_R, E_L; B, C) = & -\mu_B^{(0)2}(B^*B) - \mu_C^{(0)2}(C^*C) + \lambda_{B1}^{(0)}[(B^*B)^2 + (C^*C)^2] + \lambda_{B2}^{(0)}(B^*B)(C^*C) \\ & - \mu_{E_R}^{(0)2}E_R^*E_R - \mu_{E_L}^{(0)2}E_L^*E_L + \lambda_{E1}^{(0)}[(E_R^*E_R)^2 + (E_L^*E_L)^2] + \lambda_{E2}^{(0)}(E_R^*E_R)(E_L^*E_L) \\ & + \kappa_s^{(0)}(E_R^*E_R + E_L^*E_L)(B^*B + C^*C) + \kappa_a^{(0)}(E_R^*E_R - E_L^*E_L)(B^*B - C^*C). \end{aligned} \quad (2)$$

We do not exhibit the presence of other fields such as $A = (2, \underline{2}, Y=0)$ which must be present to give masses to fermions. The presence of such fields does not influence the issue of naturalness. The terms

$$\lambda_{E2}^{(0)}[(E_R^* t_i E_R)(E_R^* t_i E_R) + R - L]$$

and

$$\lambda_{EB}^{(0)}[(E_R^* t_i E_R)(B^* \tau_i B) + (E_L^* t_i E_L)(C^* \tau_i C)]$$

are dropped for ease of writing. These would not contribute to the extremum conditions $\partial V / \partial B^* = 0$, $\partial V / \partial C^* = 0$ upon substitutions for the vacuum expectation values for $E_{L,R}$.

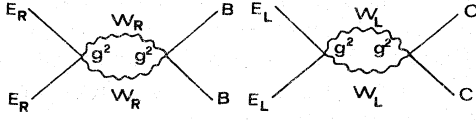
Insisting on complete $L \leftrightarrow R$ symmetry in the basic Lagrangian, one must set the scalar mass terms to be $L \leftrightarrow R$ symmetric ($\mu_B^{(0)} = \mu_C^{(0)}$ and $\mu_{E_R}^{(0)} = \mu_{E_L}^{(0)}$). It can be shown following Ref. 13 that even with a completely $L \leftrightarrow R$ symmetric potential involving all four fields

(B, C, E_L, E_R) , it is possible to obtain a solution $\langle E_R \rangle \neq \langle E_L \rangle$ and thereby $m_{w_R^+} \neq m_{w_L^+}$ for a range of values of the parameters subject to $\mu_i^2 > 0$ ($i=B, C, E_R, E_L$) and $\lambda_{E2}^{(0)} > 2\lambda_{E1}^{(0)}$. Thus to proceed, let us set $\mu_B^{(0)} = \mu_C^{(0)}$; $\mu_{E_R}^{(0)} = \mu_{E_L}^{(0)}$. (As it will be clear later, our conclusions will not depend upon this restriction.)

We are asking the question: Is the following pattern of zeroth-order vacuum expectation values:

$$\begin{aligned} \langle B \rangle = \langle C \rangle &= \begin{pmatrix} 0 \\ b \end{pmatrix} \neq 0, \\ \langle E_R \rangle &= \begin{pmatrix} 0 \\ \epsilon_R \end{pmatrix} \quad \text{and} \quad \langle E_L \rangle = \begin{pmatrix} 0 \\ \epsilon_L \end{pmatrix} \neq \langle E_R \rangle \end{aligned} \quad (3)$$

radiatively stable and therefore a "natural" solution for the minimum of the potential for a range of values of the parameters defining the zeroth-

FIG. 1. Radiatively induced $E_R^2 B^2$ and $E_L^2 C^2$ terms.

order potential?

To answer this question first write the extremum conditions for the zeroth-order potential ($\partial V/\partial B^* = 0$; $\partial V/\partial C^* = 0$):

$$[-\mu_b^{(0)2} + 2\lambda_{B1}^{(0)} B^* B - \lambda_{B2}^{(0)*} (C^* C) + \kappa_s^{(0)} (E_R^* E_R + E_L^* E_L) + \kappa_a^{(0)} (E_R^* E_R - E_L^* E_L)] B = 0, \quad (4)$$

$$[-\mu_c^{(0)2} + 2\lambda_{C1}^{(0)} C^* C - \lambda_{C2}^{(0)} (B^* B) + \kappa_s^{(0)} (E_R^* E_R + E_L^* E_L) - \kappa_a^{(0)} (E_R^* E_R - E_L^* E_L)] C = 0. \quad (5)$$

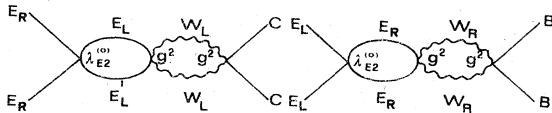
Substituting the pattern of vacuum expectation values (3) into (4) and (5) and taking the difference between the two equations, we obtain (with $\mu_B^{(0)2} = \mu_C^{(0)2}$)

$$\kappa_a^{(0)} (\epsilon_R^2 - \epsilon_L^2) = 0. \quad (6)$$

Since $\epsilon_R \neq \epsilon_L$, we see that a *necessary condition* for the pattern $\langle B \rangle = \langle C \rangle \neq 0$ with $\langle E_R \rangle \neq \langle E_L \rangle$ is that

$$\kappa_a^{(0)} = 0, \quad (7)$$

i.e., the term $(E_R^* E_R - E_L^* E_L)(B^* B - C^* C)$ must be absent in the bare Lagrangian. This term, though odd under the interchange $B \leftrightarrow C$, is even under the simultaneous interchange $(B \leftrightarrow C, E \leftrightarrow F)$, and thus allowed by discrete $L \leftrightarrow R$ symmetry. It is, of course, also allowed by the gauge symmetry $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$. Thus, as might be expected,¹² even if one did not introduce such a term into the bare Lagrangian, it is induced by loop diagrams calculated perturbatively with respect to the symmetric vacuum (see Figs. 1 and 2). Note that both Figs. 1 and 2 are logarithmically divergent. Thus they generate (since their strengths are unequal) both the $B \leftrightarrow C$ symmetric $(B^* B + C^* C)(E_R^* E_R + E_L^* E_L)$ as well as the $B \leftrightarrow C$ antisymmetric term $(B^* B - C^* C)(E_R^* E_R - E_L^* E_L)$ with infinite strengths. The infinities can be absorbed only if we allow the presence of corresponding counter terms in the bare Lagrangian. Hence, insisting on renormalizability, we must

FIG. 2. Radiatively induced $E_R^2 C^2$ and $E_L^2 B^2$ terms.

choose $\kappa_s^{(0)} \neq 0$, $\kappa_a^{(0)} \neq 0$ in the bare Lagrangian. The renormalized value of κ_a is thus a free parameter in the theory which cannot be computed. To this extent the renormalized value of $(\langle B \rangle^2 - \langle C \rangle^2)/\langle C \rangle^2$ as well is not calculable. It thus follows that the zeroth-order solution $\langle B \rangle = \langle C \rangle \neq 0$ together with $\langle E_R \rangle \neq \langle E_L \rangle$ is not a "natural" solution of the theory (in the technical sense).

Note that the same conclusion is reproduced if we examine the minimum of the *effective potential* calculated with respect to the symmetric vacuum by including the effect of all one-loop corrections to order g^4 , which inevitably reproduces the κ_a term through Fig. 1.

Note, for the sake of generality, that if we had chosen $\mu_B^{(0)} \neq \mu_C^{(0)}$ (and even if this had permitted $\langle B \rangle = \langle C \rangle \neq 0$) we would obtain¹⁴ from the difference between (4) and (5) the equation

$$(\mu_b^{(0)2} - \mu_c^{(0)2}) + 2\kappa_a^{(0)} (\epsilon_R^2 - \epsilon_L^2) = 0$$

instead of (6). This can only be satisfied for a *specific value* of the parameter

$$\kappa_a^{(0)} = -[\mu_b^{(0)2} - \mu_c^{(0)2}] / (\epsilon_R^2 - \epsilon_L^2).$$

Thus one more parameter is needed for the calculation of $(\langle B \rangle^2 - \langle C \rangle^2)/\langle C \rangle^2$. This is contrary to the conventional concept of naturalness.

Now assume that with continuing improvements in experimental measurements and theoretical calculations, it is established that the effective strength of parity violation in atoms is not just one but *two* orders of magnitude smaller than G_F . This observation can, of course, be accommodated within the left-right-symmetric theory⁴ by assuming that the renormalized values of the parameters $\langle B \rangle$ and $\langle C \rangle$ are nearly equal. Correspondingly, there would be several testable predictions (in particular those involving e^-e^+ forward-backward asymmetry measurements¹⁵ and likewise measurements involving dilepton production by hadrons¹⁶). However, one could face a dilemma calling for a natural understanding of this dramatic situation. Below we present briefly a possible resolution of such a possible dilemma.

We have so far followed the pattern of spontaneous symmetry breaking proposed in earlier works^{4,7,9,13} and have posed the question of whether within such a pattern the *zeroth-order* parity-conserving solution $\langle B \rangle = \langle C \rangle \neq 0$ is radiatively stable with $\langle E_R \rangle \neq \langle E_L \rangle$. Note the distinctive feature of this pattern that all gauge particles (charged as well as neutral) acquire mass in the zeroth order.

Now consider an alternative solution. Allowing for *all* possible invariant terms in the potential [Eq. (2)] consistent with renormalizability and discrete $L \leftrightarrow R$ symmetry,⁴ choose the signs of

B - and C -(mass)² terms, so that in the *zeroth-order*, minimization of the potential yields

$$\langle B \rangle = \langle C \rangle = 0; \quad \langle E_L \rangle = 0.$$

But

$$\langle E_R \rangle \neq 0. \quad (8)$$

Note that the vanishing of the $\kappa_a^{(0)}$ term is no longer necessary [see Eqs. (4) and (5)] once $\langle B \rangle = \langle C \rangle = 0$ (rather than $\langle B \rangle = \langle C \rangle \neq 0$). The solution (8) implies that in the zeroth order of spontaneous symmetry breaking (i.e., barring loop corrections) only the charged W_R^\pm acquire a mass, all other gauge particles (W_L^\pm, W_L^3, W_R^3 , as well as the U(1) field remain massless. The symmetry $\mathfrak{g} = \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{L+R} \times (P)$ thereby descends to $\text{SU}(2)_L \times \text{U}(1)_R \times \text{U}(1)_{L+R}$ (where P denotes discrete $L \leftrightarrow R$ symmetry).

But now allowing for radiative corrections,¹⁷ both $\langle B \rangle$ and $\langle C \rangle$ can develop, at the one-loop level, nonzero vacuum expectation values. However, this time there is the important bonus that both $\langle B \rangle^2$ and $\langle C \rangle^2$ are calculable¹² and $O(\alpha)$ compared to $\langle E_R \rangle^2$. In turn the (mass)² of the left-handed

gauge particles (W_L^\pm) mediating $(V-A)$ interactions and the (mass)² of the two neutral gauge particles (N_1 and N_2) are calculable¹⁸ and of order αm_{WR}^2 . The difference $(\langle B \rangle^2 - \langle C \rangle^2)$, however, it may easily be seen, is

$$\{[O(\alpha^2) + O(\kappa_a^{\text{ren}}, \lambda_i^{\text{ren}})] / (2\lambda_{B1}^{\text{ren}} - \lambda_{B2}^{\text{ren}})\} \langle E_R \rangle^2.$$

The $O(\alpha)$ contribution to $(\langle B \rangle^2 - \langle C \rangle^2)$ vanishes in this case due to the left-right symmetry of the basic lagrangian. The parity violating parameter¹⁵ $x \equiv (b^2 - c^2)/c^2$ from this mechanism is expected to be naturally small,¹⁸ implying a small atomic parity violation compared to the $\text{SU}(2)_L \times \text{U}(1)$ value and a light neutral gauge particle¹⁵ N_1 (with mass $\approx m_{W_L}$). Such a picture may provide an attractive possibility for a natural hierarchy for the gauge masses¹⁹ and deserves a study in its own right. This will be pursued in a subsequent paper.

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¹P. Baird, M. Brimicombe, G. Roberts, P. Sandars, D. Soreide, E. Fortson, L. Lewis, E. Lindahl, and D. C. Soreide, *Nature* **264**, 529 (1976). Recent results by these authors have appeared as Washington and Oxford reports (unpublished) showing considerably higher accuracy and a reduced value of the atomic-parity-violation parameter compared with earlier values quoted in the above reference. We thank Dr. Baird and Dr. Sandars for early communication of their results.

²I. B. Khriplovich, *Zh. Eksp. Teor. Fiz. Pis'ma Red.* **20**, 686 (1974) [*JETP Lett.* **20**, 315 (1974)]; E. M. Henley and L. Willets, *Phys. Rev. A* **19**, 1911 (1976); N. W. S. M. Brimicombe, C. E. Loving, and P. G. H. Sandars, *J. Phys. B* **9**, L237 (1976).

³S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967); Abdus Salam, in *Elementary Particle Physics: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Almqvist and Wiksells, Stockholm, 1968), p. 367; S. L. Glashow, J. Iliopoulos, and L. Maiani, *Phys. Rev. D* **2**, 1285 (1970). For earlier work see S. L. Glashow, *Nucl. Phys.* **22**, 579 (1961); and Abdus Salam and J. C. Ward, *Phys. Lett.* **13**, 168 (1964).

⁴The first suggestion of the left-right-symmetric theory $\text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{L+R}$ comprising all matter (quarks as well as leptons) was made by J. C. Pati and Abdus Salam, *Phys. Rev. Lett.* **31**, 661 (1973); *Phys. Rev. D* **10**, 275 (1974). In this work two alternative patterns of spontaneous symmetry breaking were proposed, one of them permitting the possibility of *two* relatively light neutral gauge bosons. It is this alternative, which is of relevance to present atomic parity experiments (Ref. 1), and has been pursued recently by several authors (Ref. 7).

⁵J. C. Pati and Abdus Salam, *Phys. Lett.* **58B**, 333 (1975).

⁶See Sec. IV and footnote 21 of Ref. 4.

⁷The consequences of the zeroth-order solution $\langle B \rangle = \langle C \rangle \neq 0$ arising within the model proposed in Ref. 4 has been examined by H. Fritzsch and P. Minkowski, *Nucl. Phys. B* **103**, 61 (1976), and more recently by R. N. Mohapatra and D. P. Sidhu, *Phys. Rev. Lett.* **38**, 667 (1977); *Phys. Rev. D* **16**, 2843 (1977). The more general case comprising $\langle B \rangle \sim \langle C \rangle$ and $\langle B \rangle = \langle C \rangle$ has been examined by J. C. Pati, S. Rajpoot, and Abdus Salam, *Phys. Rev. D* **17**, 131 (1978); *Phys. Lett.* **71B**, 387 (1977).

For a somewhat different treatment of spontaneous symmetry breaking of the group structure proposed in Ref. 4 see A. De Rújula, H. Georgi, and S. L. Glashow, Harvard report, 1977 (unpublished).

⁸R. N. Mohapatra and J. C. Pati, *Phys. Rev. D* **11**, 566 (1975); *D* **11**, 2558 (1975). For a calculation of such $O(\alpha)$ radiative corrections, see Q. Shafi and Ch. Wetterich, University of Freiburg Report No. THEP 77/3 (unpublished).

⁹A vectorlike $\text{SU}(2) \times \text{U}(1)$ model was first proposed by M. A. B. Ég and A. Zee, *Phys. Rev. Lett.* **30**, 675 (1973). For a list of references on other vectorlike models, see R. M. Barnett, in *Particles and Fields '76*, proceedings of the Annual Meeting of the Division of Particles and Fields of the APS, edited by H. Gordon and R. F. Peirls (BNL, Upton, New York, 1977), p. D77.

¹⁰It needs to be stressed that contrary to common impression ($\sigma_{\nu_L p} \neq \sigma_{\bar{\nu}_R p}$)_{NC} does *not* imply parity nonconservation since $\nu_L \not\leftrightarrow \bar{\nu}_R$ under parity. Such a distinction between ν_L and $\bar{\nu}_R$ cross sections eliminates only the class of parity-conserving theories which are vector-

like with no AA piece.

- ¹¹A. Benvenuti *et al.*, Phys. Rev. Lett. 37, 1039 (1976); J. Blietschau *et al.*, Nucl. Phys. B118, 2181 (1977); B. C. Barish, Caltech. Report No. CALT-68-544 (unpublished). The clearest distinction is shown by measurements of $\sigma(\bar{\nu}_R p \rightarrow \bar{\nu}_R p)$ versus $\sigma(\nu_L p \rightarrow \nu_L p)$. See D. Clive *et al.*, Phys. Rev. Lett. 37, 648 (1976); 37, 648 (1976); and W. Lee *et al.*, *ibid.* 37, 186 (1976).
- ¹²That the left-right-symmetric gauge structure $SU(2)_L \times SU(2)_R \times U(1)$ provides a solution for natural conservation of isospin was first observed by S. Weinberg, Phys. Rev. Lett. 29, 1698 (1972), who introduced as a semirealistic model the symmetric gauge structure for quarks, but not for leptons.
- ¹³G. Senjanovic and R. N. Mohapatra, Phys. Rev. D 12, 1502 (1975).
- ¹⁴Whether $\langle B \rangle = \langle C \rangle \neq 0$ is an allowed zeroth-order solution for the *minimum* of the potential with $\mu_b^{(0)2} \neq \mu_c^{(0)2}$, needs to be examined. But this is not relevant to our conclusion.
- ¹⁵Pati, Rajpoot, and Salam, Ref. 7.
- ¹⁶H. S. Mani, J. C. Pati, S. Rajpoot, and Abdus Salam, Phys. Lett. 72B, 75 (1977).

¹⁷S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).

- ¹⁸Such a calculation is in progress. The quantities $\langle B \rangle^2$ and $\langle C \rangle^2$ would be of order $(\alpha m_{W_R}^{\pm 2})$ multiplied by appropriate logarithmic factors, if $m_{W_R}^{\pm 2}$ is the controlling heaviest mass in the theory. Given that the (mass)² terms for B and C fields acquire the "desirable" sign through $O(\alpha)$ radiative corrections to acquire nonzero vacuum expectation values, it may be argued that $(\langle B \rangle^2 - \langle C \rangle^2)$ should vanish for some critical value of the coupling constant $g^4 = g_c^4 = o(\kappa_a, \lambda_i)$, i.e. $(\langle B \rangle^2 - \langle C \rangle^2) \propto (g^4 - g_c^4)$ with $0 \leq g_c^4 \leq g^4$ implying that $[(\langle B \rangle^2 - \langle C \rangle^2)/\langle C \rangle^2]$ would in this case be naturally $O(\alpha)$. This is subject to the inclusion of only the one-loop correction; stability of the minimum of the potential in the presence of higher-order corrections is not yet ascertained. Note that the ratio $(m_{W_L}^{\pm 2}/m_{W_R}^{\pm 2})^2$ being of order α provides a natural explanation of the observed smallness of $|\eta_{\pm}| \sim 1/600$, since within left-right-symmetric theories $|\eta_{\pm}| \sim (\sin \gamma) (m_{W_L}^{\pm 2}/m_{W_R}^{\pm 2})^2$ (see Ref. 8).
- ¹⁹Also, this question may acquire a new complexion if we embed $SU(2)_L \times SU(2)_R \times U(1)_{L+R}$ or $SU(2)_L \times SU(2)_R \times SU(4)_{L+R}$ into a bigger group like $[SU(4)]^4$.