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Comments and Addenda

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Some rare decay modes of the K meson in the modified baryon-loop model. $K_L \to \pi^0\gamma\gamma$ and $K_L \to \pi^0 e^+ e^-$ decays

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The $K_L \to \pi^0\gamma\gamma$ decay rate calculated in the zero-parameter modified baryon-loop model is found to be 38 and 43 sec^{-1} for mean vector-meson masses of 0.9 and 0.8 GeV respectively. Using unitarity, a lower bound (and estimate) is obtained on the decay rate for $K_L \to \pi^0 e^+ e^-$ which goes via $K_L \to \pi^0$ (virtual)γ(virtual)γ in the modified loop model. We find $[\Gamma(K_L \to \pi^0 e^+ e^-)/\Gamma(K_L \to \text{all})] \gtrsim 2.1 \times 10^{-13}$.

I. INTRODUCTION

It is almost unnecessary to emphasize that the rare decay modes of the K meson provide the testing ground for the various weak-interaction theories.[1] The search for many of these rare decays is under way at present[2] so that one can look forward to the improvement of some of the associated upper bounds in the near future. In this brief paper we round out our extensive exploration of the predictive powers of the *zero-parameter* modified baryon-loop[3] model with a presentation of the results of the calculation of the rate for $K_L \to \pi^0\gamma\gamma$ and, using unitarity, the rate for $K_L \to \pi^0 e^+ e^-$ as well.

II. $K_L \to \pi^0\gamma\gamma$ DECAY

The model[3] is characterized by a parity-conserving (pc) weak nonleptonic Hamiltonian density,[3]

$$\mathcal{H}_W(\text{pc}) = -\sqrt{2}F\,\text{Tr}([\bar{B},B]\lambda_6) + \sqrt{2}D\,\text{Tr}(\{\bar{B},B\}\lambda_6), \tag{1}$$

with

$$F = 4.7\times 10^{-5}\,\text{MeV}, \tag{2}$$

$$D/F = -0.85, \tag{3}$$

and a strong plus electromagnetic interaction Hamiltonian density,[3]

$$\mathcal{H}_{\text{strong}} + \mathcal{H}_{\text{em}} = \sqrt{2}gf\,\text{Tr}([\bar{B}i\gamma_5, B]M) - \sqrt{2}gd\,\text{Tr}(\{\bar{B}i\gamma_5, B\}M) + \sqrt{2}\phi\,\text{Tr}([\bar{B}\gamma_\mu, B]V^\mu) - \sqrt{2}\delta\,\text{Tr}(\{\bar{B}\gamma_\mu, B\}V^\mu)$$
$$+ i(g_\rho/\sqrt{2})\,\text{Tr}([M, \partial_\mu M]V^\mu) + \tfrac{1}{2}eA^\mu\,\text{Tr}\{([B\gamma_\mu, B] + 2i[M, \partial_\mu M])Q\}. \tag{4}$$

The graphs contributing to the amplitude for $K_L \to \pi^0\gamma\gamma$ decay comprise a baryon-loop "contact" term (Fig. 1) for which there are 24 topologically distinct graphs and a less important vector-pole contribution (Fig. 2). The technical problems entailed in the reduction of these contributions have been discussed at length in the earlier communications[3] so it is enough to give here a brief summary. We find

$$A_{\text{contact}}(K_L \to \pi^0\gamma\gamma) = -i4\sqrt{2}(ge)^2[D(d^2 - 3f^2) - 2Fdf](\partial/\partial m)I(m), \tag{5}$$

where

$$I(m)=8\int_0^1 dx\int_0^1 y\,dy\int\frac{d^4t}{(2\pi)^4}\{t^2-m^2-[k_1xy-k_2(1-y)]^2\}^{-3}$$
$$\times(\epsilon_1\cdot\epsilon_2\{m^2-k_1\cdot k_2[1-2xy(1-y)]\}+k_1\cdot\epsilon_2k_2\cdot\epsilon_1[1-4xy(1-y)]). \quad (6)$$

and thus to lowest order in external invariants,[3] and consistent with the requirements of gauge invariance,

$$A_{\text{contact}}(K_L\to\pi^0\gamma\gamma)=\frac{8\sqrt{2}}{3\pi m^3}g^2\alpha[D(3f^2-d^2)+2Fdf](\epsilon_1\cdot\epsilon_2k_1\cdot k_2-\epsilon_1\cdot k_2\epsilon_2\cdot k_2); \quad (7)$$

m is a "mean" baryon mass of 1 GeV,[3] and the effective coupling $\mathfrak{F}$ has the value

$$\mathfrak{F}=\frac{8\sqrt{2}}{3\pi m^3}g^2\alpha[D(3f^2-d^2)+2Fdf]=3.66\times10^{-8}\ \text{GeV}^{-2}. \quad (8)$$

In evaluating the vector-pole contribution to $K_L\to\pi^0\gamma\gamma$ we use the effective couplings of the baryon-loop model.[3] A straightforward albeit tedious calculation finds

$$A_{\text{vector poles}}(K_L\to\pi^0\gamma\gamma)=\frac{8\sqrt{2}g^2\alpha}{3\pi^3m^3}(\delta f+d\phi)[\Gamma(3,6)-\Gamma(6,3)]$$
$$\times\{[K^2(k_1\cdot\epsilon_2k_2\cdot\epsilon_1-\epsilon_1\cdot\epsilon_2k_1\cdot k_2)$$
$$-(K\cdot\epsilon_1K\cdot k_2k_1\cdot\epsilon_2+K\cdot\epsilon_2K\cdot k_1k_2\cdot\epsilon_1-\epsilon_1\cdot\epsilon_2K\cdot k_1K\cdot k_2-k_1\cdot k_2K\cdot\epsilon_1K\cdot\epsilon_2)$$
$$-K\cdot k_1(k_1\cdot\epsilon_2k_2\cdot\epsilon_1-\epsilon_1\cdot\epsilon_2k_1\cdot k_2)]1/[(K-k_1)^2-m_V^2]+(k_1,\epsilon_1\longleftrightarrow k_2,\epsilon_2)\}, \quad (9)$$

where K^μ is the kaon four-momentum and[3]

$$\Gamma(3,6)=\tfrac{3}{4}[3\phi(Dd+Ff)-\delta(Df+Fd)]$$
$$=-4.7\times10^{-8}\ \text{GeV}, \quad (10)$$

$$\Gamma(6,3)=\tfrac{3}{4}[3f(D\delta+F\phi)-d(D\phi-F\delta)]$$
$$=28.4\times10^{-8}\ \text{GeV}. \quad (11)$$

The introduction of an average vector-meson mass m_V leads to a great simplicity in the transcription of the result in Eq. (9); moreover, our numerical results are seen to be relatively insensitive to the value taken for m_V. Note that $A_{\text{vector poles}}$, like A_{contact}, is gauge invariant by itself. For the effective coupling constant one has the numerical value

$$\frac{8\sqrt{2}g^2}{3\pi^3m^3}(\delta f+d\phi)=0.253\ \text{GeV}^{-3}. \quad (12)$$

The expression for the decay rate

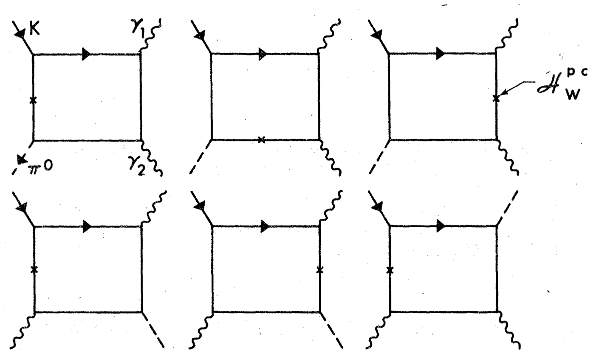

FIG. 1. Baryon-loop graphs for contact contribution to $K_L\to\pi^0\gamma\gamma$ decay.

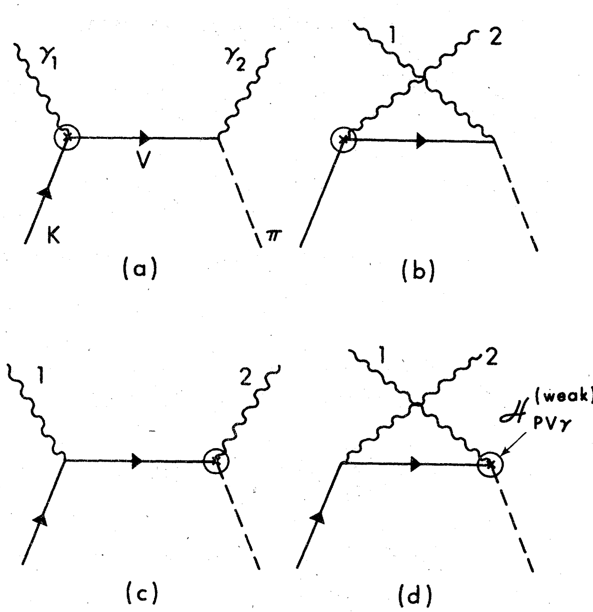

FIG. 2. Vector-pole contributions to $K_L\to\pi^0\gamma\gamma$ decay.

$$\Gamma = [16m_K(2\pi)^3]^{-1} \int \sum_{\text{pol}} |A(K_L \to \pi^0\gamma\gamma)|^2 \times dp^0 dk_1^0 \qquad (13)$$

yields the result

$$\Gamma(K_L \to \pi^0\gamma\gamma) = 2.53 \times 10^{-23}\ \text{GeV (for } m_V = 0.9\ \text{GeV)}$$
$$= 2.86 \times 10^{-23}\ \text{GeV (for } m_V = 0.8\ \text{GeV)}. \qquad (14)$$

Since[4] $(K_L \to \text{all}) = 1.27 \times 10^{-17}$ GeV, we have

$$\frac{K_L \to \pi^0\gamma\gamma}{K_L \to \text{all}} = 2.0 \times 10^{-6}\ \text{(for } m_V = 0.9\ \text{GeV)}$$
$$= 2.25 \times 10^{-6}\ \text{(for } m_V = 0.8\ \text{GeV)}. \qquad (15)$$

Note that the experimental upper bound[4] on the branching ratio is

$$\Gamma(K_L \to \pi^0\gamma\gamma)/\Gamma(K_L \to \text{all}) < 2.4 \times 10^{-4}.$$

The alternative rates of Eq. (20) imply 38 and 43 decays per second respectively, well within the upper bound[4] of 4.6×10^2 decays per second. We find that A_{contact} yields a branching ratio of 1.65 $\times 10^{-6}$ by itself, *with the interference term between* A_{contact} *and* $A_{\text{vector poles}}$ *making a negligible contribution to the rate*. By way of comparison with earlier theoretical treatments, we mention that recently Intemann[5] obtained rates of $27^{+2.3}_{-2.2}$ and $0.7^{+1.5}_{-0.6}$ per second in a calculation within the framework of the Moshe-Singer[6] model of radiative K-meson decay, which is characterized by a phenomenological Lagrangian and with gauge fields, current mixing, and SU(3)-symmetry breaking. An earlier estimate by Sehgal[7] based on dispersion relations and a model determining the amplitude for $K_L \to \pi^0\gamma\gamma$ decay in terms of that for $K_L \to \pi^+\pi^-\pi^0$ found a rate of 13 per second.

III. UNITARITY ESTIMATE OF $K_L \to \pi^0 e^+ e^-$ RATE

It is straightforward to show that the decay $K_L \to \pi^0 e^+ e^-$ cannot proceed via one virtual photon in our modified loop model[8]; instead one must depend on the mechanism of *virtual* $K_L \to \pi^0\gamma\gamma$ decay. However, we choose to avoid the rather formidable calculation this implies and approach this calculational problem by exploiting the unitarity relation for $A(K_L \to \pi^0 e^+ e^-)$,

$$\text{Im}A(K_L \to \pi^0 e^+ e^-) = \tfrac{1}{2} \sum \langle \gamma(k_1\epsilon_1)\gamma(k_2\epsilon_2) | T | e^+(q_+) e^-(q_-)\rangle^* \times (4q_+^0 q_-^0)^{-1} \langle \gamma(k_1\epsilon_1)\gamma(k_2\epsilon_2) | T | K_L(K)\pi^0(-p)\rangle (2\pi)^4 \delta^4(K - p - k_1 - k_2), \qquad (16)$$

where we make the reasonable approximation,

$$\langle \gamma(k_1\epsilon_1)\gamma(k_2\epsilon_2) | T | K_L(K)\pi^0(-p)\rangle \simeq A_{\text{contact}}(K_L \to \pi^0\gamma\gamma). \qquad (17)$$

One finds

$$\text{Im}A(K_L \to \pi^0 e^+ e^-) \simeq -[\alpha \mathfrak{F} m_e \sqrt{s} H(s)/4]\, \bar{u}(q_-) v(q_+), \qquad (18)$$

where s is the invariant energy squared of the e^+e^- system and

$$H(s) = (1/|\vec{q}_-|) \ln[(q_-^0 + |\vec{q}_-|)/(q_-^0 - |\vec{q}_-|)] \qquad (19)$$

with $q_-^\mu = (q_-^0, \vec{q}_-)$ evaluated in the e^+e^- center-of-mass system. Finding the decay amplitude to be largely absorptive,[9] we calculate the $K_L \to \pi^0 e^+ e^-$ decay rate,

$$\Gamma(K_L \to \pi^0 e^+ e^-) = \frac{m_e^2}{2m_K} (2\pi)^{-5} \int \sum_{\text{spins}} |A(K_L \to \pi^0 e^+ e^-)|^2 \frac{d^3p}{2p^0} \frac{d^3q_+}{q_+^0} \frac{d^3q_-}{q_-^0} \delta(K - q_+ - q_- - p), \qquad (20)$$

inserting ImA for A. *Thus we obtain an estimate for* $\Gamma(K_L \to \pi^0 e^+ e^-)$, *which is also a lower bound.* We find

$$\Gamma(K_L \to \pi^0 e^+ e^-) = 2.72 \times 10^{-30}\ \text{GeV} \qquad (21)$$

with

$$(K_L \to \pi^0 e^+ e^-)/\Gamma(K_L \to \text{all}) = 2.1 \times 10^{-13}. \quad (22)$$

Note that the only known bound[2] on this branching ratio is $\leqslant 1.4 \times 10^{-5}$.

ACKNOWLEDGMENT

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[1]For a useful review of the field we refer the reader to G. J. Donaldson, thesis (Stanford University, 1974), SLAC Report No. 184 (unpublished).

[2]L. Littenberg, in *Proceedings of XVIII International Conference on High Energy Physics, Tbilisi, 1976,* edited by N. N. Bogolubov *et al.* (JINR, Dubna, U.S.S.R., 1977), Vol. II, p. B171.

[3]R. Rockmore and T. F. Wong, Phys. Rev. Lett. 28, 1736 (1972); Phys. Rev. D 7, 3425 (1973); R. Rockmore, J. Smith, and T. F. Wong, *ibid.* 8, 3224 (1973); R. Rockmore, *ibid.* 8, 3226 (1973); A. N. Kamal and R. Rockmore, *ibid.* 9, 752 (1974); 10, 2091 (1974). The modified loop model has had reasonable success in describing the weak radiative kaon decays except in the case of $K^+ \to \pi^+ e^+ e^-$ decay where the prediction for the branching ratio $R = \Gamma(K^+ \to \pi^+ e^+ e^-)/\Gamma(K^+ \to \text{all}) = 16 \times 10^{-7}$ (for $m = 1$ GeV) is in sharp disagreement with the recent experimental upper limit of R. J. Cence *et al.* [Phys. Rev. D 10, 776 (1974)] of $R < 2.7 \times 10^{-7}$. However, in view of the sensitive dependence of $\Gamma(K^+ \to \pi^+ e^+ e^-)$ on m ($\Gamma \propto m^{-6}$) this manifest discrepancy can be reduced somewhat, although it still poses something of a problem to us. Note that the recent calculation of this branching ratio in the gauge model by M. K. Gaillard and B. W. Lee [Phys. Rev. D 10, 897 (1974)] finds for R acceptable values ranging from 30×10^{-7} to 5×10^{-7} so that here as well the experimental result is a worrisome challenge to present theory.

[4]Particle Data Group, Rev. Mod. Phys. 48, S1 (1976).

[5]G. W. Intemann, Phys. Rev. D 12, 654 (1976).

[6]M. Moshe and P. Singer, Phys. Rev. D 6, 1379 (1972); Phys. Lett. 51B, 367 (1974).

[7]L. M. Sehgal, Phys. Rev. D 6, 367 (1972).

[8]One finds that in the modified baryon-loop model the contribution $O(G_W \alpha)$ vanishes from a pairwise cancellation of the relevant six graphs using the properties of the Dirac matrices under charge conjugation. [T. F. Wong (private communication)].

[9]Re$A(K_L \to \pi^0 e^+ e^-)$ diverges linearly; on evaluating it with a regularized lepton propagator (with a regulator mass of 1 GeV), we found its contribution to the rate to be two orders of magnitude lower than that of ImA.