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Diffractive and nondiffractive mechanisms and the universality of multiplicity distributions

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Using the universality of multiplicity distributions as a guide we motivate a simple physical picture for the multiparticle structure of both diffractive and nondiffractive regimes. We assume that hadronic bremsstrahlung is the common underlying mechanism.

A striking regularity which hadron phenomena seem to possess is the universality of topological cross sections. Empirically, the entire multiplicity distribution of a mass recoiling from a leading hadron appears to be the same as that of the overall reaction at an equivalent c.m. energy.[1] This regularity may even hold in a wider context as the hadronic multiplicities produced in e^+e^- annihilation and deep-inelastic processes (not considered here) do not seem to differ greatly from those of hadron-induced reactions.[2]

It is hoped that such a universal feature of the data should be a manifestation of a unifying and fundamental microscopic description of multihadron production.[2] In particular, considering the dependence on the missing mass M^2 of the system X in $pp \to pX$, the same basic mechanism should operate at both small M^2 (diffractive region) and larger M^2 (nondiffractive). One may thus expect that alternative *elementary* physical lines of reasoning should be possible leading to the same conclusion, i.e., a common mechanism underlying diffractive and nondiffractive processes may exist. This work motivates the crude and simple phenomenological model of Ref. 3 in which a sole mechanism of this kind is present. This model may therefore offer an alternate plausible unifying view relevant to the diffractive and nondiffractive domains.

We propose to use the universality of prong distributions as a guide. Several points shall therefore need to be emphasized in the present approach: (1) The bremsstrahlung model of Stodolsky[4] predicts, in a remarkably simple way, a universal multiplicity distribution as a function of missing mass. (2) The predicted prong distribution vs M^2 gives a satisfactory fit to the data, but the model is not applicable in the diffractive region. (3) In order to extend the model into this retion as well, a simple and intuitively appealing generalization of the bremsstrahlung model naturally emerges. Such a generalization preserves universality in the diffractive domain. Thus the extended bremsstrahlung model is to be contrasted with previous models which either use the universality of multiplicity distributions as input[5] or else universality is not satisfied in the diffractive regime (e.g., Ref. 6).

In the bremsstrahlung model, a leading hadron is slowed down similarly to the case of an electron radiating photons in bremsstrahlung. The leading particle spectrum predicted by the model is[4]

$$d\sigma/dx = \sigma\lambda(1-|x|)^{\lambda-1}, \tag{1}$$

where $|x| \simeq 1 - M^2/s$ is the Feynman scaling variable, and for the semiinclusive leading particle distribution one has[3]

$$\frac{d\sigma_N}{dx} = \frac{d\sigma}{dx}\,\frac{[\lambda \ln(M^2/s_0)]^{N-1}}{(N-1)!}\exp\left(-\lambda \ln \frac{M^2}{s_0}\right), \tag{2}$$

where N is the number of primarily radiated objects. As the missing mass squared cannot be less than s_0, the effective threshold-mass-squared of the primarily emitted unit, both (1) and (2) are valid for[7]

$$|x| \lesssim 1 - \frac{s_0}{s}, \tag{3}$$

and normalized to the inelastic cross section

$$\sigma\left(\frac{s_0}{s}\right)^\lambda + \int_0^{1-s_0/s} dx\, \frac{d\sigma}{dx} = \sigma. \tag{4}$$

The N-particle production cross section will be

$$\sigma_N(s) = \int_0^{1-s_0/s} dx\, \frac{d\sigma_N}{dx} = \sigma \exp\left(-\lambda \ln \frac{s}{s_0}\right)\frac{[\lambda \ln(s/s_0)]^N}{N!}, \tag{5a}$$

$$\bar{N}(s) = \lambda \ln(s/s_0). \tag{5b}$$

Thus the following bootstrap relation

$$\sigma_N(s) = \int_0^{1-s_0/s} dx\, \sigma^{-1}(d\sigma/dx)\sigma_{N-1}(s(1-x)), \tag{6}$$

is automatically satisfied with a kernel, Eq. (1), also given by the model. Integral equations such as (6) have been conjectured before.[5] In the present case, however, one finds that the bremsstrahlung model predicts this relation, i.e., a universal character of multiplicity distributions, in a simple way. Equation (6) also occurs in a very simple multiperipheral model. However, such a model does not lead to universality in the diffractive region, contrary to the case of the bremsstrahlung model as indicated below. This point is further clarified in the Appendix.

The Poisson distribution (5a) leads to the identification of the radiated objects as the central clusters of multihadron production.[8] As the average cluster density in rapidity is $\lambda \simeq 1$, Eq. (1) predicts[4,8] a flat leading spectrum for $|x| \lesssim 1 - s_0/s$ in agreement with a remarkable feature of the data. With $\lambda = 1$ and a mean of two charged hadrons which decay from a cluster,[9] accordance of (5b) with the average charged multiplicity data requires $s_0 \sim 20$ GeV^2. The interval (3) is now fixed. Notice that this region of x explicitly excludes the situation where there is no radiation at all ($|x| \simeq 1$), hence the inclusion of $\sigma_{N=0}(s)$ in the normalization (4). Thus, for any $|x| \lesssim 1 - s_0/s$, one cluster, at least, must have been radiated which implies that $d\sigma_N/dx$ in (2) is a function of $N-1$. Furthermore, at present Fermilab energies the region $|x| \gtrsim 1 - s_0/s \sim 0.95$, which is essentially diffractive, falls outside the applicability of the model. We shall return to this point below.

Consider now the model's prediction for the multiplicity distribution of the recoiling mass in $pp \to pX$ at 205 GeV/c. Assume, for simplicity, that a cluster always decays into either two charged particles or neutrals and, for definiteness, $s_0 = 15$ GeV^2 and $\lambda = 1$. The $d\sigma_{n_c}/dx$ experimental results at 205 GeV/c (Ref. 10) are given in the form

$$P_{n_c} \equiv (M_2^2 - M_1^2)^{-1} \int_{M_1^2}^{M_2^2} dM^2 \, \frac{1}{\sigma} \frac{d\sigma_{n_c}}{dx} , \qquad (7a)$$

where $n_c = 1, 3, \ldots$ is the number of final-state charged hadrons. Standard handling of Eq. (2) with the assumptions stated above yields

$$P_{n_c} = s \left[(M_2^2/s)^\lambda \exp\left(-\tfrac{1}{2}\lambda \bar{N}_c \ln \frac{M_2^2}{s_0}\right) \left(\tfrac{1}{2}\lambda \bar{N}_c \ln \frac{M_2^2}{s_0}\right)^n \Big/ n! - (M_2^2 \to M_1^2) \right] (M_2^2 - M_1^2)^{-1} ,$$
$$n_c = 2n + 1. \qquad (7b)$$

With $\bar{N}_c \simeq 1.70$, $\bar{N}_c$ = mean charged multiplicity per cluster, the agreement with experiment is satisfactory[11] [a slight improvement over the fit shown in Figs. 1(c)–1(e) of Ref. 3]. The predicted spectrum and average charged multiplicity at this same energy, $\sigma^{-1} d\sigma/dx \simeq 1$ and $\bar{n}_c(s) = 2 + \bar{N}_c \ln(s/s_0) \simeq 7.5$, also agree well with the experimental results[12,13]: $\sigma^{-1} d\sigma^{\text{exp}}/dx \simeq 0.92 \pm 0.10$ for $|x| \lesssim 0.95$, normalized to one proton per inelastic collision as in the model, and $\bar{n}_c^{\text{exp}} \simeq 7.68 \pm 0.07$.

Let us proceed to explore the possibility of extending the model into the diffractive domain. Suppose that two distinct kinds of objects, λ_1 and λ_2 with effective threshold masses $s_0(\lambda_1) \ll s_0(\lambda_2)$, can be incoherently radiated by the colliding hadrons. It follows immediately that in the small-M^2 diffractive region, radiation of the light-mass species is allowed provided $s_0(\lambda_1) \lesssim M^2$. The model is thus naturally extended into the diffractive domain. Accordingly, (1) is now replaced by[7]

$$\frac{1}{\sigma} \frac{d\sigma}{dx} = \begin{cases} \dfrac{\tilde{\sigma}_1}{\sigma} \lambda_1 (1 - |x|)^{\lambda_1 - 1}, & 1 - \dfrac{s_0(\lambda_2)}{s} \lesssim |x| \lesssim 1 - \dfrac{s_0(\lambda_1)}{s} , \\ \displaystyle\sum_{i=1}^{2} \frac{\tilde{\sigma}_i}{\sigma} \lambda_i (1 - |x|)^{\lambda_i - 1}, & |x| \lesssim 1 - \dfrac{s_0(\lambda_2)}{s} \end{cases} \qquad (8)$$

where $\tilde{\sigma}_1, \tilde{\sigma}_2$ are inelastic cross sections corresponding to each component and, as previously, $\lambda_2 \sim 1$ while $\lambda_1 \ll \lambda_2$,[14] as the second component must dominate in the larger $-M^2$ region. One then obtains for both diffractive and nondiffractive regimes $\sigma^{-1} d\sigma/dx \sim (1-x)^{-1}$ and $\sigma^{-1} d\sigma/dx \sim$ const, respectively, in agreement with the most striking characteristics of the leading spectrum at Fermilab energies.

Since the heavy component is identified with the central clusters of multiparticle production, $s_0(\lambda_2) \sim 20$ GeV^2 as before; the other component presumably represents a rough average which may include direct-π radiation and low-mass resonances, then $s_0(\lambda_1) \ll 20$ GeV^2. One has in fact recovered a special version of the standard two-component model of multihadron production although motivated rather differently from the usual case.[15] In particular, the model is now expected to hold for a wider range in missing mass as two-component models allow one to come to terms with essentially all features of the currently available multiplicity distribution data. Moreover, universality is kept in the sense that each cluster component obeys a bootstrap relation as in (6) and, instead of having two distinct mechanisms at work, they are a manifestation of the same basic physical process.

An intriguing point may be raised if one is willing to assume that at *asymptotic* energies an infinite number of species is radiated for any $|x|$ not extremely close to unity. In this case, for all such $|x|$, (8) and (5a) become

$$\frac{1}{\sigma}\frac{d\sigma}{dx}=\sum_{i=1}^{\infty}\frac{\tilde{\sigma}_i}{\sigma}\lambda_i(1-|x|)^{\lambda_i-1}$$

$$\to\int_0^{\infty}d\lambda\,\frac{\sigma(\lambda)}{\sigma}\lambda(1-|x|)^{\lambda-1}\,, \tag{9a}$$

$$\sigma_N(s)\to\sigma\int_0^{\infty}d\lambda\,\frac{\sigma(\lambda)}{\sigma}\,\frac{\{\lambda\ln[s/s_0(\lambda)]\}^N}{N!}\times\exp\left(-\lambda\ln\frac{s}{s_0(\lambda)}\right). \tag{9b}$$

Since *asymptotically* the Poissonian λ components in (9b) are δ functions, and if $\ln s_0(\lambda)$ varies smoothly with λ, by the arguments of Ref. 16

$$\sigma(\lambda,s)/\sigma(s)\xrightarrow[s\to\infty]{}\sigma(\lambda)/\sigma=\psi(\lambda)$$

where $\psi(\lambda)$ is the scaling Koba-Nielsen-Olesen (KNO) function, $\bar{N}\sigma_N/\sigma=\psi(N/\bar{N})$. Thus the infinite-component generalization of the bremsstrahlung model[16] emerges, in this manner, as an *asymptotic* limit. Under this point of view a number of conclusions can be drawn at once. (i) The physical meaning of the infinite-component incoherent continuum (9) is now obvious. This physical meaning was unknown so far, although some alternatives in this regard had been mentioned before.[17] (ii) Two-component, or several-component, schemes are a useful but temporary way of representing the data at present energies. (iii) The agreement found[16] between the infinite-component model and the 400-GeV/c $d\sigma/dx$ data must be, to a large extent, related to the approximate experimental scaling of the leading spectrum [it is meaningful to use the asymptotic form 9(a), with $\sigma(\lambda)/\sigma=\psi(\lambda)$, down to Fermilab energies]. But for the s-dependent P_{n_c} distribution (7) the situation is different. That is, the prediction of the infinite-component model for the 205 GeV/c P_{n_c} distribution, $\int_0^\infty d\lambda\psi(\lambda)P_{n_c}(\lambda)$ with $s_0=15$ GeV^2, $\bar{N}_c=1.70$ and $P_{n_c}(\lambda)$ given in (7b), does not seem compatible with experiment.[18] Furthermore, the $d\sigma/dx$ distribution (9a) could not be used, as already stated, inside the diffractive region at 205 GeV/c. Within the present interpretation these difficulties vanish as the infinite-component model should only be applicable at asymptotic energies.

In Ref. 3 we showed that for both small and larger M^2 the gross features of the data: topological cross sections, leading spectrum, and average multiplicities could be described by a simple model of the type represented in Eq. (8). On the other hand, the question naturally arises of whether other aspects of the diffractive and nondiffractive data, e.g., t and s dependences, factorization, etc., can still be approximately understood by a common physical mechanism of this sort. It is therefore of interest to study further all these possibilities.

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APPENDIX

The bootstrap relation (6) results from the combination of three ingredients which enter in Eqs. (1), (2), and (5): $\sigma_N(s)$ is a Poisson distribution in N, $d\sigma_N/dx$ is a Poisson distribution in $N-1$, and the coefficient of the logarithmic terms in (2) and (5a) are correlated in a special manner with the exponent of the kernel in Eq. (1). As a consequence of this a very simple model, such as the one-dimensional multiperipheral model, fails to predict universality in the diffractive regime.

In the longitudinal Chew-Pignotti model with ordinary Reggeon exchange along the chain (nondiffractive component) one has[19]

$$\sigma_N^{\mathrm{nd}}(s)=\sigma_{nd}\frac{[g^2\ln(s/s_0)]^N}{N!}\exp\left(-g^2\ln\frac{s}{s_0}\right), \tag{A1}$$

where g is the R-R-particle coupling and to obtain a constant total cross section

$$\alpha_P=2\alpha_R-1+g^2=1. \tag{A2}$$

For the leading-particle semi-inclusive distribution,

$$\frac{d\sigma_N^{\mathrm{nd}}}{dx}=\frac{1}{\sigma_{\mathrm{nd}}}\frac{d\sigma_{\mathrm{nd}}}{dx}\sigma_{\mathrm{nd}}\frac{[g^2\ln(M^2/s_0)]^{N-1}}{(N-1)!}\times\exp\left(-g^2\ln\frac{M^2}{s_0}\right) \tag{A3}$$

is easily derived where the leading spectrum is given by a PRR triple-Regge term

$$\frac{1}{\sigma_{\mathrm{nd}}}\frac{d\sigma_{\mathrm{nd}}}{dx}\propto(1-x)^{\alpha_P-2\alpha_R}. \tag{A4}$$

It then follows from (A2) that the exponent of the kernel (A4) is g^2-1 and thus the bootstrap relation (6) is satisfied. Universality holds for the nondiffractive component in this model.

In order to accommodate a diffractive contribution, this model can be trivially extended (see, e.g., Refs. 6 and 15). It amounts to incoherently adding to the previous terms another component in which Pomeron exchange occurs at either end on the chain. In this case we then obtain

$$\sigma_N^d(s) = \sigma_d \int_0^{\ln(s/s_0)} dz \, \frac{(g^2 z)^{N-1}}{(N-1)!} \exp(-g^2 z)$$

$$= \frac{\sigma_d}{g^2}\left[1 - \sum_{j=0}^{N-1} \frac{[g^2 \ln(s/s_0)]^j}{j!} \exp\left(-g^2 \ln \frac{s}{s_0}\right)\right], \tag{A5}$$

$$\frac{d\sigma_N^d}{dx} = \frac{1}{\sigma_d}\frac{d\sigma_d}{dx}\, \sigma_d \frac{[g^2 \ln(M^2/s_0)]^{N-1}}{(N-1)!} \exp\left(-g^2 \ln \frac{M^2}{s_0}\right), \tag{A6}$$

and the leading distribution is now given by a *PPP* triple-Pomeron term

$$\sigma_d^{-1} d\sigma_d/dx \propto (1-x)^{\alpha_P - 2\alpha_P} = (1-x)^{-1}. \tag{A7}$$

We can see that the diffractive component (A5)–(A7) does not lead to a bootstrap relation and universality is not satisfied.

There are two simple but important differences, therefore, between the longitudinal multiperipheral model and the model of this work:

(1) Only the extended bremsstrahlung model leads to universality for both diffractive and nondiffractive contributions. Experimentally, the data of Ref. 10 suggest that universality holds in both regions.

(2) In the multiperipheral model the same constants s_0 and g occur for both diffractive and nondiffractive multiparticle states. This is not the case for the extended bremsstrahlung model. Our choice $s_0(\lambda_1) \ll s_0(\lambda_2)$ and $\lambda_1 \ll \lambda_2$, which was physically motivated, is compatible with the data even though the data apparently do not show striking differences in the diffractive and nondiffractive regions. Differences nevertheless exist.[3] Thus we find agreement with the inclusive leading-particle spectrum, as discussed before, and also with the semi-inclusive leading-particle distribution data.[3]

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[7] Since one is interested in the x region away from thresholds, we ignore a threshold factor present in (1) which makes $d\sigma/dx$ vanish as $|x| \to 1 - s_0/s$. The same comment shall apply later to (8).

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