Symmetry breaking and naturalness of parity conservation in weak neutral currents in left-right–symmetric gauge theories

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We discuss the symmetry-breaking patterns and the naturalness of parity conservation in weak neutral-current interactions in the left-right–symmetric $SU(2)_L \otimes SU(2)_R \otimes U(1)$ gauge theories. Two Higgs systems are discussed which enable us to achieve the desired breaking of the gauge symmetry. We show by a detailed analysis of the Higgs potential that the more economical of the two Higgs systems, that involving left and right Higgs scalar doublets, fails to meet the criteria of naturalness of parity conservation in neutral-current interactions. We find that with the second Higgs system neutral currents conserve parity naturally regardless of the structure of the physical charged-current weak interactions. Implications of this for the computability of induced parity violation in higher orders and the search for parity nonconservation in atoms are also discussed.

I. INTRODUCTION

Recently, gauge theories based on the group $SU(2)_L \otimes SU(2)_R \otimes U(1)$ have been proposed[1-3] as serious candidates for the unified description of the weak and electromagnetic interactions. Such theories have a number of attractive features which are not shared by the standard $SU(2) \otimes U(1)$ theories.[4] These theories are parity conserving before spontaneous symmetry breaking and also afterwards at asymptotic energies. The asymmetry in the low-energy charged-current weak interactions, i.e., predominance of the left-handed interactions over the right-handed ones, is a consequence of the symmetry breaking thus leading to a conceptually different picture of parity violation in weak interactions at low energies.

It was shown in Ref. 1 that one of the symmetry-breaking schemes in $SU(2)_L \otimes SU(2)_R \otimes U(1)$ gauge theories leads to a very interesting structure of the weak neutral-current interaction such that one massive neutral vector boson (Z_A) couples only to axial-vector currents and the other (Z_V) couples only to vector currents.[5] Specifically, the neutral-current interaction Lagrangian is of the form

$$\mathcal{L}_{\rm int}{}^{\rm NC} = \frac{ie}{2\sin\theta} Z_{A\mu}(a_e \bar{e}\gamma_\mu\gamma_5 e + a_\nu \bar{\nu}\gamma_\mu\gamma_5\nu + a_q \bar{q}\gamma_\mu\gamma_5 q) - \frac{ie}{\sin 2\theta} Z_{V\mu}(v_e \bar{e}\gamma_\mu e + v_\nu \bar{\nu}\gamma_\mu \nu + v_q \bar{q}\gamma_\mu q) + (\mu \to e)\,, \qquad (1.1)$$

where $q = u, d, c, s, \ldots$. This form of the interaction will yield an effective neutral-current Hamiltonian which is clearly parity conserving having the form $\mathcal{L}_{\rm NC} = VV + AA$ with no terms of the type $VA + AV$ which involve vector–axial-vector interference. However, one of the notable features of the model is that in spite of the parity-conserving neutral-current interaction, the neutrino and antineutrino neutral-current cross sections are predicted to be unequal. The inequality here of the two cross sections is a direct consequence of the fact that the two massive neutral vector bosons have definite and opposite charge conjugation. This ia a counterexample to the statement that $\sigma_{\rm NC}(\nu_\mu N) \neq \sigma_{\rm NC}(\bar{\nu}_\mu N)$ necessarily implies that neutral currents are parity violating. Note that in the neutral-current interaction Lagrangian of Eq. (1.1), there is no parity-violating electron-nucleon coupling and, hence, this model predicts that there be no parity violation in atomic physics to $O(G_F)$. The results of two recently completed atomic-physics experiments[6] are such that the present upper limits on the parity-violating eN interaction are an order of magnitude below the prediction of the Weinberg-Salam model.[4]

Parity conservation in neutral-current interactions was obtained in Ref. 1 by a particular choice of the vacuum expectation values of the Higgs fields. It is important to know whether this choice is "natural," i.e, whether it can be obtained for a finite range of the parameters of the Higgs potential and is stable under renormalization. In this paper we show that the Higgs system proposed in Ref. 1 does not lead naturally to parity-conserving neutral currents. However, there is another Higgs system which does naturally give parity-conserving neutral currents.

The plan of the rest of this paper is as follows: In Sec. II we define the various Higgs multiplets and discuss the structure of the neutral currents. In Sec. III we examine the implications of left-right symmetry for the fermions. We find that the original definition of left-right symmetry leads to left and right generalized Cabibbo rotations which are related by complex conjugation, but that the definition can be modified to allow the two rotations to be independent. In Sec. IV we prove that the Higgs system proposed in Ref. 1 does not lead naturally to parity conservation in neutral currents. In Sec. V we show that with a different Higgs system such parity conservation can be achieved naturally. Finally, in Sec. VI we discuss the implications of our results.

II. HIGGS SYSTEMS AND THE STRUCTURE OF NEUTRAL CURRENTS

We consider a left-right–symmetric gauge theory based on the group $SU(2)_L \otimes SU(2)_R \otimes U(1)$ with generators $\vec{T}_L$, $\vec{T}_R$, and Y corresponding to the three subgroups. The electric charge is defined as

$$Q = T_{3L} + T_{3R} + \tfrac{1}{2} Y. \tag{2.1}$$

Here, by left-right symmetry we mean the symmetry of theory which yields $g_L = g_R$ naturally. The left- (right-) handed fermions are assigned to doublets under T_L (T_R). To generate masses for the fermions, we need a Higgs multiplet of the type

$$\Phi_1: \ (\tfrac{1}{2}, \tfrac{1}{2}, 0), \tag{2.2}$$

where the numbers in the parentheses are the values of (T_L, T_R, Y). Corresponding to each Φ_1 multiplet, there is a Φ_2 multiplet defined as

$$\Phi_2 = \tau_2 \Phi_1^* \tau_2: \ (\tfrac{1}{2}, \tfrac{1}{2}, 0), \tag{2.3}$$

which transform like Φ_1. To obtain a general mass matrix for the fermions, it may be necessary to introduce more than one Φ_1-like Higgs multiplet; this is an inessential complication. Fermions acquire masses after the symmetry is spontaneously broken by given nonzero vacuum expectation value (VEV) to the Φ_1 $(\frac{1}{2}, \frac{1}{2}, 0)$ Higgs field. The most general VEV is of the form

$$\langle \Phi_1 \rangle = \begin{pmatrix} \mathcal{K} e^{i\alpha} & 0 \\ 0 & \mathcal{K}' e^{i\alpha'} \end{pmatrix}. \tag{2.4}$$

After this first step, the symmetry of the theory is broken down to $U(1)_{L+R} \times U(1)_V$, i.e., there are two massless neutral vector bosons. We are interested in breaking the symmetry further down to U(1) so that we have only one massless neutral vector boson, the photon. There are two alternative ways to achieve the above objective of symmetry breaking down to U(1) of electromagnetism, differing in the selection of the Higgs fields used to trigger the second step of symmetry breaking.

Case (i): $\chi_L(\frac{1}{2}, 0, 1)$ *and* $\chi_R(0, \frac{1}{2}, 1)$ *Higgs fields.* The choice of VEV which breaks the symmetry down to U(1) and also leads to the parity-conserving structure of the neutral current is

$$\langle \chi_L \rangle = \langle \chi_R \rangle = \begin{pmatrix} 0 \\ \lambda \end{pmatrix}. \tag{2.5}$$

In Sec. IV we shall show that $\langle \chi_L \rangle = \langle \chi_R \rangle$ does not hold naturally, at least for the particular definitions of left-right symmetry which we have considered. Thus in general the neutral-vector-boson eigenstates are linear combinations of Z_V and Z_A, and parity is not conserved.

To construct phenomenologically viable models, it is sometimes necessary to include other Higgs fields such as $\delta_L(1, 0, 0)$ and $\delta_R(0, 1, 0)$ with the following VEV's,

$$\langle \delta_L \rangle = 0,$$

$$\langle \delta_R \rangle = \begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix}. \tag{2.6}$$

This type of VEV's contributes only to the charged-gauge-bosons (mass)^2 matrix and make the right-handed charged W bosons heavier than the left-handed ones. This is certainly required in the four-quark model of Ref. 1 in order to suppress the unwanted right-handed currents. However, in the multiquark models, one may dispense with δ_L and δ_R Higgs multiplets if $W_R^{\pm}$ connect the known, light fermions only to very heavy ones.

Case (ii): $\rho(\frac{1}{2}, \frac{1}{2}, 2)$ *and* $\rho'(\frac{1}{2}, \frac{1}{2}, -2)$ *Higgs fields.* The following set of VEV's of these fields[7] breaks the symmetry down to U(1) and gives the desired structure of the neutral current,

$$\langle \rho \rangle = \begin{pmatrix} 0 & 0 \\ a & 0 \end{pmatrix}$$

and

$$\langle \rho' \rangle = \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix}. \tag{2.7}$$

Note that because of the nonzero hypercharge, ρ and ρ' do not contribute to the fermion masses. Just as in case (i), one may still need triplet Higgs fields $\delta_L(1, 0, 0)$ and $\delta_R(0, 1, 0)$ with the VEV's given in Eq. (2.6) to make the model satisfactory phenomenologically.

Turning to the structure of the neutral-current interactions, we note that there are three neutral gauge bosons in $SU(2)_L \otimes SU(2)_R \otimes U(1)$ gauge theories denoted by W_L^3, W_R^3, and B coupled to the neutral generators T_{3L}, T_{3R}, and Y, respectively. The (mass)^2 matrix for these bosons receives contributions from the Φ_i and from χ_L and χ_R or ρ and ρ'; there is no contribution from δ_L and δ_R. If we have either the χ_L, χ_R Higgs system with $\langle\chi_L\rangle = \langle\chi_R\rangle$ or the ρ, ρ' Higgs system, then the mass of eigenstates are

$$A_\mu = \frac{\sin\theta}{\sqrt{2}}(W_{L\mu}^3 + W_{R\mu}^3) + \cos\theta B_\mu\,,$$

$$Z_{V\mu} = \frac{\cos\theta}{\sqrt{2}}(W_{L\mu}^3 + W_{R\mu}^3) - \sin\theta B_\mu\,, \tag{2.8}$$

$$Z_{A\mu} = \frac{1}{\sqrt{2}}(W_{L\mu}^3 - W_{R\mu}^3)\,,$$

where $g_L = g_R = g = \sqrt{2}e/\sin\theta = g' = e/\cos\theta$. For the neutral weak interactions to be parity conserving we require that the massive neutral vector boson $Z_{V\mu}$ ($Z_{A\mu}$) couples to a purely vector (axial-vector) current. We expect this to be true for each elementary fermion field in the Lagrangian including the neutrinos. As can be easily seen by writing down the interaction Lagrangian, each species of fermions ψ_i must have definite values of T_{3L} and T_{3R} with

$$T_{3L}(\psi_i) = T_{3R}(\psi_i)\,. \tag{2.9}$$

This requirement is met in the four-quark model but not in the six-quark model of Ref. 1. If it is satisfied then the neutral-current interaction Lagrangian involving the fermion fields ν_μ, e, u, and d is

$$\begin{aligned}\mathcal{L}_{\rm int}^{\rm NC} = \frac{ie}{2\sin\theta} Z_{A\mu}(\bar{e}\gamma_\mu\gamma_5 e - \bar{\nu}\gamma_\mu\gamma_5\nu + \bar{d}\gamma_\mu\gamma_5 d - \bar{u}\gamma_\mu\gamma_5 u)\\ - \frac{ie}{\sin 2\theta} Z_{V\mu}[-\cos 2\theta\bar{e}\gamma_\mu e + \bar{\nu}\gamma_\mu\nu\\ - \tfrac{1}{3}(1 - 4\cos^2\theta)\bar{u}\gamma_\mu u\\ - \tfrac{1}{3}(1 + 2\cos^2\theta)\bar{d}\gamma_\mu d]\,.\end{aligned} \tag{2.10}$$

III. LEFT-RIGHT SYMMETRY AND FERMIONS

We define a left-right symmetry as any discrete symmetry of the Lagrangian which requires the equality of the $SU(2)_L$ and $SU(2)_R$ coupling constants. The equality is then naturally preserved under spontaneous symmetry breaking except for finite and calculable corrections.[8] Evidently it is necessary only that the symmetry interchange $W_{L\mu}$ and $W_{R\mu}$ and that it be consistent with gauge invariance. The particular symmetry considered in Ref. 1 is

$$\begin{aligned} &W_{L\mu} \leftrightarrow W_{R\mu}\,,\\ &\psi_{iL} \leftrightarrow \psi_{iR}\,,\\ &\Phi_i \leftrightarrow \Phi_i^\dagger\,,\\ &\chi_L \leftrightarrow \chi_R\,,\\ &\delta_L \leftrightarrow \delta_R\,.\end{aligned} \tag{3.1}$$

If ρ and ρ' are used in place of χ_L and χ_R, then they transform as

$$\rho \leftrightarrow \rho'^\dagger\,. \tag{3.2}$$

Left-right symmetry must be broken to account for the observed parity violation in charged currents. Two patterns of symmetry breaking can be considered. In the first, $W_R^\pm$ are made much heavier than $W_L^\pm$. (This requires the presence of the Higgs multiplets δ_L and δ_R, with $\langle\delta_R\rangle \gg \langle\delta_L\rangle$.) Then the right-handed fermion multiplets enter only in the neutral currents except at very high energy. One possibility of this type is manifest left-right symmetry, in which the left and right generalized Cabibbo rotations are identical.

In the second pattern of symmetry breaking, $W_L^\pm$ and $W_R^\pm$ have comparable masses. Then it is necessary to put the light quarks in right-handed multiplets with new, heavy quarks, so that parity is violated at low energy. We shall see that this pattern is not possible with the left-right symmetry in Eq. (3.1) but that it can be obtained with a different definition of left-right symmetry if desired.

The most general gauge-invariant and renormalizable Higgs-boson–fermion interaction is

$$\mathcal{L}_{\rm H} = \sum_n F_{ij}^{(n)}\bar{\psi}_{iL}\Phi_n\psi_{jR} + {\rm H.c.} \tag{3.3}$$

We assume that CP is conserved before spontaneous symmetry breaking, so that the $F_{ij}^{(n)}$ are real. Then invariance under the left-right transformation of Eq. (3.1) implies

$$F_{ij}^{(n)} = F_{ji}^{(n)} \tag{3.4}$$

for each n. Fermion masses are generated by spontaneous symmetry breaking as usual. The fermion mass matrix is

$$\mathcal{L}_{\rm mass} = \bar{\psi}_{iL}M_{ij}\psi_{jR} + {\rm H.c.}\,, \tag{3.5}$$

where

$$M_{ij} = \sum_n F_{ij}^{(n)}\langle\Phi_n\rangle\,. \tag{3.6}$$

We shall show in the following section that there is a finite range of the parameters in the Higgs potential for which the $\langle\Phi_n\rangle$ are real and another finite range for which they are complex. Thus M can be either real or complex, but in either case it is symmetric because of Eq. (3.4).

For any matrix M we can write

$$M = U_L D U_R^\dagger\,, \tag{3.7}$$

where U_L and U_R are unitary and D is diagonal and has real, non-negative elements. If M is real and symmetric, then $U_L = U_R$ is a real orthogonal matrix. Hence the left and right generalized Cabibbo rotations are equal, and one has manifest left-right symmetry.[9]

If M is complex and symmetric, then U_L and U_R are not equal, but if there are no degenerate masses, then

$$U_R = U_L^*, \tag{3.8}$$

which means that the left and right generalized Cabibbo rotations are equal but the phases are equal and opposite in sign, and one has pseudo-manifest left-right symmetry. To see this we set $M = M^T$ and use Eq. (3.7) to obtain

$$(U_R^T U_L) D = D (U_R^T U_L)^T . \tag{3.9}$$

The Hermitian conjugate of this gives

$$(U_R^T U_L)^T D = D (U_R^T U_L) . \tag{3.10}$$

But

$$D_{ij} = M_i \delta_{ij} , \quad M_i \geq 0 \tag{3.11}$$

so that

$$\begin{aligned} (U_R^T U_L)_{ij} M_j &= M_i (U_R^T U_L)_{ij} , \\ (U_R^T U_L)_{ji} M_j &= M_i (U_R^T U_L)_{ji} , \end{aligned} \tag{3.12}$$

with no sum on i or j. For $M_i \neq M_j$, $i \neq j$, it follows that

$$(U_R^T U_L)_{ij} = 0, \quad i \neq j , \tag{3.13}$$

which implies that $U_R = U_L^*$.

Thus, making $\langle \Phi_n \rangle$ complex does not substantially change the nature of the model. In particular it is still necessary to make $W_R^\pm$ much heavier than $W_L^\pm$ to suppress the right-handed charged currents at low energy. However, it may be useful to take $\langle \Phi_n \rangle$ to be complex in order to introduce CP violation in the fermion mass matrix.[10]

If one wants the left and right generalized Cabibbo rotations to be independent, then it is necessary to change the definition of left-right symmetry. The simplest change is to require invariance under

$$\Phi_1 \leftrightarrow \Phi_2^\dagger \tag{3.14}$$

while keeping the rest of Eq. (3.1) the same. Then the Higgs-fermion couplings in Eq. (3.3) must satisfy

$$F_{ij}^{(1)} = F_{ji}^{(2)} . \tag{3.15}$$

If $\langle \Phi_1 \rangle \neq \langle \Phi_2 \rangle$ this provides enough freedom to construct an arbitrary mass matrix. Another set of Higgs fields, Φ_3 and

$$\Phi_4 = \tau_2 \Phi_3^* \tau_2 , \tag{3.16}$$

is needed to make the mass matrices for the upper and lower fermions independent, but this is an inessential complication.

IV. MODELS WITH χ AND δ HIGGS FIELDS

In this section, as in Ref. 1, we consider models in which the Higgs multiplets are $\Phi_1, \Phi_2, \chi_L, \chi_R$ and perhaps δ_L, δ_R, the notation being that defined in Sec. II. For the neutral currents to be parity conserving it is necessary that

$$\langle \chi_L \rangle = \langle \chi_R \rangle , \tag{4.1}$$

We shall see that for either of the definitions of left-right symmetry considered in the previous section, this condition cannot be satisfied naturally, although of course it can be arranged by adjusting the parameters in the Higgs potential.

We first consider the definition of left-right symmetry given in Eq. (3.1), i.e., that with $\Phi_i \leftrightarrow \Phi_i^\dagger$. Then since the left and right generalized Cabibbo rotations are related by

$$U_R = U_L^* , \tag{4.2}$$

it is necessary to use δ_L and δ_R to make $W_R^\pm$ much heavier than $W_L^\pm$. For simplicity we impose additional discrete symmetries under

$$\delta_L \to -\delta_L \text{ and } \delta_R \to -\delta_R . \tag{4.3}$$

Then the most general gauge-invariant, CP-invariant, and renormalizable Higgs potential is

$$\begin{aligned} V = &- \sum_{ij=1,2} \mu_{ij}^2 \operatorname{Tr}(\Phi_i^\dagger \Phi_j) - \mu_1^2 (\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R) - \mu_2^2 (\delta_L^\dagger \delta_L + \delta_R^\dagger \delta_R) + \sum_{i=1,2} \mu_i' (\chi_L^\dagger \Phi_i \chi_R + \chi_R^\dagger \Phi_i^\dagger \chi_L) \\ &+ \sum_{ijkl} [\lambda_{ijkl} \operatorname{Tr}(\Phi_i^\dagger \Phi_j) \operatorname{Tr}(\Phi_k^\dagger \Phi_l) + \lambda_{ijkl}' \operatorname{Tr}(\Phi_i^\dagger \Phi_j \Phi_k^\dagger \Phi_l)] + \lambda_1 (\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R)^2 + \lambda_2 (\chi_L^\dagger \chi_L)(\chi_R^\dagger \chi_R) \\ &+ \sum_{ij=1,2} [\lambda_{3ij} (\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R) \operatorname{Tr}(\Phi_i^\dagger \Phi_j) + \lambda_{3ij}' (\chi_L^\dagger \Phi_i \Phi_j^\dagger \chi_L + \chi_R^\dagger \Phi_i^\dagger \Phi_j \chi_R)] \\ &+ \lambda_4 (\delta_L^\dagger \delta_L + \delta_R^\dagger \delta_R)^2 + \lambda_5 (\delta_L^\dagger \delta_L)(\delta_R^\dagger \delta_R) + \sum_{ij=1,2} \lambda_{6ij} (\delta_L^\dagger \delta_L + \delta_R^\dagger \delta_R) \operatorname{Tr}(\Phi_i^\dagger \Phi_j) \\ &+ \lambda_7 (\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R)(\delta_L^\dagger \delta_L + \delta_R^\dagger \delta_R) + \lambda_8 [(\chi_L^\dagger \chi_L)(\delta_L^\dagger \delta_L) + (\chi_R^\dagger \chi_R)(\delta_R^\dagger \delta_R)] , \end{aligned} \tag{4.4}$$

where all of the couplings are real and satisfy

$$\begin{aligned}&\mu_{ij}{}^2=\mu_{ji}{}^2\,,\\&\lambda_{ijkl}=\lambda_{ijlk}=\lambda_{jikl}=\lambda_{jilk}\,,\\&\lambda'_{ijkl}=\lambda'_{lijk}=\lambda'_{klij}=\lambda'_{jkli}\,,\\&\lambda_{3ij}=\lambda_{3ji}\,,\quad \lambda'_{3ij}=\lambda'_{3ji}\,,\quad \lambda_{6ij}=\lambda_{6ji}\,.\end{aligned}\tag{4.5}$$

Note that the terms involving $\chi_L^\dagger\Phi_i\chi_R+\chi_R^\dagger\Phi_i^\dagger\chi_L$ might be eliminated by imposing another discrete symmetry under $\chi_L\to-\chi_L$ or $\chi_R\to-\chi_R$, but then V would have an extra U(1) symmetry acting on χ_L or χ_R. This symmetry would be broken by the vacuum expectation value, giving rise to an unwanted Goldstone boson.

To show that $\langle\chi_L\rangle=\langle\chi_R\rangle$ cannot be satisfied naturally, it is sufficient to study V with $\Phi_i=0$; the cross terms involving Φ_i will not affect the conclusion. It is then straightforward to show that at the minimum of V,

$$2\lambda_1(\langle\chi_L^\dagger\rangle\langle\chi_L\rangle-\langle\chi_R^\dagger\rangle\langle\chi_R\rangle)+\lambda_8(\langle\delta_L^\dagger\rangle\langle\delta_L\rangle-\langle\delta_R^\dagger\rangle\langle\delta_R\rangle)=0\,.\tag{4.6}$$

Since we must have $\langle\delta_R\rangle\gg\langle\delta_L\rangle$ to make $W_R^\pm$ much heavier than $W_L^\pm$, we can have $\langle\chi_L\rangle=\langle\chi_R\rangle$ only if $\lambda_8=0$.

When higher-order graphs are considered, an additional problem arises. Since the left-right symmetry is spontaneously broken, one is guaranteed that there are no infinite counterterms of dimension four which violate the symmetry.[8] Thus the equality of the $SU(2)_L$ and $SU(2)_R$ coupling constants and the symmetries of the four-Higgs-fields couplings are preserved except for finite corrections. However, there may be counterterms of lower dimension which violate the symmetry, and in fact the χ_L and χ_R masses are separately renormalized. Consider the graphs shown in Fig. 1. For small $\mathcal{K}\mathcal{K}'$, $W_L^\pm$ and $W_R^\pm$ are approximate mass eigenstates. Using dimensional regularization, we find for the difference of the χ_L and χ_R self-energies

$$\Gamma_{\chi_L\chi_L}(p)-\Gamma_{\chi_R\chi_R}(p)=\frac{ig^2}{8\pi^2}\frac{1}{n-4}(m_{W_L}{}^2-m_{W_R}{}^2)+\text{reg}\,,\tag{4.7}$$

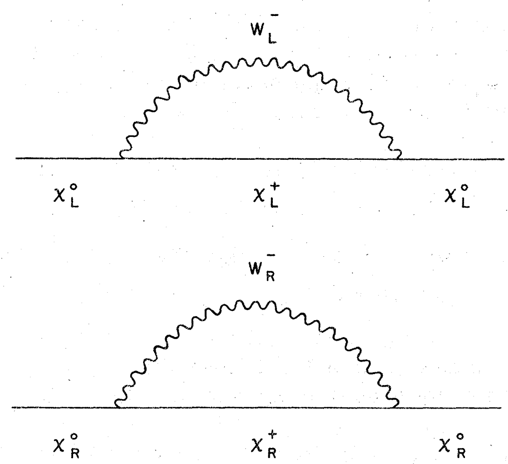

FIG. 1. Graphs contributing to the infinite renormalization of $M_{\chi_L}{}^2-M_{\chi_R}{}^2$.

where reg denotes terms which are finite as $n\to4$. Thus the χ_L and χ_R mass counterterms must be adjusted separately in each order of perturbation theory if the equality of the masses presumed in Eq. (4.4) is to be maintained.

The present experimental limits on parity violation in atoms are not so stringent as to make the possibility that $\langle\chi_L\rangle$ and $\langle\chi_R\rangle$ are only approximately equal unreasonable. We discuss this possibility in the Appendix.

To see that $\langle\Phi_i\rangle$ can be either real or complex, we note that the most general neutral vacuum expectation value for the Higgs fields is

$$\begin{aligned}&\langle\chi_L\rangle=\begin{pmatrix}0\\ \chi\cos\beta\end{pmatrix},\quad\langle\chi_R\rangle=\begin{pmatrix}0\\ \chi\sin\beta\end{pmatrix},\\&\langle\Phi_1\rangle=\begin{pmatrix}\mathcal{K}e^{i\alpha}&0\\0&\mathcal{K}'e^{i\alpha'}\end{pmatrix},\quad\langle\Phi_2\rangle=\begin{pmatrix}\mathcal{K}'e^{-i\alpha'}&0\\0&\mathcal{K}e^{-i\alpha}\end{pmatrix},\\&\langle\delta_L\rangle=\begin{pmatrix}0\\ \delta_L\\ 0\end{pmatrix},\quad\langle\delta_R\rangle=\begin{pmatrix}0\\ \delta_R\\ 0\end{pmatrix},\end{aligned}\tag{4.8}$$

with $\langle\chi_L\rangle=\langle\chi_R\rangle$ corresponding to $\beta=\pi/4$. When these expressions are substituted in V, the terms involving α and α' are

$$\begin{aligned}V=&-4\mu_{12}{}^2\mathcal{K}\mathcal{K}'\cos(\alpha+\alpha')+\mu_1'\chi^2\sin2\beta\,\mathcal{K}'\cos\alpha'+\mu_2'\chi^2\sin2\beta\,\mathcal{K}\cos\alpha\\&+\bar\lambda_1(\mathcal{K}^2+\mathcal{K}'^2)\mathcal{K}\mathcal{K}'\cos(\alpha+\alpha')+\bar\lambda_2\mathcal{K}^2\mathcal{K}'^2\cos2(\alpha+\alpha')+(8\lambda_{3,12}+4\lambda'_{3,12})\chi^2\mathcal{K}\mathcal{K}'\cos(\alpha+\alpha')\\&+2\lambda_{6,12}(\delta_L{}^2+\delta_R{}^2)\mathcal{K}\mathcal{K}'\cos(\alpha+\alpha')\,,\end{aligned}$$

$$\begin{aligned}&\bar\lambda_1=4(\lambda_{1112}+\lambda_{1121}+\lambda_{2212}+\lambda_{2221})+2(\lambda'_{1112}+\lambda'_{2221})\,,\\&\bar\lambda_2=8(\lambda_{1212}+\lambda_{1221}+\lambda_{2112}+\lambda_{2121})+4(\lambda'_{1212}+\lambda'_{2121})\,.\end{aligned}$$

Evidently the first derivatives vanish at $\alpha=\alpha'=0$, and the signs can be chosen to make this point either a maximum or a minimum for a finite range of the parameters.

We now consider the second definition of left-right symmetry given in the previous section, i.e., that with $\Phi_1 \longleftrightarrow \Phi_2^\dagger$. For appropriately chosen fermion mixings, $W_L^\pm$ and $W_R^\pm$ can be comparable in mass, so that δ_L and δ_R are not needed. Then the most general Higgs potential is

$$\begin{aligned} V = &- \mu_1{}^2(\chi_{L1}^\dagger\chi_L + \chi_R^\dagger\chi_R) + \mu'(\chi_L^\dagger\Phi_1\chi_R + \chi_R^\dagger\Phi_2\chi_L + \chi_R^\dagger\Phi_1\chi_L + \chi_L^\dagger\Phi_2\chi_R) + \lambda_1(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R)^2 + \lambda_2(\chi_L^\dagger\chi_L)(\chi_R^\dagger\chi_R) \\ &+ \lambda_3(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R)\,\mathrm{Tr}(\Phi_1^\dagger\Phi_1) + \lambda_3'(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R)[\mathrm{Tr}(\Phi_1^\dagger\Phi_2) + \mathrm{Tr}(\Phi_2^\dagger\Phi_1)] \\ &+ \lambda_4(\chi_L^\dagger\Phi_1\Phi_1^\dagger\chi_L + \chi_R^\dagger\Phi_2^\dagger\Phi_2\chi_R) + \lambda_4'(\chi_L^\dagger\Phi_2\Phi_2^\dagger\chi_L + \chi_R^\dagger\Phi_1^\dagger\Phi_1\chi_R) \\ &+ \lambda_4''(\chi_L^\dagger\Phi_1\Phi_2^\dagger\chi_L + \chi_R^\dagger\Phi_2^\dagger\Phi_1\chi_R + \chi_L^\dagger\Phi_2\Phi_1^\dagger\chi_L + \chi_R^\dagger\Phi_1^\dagger\Phi_1\chi_R) + F(\Phi_1, \Phi_2)\,, \end{aligned} \tag{4.10}$$

where $F(\Phi_1, \Phi_2)$ contains all the terms not involving χ_L and χ_R.

From this form of V it follows that $\langle\chi_L\rangle = \langle\chi_R\rangle$ can hold naturally only if $\mathcal{K} = \mathcal{K}'$ and $\alpha = \pm\alpha'$, if at all. To see this, we substitute the vacuum expectation values, Eq. (4.8), in V,

$$\begin{aligned} V = &-\mu_1{}^2\chi^2 + \mu'\chi\sin 2\beta(\mathcal{K}\cos\alpha + \mathcal{K}'\cos\alpha') + \lambda_1\chi^4 + \tfrac{1}{4}\lambda_2\chi^4\sin^2 2\beta + \lambda_3\chi^2(\mathcal{K}^2 + \mathcal{K}'^2) + 4\lambda_3'\chi^2\mathcal{K}\mathcal{K}'\cos(\alpha + \alpha') \\ &+ \lambda_4\chi^2(\mathcal{K}'^2\cos^2\beta + \mathcal{K}^2\sin^2\beta) + \lambda_4'\chi^2(\mathcal{K}^2\cos^2\beta + \mathcal{K}'^2\sin^2\beta) + 2\lambda_4''\chi^2\mathcal{K}\mathcal{K}'\cos(\alpha + \alpha') + F(\Phi_1, \Phi_2)\,. \end{aligned} \tag{4.11}$$

Evidently $\beta = \pi/4$ is a stationary point for arbitrary λ_4 and λ_4' only if $\mathcal{K} = \mathcal{K}'$. For $\mu' \neq 0$, $\mathcal{K} = \mathcal{K}'$ can be a stationary point only if $\alpha' = \pm\alpha$, since every other term is symmetric under $\mathcal{K} \longleftrightarrow \mathcal{K}'$. But then the fermion mass matrix is either Hermitian or symmetric, δ_L and δ_R are again required, and the natural equality of $\langle\chi_L\rangle$ and $\langle\chi_R\rangle$ fails as before.

We have not attempted to prove that there exists no definition of left-right symmetry for which $\langle\chi_L\rangle = \langle\chi_R\rangle$ holds naturally. However, it seems simpler to use the ρ, ρ' Higgs system, which we discuss in the following section.

V. MODELS WITH $\rho\,(\frac{1}{2}, \frac{1}{2}, 2)$ AND $\rho'\,(\frac{1}{2}, \frac{1}{2}, -2)$ HIGGS FIELDS

In this section, we discuss the second Higgs system introduced in Sec. II (case ii) which can be used to break the residual symmetry $\mathrm{U}(1)_{L+R} \otimes \mathrm{U}(1)$ after $\Phi(\frac{1}{2}, \frac{1}{2}, 0)$ field picks up a nonzero VEV given in Eq. (2.4). In some sense, the $\rho(\frac{1}{2}, \frac{1}{2}, 2)$ field which carries nonzero Y can be thought of as a clever combination of the two hypercharge carrying Higgs fields $\chi_L(\frac{1}{2}, 0, 1)$ and $\chi_R(0, \frac{1}{2}, 1)$. With this Higgs system not only the nonmanifest but also the manifest left-right-symmetric theories can possess naturally parity-conserving weak neutral-current interactions. Models could contain as few quarks as four. In the manifest case, one must introduce triplet Higgs fields $\delta_L(1,0,0)$ and $\delta_R(0,1,0)$ with VEV's given in Eq. (2.6) in order to make the model viable phenomenologically but the inclusion of such fields does not change the naturalness of parity conservation in neutral-current interactions.

The $\rho(\frac{1}{2}, \frac{1}{2}, 2)$ and $\rho'(\frac{1}{2}, \frac{1}{2}, 2)$ fields can be written as in matrix form as

$$\rho = \begin{pmatrix} \rho_1^+ & \rho_1^{++} \\ \rho_2^0 & \rho_2^+ \end{pmatrix}, \qquad \rho' = \begin{pmatrix} \rho_1'^- & \rho_1'^0 \\ \rho_2'^{--} & \rho_2'^- \end{pmatrix}. \tag{5.1}$$

The only allowed pattern of VEV consistent with the charge conservation is

$$\langle\rho\rangle = \begin{pmatrix} 0 & 0 \\ c & 0 \end{pmatrix} \quad \text{and} \quad \langle\rho'\rangle = \begin{pmatrix} 0 & d \\ 0 & 0 \end{pmatrix}. \tag{5.2}$$

The neutral vector bosons $(\text{mass})^2$ matrix receives contribution from $\langle\Phi_i\rangle$, $\langle\rho\rangle$, and $\langle\rho^{\dagger\prime}\rangle$ and has the form

$$\begin{array}{c} \\ W_L^3 \\ W_R^3 \\ B \end{array} \begin{array}{c} \begin{array}{ccc} W_L^3 & W_R^3 & B \end{array} \\ \begin{pmatrix} g^2(x+y) & -g^2(x-y) & -2gg'y \\ -g^2(x-y) & g^2(x+y) & -2gg'y \\ -2gg'y & -2gg'y & 4g'^2y \end{pmatrix} \end{array}, \tag{5.3}$$

where $x = \frac{1}{2}(\mathcal{K}^2 + \mathcal{K}'^2)$ and $y = \frac{1}{2}(c^2 + d^2)$.

For arbitrary values of x and y the eigenstates of this matrix are just the A, Z_V, and Z_A defined in Eq. (2.8) with masses

$$\begin{aligned} m_A{}^2 &= 0\,, \\ m_V{}^2 &= 2(g^2 + 2g'^2)\,y\,, \\ m_A{}^2 &= 2g^2x\,. \end{aligned} \tag{5.4}$$

Since A, Z_V, and Z_A do not depend on x and y, they are in fact the eigenstates for the most general (neutral) vacuum expectation values $\langle\Phi_i\rangle$, $\langle\rho\rangle$, and $\langle\rho'\rangle$ and thus are not dependent on any detailed properties of the Higgs potential.

Having the neutral bosons given by Eq. (2.8) is not sufficient to ensure parity conservation; we must also have $g_L = g_R = g$. This is guaranteed naturally up to finite and calculable higher-order corrections by the left-right symmetry defined in Eqs. (3.1) and (3.2). To see that this symmetry can be imposed consistently, we consider the gauge-invariant kinetic energy terms for ρ and ρ',

$$\mathcal{L}_{\rho\rho'} = -\mathrm{Tr}[(\mathcal{D}_\mu\rho)^\dagger(\mathcal{D}^\mu\rho)] - \mathrm{Tr}[(\mathcal{D}_\mu\rho')^\dagger(\mathcal{D}^\mu\rho')]\,, \tag{5.5}$$

where

$$\mathcal{D}_\mu\rho = \partial_\mu\rho - \tfrac{1}{2}ig\vec{\mathrm{W}}_{\mu L}\cdot\vec{\tau}\rho + \tfrac{1}{2}ig\rho\vec{\tau}\cdot\vec{\mathrm{W}}_{\mu R} - ig'B_\mu\rho\,,$$
$$\mathcal{D}_\mu\rho' = \partial_\mu\rho' - \tfrac{1}{2}ig\vec{\mathrm{W}}_{\mu L}\cdot\vec{\tau}\rho' + \tfrac{1}{2}ig\rho'\vec{\tau}\cdot\vec{\mathrm{W}}_{\mu R} + ig'B_\mu\rho'\,. \tag{5.6}$$

Under left-right symmetry $\rho \to \rho'^\dagger$ and $\vec{\mathrm{W}}_{\mu L} \leftrightarrow \mathrm{W}_{\mu R}$, so

$$\mathcal{D}_\mu\rho \to \partial_\mu\rho'^\dagger - \tfrac{1}{2}ig\vec{\mathrm{W}}_{\mu R}\cdot\vec{\tau}\rho'^\dagger + \tfrac{1}{2}ig\rho'^\dagger\vec{\tau}\cdot\vec{\mathrm{W}}_{\mu L} - igB_\mu\rho'^\dagger = (\mathcal{D}_\mu\rho')^\dagger\,. \tag{5.7}$$

Hence $\mathcal{L}_{\rho\rho'}$ is invariant by the cyclic property of traces. The only other terms in the Lagrangian involving ρ and ρ' are terms in the Higgs potential, and these can clearly be chosen to be invariant. Since we have already noted that the neutral boson eigenstates are independent of the exact form of the Higgs potential, we need not study it in detail.

The parity-conserving neutral-current interaction in Eq. (2.10) follows immediately from $g_L = g_R = g$ together with Eq. (2.8). Parity conservation in this model is evidently natural, depending only on left-right symmetry and on the representations of the Higgs mesons, which force the neutral-vector-boson (mass)^2 matrix to have the form given in Eq. (5.3).

The discussion of manifest vs nonmanifest symmetry breaking in Secs. II and III applies equally to the χ_L, χ_R and the ρ, ρ' models since neither set of Higgs fields contributes to the fermion mass matrix. In particular, additional Higgs mesons δ_L and δ_R will be needed in the manifest case to make $W_R^\pm$ heavy, but these do not affect the neutral-current sector of the model.

VI. CONCLUSIONS

In the context of $SU(2)_L \times SU(2)_R \times U(1)$ gauge theories several conditions are necessary for the neutral-current weak interactions to conserve parity naturally:

(i) There must be a spontaneously broken left-right symmetry to ensure that the coupling constants for $SU(2)_L$ and $SU(2)_R$ are naturally equal. Depending on the choice of this symmetry, the generalized Cabibbo rotations for the left-handed and the right-handed fermions may be either related by complex conjugation or independent.

(ii) Each fermion must appear in a left-handed and a right-handed multiplet and have $T_{3L} = T_{3R}$.

(iii) The Higgs system must be chosen so that the neutral vector bosons have definite parity. The ρ, ρ' Higgs system described in Sec. V achieves this naturally. The χ_L, χ_R Higgs system originally proposed in Ref. 1 does not, at least for the definitions of left-right symmetry which we have considered.

If parity violation is naturally absent in lowest order, then it must be finite and calculable in higher order. We expect the parity-violating effective interaction to be of order αG_F in models containing δ_L and δ_R and having $W_R^\pm$ much heavier than $W_L^\pm$. In models without δ_L and δ_R, parity would be conserved in the limit of equal quark masses, so we expect the parity-violating effective interaction to be of order $G_F{}^2\Delta m_q{}^2$, where $\Delta m_q{}^2$ is a difference of quark masses. The latter possibility is analogous to the Glashow-Iliopoulos-Maiani[11] mechanism for the suppression of strangeness-changing neutral currents.

The absence of parity violation in heavy atoms can also be accommodated in $SU(2) \times U(1)$ gauge theories.[12] One must put the electron in a right-handed doublet so that its coupling to Z_μ is pure vector. Then the parity-violating effective interaction involves the axial nucleon current; hence it is small for heavy atoms but is comparable to that in the Weinberg-Salam model for hydrogen. An experiment now in progress to measure the parity violation in hydrogen should therefore distinguish the two models.

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APPENDIX

We have seen that in $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models with $\chi_L(\tfrac{1}{2}, 0, 1)$ and $\chi_R(0, \tfrac{1}{2}, 1)$ Higgs fields,

$\langle\chi_L\rangle$ and $\langle\chi_R\rangle$ are not naturally equal. In such models the mixing between the vector and axial neutral bosons is renormalized and hence the parity violation in atoms is not calculable. However, we can parametrize this parity violation by its value at the tree level as a function of $\langle\chi_L\rangle$ and $\langle\chi_R\rangle$.[13] Then such models will remain acceptable provided that the difference between $\langle\chi_L\rangle$ and $\langle\chi_R\rangle$ does not have to be unreasonably small.

For definiteness we consider the four-quark and the six-quark models of Ref. 1. We take

$$\langle\chi_L\rangle=\begin{pmatrix}0\\ \lambda_L\end{pmatrix},\quad \langle\chi_R\rangle=\begin{pmatrix}0\\ \lambda_R\end{pmatrix},$$

$$\langle\Phi\rangle=\begin{pmatrix}\mathcal{K}e^{i\alpha} & 0\\ 0 & \mathcal{K}'e^{i\alpha'}\end{pmatrix} \tag{A1}$$

and define

$$\epsilon=\frac{2\mathcal{K}^2+2\mathcal{K}'^2}{\lambda_L{}^2+\lambda_R{}^2+2\mathcal{K}^2+2\mathcal{K}'^2}\,. \tag{A2}$$

For $\lambda_L=\lambda_R$ the eigenstates are

$$A_\mu=\frac{\sin\theta}{\sqrt{2}}(W^3_{L\mu}+W^3_{R\mu})+\cos\theta B_\mu\,,$$
$$Z_V=\frac{\cos\theta}{\sqrt{2}}(W^3_{L\mu}+W^3_{R\mu})-\sin\theta B_\mu\,, \tag{A3}$$
$$Z_A=\frac{1}{\sqrt{2}}(W^3_{L\mu}-W^3_{R\mu})\,,$$

where Z_V and Z_A have pure vector and axial-vector couplings. For $\lambda_L\neq\lambda_R$, Z_V and Z_A are mixed, the new eigenstates being

$$Z_1=(\cos\zeta)Z_A-(\sin\zeta)Z_V\,,$$
$$Z_2=(\sin\zeta)Z_A+(\cos\zeta)Z_V\,, \tag{A4}$$

where

$$\tan\zeta=\frac{2\rho}{1+(1+4\rho^2)^{1/2}}\,,$$
$$\rho=\frac{(1-\epsilon)\cos\theta}{(1-\epsilon)\sin^2\theta-2\epsilon\cos^2\theta}\left(\frac{\lambda_L{}^2-\lambda_R{}^2}{\lambda_L{}^2+\lambda_R{}^2}\right). \tag{A5}$$

The corresponding masses are

$$m_{Z_1,Z_2}{}^2=\tfrac{1}{2}(\lambda_L{}^2+\lambda_R{}^2)\left[\left(\frac{1+\cos^2\theta}{\cos^2\theta}+\frac{2\epsilon}{1-\epsilon}\right)\pm\left(\tan\theta^2-\frac{2\epsilon}{1-\epsilon}\right)(1+4\rho^2)^{1/2}\right]. \tag{A6}$$

In a heavy atom the dominant parity-violating term comes from the axial electron current times the vector nucleon current. The sum of the Z_1 and the Z_2 graphs gives for this term

$$H_{\text{eff}}=\frac{e^2}{4\sin^2\theta\cos\theta}(\bar{e}\gamma^\mu\gamma_5 e)\left[\tfrac{1}{3}(4\cos^2\theta-1)\bar{u}\gamma_\mu u-\tfrac{1}{3}(2\cos^2\theta+1)\bar{d}\gamma_\mu d\right]\sin\zeta\cos\zeta\left(\frac{1}{m_{Z_2}{}^2}-\frac{1}{m_{Z_1}{}^2}\right). \tag{A7}$$

To calculate the weak charge Q_W we need the Fermi constant. For both of the models in Ref. 1,

$$\frac{G_F}{\sqrt{2}}=\frac{1}{2(1+\epsilon)(\lambda_L{}^2+\lambda_R{}^2)}\,. \tag{A8}$$

Then

$$Q_W=\sin\zeta\cos\zeta\,\frac{2(1-\epsilon^2)\cos\theta[(1-\epsilon)\sin^2\theta-2\epsilon\cos^2\theta](1+4\rho^2)^{1/2}}{(1-\epsilon^2)\cos^2\theta-\rho^2[(1-\epsilon)\sin^2\theta-2\epsilon\cos^2\theta]^2}[Z(1-2\cos^2\theta)+N]\,. \tag{A9}$$

For small $\lambda_L{}^2-\lambda_R{}^2$ this becomes

$$Q_W\approx 2(1-\epsilon)\left(\frac{\lambda_L{}^2-\lambda_R{}^2}{\lambda_L{}^2+\lambda_R{}^2}\right)[Z(1-2\cos^2\theta)+N]\,. \tag{A10}$$

Note that Q_W is positive (negative) for $\lambda_L{}^2>\lambda_R{}^2$ ($\lambda_L{}^2<\lambda_R{}^2$).

The present experimental limit on parity violation in bismuth corresponds to[6]

$$Q_W(\text{Bi})=40\pm 32\ \text{(Oxford)}$$
$$=-42\pm 16\ \text{(Washington)}\,. \tag{A11}$$

Then using the parameters of Ref. 1,

$$\left|\frac{\lambda_L{}^2-\lambda_R{}^2}{\lambda_L{}^2+\lambda_R{}^2}\right|<0.15\ \text{(four-quark model)}$$
$$<0.36\ \text{(six-quark model)}\,. \tag{A12}$$

While the difference between $\lambda_L{}^2$ and $\lambda_R{}^2$ has to be fairly small, it seems premature to rule out such models definitely.

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