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Neutrino oscillations in matter

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The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.

I. INTRODUCTION

There exists considerable interest in the possibility that one type of neutrino may transform into another type while propagating through the vacuum.[1] A number of experiments have been proposed to search for such oscillations.[2,3] In order for such *vacuum oscillations* to occur, it is necessary that at least one neutrino have a nonzero mass and that the neutrino masses be not all degenerate. In addition, there must be a nonconservation of the separate lepton numbers (like electron number and muon number) so that the different neutrino types as defined by the weak charged current are mixtures of the mass eigenstates.

In this paper we show that even if all neutrinos are massless it is possible to have oscillations occur when neutrinos pass through matter. This can happen as a result of coherent forward scattering provided that this scattering is partially off diagonal in neutrino type. The phenomenon is analogous to the regeneration of K_S from a K_L beam passing through matter, A simple model is given in Sec. II from which it is seen that the oscillation length in matter of normal density is of the order 10^9 cm or larger. One of the proposed experiments[2] to test the hypothesis of vacuum oscillations involves the detection of neutrinos 1000 km distant from their source at Fermilab. Since the neutrinos would pass through the earth, we show that this experiment could possibly also test the hypothesis that the neutral current changes neutrinos from one type to another.

If neutrinos have a mass and vacuum oscillations do occur, these oscillations may be modified when neutrinos pass through matter. For the case of electron-type neutrinos ν_e such a modification takes place even if the neutrino scattering is described by a standard theory. The effect of the medium in this case, discussed in Sec. III, arises from the coherent forward scattering of ν_e as a result of its charged-current interaction with electrons. It is shown that the modification is large only if the vacuum oscillation length is larger than 10^9 cm.

Our results are summarized in Sec. IV and their significance with respect to gauge models of current interest is discussed.

II. OSCILLATIONS FOR MASSLESS NEUTRINOS

We consider here a simple model involving two types of massless neutrinos for which the effective neutral-current Hamiltonian is

$$H_w = \frac{G}{\sqrt{2}} L_\lambda J_\lambda , \qquad (1)$$

$$L_\lambda = \cos^2\alpha[\bar{\nu}_a\gamma_\lambda(1+\gamma_5)\nu_a + \bar{\nu}_b\gamma_\lambda(1+\gamma_5)\nu_b] + \sin^2\alpha[\bar{\nu}_a\gamma_\lambda(1+\gamma_5)\nu_b + \bar{\nu}_b\gamma_\lambda(1+\gamma_5)\nu_a] , \qquad (2a)$$

$$J_\lambda = g_p \bar{p}\gamma_\lambda p + g_n \bar{n}\gamma_\lambda n + \bar{g}_e \bar{e}\gamma_\lambda e + \cdots , \qquad (2b)$$

where the ellipses represent terms in J_λ of no interest for our present considerations. The essential term in H_w is the off-diagonal term proportional to $\sin^2\alpha$, where ν_a and ν_b are neutrino types defined by the charged-current interaction. We have also assumed $\nu_a - \nu_b$ symmetry, which simplifies the discussion and maximizes the effect of interest.

The neutrino current may be rewritten

$$L_\lambda = \cos^2\alpha(\bar{\nu}_1\nu_1 + \bar{\nu}_2\nu_2) + \sin^2\alpha(\bar{\nu}_1\nu_1 - \bar{\nu}_2\nu_2) , \qquad (3)$$

$$|\nu_1\rangle = (|\nu_a\rangle + |\nu_b\rangle)/\sqrt{2} , \quad |\nu_2\rangle = (|\nu_a\rangle - |\nu_b\rangle)/\sqrt{2} , \qquad (4)$$

where we have omitted the Lorentz indices. A beam originally ν_a propagating through matter is described by

$$|\nu_a(x)\rangle = (|\nu_1\rangle e^{ikn_1x} + |\nu_2\rangle e^{ikn_2x})/\sqrt{2} , \qquad (5)$$

where the indices of refraction,

$$n_i = 1 + (2\pi N/k^2) f_i(0) , \qquad (6)$$

are different because of the $\sin^2\alpha$ term. Because the interaction is weak, for practical purposes we can consider $f_i(0)$ as real; even though the absorption is negligible, the real part of the index of refraction may yield interesting effects. The probability that ν_a emerges as ν_a (that is, not transformed to ν_b) is

$$|\langle\nu_a|\nu_a(x)\rangle|^2=\tfrac{1}{2}[1+\cos k(n_1-n_2)x]. \qquad (7)$$

A direct calculation gives the simple result

$$k(n_1-n_2)=2G\sin^2\alpha(g_pN_p+g_nN_n+g_eN_e), \qquad (8)$$

where N_p is the number of protons per unit volume, etc. Writing $N_i=\rho_iN_0$, where $N_0=6\times10^{23}$ cm^{-3},

$$k(n_1-n_2)\approx5\times10^{-9}\text{ cm}^{-1}[\sin^2\alpha(g_p\rho_p+g_n\rho_n+g_e\rho_e)]. \qquad (9)$$

The effective oscillation length l, defined by $|k(n_1-n_2)l|=2\pi$, has a minimum value of the order 10^9 cm, provided ρ_ig_i is of the order unity.

In contrast to the case of vacuum oscillations this length is independent of neutrino energy. This can be seen from the relation

$$|k(n_1-n_2)|\propto\left|\frac{f(0)}{k}\right|=\left|\frac{d\sigma/d(\cos\theta)(0)}{2\pi k^2}\right|^{1/2}=\left|\frac{d\sigma/dq^2(0)}{2\pi}\right|^{1/2}.$$

For neutrino elastic scattering it is well known that $d\sigma/dq^2$ is independent of energy for $q^2=0$.

As a specific example, consider $\nu_a=\nu_\mu$ and ν_b as some new neutrino such as the postulated[4] ν_τ. The extreme case of $\sin^2\alpha=1$ corresponds to the idea that in neutral-current scattering the outgoing neutrino is never the same as the incoming one. In our previous discussion[5] it seemed impossible to check this idea unless the hadronic neutral current had a special non-Hermitian character. Here we see that a proposed experiment[2] looking for ν_μ 1000 km distant from their origin could be considered a test of this idea. Assuming a density of 4 g/cm^3 ($\rho_n\approx\rho_p=\rho_e=2$) for the rock through which the neutrinos pass, we have from Eqs. (7) and (9), with $\sin^2\alpha=1$ and $x=10^8$ cm,

$$\begin{aligned}|\langle\nu_\mu|\nu_\mu(x)\rangle|^2&=\tfrac{1}{2}[1+\cos(g_p+g_n+g_e)],\\|\langle\nu_\tau|\nu_\mu(x)\rangle|^2&=\tfrac{1}{2}[1-\cos(g_p+g_n+g_e)].\end{aligned} \qquad (10)$$

If $(g_p+g_n+g_e)$ is less than unity we have approximately

$$|\langle\nu_\tau|\nu_\mu(x)\rangle|^2\approx\tfrac{1}{4}(g_p+g_n+g_e)^2. \qquad (11)$$

TABLE I. Transformation probabilities $P=|\langle\nu_b|\nu_\mu(x)\rangle|^2$ for ν_μ passing through 1000 km of rock for various choices of neutral currents with $\sin^2\alpha=1$. Within accuracy shown results hold for $\nu_b=\nu_\tau$ [Eq. (10)] or $\nu_b=\nu_e$ [Eqs. (18), (15), (16)]. A, B stand for solutions in Ref. 6 for g_p and g_n; WS stands for Weinberg-Salam values. For the same cases the transmission probability f for neutrinos from the sun is shown.

g_p, g_n	g_e	$g_p+g_n+g_e$	P	f
A	0.45	−0.6	0.09	0.98
A	−0.45	−1.5	0.46	0.61
B	0.45	0.8	0.15	0.60
B	−0.45	−0.1	0.002	0.97
WS	WS	−0.5	0.06	0.97

The quantities g_p, g_n, and g_e must be determined from neutral-current scattering experiments. In Table I we give results for the transformation probability given by Eq. (10) using values of g_p and g_n derived[6] from ν_μ scattering experiments and setting $|g_e|=0.45$, a value in the middle of the allowed range of values.[7] We also show results when $g_p+g_n+g_e$ has the value it has in the Weinberg-Salam theory, a value which is independent of $\sin^2\theta_W$. Transformation probabilities anywhere between 0 and 50% are seen to be possible. If such transformations were found, the model discussed here could be distinguished from the original idea of vacuum oscillations by a study of the dependence on neutrino energy as noted above.

We now discuss the modification required when one of the neutrinos (ν_a) is ν_e. In this case there is a contribution to the coherent scattering from the charged-current term[8]

$$\frac{G}{\sqrt2}\bar\nu_e\gamma_\lambda(1+\gamma_5)e\bar e\gamma_\lambda(1+\gamma_5)\nu_e=\frac{G}{\sqrt2}\bar\nu_e\gamma_\lambda(1+\gamma_5)\nu_e(\bar e\gamma_\lambda e+\cdots), \qquad (12)$$

where the second line is obtained via the Fierz transformation, and we have left out the axial-vector electron current which is of no interest here. The Hamiltonian contributing to coherent scattering can now be written

$$H=G/\sqrt2\{\tfrac{1}{2}[\bar\nu_e\gamma_\lambda(1+\gamma_5)\nu_e+\bar\nu_b\gamma_\lambda(1+\gamma_5)\nu_b]\bar e\gamma_\lambda e+\sin^2\alpha[\bar\nu_e\gamma_\lambda(1+\gamma_5)\nu_b+\bar\nu_b\gamma_\lambda(1+\gamma_5)\nu_e](g_p\bar p\gamma_\lambda p+g_n\bar n\gamma_\lambda n+g_e\bar e\gamma_\lambda e)+\cdots\}, \qquad (13)$$

where the ellipses include diagonal terms symmetric under the interchange of ν_e and ν_b which do not contribute to the difference $n_e - n_b$. The eigenstates for propagation through matter are no longer given by Eq. (3) but are

$$|\nu_1\rangle = \cos\theta\,|\nu_e\rangle + \sin\theta\,|\nu_b\rangle\,, \qquad (14)$$
$$|\nu_2\rangle = -\sin\theta\,|\nu_e\rangle + \cos\theta\,|\nu_b\rangle\,,$$

$$\tan 2\theta = 2\sin^2\alpha(g_p\rho_p + g_n\rho_n + g_e\rho_e)/\rho_e\,. \qquad (15)$$

Equation (9) which determines the oscillation length l is also modified,

$$2\pi/l = k(n_1 - n_2)$$
$$= 5\times 10^{-9}\ \mathrm{cm}^{-1}\rho_e\left(\frac{1+\tan^2 2\theta}{4}\right)^{1/2}. \qquad (16)$$

Thus for matter of normal density ($\rho_e = 1$) there is a maximum oscillation length of about 2.5×10^9 cm. In place of Eq. (7) the oscillation probability is determined by

$$|\langle\nu_e|\nu_e(x)\rangle|^2 = \cos^4\theta + \sin^4\theta + 2\cos^2\theta\sin^2\theta\cos[k(n_1 - n_2)x]\,. \qquad (17)$$

For the application discussed above involving ν_μ going through 1000 km of earth with now ν_e taking the role of ν_τ, the quantitative results are changed very little. Starting with an equation for ν_μ such as Eq. (17) we have for the transformation probability

$$|\langle\nu_e|\nu_\mu(x)\rangle|^2 = \tfrac{1}{2}\sin^2 2\theta\{1 - \cos[k(n_1 - n_2)x]\}\,. \qquad (18)$$

If the argument of the cosine is less than unity, we find, using Eqs. (15) and (16) and setting $\sin^2\alpha = 1$, $x = 10^8$ cm, $\rho_e = \rho_p = \rho_n = 2$,

$$|\langle\nu_e|\nu_\mu(x)\rangle|^2 \approx \tfrac{1}{4}(g_p + g_n + g_e)^2\,,$$

which is the same result as Eq. (11). The effects of the decrease in the oscillation length and the decrease in the amplitude of oscillation approximately cancel each other as long as the transformation probability is not too large. For the cases discussed before shown in Table I, the results from Eq. (18) agree with those of Eq. (10) within 2%. On the other hand, if we consider the minimum in the oscillation, where $[k(n_1 - n_2)x] = \pi$, we find from Eq. (17) that this minimum is $\cos^2 2\theta$ in contrast to Eq. (7) which has a minimum of zero. From Eq. (15), setting $\rho_p = \rho_n = \rho_e$ and $\sin^2\alpha = 1$, this minimum value is $[1 + 4(g_p + g_n + g_e)^2]^{-1}$.

A very interesting use of the vacuum oscillation theory is for the problem of the apparent deficiency of solar neutrinos in terrestrial experiments.[9] According to that theory, ν_e may oscillate into one or more other neutrino types during the trip from sun to earth. Here we consider the possibility that ν_e may transform into ν_b while passing from the interior of the sun to the surface as a result of the coherent off-diagonal scattering. Since the oscillation length is less than $2.5\rho_e^{-1}\times 10^9$ cm from Eq. (16), the oscillating term in Eq. (17) will average to zero as a result of averaging over the radial position of the neutrino source. Thus the fraction of ν_e emerging from the solar surfaces is

$$f = \cos^4\theta + \sin^4\theta = \tfrac{1}{2}[1 + (1 + \tan^2 2\theta)^{-1}]\,. \qquad (19a)$$

From Eq. (15)

$$\tan 2\theta = +2\sin^2\alpha(g_p + g_e + y\,g_n)\,, \qquad (19b)$$

where y is the ratio of neutrons to protons. The value of y changes from a value of about 0.41 at the solar center to about 0.14 at the solar surface as X, the hydrogen mass fraction, changes from 0.42 to 0.75. Using an appropriate average value of y and assuming the extreme case discussed above with $\sin^2\alpha = 1$, we have calculated the values of f shown in Table I for various possible neutral-current couplings. The most extreme case gives a reduction of about 40% in ν_e flux. This is in contrast to the vacuum oscillation theory which gives a reduction of 50% for a large range of parameters and an even larger reduction for a very special choice of parameter.[10] In the case of the vacuum oscillation theory a reduction well below 50% is also possible by the introduction of additional types of neutrinos[11]; in our model this does not give much further reduction since the minimum is determined by the ratio of the effective neutral off-diagonal current to the charged current. A further reduction could be obtained if the diagonal neutral current were not symmetric with respect to ν_e and ν_b, but this alternative requires particularly artificial assumptions.

So far we have assumed the neutrinos defined by the charged current are orthogonal to each other. However, in many models involving violation of lepton number there also occurs a nonorthogonality among the neutrinos associated with different charged leptons.[12] To be specific, we label as ν_e the neutrino associated with the electron and let

$$|\nu'_\mu\rangle = \cos\phi\,|\nu_b\rangle + \sin\phi\,|\nu_e\rangle$$

be the neutrino associated with the muon. Here ν_b and ν_e are the basic orthogonal neutrinos so that

$$\langle\nu_e|\nu'_\mu\rangle = \sin\phi\,.$$

Assuming[13] the Hamiltonian (13) we can express ν'_μ in terms of the eigenstates (14) for propagation in matter as

$$|\nu_\mu'\rangle = \cos(\theta+\phi)|\nu_2\rangle + \sin(\theta+\phi)|\nu_1\rangle .$$

The oscillations of ν_μ' are determined by

$$|\langle\nu_\mu'|\nu_\mu'(x)\rangle|^2 = \cos^4(\theta+\phi) + \sin^4(\theta+\phi) + 2\cos^2(\theta+\phi)\sin^2(\theta+\phi)\cos[k(n_1-n_2)x] . \quad (20)$$

It is interesting to note that oscillations occur even in the absence of any nondiagonal neutral current, that is, for the case $\sin^2\alpha = 0$ which gives, from Eq. (15), $\theta = 0$. The minimum in the transmission in this case is given by $(1-\sin^2 2\phi)$; since ϕ^2 is limited to about 0.01 by the limits on the nonorthogonality of ν_e and ν_μ', the oscillation amplitude must be very small. From Eq. (16) the minimum occurs at a distance of about 5×10^8 cm of terrestrial matter. Another way to look at this oscillation (the case with $\theta = 0$) is to consider it as resulting from nondiagonal charged-current scattering. To see this we choose $|\nu_\mu'\rangle$ as one of the basic vectors, in which case $|\nu_e\rangle$ must be written as

$$|\nu_e\rangle = \sin\phi|\nu_\mu'\rangle + \cos\phi|\nu_c\rangle ,$$

where $|\nu_c\rangle$ is the other basic vector. The charged-current coupling of ν_e to electrons now includes a term which transforms ν_μ' to ν_c, which is proportional to $\sin 2\phi$.

III. MODIFICATION OF VACUUM OSCILLATIONS IN MATTER

In this section we shall adopt the assumptions of the vacuum-oscillation theory and consider how these oscillations appear when neutrinos pass through matter. We further assume that the neutral current has the same general form as in the standard model; this means it is diagonal and symmetric with respect to neutrino types. Considering just two types of neutrinos, it is then given by Eqs. (1) and (2) with $\sin^2\alpha = 0$.

With these assumptions the only interesting case is that in which one of the neutrinos is ν_e. Considering the case of ν_e and ν_μ, vacuum oscillations require that the eigenstates in vacuum are mixtures

$$\begin{aligned} |\nu_1\rangle &= |\nu_e\rangle\cos\theta_v - |\nu_\mu\rangle\sin\theta_v ,\\ |\nu_2\rangle &= |\nu_e\rangle\sin\theta_v + |\nu_\mu\rangle\cos\theta_v , \end{aligned} \quad (21)$$

with distinct masses m_1 and m_2 ($m_1 > m_2$). Neutrino oscillations result from the difference in the phase factors governing the time dependence of ν_1 and ν_2,

$$|\nu_i t\rangle \sim \exp(-itm_i^2/2k) .$$

The characteristic oscillation length in the vacuum is $l_v(k) = 4\pi k/(m_1^2 - m_2^2)$. For propagation through matter we must also consider the phase factors arising from coherent scattering. Since the neutral-current scattering is diagonal and symmetric, it just causes an overall phase shift of no physical importance. However, we must include the charged-current scattering which singles out ν_e. As a result we have in the ν_1-ν_2 representation, omitting the neutral current,

$$i\frac{d}{dt}\begin{bmatrix}\nu_1\\ \nu_2\end{bmatrix} = \begin{bmatrix}\frac{m_1^2}{2k} - GN_e\cos^2\theta_v & -GN_e\sin\theta_v\cos\theta_v \\ -GN_e\sin\theta_v\cos\theta_v & \frac{m_2^2}{2k} - GN_e\sin^2\theta_v\end{bmatrix} \times \begin{bmatrix}\nu_1\\ \nu_2\end{bmatrix}, \quad (22)$$

where $GN_e = k(n_e - 1)$ and n_e is the index of refraction associated with the charged-current scattering. The eigenstates for propagation in matter are

$$|\nu_{1m}\rangle = |\nu_e\rangle\cos\theta_m - |\nu_\mu\rangle\sin\theta_m ,$$

$$|\nu_{2m}\rangle = |\nu_e\rangle\sin\theta_m + |\nu_\mu\rangle\cos\theta_m ,$$

$$\tan 2\theta_m = \tan 2\theta_v\left(1 - \frac{l_v}{l_0}\sec 2\theta_v\right)^{-1} , \quad (23a)$$

$$l_0 \equiv 2\pi/GN_e = 2.5\times 10^9 \text{ cm}/\rho_e . \quad (23b)$$

The oscillation length in matter is

$$l_m(k) = l_v(k)\left[1 + \left(\frac{l_v(k)}{l_0}\right)^2 - 2\cos 2\theta_v\left(\frac{l_v(k)}{l_0}\right)\right]^{-1/2} , \quad (24a)$$

and the transformation probability is given by

$$|\langle\nu_e|\nu_\mu(x)\rangle|^2 = \tfrac{1}{2}\sin^2(2\theta_v)(l_m/l_v)^2 \times [1 - \cos(2\pi x/l_m)] . \quad (24b)$$

As long as $l_v \ll l_0$, it is seen from Eqs. (23) and (24) that $l_m \approx l_v$, $\theta_m \approx \theta_v$, and therefore the oscillations in the medium will be very much the same as in the vacuum. For $l_v \gg l_0$ it is seen that $l_m \approx l_0$ independent of θ_v and therefore from Eq. (24b) the amplitude of the oscillation is very small. Some examples of the effect of the medium for the intermediate case $l_v = l_0$ are illustrated in Table II. Independent of the value of l_v/l_0, it follows from Eq. (24b) that as long as $(2\pi x/l_m) < 1$, the oscilla-

TABLE II. Transformation probabilities $|\langle\nu_e|\nu_\mu(x)\rangle|^2$ for a vacuum oscillation length $l_v = l_0 = (2.5\times10^9\ \mathrm{cm})\rho_e^{-1}$ for three values of the vacuum mixing angle θ_v. Results are shown for vacuum oscillations (vac) and oscillations for neutrinos passing through matter (mat) with an electron number density of $6\times10^{23}\rho_e\ \mathrm{cm}^{-3}$.

	$\theta_v=45°$		$\theta_v=60°$		$\theta_v=15°$	
x/l_0	vac	mat	vac	mat	vac	mat
0.1	0.095	0.093	0.072	0.067	0.024	0.025
0.2	0.345	0.301	0.259	0.196	0.086	0.095
0.3	0.655	0.472	0.491	0.249	0.164	0.205
0.4	0.905	0.479	0.679	0.169	0.226	0.342
0.5	1.000	0.317	0.750	0.041	0.250	0.492

tion probability in the medium is approximately the same as in vacuum.

IV. DISCUSSION AND SUMMARY

This paper has demonstrated the possible importance of coherent forward scattering when neutrinos pass through matter. It follows from our discussion that for matter of normal density the coherent scattering produces a phase change of the order of π after neutrinos have traversed a distance of the order of 10^9 cm of normal matter. This distance, which is independent of energy, is much less than the mean free path, which is of the order 10^{14} cm for a 1-GeV neutrino and varies inversely as the energy. The reason for this large difference, of course, is that the coherent scattering effect depends on the scattering amplitude proportional to G whereas the mean free path depends on the cross section proportional to G^2.

In Sec. II we considered a test for the extreme hypothesis that the neutral current always changed ν_μ to another type of neutrino. In this case a significant fraction of ν_μ may be transformed to the other type of neutrino after passing through 1000 km or more of terrestrial rock. The quantitative results depend on the detailed form of the neutral current and in general the fraction is somewhat smaller when the other neutrino is ν_e. When this extreme hypothesis is applied to the passage of neutrinos from the center of the sun, it is found that up to 40% of these ν_e may have transformed to another type of neutrino on arrival at the surface of the sun. At best this extreme hypothesis would provide only a partial answer to the deficiency of solar neutrinos.

Nondiagonal neutral currents of the form assumed in Eq. (2) occur in many gauge models that extend the standard Weinberg-Salam model to heavy leptons and right-handed currents. Indeed, it has been shown on general grounds[14] that only a very special class of gauge theories can avoid having flavor-changing neutral currents; the nondiagonal neutral current in Eq. (2) is an example of a current which changes "lepton flavor." However, in any realistic model we can think of, the value of the parameter $\sin^2\alpha$ in Eq. (2) would be much less than unity (of the order 0.1 or less). This follows from the empirical constraints of hadron-lepton universality and limits on lepton-number-nonconserving processes. The lower value of $\sin^2\alpha$ has two important phenomenological consequences relative to the extreme hypothesis discussed above:

(1) The characteristic oscillation length in normal matter (when ν_e is not involved) determined from Eq. (9) now has a minimum value of 10^{10} cm. Thus, the effect of these oscillations on an experiment using a path length of 1000 km of terrestrial rock will be negligible.

(2) The amplitude of oscillations when ν_e is involved as determined from Eqs. (15) and (17) will be small. Thus the effect of the oscillations on terrestrial experiments with a path length of 1000 km or on solar neutrinos will be negligible.

An example of a gauge model which has such a term is the model with right-handed charged leptons coupled to heavy neutral leptons.[15] In this model the value of $\sin^2\alpha$ for ν_i coupling to ν_j is $m_i m_j/m^2$ where m_i is the mass of the charged lepton associated with ν_i and m is some mean mass of the neutrals. Thus for ν_μ coupled to ν_τ, $\sin^2\alpha$ would be of the order $m_\mu m_\tau/m^2$, which is less than or of the order of m_μ/m_τ. In this model the different diagonal terms have different coefficients; thus even for the case of ν_μ and ν_τ the mixing is not complete and the eigenstates in matter are given by Eq. (12) with a small value of θ instead of by Eq. (3). Thus, in this model, as in other realistic gauge models, the phenomenological consequences of the nondiagonal coherent scattering appear to be very hard to detect.

In Sec. III we assumed that vacuum oscillations occurred. This means that the eigenstates of the neutrino mass matrix are mixtures of different neutrino types as defined by the charged current and that the eigenvalues are nondegenerate. On the other hand, we assumed that the neutral current was diagonal or that the nondiagonal pieces were negligible. With these assumptions the oscillation phenomenon in matter differs from that in vacuum when ν_e is involved because the charged-current ν_e-electron scattering gives ν_e a different index of refraction from other neutrinos. The qualitative conclusions are:

(1) For characteristic vacuum oscillation lengths much smaller than 10^9 cm, oscillations in normal matter are essentially the same as in vacuum.

(2) For characteristic vacuum oscillation lengths

much larger than 10^9 cm. oscillations involving ν_e in normal matter have a much smaller amplitude than in vacuum.

(3) For characteristic vacuum oscillation lengths of the order of 10^9 cm. the quantitative results in matter are quite different from in vacuum as illustrated in Table II.

In general, if one is considering the possibility of large vacuum oscillation lengths, as in the discussion of solar neutrinos, the oscillations should be calculated for the actual vacuum path[16] ignoring the passage through matter. Thus, in the detailed solar neutrino calculations[10] the effective distance over which neutrino oscillations take place is from the solar surface to the earth's surface; there are no significant oscillations inside the sun or in traversals through the earth.

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