Three-body nonleptonic and four-body semileptonic $\Delta C = \Delta S$ decays of charmed hadrons

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Tests of the isospin and SU(3) transformation properties of the $\Delta C = \Delta S$ current and nonleptonic interaction are given for the three-body nonleptonic and four-body semileptonic decays of a charmed hadron belonging to the $\underline{3}^*$ representation of SU(3). The relations obtained can also be used for quasi-three-body nonleptonic and quasi-four-body semileptonic decays thus covering to some extent higher multibody decays of both kinds.

I. INTRODUCTION

The recent discovery[1] of a neutral and a charged boson of mass around 1.87 GeV seems to indicate that two of the lowest charmed mesons D^0 and D^+ with charm $C=1$ belonging to the $\underline{3}^*$ representation of SU(3) have been found. In the charm model[2] an SU(4) symmetry group,[3] containing the usual SU(3) symmetry group, underlies the strong interactions of the hadrons while their weak decays are given by the scheme proposed by Glashow, Iliopoulos, and Maiani (GIM).[4] For the purpose of this note we will assume that the observed particles decay via the $\Delta C = \Delta S$ nonleptonic interaction of the GIM scheme and that (D^+, D^0, F^+) form the $C=1$, pseudoscalar $\underline{3}^*$ representation of SU(3) and denote it by $P(3^*)$.

Many authors[5] have already considered the consequences of the GIM scheme for the two- and three-body nonleptonic and the simple semileptonic decays of the low-lying charmed hadrons. Recently we[6] have given relations for the inclusive semileptonic and nonleptonic decays of a charmed hadron belonging to $\underline{3}^*$, which would provide simple tests of the isospin and SU(3) structure of the $\Delta C = \Delta S$ GIM interaction. However, the exclusive $\Delta C = \Delta S$ nonleptonic and semileptonic decay modes may provide further tests which are more likely to be verified in the near future. Consequently we consider the following $\Delta C = \Delta S$ decays:

$$P(3^*) \to P(8) + P(8) + P(8), \qquad (1)$$

$$P(3^*) \to P(9) + V(9) + l^+ + \nu_l\,, \qquad (2a)$$

$$\to P(9) + P(9) + l^+ + \nu_l\,, \qquad (2b)$$

where $P(8)$ and $V(9)$ are the usual pseudoscalar octet and vector nonet, respectively, and $l^+ = e^+$ or μ^+. Our analysis is at the SU(3) level so that our results will also apply for the decays of other charmed hadrons $\underline{3}^*$. The process (1) has been considered by Einhorn and Quigg,[5] but under the rather special assumption that the three $P(8)$'s are in a totally symmetric state, and they further assume sextet dominance[7] for the $\Delta C = \Delta S$ nonleptonic Hamiltonian, H_{CS}, in the GIM scheme. We give separately relations for the full H_{CS} as well as with sextet dominance but make no restriction on the nature of the final state. As a result our relations for sextet dominance are more general. The processes in (2) have not been considered by anyone, in detail, so far.

In Sec. II the I-spin, V-spin, and U-spin and SU(3) transformation properties[6] of H_{CS} in the GIM scheme are exploited to obtain relations between the nonleptonic amplitudes in (1) which will provide tests of its isospin and SU(3) transformation properties. In addition we give the modification of our results so as to apply to the decays of the $C=1$ baryon $B(3^*)$, in particular to $B(3^*) \to B(8)P(8)V(9)$ as well as $P(3^*) \to P(8)P(8)V(9)$. The SU(3) relations for the semileptonic decays in (2) and their specialization to the decays $B(3^*) \to B(8) + P(9) + l^+ + \nu_l$ are given in Sec. III. Finally Sec. IV is devoted to discussion.

II. THREE-BODY NONLEPTONIC DECAYS

For the process (1) there are 20 decay amplitudes $M_1, \ldots, M_{20}$ which satisfy $\Delta C = \Delta S$ and which are defined in Table I. Owing to the identity of the particles in the final state one has to be careful in doing the isospin, etc. analysis. We adopt the convention that the particles in the final state have

TABLE I. The three-body nonleptonic $\Delta C = \Delta S$ decay amplitudes for $P(3^*) \to P(8) + P(8) + P(8)$.

$M_1 = A(D^+ \to K^-\pi^+\pi^+)$	$M_{11} = A(D^0 \to \bar{K}^0\bar{K}^0K^0)$
$M_2 = A(D^+ \to \bar{K}^0\pi^+\pi^0)$	$M_{12} = A(D^0 \to \bar{K}^0K^-K^+)$
$M_3 = A(D^+ \to \bar{K}^0\pi^+\eta)$	$M_{13} = A(F^+ \to \pi^-\pi^+\pi^+)$
$M_4 = A(D^+ \to K^+\bar{K}^0\bar{K}^0)$	$M_{14} = A(F^+ \to \pi^+\pi^0\pi^0)$
$M_5 = A(D^0 \to K^-\pi^+\pi^0)$	$M_{15} = A(F^+ \to \pi^+\pi^0\eta)$
$M_6 = A(D^0 \to \bar{K}^0\pi^0\pi^0)$	$M_{16} = A(F^+ \to \pi^+\eta\eta)$
$M_7 = A(D^0 \to \bar{K}^0\pi^+\pi^-)$	$M_{17} = A(F^+ \to \pi^+\bar{K}^0K^0)$
$M_8 = A(D^0 \to K^-\pi^+\eta)$	$M_{18} = A(F^+ \to \pi^+K^-K^+)$
$M_9 = A(D^0 \to \bar{K}^0\pi^0\eta)$	$M_{19} = A(F^+ \to \bar{K}^0K^+\pi^0)$
$M_{10} = A(D^0 \to \bar{K}^0\eta\eta)$	$M_{20} = A(F^+ \to \bar{K}^0K^+\eta)$

momenta p, q, k, in order of their appearance in the amplitude. The simple M_i are reserved for the ordering given in Table I; thus, e.g., $M_2 \equiv A(D^+ \to \bar{K}^0(p)\pi^+(q)\pi^0(k))$ is different from $A(D^+ \to \bar{K}^0(p)\pi^0(q)\pi^+(k))$. The latter will be denoted by simply $M_2(\bar{K}^0\pi^0\pi^+)$ according to our convention. Note that for the particular assumption of a purely symmetric final state in the three $P(8)$'s, as in Einhorn and Quigg,[5] the amplitudes will be symmetric in the momenta also so that $M_2 = M_2(\bar{K}^0\pi^0\pi^+)$, etc. and the relations obtained would be less general.

The relation for the full H_{CS}, since it satisfies a pure $|\Delta\vec{I}|=1$ and $|\Delta\vec{U}|=1$ rule, can be obtained from I-spin and U-spin analysis. We give the relations under the subheads of type $I=I_c \to I_1+I_2+I_3$ or $U=U_c \to U_1+U_2+U_3$, where I_c (U_c) is the isospin (U spin) of the decaying charmed particle and I_i (U_i), $i=1,2,3$ the isospin (U spin) of the particles in the final state. We further separate out relations between amplitudes or sum rules from other relations called the symmetry relations. The latter follow from the identity of the particles as in process (1) while the sum rules will be valid for any decay of the type in the subhead. The symmetry relations in general would become relations between different decay amplitudes in the case of nonidentical particles.

(a) Isospin relations. We give all the isospin relations as these are fairly simple. Further, the symmetry relations involve the interchange of particles in the same isospin multiplet (i.e., particles of nearly the same mass), and they are statements about the connection between different parts of the Dalitz plot of the various decays and would be verifiable. These are summarized in Table II. The amplitude relations in Table II would lead to obvious inequalities among $|M_i|^2$, the transition rates at a given distribution of momenta, which are related to the density of points in the corresponding region of the Dalitz plot. However, for the F^+ decays into $K\bar{K}\pi$, in addition to such inequalities, one has directly the result

$$|M_{17}|^2+|M_{18}|^2 \geq |M_{19}|^2 \,. \tag{3}$$

(b) U-spin relations. As pointed out earlier, these relations are also valid for the full H_{CS}; however, of the many relations we only give in Table II the simpler ones, i.e., those involving at most four or five decays. The symmetry relations in this case, unlike isospin, will not in general provide useful tests since the particles in a given U-spin multiplet have large mass differences.

(c) V-spin relations. For the full H_{CS} the relations obtained using V-spin analysis are not simple. Members of V-spin multiplets (like U spin) have large mass differences, so that the inequal-

TABLE II. I-spin and U-spin relations valid for the full H_{CS}. The superscripts (a) and (b) denote, respectively, the amplitude and symmetry relations.

$I_c=\frac{1}{2} \to \frac{1}{2}+1+1$:

$$\sqrt{2}\,(M_6-M_7)=M_2-M_5\,, \tag{II.1}^{(a)}$$

$$\begin{aligned} M_1 &= -\sqrt{2}\,[M_2+M_2(\bar{K}^0\pi^0\pi^+)] \\ &= \sqrt{2}\,[M_5+M_5(K^-\pi^0\pi^+)] \\ &= -2M_6+M_7+M_7(\bar{K}^0\pi^-\pi^+)\,, \end{aligned} \tag{II.2}^{(b)}$$

$$-\sqrt{2}M_2(\bar{K}^0\pi^0\pi^+)=\sqrt{2}M_5-M_7+M_7(\bar{K}^0\pi^-\pi^+)\,. \tag{II.3}^{(b)}$$

$I_c=\frac{1}{2} \to \frac{1}{2}+1+0$:

$$M_3=M_8+\sqrt{2}M_9\,. \tag{II.4}^{(a)}$$

$I_c=\frac{1}{2} \to \frac{1}{2}+\frac{1}{2}+\frac{1}{2}$:

$$M_{12}(\bar{K}^0K^-K^+)+M_{12}(K^-\bar{K}^0K^+)=M_{11}(\bar{K}^0\bar{K}^0K^0)+M_4(\bar{K}^0\bar{K}^0K^+)\,. \tag{II.5}^{(b)}$$

$I_c=0 \to 1+\frac{1}{2}+\frac{1}{2}$:

$$M_{17}-M_{18}=\sqrt{2}M_{19}(\pi^0\bar{K}^0K^+)\,. \tag{II.6}^{(a)}$$

$U_c=0 \to 1+\frac{1}{2}+\frac{1}{2}$:

$$\sqrt{2}M_7=M_5(\pi^0\pi^+K^-)-\sqrt{3}M_8(\eta\pi^+K^-)+\sqrt{2}M_{12}(\bar{K}^0K^+K^-)\,. \tag{II.7}^{(a)}$$

$U_c=\frac{1}{2} \to \frac{1}{2}+\frac{1}{2}+\frac{1}{2}$:

$$M_1+M_{13}=M_{18}(K^-\pi^+K^+)+M_{18}(K^-K^+\pi^+)\,. \tag{II.8}^{(a)}$$

TABLE III. V-spin relations valid under sextet dominance. The superscripts (a) and (b) have the same meaning as in Table I.

$V_c=0 \to 1+\frac{1}{2}+\frac{1}{2}$:

$$\begin{aligned} -\sqrt{2}M_1(K^-\pi^+\pi^+) &= \sqrt{2}M_4(K^+\bar{K}^0\bar{K}^0) \\ &= M_2(\pi^0\bar{K}^0\pi^+)+\sqrt{3}M_3(\eta\bar{K}^0\pi^+)\,. \end{aligned} \tag{III.1}^{(a)}$$

$$\sqrt{3}M_3(\bar{K}^0\pi^+\eta)-\sqrt{3}M_3(\pi^+\bar{K}^0\eta)=M_2(\pi^+\bar{K}^0\pi^0)-M_2(\bar{K}^0\pi^+\pi^0)\,. \tag{III.2}^{(a)}$$

$V_c=\frac{1}{2} \to \frac{1}{2}+\frac{1}{2}+\frac{1}{2}$:

$$M_{11}(\bar{K}^0\bar{K}^0K^0)=-M_{13}(\pi^+\pi^+\pi^-)\,, \tag{III.3}^{(a)}$$

$$M_7(\bar{K}^0\pi^+\pi^-)=-M_{17}(\pi^+\bar{K}^0K^0)\,, \tag{III.4}^{(a)}$$

$$-M_{13}(\pi^+\pi^+\pi^-)=M_7(\bar{K}^0\pi^+\pi^-)+M_7(\pi^+\bar{K}^0\pi^-)\,. \tag{III.5}^{(b)}$$

$V_c=\frac{1}{2} \to \frac{1}{2}+1+1$:

$$M_5(\pi^+K^-\pi^0)+\sqrt{3}M_8(\pi^+K^-\eta)=M_{19}+\sqrt{3}M_{20}\,, \tag{III.6}^{(a)}$$

$$M_{12}(\bar{K}^0K^-K^+)=-M_{18}(\pi^+K^+K^-)\,, \tag{III.7}^{(a)}$$

$$M_{19}+\sqrt{3}M_{20}=\sqrt{2}M_{18}(\pi^+K^-K^+)-\sqrt{2}M_{18}(\pi^+K^+K^-)\,. \tag{III.8}^{(b)}$$

$V_c=\frac{1}{2} \to \frac{1}{2}+1+0$:

$$\sqrt{3}M_5(\pi^+K^-\pi^0)-M_8(\pi^+K^-\eta)=-\sqrt{3}M_{19}+M_{20}\,. \tag{III.9}^{(a)}$$

ities for $|M_i|$ obtained from amplitude relations, though true in the exact-SU(3) limit, are bound to be rather approximate. We thus do not present any relation for the full H_{CS}; however, simple relations are obtained for sextet dominance[7] in which case H_{CS} becomes a V-spin singlet, and these are listed in Table III.

It should be emphasized that the sextet-dominance relations in Table III are more general than those derived by Einhorn and Quigg,[5] who made the restrictive assumption that the 3 $P(8)$'s are in a completely symmetric state. For example they would satisfy (III.6) to (III.9) with $M_5=-\sqrt{3}\,M_8=-M_{19}=\sqrt{3}\,M_{20}$ and $M_{18}(\pi^+K^-K^+)=-M_{18}(\pi^+K^+K^-)$. Furthermore an important advantage of the present general analysis is that the relations obtained can be easily translated to the cases where the final state does not involve identical nonets. This is illustrated below in terms of the $B(3^*)$ decays.

(d) $B(3^*)\to B(8)P(8)V(9)$. We consider the extension of our results to these three-body decays since the relations obtained for them may be useful for the four-body nonleptonic decays because the vector mesons in $V(9)$ will decay into two pseudoscalars. Of course this will also cover the cases of $B(3^*)\to B(8)P(8)P(8)$ as well as $P(3^*)\to P(8)P(8)V(9)$. The number of different decays for $B(3^*)\to B(8)P(8)V(9)$ is over 100 while for the case $P(3^*)\to P(8)P(8)V(9)$ there are 62 decays, clearly a tedious task even to tabulate them. However, instead of working out all the relations in detail one can obtain the simple relations for these decays from the relations given above for process (1). This is possible as the relations have been given under subheads such as $I_c=I\to I_1+I_2+I_3$, etc. so that they can be translated for the case of other particles having the same isospins, etc. We illustrate this by an example for the $B(3^*)\to B(8)P(8)V(9)$ decays. Let us denote the members of $B(3^*)$ by (A^+,A^0,C_0^+) in the notation of Gaillard, Lee, and Rosner.[3] Consider the U-spin relation (II.8), between D^+ and F^+ decays in Table II. We choose this example as it involves only charged particles in the final state and would be easier to check experimentally. Now, corresponding to the single decay amplitude M_1, A^+ will have three decays, namely $\Xi^-\pi^+\rho^+$, $\Sigma^+K^-\rho^+$, and $\Sigma^+\pi^+K^{*-}$. Similarly, corresponding to the amplitudes M_{13} and M_{18}, C_0^+ will have 3 and 6 different decays, respectively. Of these 12 decays three sets of four amplitudes correspond to the transition $U_c=\frac{1}{2}\to\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$. Thus, corresponding to (II.8) for identical particles, one immediately has the three sum rules

$$A(A^+\to\Xi^-\pi^+\rho^+)+A(C_0^+\to\Sigma^-\pi^+\rho^+)=A(C_0^+\to\Xi^-\pi^+K^{*+})+A(C_0^+\to\Xi^-K^+\rho^+),\quad(4a)$$

$$A(A^+\to\Sigma^+K^-\rho^+)+A(C_0^+\to\Sigma^+\pi^-\rho^+)=A(C_0^+\to\Sigma^+K^-K^{*+})+A(C_0^+\to pK^-\rho^+),\quad(4b)$$

$$A(A^+\to\Sigma^+\pi^+K^{*-})+A(C_0^+\to\Sigma^+\pi^+\rho^-)=A(C_0^+\to\Sigma^+K^+K^{*-})+A(C_0^+\to p\pi^+K^{*-}).\quad(4c)$$

In a similar fashion one can convert the other relations given above for process (1) for $B(3^*)$ and other $P(3^*)$ decays.

III. FOUR-BODY $\Delta C=\Delta S$ SEMILEPTONIC DECAYS

We now consider the decays given in process (2a) which are analogous to the K_{l4} decays. To obtain SU(3) relations we consider $J_3^4+P(3^*)\to P(9)+V(9)$, where the $\Delta C=\Delta S$ current J_3^4 is treated as a spurion and transforms as an SU(3) triplet. The 27 decay amplitudes are given in Table IV in terms of the three SU(3) parameters $g_{S,A}=(\underline{8}\to\underline{8}_{S,A}(PV))$ and $g_1=(\underline{1}\to\underline{1}(PV))$, where $\underline{8}_S(PV)$, etc. mean the symmetric octet, etc. made out of P and V. Of the 24 SU(3) relations expected 14 are isospin relations which follow from the isosinglet property of the current, and these are shown in Table IV. There are four SU(3) relations among the D^+ decays alone and four SU(3) relations among the F^+ decays. Of these the two simplest involving four amplitudes are (suppressing the lepton pair)

$$2A(D^+\to\pi^+K^{*-})-A(D^+\to K^-\rho^+)=-\sqrt{6}\,(\cos\theta_V A(D^+\to\overline{K}{}^0\phi)+\sin\theta_V A(D^+\to\overline{K}{}^0\omega),\quad(5a)$$

$$\tan(\theta_V-\theta_P)[A(F^+\to\eta\phi)+A(F^+\to\eta'\omega)]=A(F^+\to\eta\omega)-A(F^+\to\eta'\phi).\quad(5b)$$

The remaining two SU(3) sum rules between the D^+ and F^+ decays are particularly simple, viz.

$$A(F^+\to\pi^+\rho^-)=A(D^+\to\pi^+K^{*-})-A(F^+\to K^+K^{*-})=A(D^+\to K^-\rho^+)-A(F^+\to K^-K^{*+}),\quad(6)$$

and may be verifiable experimentally. In addition to amplitude relations Table IV also yields directly relations between rates, for example the first four D^+ decays satisfy the equality

$$|A(D^+\to\overline{K}{}^0\phi)|^2+|A(D^+\to\overline{K}{}^0\omega)|^2=|A(D^+\to\pi^+K^{*-})|^2+\tfrac{1}{2}|A(D^+\to K^-\rho^+)|^2.\quad(7)$$

It is interesting to note that for the ideally mixed vector nonet ($\sin\theta_V=1/\sqrt{3}$) one would expect the F^+ decays involving $\pi\rho$, $\eta\omega$, and $\eta'\omega$ to be suppressed or small by the Okubo-Zweig-Iizuka (OZI) rule as they cannot occur through a single quark transition. This suppression would imply $\sqrt{2}g_S-3g_1\approx 0$ for arbitrary θ_P and convert (5b) and (6) into effectively two amplitude relations which could give a measure of this suppression.

TABLE IV. The four-body semileptonic decay amplitudes for $P(3^*) \to P(9)+V(9)+l^+ + \nu_l$. For compactness we have suppressed the lepton pair $l^+\nu_l$ in the amplitudes in the table and in the text. θ_V and θ_P are the mixing angles for the vector and the pseudoscalar nonets, respectively.

Amplitude	g_S	g_A	g_1	I-spin-related amplitudes
$A(D^+ \to \pi^+ K^{-*})$	$1/\sqrt{2}$	$1/\sqrt{2}$	0	$-\sqrt{2}A(D^+ \to \pi^0 \bar{K}^{*0})$ $=A(D^0 \to \pi^- \bar{K}^{*0})$ $=\sqrt{2}A(D^0 \to \pi^0 K^{*-})$
$A(D^+ \to K^- \rho^+)$	$1/\sqrt{2}$	$-1/\sqrt{2}$	0	$-\sqrt{2}A(D^+ \to \bar{K}^0 \rho^0)$ $=A(D^0 \to \bar{K}^0 \rho^-)$ $=\sqrt{2}A(D^0 \to K^- \rho^0)$
$A(D^+ \to \bar{K}^0 \phi)$	$\frac{-\cos\theta_V}{2\sqrt{3}} - (\frac{2}{3})^{1/2}\sin\theta_V$	$-\frac{\sqrt{3}}{2}\cos\theta_V$	0	$A(D^0 \to K^- \phi)$
$A(D^+ \to \bar{K}^0 \omega)$	$(\frac{2}{3})^{1/2}\cos\theta_V - \frac{1}{2\sqrt{3}}\sin\theta_V$	$-\frac{\sqrt{3}}{2}\sin\theta_V$	0	$A(D^0 \to K^- \omega)$
$A(D^+ \to \eta \bar{K}^{*0})$	$-\frac{1}{2\sqrt{3}}\cos\theta_P - (\frac{2}{3})^{1/2}\sin\theta_P$	$\frac{\sqrt{3}}{2}\cos\theta_P$	0	$A(D^0 \to \eta K^{*-})$
$A(D^+ \to \eta' \bar{K}^{*0})$	$(\frac{2}{3})^{1/2}\cos\theta_P - \frac{1}{2\sqrt{3}}\sin\theta_P$	$\frac{\sqrt{3}}{2}\sin\theta_P$	0	$A(D^0 \to \eta' K^{*-})$
$A(F^+ \to \pi^+ \rho^-)$	$-\sqrt{2}/3$	0	1	$A(F^+ \to \pi^- \rho^+)$ $A(F^+ \to \pi^0 \rho^0)$
$A(F^+ \to K^+ K^{*-})$	$1/3\sqrt{2}$	$1/\sqrt{2}$	1	$A(F^+ \to K^0 \bar{K}^{*0})$
$A(F^+ \to K^- K^{*+})$	$1/3\sqrt{2}$	$-1/\sqrt{2}$	1	$A(F^+ \to \bar{K}^0 K^{*0})$
$A(F^+ \to \eta\phi)$	$\frac{\sqrt{2}}{3}\cos\theta_V\cos\theta_P + \frac{2}{3}\sin(\theta_V+\theta_P)$	0	$\cos(\theta_V-\theta_P)$	
$A(F^+ \to \eta\omega)$	$\frac{\sqrt{2}}{3}\sin\theta_V\cos\theta_P - \frac{2}{3}\cos(\theta_V+\theta_P)$	0	$\sin(\theta_V-\theta_P)$	
$A(F^+ \to \eta'\phi)$	$\frac{\sqrt{2}}{3}\cos\theta_V\sin\theta_P - \frac{2}{3}\cos(\theta_V+\theta_P)$	0	$-\sin(\theta_V-\theta_P)$	
$A(F^+ \to \eta'\omega)$	$\frac{\sqrt{2}}{3}\sin\theta_V\sin\theta_P - \frac{2}{3}\sin(\theta_V+\theta_P)$	0	$\cos(\theta_V-\theta_P)$	

We briefly remark on the other cases covered by our analysis.

$B(3^*) \to B(8)P(9)l^+\nu_l$ decays. These B_{l4} decays cover the case where $B(8)$ can be the usual $J^P=\frac{1}{2}^+$ baryon octet or a baryon-resonance octet $B^*(8)$, and $P(9)$ can be any meson nonet. Table IV can be used for such decays by replacing $V(9)$ by $B(8)$, $\theta_V=0$, and noting that there are no B_{l4} amplitudes corresponding to the amplitudes involving ω since $B(8)$ is an octet. Translation of the sum rules for $B(3^*) \to B^*(8)V(9)l^+\nu_l$ would be useful if B_{l6} decays are quasi-B_{l4} decays.

$P(3^*) \to P(9)P(9)l^+\nu_l$ decays. These P_{l4} decays are interesting as they involve identical nonets, and as a result there are only 15 decay amplitudes, which can be obtained by replacing $V(9)$ by $P(9)$ in Table IV. In doing this one should recall that the corresponding SU(3) amplitudes g_S and g_1 are symmetric while g_A is antisymmetric in the momenta of the two final hadrons. There are seven isospin relations and five SU(3) sum rules which are fairly simple and amenable to experimental test. The only decay suppressed by the OZI rule is $F^+ \to \pi^+\pi^- l^+\nu_l$ and this would again imply $\sqrt{2}\, g_S \approx 3g_1$. Finally, if P_{l6} decays are quasi-four-body decays of the type $P(3^*) \to V(9)V(9)l^+\nu_l$, then amplitude and rate relations for them can be easily obtained from Table IV.

IV. DISCUSSION

In the foregoing sections, we have presented an SU(3) analysis of the three-body and quasi-three-body nonleptonic and four-body and quasi-four-body semileptonic $\Delta C = \Delta S$ decays of a charmed hadron belonging to the 3* of SU(3).

It may be worthwhile to summarize the new features in the present analysis *vis-á-vis* some earlier work.[5] The relations for nonleptonic decays are cataloged in terms of I-spin, U-spin, and V-spin properties so that it is easy to delineate the sum rules which are valid for the full H_{CS} from those which hold only under the assumption of sextet dominance. No assumption such as complete symmetry of the final-state hadrons is made, so that the relations obtained are more general and it is easy to translate them to other cases like decays of the charmed baryons $B(3^*)$ and so on.

For a massive state, a multibody decay involving three or more particles in the final state is expected to be much more copious compared to a simple two-body mode. In view of this, the isospin and SU(3) tests for the exclusive decays considered here and the $\Delta C = \Delta S$ inclusive decays[6] of charmed particles within the GIM scheme may be useful in the near future.

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