PHYSICAL REVIEW D VOLUME 17, NUMBER 9 1 MAY 1978

Production of bound quark-antiquark systems

L. M. Jones and H. W. Wyld, Jr.
Physics Department, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801
(Received 14 November 1977; revised manuscript received 23 December 1977)

We use the technique of averaging low-order quantum-chromodynamics diagrams over the range from quark threshold to meson threshold to compute the photoproduction of ψ and Υ vector mesons and the hadroproduction of Υ. We find that the energy dependence of ψ photoproduction is reproduced quite well. The model then implies a very rapid energy variation for Υ photoproduction at Fermilab energies. Calculations for hadroproduction of Υ show that at Fermilab energies the Drell-Yan mechanism is much more important for the production of b quarks than it is for the production of charmed quarks. We compute rates for the production of Υ at Fermilab and CERN ISR energies and compare with experiment. Because the model has well defined dependence on the mass of the quark involved, we can compute the expected rates for hadroproduction of higher-mass bound systems. The variation of these rates with mass is comparable to that observed for experimental $\mu^+\mu^-$ pair production in this mass region, leading us to conclude that the signal/noise ratio for production of higher vector mesons should be roughly comparable to that for the Υ.

INTRODUCTION

Recently some effort has been devoted to calculations which use the cross sections for production of a charm-anticharm quark pair via low-order quantum-chromodynamics (QCD) graphs, and then fold this with gluon or quark distributions for the incident hadrons to obtain estimates for the total charm production in photon- or hadron-induced reactions.[1,2] Since this technique estimates the total production of charmed *quarks*, the results thus obtained should include both processes in which the quarks emerge bound to non-charmed quarks (thus producing charmed mesons and baryons), and processes in which the charmed quark and anticharmed quark emerge bound to each other in a ψ, χ, etc. If we designate by M the invariant mass of the outgoing charmed quark pair, it is then natural to associate the range $4m_c^2 \leq M^2 \leq 4m_D^2$ with "charmonium" production, and the range $4m_D^2 \leq M^2 \leq \infty$ with charmed-particle production.[3] (Here m_c is the mass of the charmed quark, and m_D is the mass of the lowest charmed meson.) This approach has been taken by Fritzsch[4] and by Glück, Owens, and Reya.[5]

We feel that it is important to study the validity of this method, since it is considerably easier to apply than other estimates for ψ production, such as those of Carlson and Suaya,[6] which depend on knowledge of the charmonium wave functions. The method has a number of essential features which may be advantages or disadvantages depending on one's point of view. These include the following:

(a) The contributions of all colors are taken into account—both quark-antiquark pairs in color singlets and in color octets are included. This is based on the assumption that the color octets will necessarily decay by emission of a (hopefully soft) gluon before the particles are observed in the form of normal color-singlet mesons. Since it is believed that this process occurs with probability 1, no specific provision for this is made in the calculation.

The Carlson-Suaya approach, on the other hand, assumes that two gluons merge in the color-singlet channel to produce the physical χ particles, which then decay via photon emission to the observed ψ.[7] By including all colors, and assuming that any number of "wee" gluons may leak off the final state, one escapes the charge-conjugation constraint that forces Carlson and Suaya into production of a χ as the initial process in the reaction. This means that ψ particles may be produced directly, and hence may be observed without an accompanying photon, or they may come from the production and subsequent decay of a χ.

(b) The integral from $4m_c^2$ to $4m_D^2$ is supposed to give the production of all charmonium states; hence, if one wishes to know cross sections for some specific state such as ψ, a correction factor must be supplied. This will have two components, (i) a guess at how the basic production is to be apportioned among the various bound systems, plus (ii) knowledge of branching ratios for decays of the higher-mass bound systems into the lower ones (which may be taken from experiment, at least for χ and ψ). The difficult part is the first component, the ratio of production of the various states. To be honest, all one can do is to assume that any given state will have a production cross section smaller than the sum of all. Fritzsch[4] and Glück, Owens, and Reya[5] advocate dividing by some integer between 2 and 8 to obtain an estimate for ψ production; this is no doubt as good as any other suggestion in the present state of ignorance.

(c) One of the most disturbing features of this model is the strong dependence of the answers on the mass of the quark. As we have noted elsewhere,[1] because the cross sections calculated fall off rapidly with M^2, any adjustment in the threshold will change the integral by a large amount. Since the exact mass of the quark is a somewhat nebulous concept, we cannot be very sure which of the many values in the literature is appropriate; in some models[8] the mass of the charmed quark is even larger than the mass of the lowest charmed meson.

When bound states of heavier quarks are considered, one can relate the mass of the quarks and the mass of the mesons by the formula[9]

$$m_0 = m_D + m_Q - m_c + \tfrac{3}{4}(1 - m_c/m_Q)(m_D{}^* - m_D) , \qquad (1)$$

which gives the mass m_0 of the lowest $Q\overline{q}$ state in terms of the mass m_Q of the heavy quark and the masses in the charmonium system.

(d) To our knowledge, there is no theorem guaranteeing that this sort of "dual averaging" should work for the charmonium states. Tests of a similar dual averaging by Poggio, Quinn, and Weinberg[10] differed in two ways from this calculation: (i) They averaged in such a way as to smooth out the effects of quark and hadron thresholds. (ii) They studied a case in which the perturbation-theory calculations were explicitly for the channel with the same quantum numbers

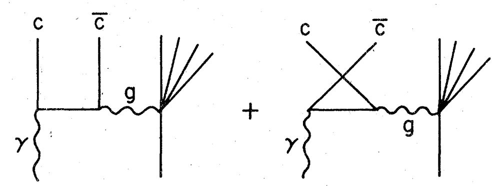

FIG. 1. The photon-gluon fusion graphs integrated over the range $4m_c{}^2 \leq M^2 \leq 4m_D{}^2$ to obtain estimates for ψ photoproduction. Formula (2) of Ref. 1(a) is used.

as the data (the single-photon propagator). Hence problems created by the quark threshold discussed in (c) and the gluon leakage discussed in (a) were not present in their case. Either one or both of these considerations may damage the usefulness of the method.

With all these caveats, one thing remains—to try the technique and see whether it actually works. Glück, Owens, and Reya[5] have done this for the hadronic production of ψ particles. They were able to achieve good agreement with the x_F distributions, at several energies, by using both the Drell-Yan and gluon-fusion mechanisms. We are thus encouraged to try the technique for other cases.

PHOTOPRODUCTION OF VECTOR MESONS

The photoproduction of ψ mesons is a good place to test these ideas because there is no Drell-Yan (quark-antiquark annihilation) diagram to consider to lowest order in α_{st} (the diagram discussed by Halzen and Scott[3] cannot produce charmonium states). Hence all the contribution should come

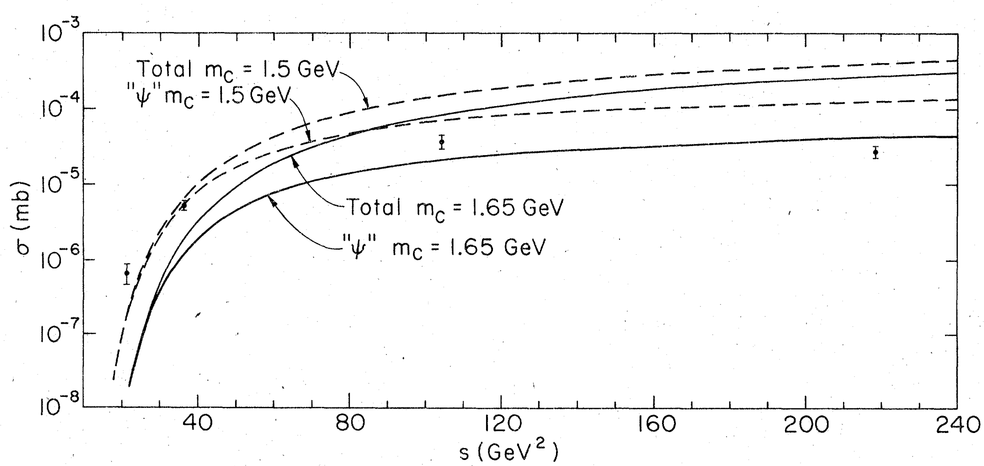

FIG. 2. Comparison of the total charm-production cross section from photon-gluon fusion with the charmonium production (labeled "ψ" here). We plot data for ψ photoproduction from Ref. 11. Values are quite dependent on the assumed mass of the charmed quark; we have shown computations for $m_c = 1.5$ and 1.65 GeV.

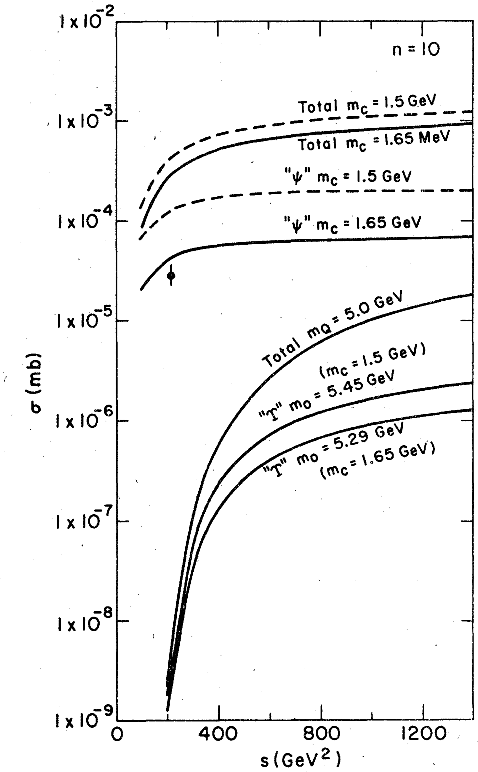

FIG. 3. Expected photoproduction of ψ and Υ in the Fermilab energy range. Values are computed for the two cases $m_c = 1.5$ and 1.65 GeV. We have assumed that the quark contained in the Υ has a mass of $m_Q = 5$ GeV, and that the lowest meson in the series has a mass m_0 derived from Eq. (1).

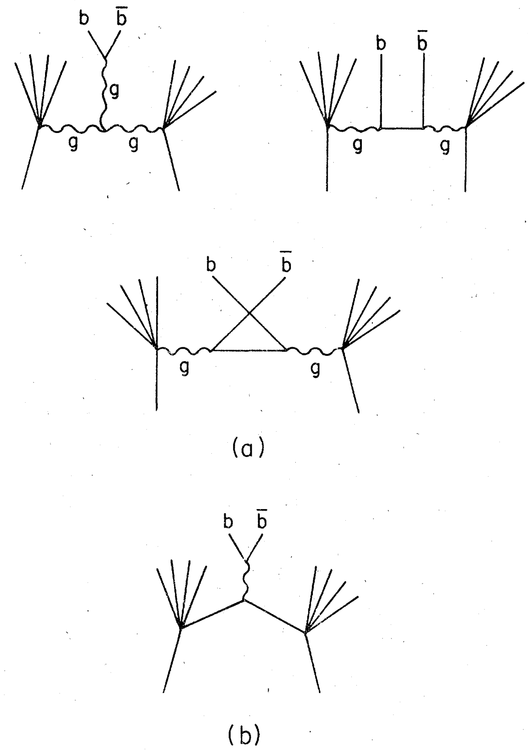

FIG. 4. The graphs integrated from $4m_Q^2$ to $4m_0^2$ to obtain estimates for Υ hadroproduction. Eqs. (1), (2), (4), and (5) of Ref. 1(b) are used to do the computations.

from the photon-gluon fusion diagrams of Fig. 1. A simple calculation [using Eq. (2) of Ref. 1(a) with the limits on the M^2 integral as discussed above] should thus yield the energy dependence of ψ photoproduction, which has been carefully measured over quite a wide energy range.[11] In Fig. 2, we show the calculation and the data; the agreement is quite good. Note that the curves marked "ψ" represent the production of all charmonium states. The reader is invited to divide by his own correction factor. All results presented in this paper have this same feature.

This leads us to calculate the predications of the same model for photoproduction of the Υ particle. These are shown in Fig. 3. Note the very rapid energy dependence over the Fermilab energy range. The values in Fig. 3 have been computed for a quark of charge $\frac{2}{3}$; they must be divided by a factor of 4 for the standard b quark of charge $-\frac{1}{3}$. Both Figs. 2 and 3 have been calculated for a gluon distribution function (in the proton) of $G(x) = (n+1)(1-x)^n/16$ with $n = 10$.

HADROPRODUCTION OF Υ AND FURTHER HEAVY BOUND SYSTEMS

As a further check on the technique, we calculate the hadroproduction of the Υ. Results have been reported[12] for the pp initial state; an experiment for the $\pi^- p$ initial state is underway.[13] As in the calculations of Glück, Owens, and Reya[5] we must estimate both the gluon-fusion graphs of Fig. 4(a) and the Drell-Yan graph of Fig. 4(b). In contrast to the case of production of charmed quarks [Ref. 1(b)] we find that when the quark mass is 5 GeV, the Drell-Yan graphs can dominate in the Fermilab energy range. Calculations of the various contributions are shown in Fig. 5(a). The solid curves represent total production of "b" quarks; the dotted curves are the "Υ" contribution. We see that for incident protons either the gluon-fusion contribution or the Drell-Yan contribution can dominate, depending on the form of the gluon distribution. For incident pions, the Drell-Yan should dominate for most gluon distributions currently under study. Owing to the very large Drell-Yan contribution, production of "Υ" by incident pions is expected to be very much more efficient than by incident protons at Fermilab energies. There are some initial indications that

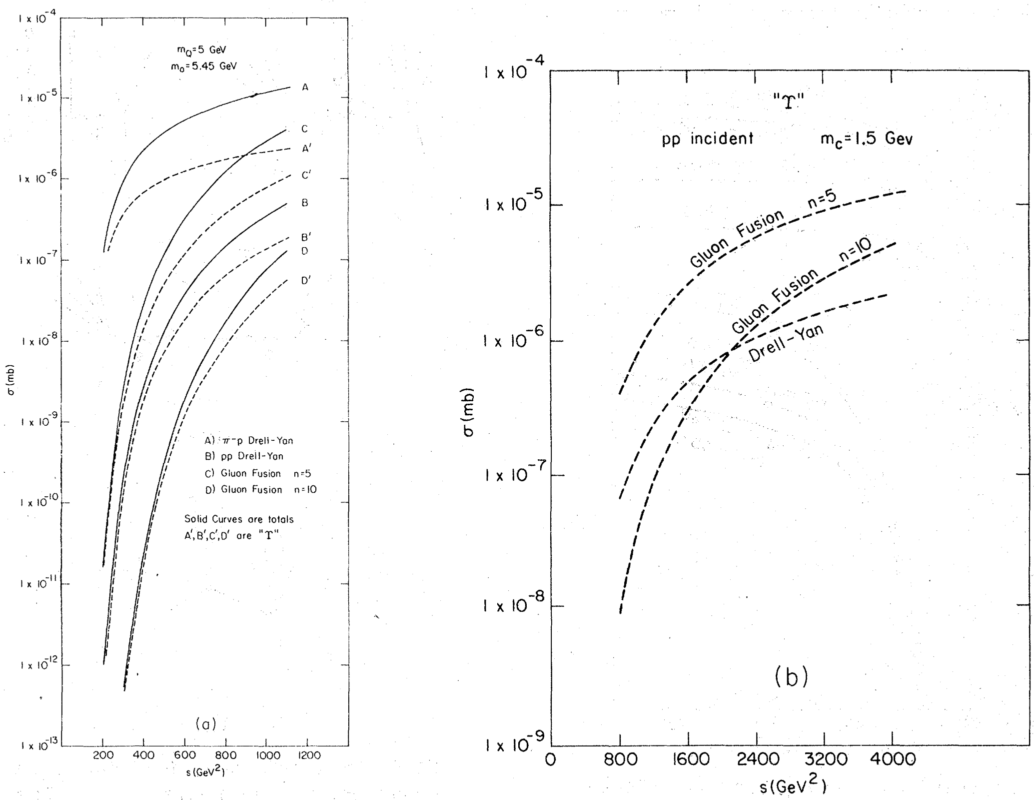

FIG. 5. (a) The Drell-Yan and gluon-fusion contributions to production of states containing b quarks. Solid curves show production of all "b" states; dashed curves show production of "Υ". The mass of the b quark is taken to be $m_Q = 5.0$ GeV. A value $m_c = 1.5$ GeV is used in Eq. (1) to obtain $m_0 = 5.45$ GeV. (b) Continuation of the "Υ" curves for incident protons, showing the ISR energy range.

this may be the case.[13]

When the energy becomes high enough, the gluon-fusion mechanism will dominate the Drell-Yan mechanism. This is shown in Fig. 5(b) for $pp \to \Upsilon + X$ in the CERN ISR energy range.

The values currently available in the literature for $pp \to \Upsilon + X$ are not for integrated cross sections, however, but for the quantity $B\, d\sigma/dy\,|_{y=0}$, where B is the (as yet unknown) branching ratio of the Υ into $\mu^+\mu^-$. The production cross section can be calculated from the formula

$$\frac{d\sigma}{dy} = \frac{1}{s}\int_{4m_Q^2}^{4m_0^2} dM^2 \left[\bar{F}\left(\frac{M}{\sqrt{s}}e^{y}\right)F\left(\frac{M}{\sqrt{s}}e^{-y}\right)\sigma_1(M^2) + G\left(\frac{M}{\sqrt{s}}e^{y}\right)G\left(\frac{M}{\sqrt{s}}e^{-y}\right)\sigma_2(M^2)\right], \tag{2}$$

where F, $\bar{F}$, and G are the appropriate quark, antiquark, and gluon distributions and where the cross sections σ_1 and σ_2 are given by formulas (2) and (5) of Ref. 1(b). The first term of Eq. (2) is for the graph in Fig. 4(b); the second term is for Fig. 4(a). We use the quark distribution functions of Feynman and Field,[14] and take the gluon distribution function of the proton $G(x)$ to be $G(x) = (n+1)(1-x)^n/16$.

At the experimental energy of $s = 800$ GeV^2, we find $d\sigma/dy\,|_{y=0} = 45$ pb for $n = 10$ and 340 pb for $n = 5$ which, with a branching ratio of 4%, gives $B\, d\sigma/dy\,|_{y=0} = 1.8$ pb for $n = 10$ and 13.6 pb for $n = 5$. This is to be compared with the experimental value 0.25 pb for the integrated area under the three Υ states.[12] Recall that the calculated value must be divided by some number larger than one to allow for the production of states not connected to lepton pairs; hence the calculation seems to be in agreement with experiment if we choose $n = 10$.

Emboldened by this agreement, we explore the production cross section for bound systems belonging to quarks of heavier mass than the one associated with the Υ. The question of interest is whether the dependence of these cross sections on the mass of the states involved is comparable with the slope in mass of the lepton-pair background. In Fig. 6 we show the production cross sections for incident pp as functions of the quark mass. The slopes are similar to that of the

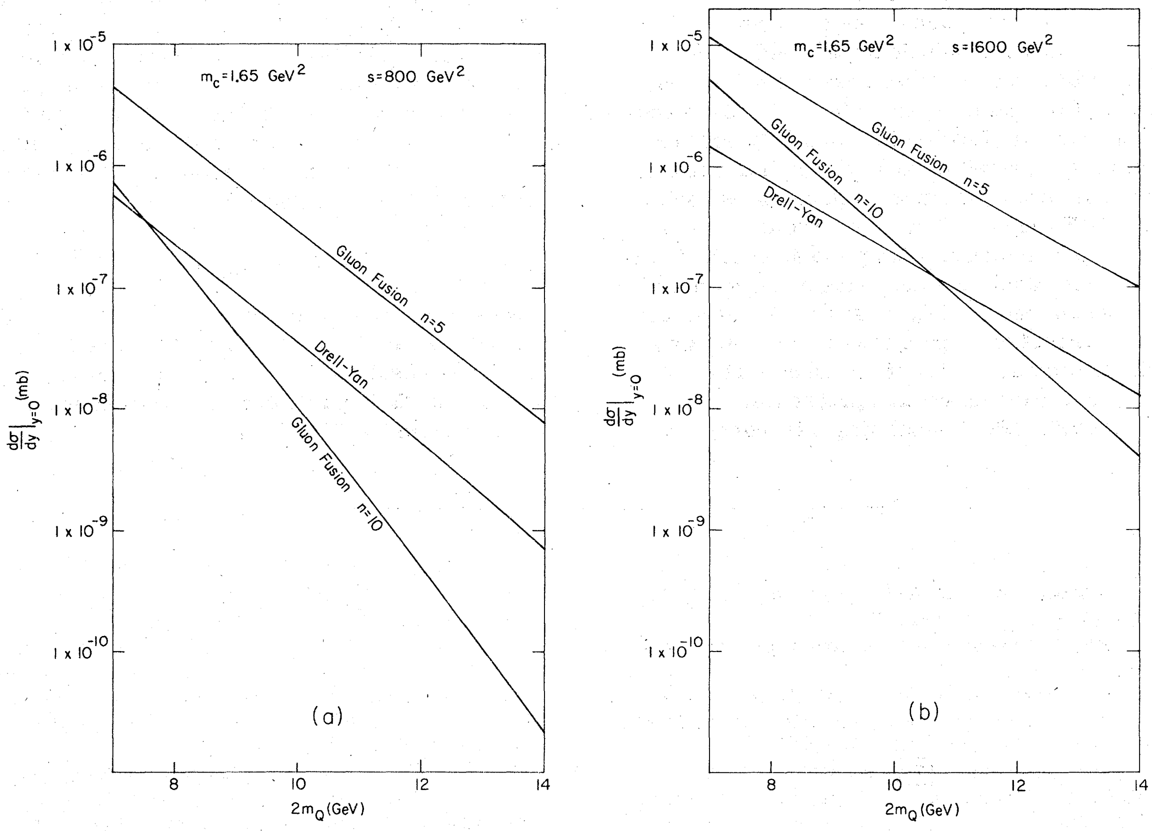

FIG. 6. Dependence of the cross section for hadroproduction of heavy vector mesons on the mass of the quark involved. Values are presented for two plausible gluon distribution functions and two energies. The Drell-Yan and gluon-fusion contributions are shown separately. The slope is to be compared with that of the $\mu^+\mu^-$ background in the Υ region reported in Ref. 12. (a) is for $s = 800\ \text{GeV}^2$, the energy of the present experiment, (b) is for the larger energy $s = 1600\ \text{GeV}^2$.

background in the Υ experiment,[12] leading to the conclusion that the signal-to-noise ratio for observation of heavier bound systems in this experiment is reasonably promising (given infinite experimental time to compensate for the slow counting rates).

Another distribution of interest is the x distribution of the produced Υ particles. This can be calculated using formula (1) of Ref. 5,

$$\frac{d\sigma^{AB}_{q\bar{q}}}{dx_F} = \sum_{q=u,d,s} \int_{4m_Q^2}^{4m_0^2} dM^2 \sigma^{q\bar{q}\to Q\bar{Q}}(M^2) \frac{1}{M^2} \frac{x_A x_B}{x_A + x_B} \times [q^A(x_A, M^2)\bar{q}^B(x_B, M^2) + q \leftrightarrow \bar{q}]\,, \tag{3}$$

with

$$x_{A,B} = \tfrac{1}{2}[\pm x_F + (x_F^2 + 4M^2/s)^{1/2}]\,,$$

for the Drell-Yan contribution, with a similar integral for the gluon-fusion contribution. As shown by Glück, Owens, and Reya for calculations of ψ production,[5] the results are quite sensitive to the form assumed for the individual parton distributions. Since the thrust of this paper is to explore the "threshold-averaging" technique rather than to study various parton distributions, we do not show x distributions for the Υ here. However, we would like to make one remark about this matter.

We have studied the behavior of the various contributions to the integrand in Eq. (3) for fixed x_F as a function of M^2. In the small region covered by the M^2 integration, the dependence of the cross section $\sigma^{q\bar{q}\to Q\bar{Q}}(M^2)$ on M^2 is quite rapid; the other functions in the integrand do not change much with M^2 for fixed x_F (our study was in the region near $M^2/s = 0.25$). Hence, we have approximately

$$\frac{d\sigma^{AB}_{q\bar{q}}}{dx_F} \approx \sum_{q=u,d,s} \frac{[q^A(x_A, M_M^2)\bar{q}^B(x_B, M_M^2) + q \leftrightarrow \bar{q}]}{s(x_F^2 + 4M_M^2/s)^{1/2}} \int_{4m_Q^2}^{4m_0^2} dM^2 \sigma^{q\bar{q}\to Q\bar{Q}}(M^2)\,, \tag{4}$$

where M_M^2 is some value between $4m_Q^2$ and $4m_0^2$. This shows that for each of the contributions (Drell-Yan and gluon-fusion) the shape of the x_F distribution of the produced heavy bound states should approximately scale with M^2/s. If we consider a physical situation in which only one contribution is expected to dominate (as for example is the case with production by pions of "ψ" below $s \approx 200$ GeV^2 and of "Υ" over the whole Fermilab range, where the Drell-Yan process should dominate in each case) then the shape of the x_F distribution should depend only on M^2/s. For example, the x_F distribution of produced Υ at $s \approx 400$ GeV^2 should be similar to that of produced ψ at $s \approx 40$ GeV^2, for a given target and projectile.

Note added. After completing this work, we received a report by H. Fritzsch and K.-H. Streng [Phys. Lett. 72B, 385 (1978)], which computes Υ and ψ photoproduction by a different technique based on the optical theorem and the gluon-photon fusion calculation for photoproduction of heavy mesons. Their results are very similar to ours. However, their technique cannot be applied to the hadroproduction of these vector mesons.

ACKNOWLEDGMENTS

We would like to thank U. E. Kruse, T. O'Halloran, J. Sullivan, J. Thaler, and A. Wattenberg for helpful discussions. This work was supported in part by the National Science Foundation under Grant No. NSF PHYS 75-21590.

[1](a) L. M. Jones and H. W. Wyld, Phys. Rev. D 17, 759 (1978); (b) *ibid.* 17, 1782 (1978).
[2]J. Babcock, D. Sivers, and S. Wolfram (private communication).
[3]Some models for charm production apply only to the region $4m_D^2 \leq M^2 \leq \infty$. See for example F. Halzen and D. M. Scott, Phys. Lett. 72B, 404 (1978).
[4]H. Fritzsch, Phys. Lett. 67B, 217 (1977).
[5]M. Glück, J. F. Owens, and E. Reya, Phys. Rev. D 17, 2324 (1978).
[6]C. E. Carlson and R. Suaya, Phys. Rev. D 14, 3115 (1976); *ibid.* 15, 1416 (1977); Phys. Rev. Lett. 39, 908 (1977).
[7]Other papers discussing this idea and its application include S. D. Ellis, M. B. Einhorn, and C. Quigg, Phys. Rev. Lett. 36, 1263 (1976); T. Hagiwara, Y. Kazama, and E. Takasugi, *ibid.* 40, 76 (1978).
[8]See for example W. Celmaster, H. Georgi, and M. Machacek, Phys. Rev. D 17, 879 (1978).
[9]E. Eichten and K. Gottfried, Phys. Lett. 66B, 286 (1977). The formula was originally derived by A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975) using the hyperfine term of the Breit interaction resulting from one-gluon exchange.
[10]E. C. Poggio, H. R. Quinn, and S. Weinberg, Phys. Rev. D 13, 1958 (1976).
[11]The data points plotted are taken from a Table given both in T. Nash *et al.*, Phys. Rev. Lett. 36, 1233 (1976) and by A. Wattenberg, in *Proceedings of the XVIII International Conference on High Energy Physics, Tbilisi, 1976*, edited by N. N. Bogolubov *et al.* (JINR, Dubna, U.S.S.R., 1977), Vol. II, p. N21.
[12]W. R. Innes *et al.*, Phys. Rev. Lett. 39, 1240 (1977); 39, 1640(E) (1977).
[13]Fermilab experiment No. 444.
[14]R. D. Field and R. P. Feynman, Phys. Rev. D 15, 2590 (1977).