Parton transverse momenta and quantum-chromodynamic effects in large-p_T hadron production

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Inclusive pion production at large transverse momenta in *pp* collisions is studied in the framework of parton-parton scattering with partons carrying transverse momentum and with quark and gluon distributions determined from exact requirements of quantum chromodynamics. CERN ISR data are fairly well accounted for, but Fermilab data somewhat exceed the predictions. Gluon effects are considered in detail.

I. INTRODUCTION

For several years it has been proposed that the dominant mechanism for hadron production at large transverse momenta (p_T) is quark-quark scattering via single-gluon exchange.[1,2]

More recently it was suggested[3] that the well known difficulty of this mechanism to reproduce the experimental p_T dependence could be removed by taking into account violations of Bjorken scaling. Such violations have been observed in deep-inelastic scattering data[4,5] and constitute one of the outstanding predictions of asymptotically free field theories, in particular of quantum chromodynamics (QCD).[6,7]

This possibility has been investigated in detail in a specific model[8] that accounts for a number of QCD requirements and leads to moments of the structure functions $\nu W_2(x, Q^2)$ asymptotically behaving like inverse powers of $\log Q^2$ (logarithmic scale violation). The essentials of the data on single-hadron ($\pi^\pm, \pi^0, K^\pm$) production and on two-hadron correlations can be understood in that model.[8]

However, a number of difficulties of, and objections to, the approach of Ref. 3, 8, and related work[9,10] should be emphasized:

(i) The quark-gluon coupling required to fit the data exceeds by factors of 2–3 the usually accepted values (resulting from applications of QCD).

(ii) The scale violation is stronger than that predicted by QCD (Refs. 8, 11, and 12).

(iii) More generally, the quark distributions were not deduced as solutions of QCD asymptotic conditions.

Very recently, quark (and gluon) distributions satisfying, to a good approximation, all QCD requirements have been deduced and applied with success to electroproduction and neutrino production data.[13-16] Clearly, it is of much interest to make use of these distributions in a confrontation with large-p_T experiments of the basic quark-quark scattering mechanism.[1,2]

On the other hand, there is now much experimental evidence indicating that the momentum $\vec{\kappa}_T$ of hadron constituents (partons) transverse to the hadron's momentum also has important effects. Such evidence results from massive-lepton-pair production showing a wide transverse-momentum distribution.[17-19] Also, it results from the lack of coplanarity observed in large-p_T events.[20-22] All these experiments suggest an average parton's transverse momentum significantly larger than the traditionally accepted $\langle\kappa_T\rangle \simeq 0.3$ GeV.

The purpose of the present work is to study the basic hadron process $pp \to \pi^0 + X$ [and $pp \to \frac{1}{2}(\pi^+ + \pi^-) + X$] in the framework of parton-parton scattering with partons carrying transverse momentum $\vec{\kappa}_T$ and parton distributions determined from strict QCD requirements.[14-16]

Certain analyses of $pp \to \pi^0 + X$ with parton distributions determined from QCD have appeared very recently.[23,24] The general conclusion of them is that at very large p_T ($\gtrsim 5$ GeV) CERN ISR data can be well accounted for; but at intermediate values ($2 \lesssim p_T \lesssim 5$ GeV) the QCD predictions fall below the data. However, in the analyses of Refs. 23 and 24 the partons' κ_T effects have not been included, and it is precisely in the intermediate-p_T region that κ_T effects are appreciable.

On the other hand, an important conclusion of Refs. 23–25 is that at $p_T \simeq 2$–4 GeV production of hadrons via quark-gluon (qg) or gluon-gluon (gg) subprocesses is very significant. In QCD determinations of quark distributions[13-16] gluon effects are inseparable and cannot be neglected. In the present work the effect of gluon subprocesses in addition to quark-quark (qq) scattering is also considered and studied in detail.

Section II presents the essential formalism of the inclusive cross section for $A + B \to C + X$ (A, B, C hadrons) when the partons' transverse momenta are included. Section III contains the parton distributions in the presence of QCD effects and disucsses the determination of the parton's fragmentation functions. Section IV presents calculational details and our general conclusions on the κ_T effects. Section V presents our conclusions on the predicted magnitude, p_T and s dependence and on the effect of changing the shape of the gluon distribution. Finally, Appendix A contains details on the kinematics of $A+B\to C+X$ with κ_T effects and Appendix B contains details on the parton distributions and fragmentation functions.

II. PARTON κ_T EFFECTS IN THE INCLUSIVE CROSS SECTION

The form of the invariant inclusive cross section for $A+B\to C+X$ with partons' transverse momenta has already been considered.[26] In our approach, however, we should properly incorporate the dependence of the parton distributions and fragmentation functions on the momentum of the probe.

The differential probability dP that a hadron A of momentum $\vec{P}_A$ is seen by a probe of four-momentum Q to contain a parton a will be written

$$dP=f_{a/A}(x,\vec{\kappa}_T,Q^2)dx\,d^2\kappa_T\,, \tag{2.1}$$

where $x\vec{P}_A$ is the longitudinal and $\vec{\kappa}_T$ is the transverse momentum of the parton relative to $\vec{P}_A$. The differential probability that a parton c of momentum $\vec{p}_c$ is seen by a probe of four-momentum Q to produce a hadron C is written

$$dP=G_{C/c}(z,\vec{\kappa}_T,Q^2)\frac{dz}{z}\,d^2\kappa_T\,, \tag{2.2}$$

where $z\vec{p}_c$ is the longitudinal and $\vec{\kappa}_T$ is the transverse momentum of the hadron C relative to $\vec{p}_c$.

It is assumed that $A+B\to C+X$ takes place via the subprocess $a+b\to c+d$, of which the differential cross section is $d\sigma/d\hat{t}$. Then the inclusive cross section for $A+B\to C+X$ with C produced at angle θ and transverse momentum p_T is

$$E\frac{d\sigma}{d^3p}(p_T,\theta,s)=\sum_{a,b,c}\int d^2\kappa_{Ta}\int d^2\kappa_{Tb}\int d^2\kappa_{Tc}\int dx_a\int dx_b\,f_{a/A}(x_a,\vec{\kappa}_{Ta},Q^2)f_{b/B}(x_b,\vec{\kappa}_{Tb},Q^2)\times\frac{1}{\pi}\frac{d\sigma}{d\hat{t}}(\hat{s},\hat{t},\hat{u})\frac{1}{z^2}G_{C/c}(z,\vec{\kappa}_{Tc},Q^2)\,. \tag{2.3}$$

The invariants $\hat{s}$, $\hat{t}$, and $\hat{u}$ are expressed in terms of x_i, $\vec{\kappa}_{Ti}$ $(i=a,b)$, and z in Appendix A. For the quark-quark scattering subprocess, $Q^2=-\hat{t}$ or $-\hat{u}$ (see end of Appendix B). The contraints determining the region of integration in (2.3) are

$$\hat{s}+\hat{t}+\hat{u}=m_a^2+m_b^2+m_c^2+m_d^2 \tag{2.4}$$

together with

$$0\le x_i\le 1\,,\quad 0\le z\le 1\,. \tag{2.5}$$

The probability function $f_{a/A}(x,\vec{\kappa}_T,Q^2)$ is known to be of the form $(1/x)F_{a/A}(x,\vec{\kappa}_T,Q^2)$ where $F_{a/A}(0,\vec{\kappa}_T,Q^2)$ may be nonzero. However, for sufficiently large $|\vec{\kappa}_T|$, x may vanish (Appendix A) causing the integrand in (2.3) to diverge. The customary modification is[27,28]

$$f_{a/A}(x,\vec{\kappa}_T,Q^2)=\left(x^2+\frac{4m_T^2}{s}\right)^{-1/2}F_{a/A}(x,\vec{\kappa}_T,Q^2)\,,$$

where $m_T^2=\vec{\kappa}_T^2+m_a^2$.

The κ_T dependence of the probability function $F_{a/A}$ is generally unknown. As usual, we proceed with the factorized ansatz,[27-30]

$$F_{a/A}(x,\vec{\kappa}_T,Q^2)=F_{a/A}(x,Q^2)D(\vec{\kappa}_T)\,, \tag{2.6}$$

subject to

$$\int d^2\kappa_T D(\vec{\kappa}_T)=1\,. \tag{2.7}$$

In the absence of any information, we use the same ansatz irrespective of whether a represents a quark or a gluon. We also use this ansatz for the fragmentation functions, as well:

$$G_{C/c}(z,\vec{\kappa}_T,Q^2)=G_{C/c}(z,Q^2)D(\vec{\kappa}_T)\,. \tag{2.8}$$

The differential cross section for the subprocess $a+b\to c+d$ is of the form

$$\frac{d\sigma}{d\hat{t}}=\frac{\pi\alpha^2}{\hat{s}^2}\,\Sigma(ab)\,. \tag{2.9}$$

The exact form of $\Sigma(ab)$ depends on whether $ab\to cd$ represents quark-quark (qq), quark-gluon (qg), or gluon-gluon (gg) scattering and is given in Appendix B. In (2.9) $\alpha=\alpha(Q^2)$ is the QCD running coupling constant with the typical value (four flavors)

$$\alpha(Q^2)=\frac{12\pi}{25\ln(Q^2/\Lambda^2)} \tag{2.10}$$

in our calculations $\Lambda=0.3$ GeV.

In the presence of parton κ_T the variables $\hat{s}$, $\hat{t}$, and $\hat{u}$ may become very small and cause (2.9) to diverge (Appendixes A and B). To avoid this we make the usual replacements[28-30]

$$\hat{s}\to\hat{s}+M^2,\quad \hat{t}\to\hat{t}-M^2,\quad \hat{u}\to\hat{u}-M^2\,,$$

with the typical hadronic mass scale $M=1$ GeV.

III. QCD EFFECTS IN DISTRIBUTION AND FRAGMENTATION FUNCTIONS

We are interested in hadron production in proton-proton collisions ($A=B=$proton). With

$\nu_u(x,Q^2)$, $\nu_d(x,Q^2)$, and $t(x,Q^2)$ the distribution of the u valence, d valence, and sea quarks inside the proton A, the probability functions $F_{a/A}$ have the form

$$F_{u/A}(x,Q^2)=2\nu_u(x,Q^2)+t(x,Q^2)\,, \tag{3.1}$$

$$F_{d/A}(x,Q^2)=\nu_d(x,Q^2)+t(x,Q^2)\,, \tag{3.2}$$

and for $a=\bar{u},\bar{d},s,\bar{s}$:

$$F_{a/A}(x,Q^2)=t(x,Q^2) \tag{3.3}$$

[SU(3)-symmetric sea]. When a is a gluon g,

$$F_{g/A}(x,Q^2)=g(x,Q^2)\,. \tag{3.4}$$

The distributions ν_u, ν_d, t, and g as functions of x and Q^2 are determined from Ref. 15 which makes a detailed account of the QCD requirements and fits old and recent data on nucleon structure functions[14,15]; they are presented in detail in Appendix B. Their Q^2 dependence is specified by the usual QCD variable

$$\bar{s}=\ln\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^{\,2}/\Lambda^2)}\,, \tag{3.5}$$

with $\Lambda=0.3$ GeV and $Q_0^{\,2}=1.8$ GeV^2.[15]

The question of Q^2 dependence of the fragmentation functions $G_{C/c}$ is rather controversial. The reciprocity relation[31] requires that at least for $z\sim 1$ $G_{C/c}$ behaves as $F_{a/A}$ for $x\sim 1$; there are other field-theoretic models and arguments suggesting scale violation for $G_{C/c}$ similar to that of $F_{a/A}$.[32] On the other hand, recent data on electroproduction of pions[33] are compatible with scaling fragmentation functions. Therefore we present complete calculations both with scaling and nonscaling $G_{C/c}$.

To determine the nonscaling form of the quark fragmentation functions we are guided by the QCD solutions of Gross[34] and Politzer,[7] which have also been used in other similar calculations.[35,24] Note that these solutions are valid for z not very small; nevertheless, most of the contribution to (2.3) comes from integrating near $z\sim 1$. Thus we take[36]

$$G_{C/c}(z,\bar{s})=g_{C/c}e^{A\bar{s}}\frac{\Gamma(1+m_{C/c}(0))}{\Gamma(1+m_{C/c}(\bar{s}))}\times(1-z)^{m_{C/c}(\bar{s})}\,. \tag{3.6}$$

The variable $\bar{s}$ is given by (3.5) with the *same* values of the parameters Λ and $Q_0^{\,2}$ determining the magnitude of the scale violation; $g_{C/c}$ are constants and

$$m_{C/c}(\bar{s})=m_{C/c}(0)+\tfrac{1}{4}G\bar{s}\,, \tag{3.7}$$

where the standard QCD model of four flavors and three colors gives $G=\frac{4}{25}$; moreover, $A=0.69G$.[34]

Our scaling form of the fragmentation functions is given by (3.7) with simply $\bar{s}=0$ ($Q^2=Q_0^{\,2}$).

The values of the constants $m_{C/c}(0)$ are determined from an analysis of hadron electroproduction data and are given in Appendix B. The functions $G_{C/c}$ are subject to the momentum conservation sum rule

$$\sum_C\int_0^1 G_{C/c}(z,\bar{s})dz=1 \tag{3.8}$$

for every species c. For $x=0$ this is satisfied if

$$\sum_C g_{C/c}[1+m_{C/c}(0)]^{-1}=1\,. \tag{3.9}$$

The values of $g_{C/c}$ are also determined from electroproduction data but are subject to (3.9) as well; they are given in Appendix B.

We note that for the nonscaling form (3.6) and with $g_{C/c}$ = constants the sum rule (3.8) cannot be satisfied for all s. In this case we are contented to satisfy (3.8) exactly for $s=0$ and notice that, owing to the weak dependence of s on Q^2 [Eq. (3.5)], the violation is $\lesssim 10\%$ for all Q^2 of interest.

When c = gluon there is practically no information on the fragmentation function $G_{C/g}$. For simplicity we proceed with a scaling form:

$$G_{C/g}(z)=g_{C/g}(1-z)^{m_{C/g}}\,. \tag{3.10}$$

This function is also subject to a sum rule such as (3.8) leading to

$$\sum_C g_{C/g}(1+m_{C/g})^{-1}=1\,. \tag{3.11}$$

The constants $m_{C/g}$ and $g_{C/g}$ are also determined in Appendix B.

IV. CALCULATIONS AND CONCLUSIONS ON κ_T EFFECTS

We present detailed calculations with a parton transverse-momentum distribution of the exponential form

$$D(\vec{\kappa}_T)=\frac{b^2}{2\pi}\exp(-b\kappa_T)\,. \tag{4.1}$$

We have taken throughout the average value

$$\langle\kappa_T\rangle=\frac{2}{b}=0.5\text{ GeV}\,. \tag{4.2}$$

This is a conservative value (e.g., 0.7 GeV is certainly acceptable), but it is not our purpose to exaggerate the κ_T effects. We have also carried calculations with the Gaussian form

$$D(\vec{\kappa}_T)=\frac{b^2}{\pi}\exp(-b^2\kappa_T^{\,2})\,, \tag{4.3}$$

where again

$$\langle\kappa_T\rangle=\frac{\sqrt{\pi}}{2b}=0.5\text{ GeV}\,; \tag{4.4}$$

as expected, the κ_T effects are somewhat (but not much) smaller. Finally, we have calculated the gluon κ_T effects using the same $D(\vec{\kappa}_T)$ and $\langle\kappa_T\rangle$.

At sufficiently large Q^2 QCD implies that $\langle\kappa_T\rangle$ increases with Q^2.[37-39] However, present data on lepton-pair production[18,19] are consistent with $\langle\kappa_T\rangle\sim$ const at large lepton-pair mass. Thus in the present work we do not investigate the effects of Q^2 dependence on $\langle\kappa_T\rangle$. Also, we do not investigate the possible x dependence of $\langle\kappa_T\rangle$.[39-41]

To show clearly our results on the κ_T effects of quarks and gluons we have separated in Fig. 1 the contributions of the subprocesses qq, qg, and gg. All results of Fig. 1 correspond to scaling fragmentation functions $[G_{C/c}=G_{C/c}(z)]$ and a gluon distribution $g(x,Q_0^2)\sim(1-x)^5$. Our conclusions can be summarized as follows:

(a) At fixed s, as p_T increases the κ_T effects always decrease. E.g., at $\sqrt{s}=52.7$ GeV and $p_T=2$ GeV the κ_T effects increase the qq contribution by a factor of ~2, but at $p_T=8$ only by ~1.1. These are typical results of other similar calculations.[27-29,42] The decrease of the κ_T effects with p_T is intuitively clear.[43,44]

(b) At fixed p_T, as s decreases, the κ_T effects increase. E.g., at $\sqrt{s}=19.4$ and $p_T=2$ they increase the qq contribution by a factor of ~3. This aspect has also been observed.[27,28]

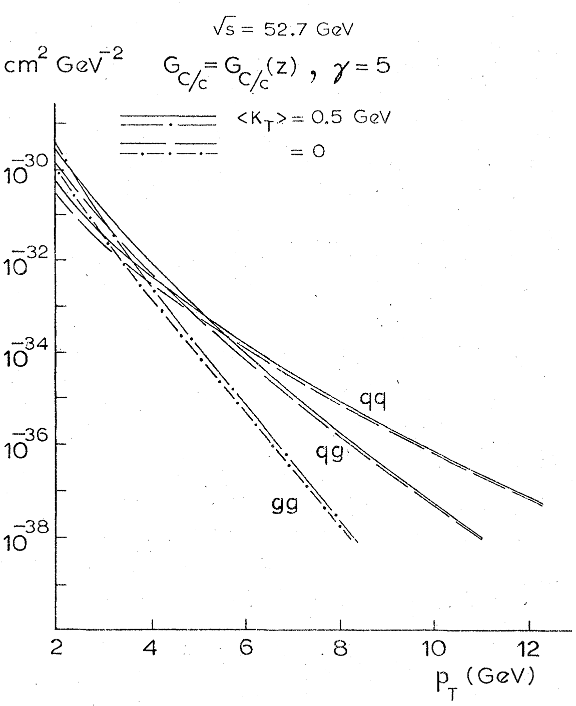

FIG. 1. Separate contributions to $pp\to\pi^0+X$ and $pp\to\frac{1}{2}(\pi^++\pi^-)+X$ at $\theta=90°$ of the subprocesses $qq\to qq$, $qg\to qg$, $gg\to gg$.

(c) Introducing transverse momentum to gluon distributions has a very important effect (at intermediate p_T). E.g., at $\sqrt{s}=52.7$ and $p_T=2$ it enhances the qg contribution by a factor 2.6 and gg by ~5. Qualitatively, this is understood as follows: In general, the stronger the p_T-dependence is of a given contribution (qq, qg, or gg) the stronger are (percentagewise) the κ_T effects. The qg, and in particular the gg, contribution has a very strong p_T dependence; this is due to the exponent of $1-x$ of $g(x,Q^2)$, which is already large at $Q^2=Q_0^2$ but increases very fast with Q^2 (see Appendix B). This results in stronger κ_T effects.

(d) Introducing transverse momentum $\vec{\kappa}_{Tc}$ in the fragmentation functions has a small effect, at least in the single-particle inclusive cross sections.

For nonscaling fragmentation functions $G_{C/c}=G_{C/c}(z,Q^2)$ or for gluon distributions $g(x,Q_0^2)\sim(1-x)^{10}$ and $\sim(1-x)^3$ (see next section) these conclusions remain qualitatively the same.

V. COMPARISON WITH EXPERIMENT AND CONCLUSIONS

The shape of the gluon distribution at $Q=Q_0$ is to a great extent unknown. Writing

$$g(x,Q_0^2)\sim(1-x)^\gamma\,, \tag{5.1}$$

the following range of values appears to be likely for the parameter γ:

$$3\leq\gamma\leq10\,. \tag{5.2}$$

This range is suggested from a model in which gluons and $q\bar{q}$ pairs are radiated from a three quark state[45] and has also been considered in other QCD applications.[14,15,38] In our calculations we have varied γ through the values $\gamma=3$, 5, and 10.

The predicted inclusive cross sections for $pp\to\pi^0+X$ and $pp\to\frac{1}{2}(\pi^++\pi^-)+X$ are presented and compared with data in Fig. 2 ($\gamma=5$, scaling $G_{C/c}$), Fig. 3 ($\gamma=5$, nonscaling quark $G_{C/c}$), and Fig. 4 ($\gamma=3$ and 10, scaling $G_{C/c}$). We stress that in all calculations the QCD coupling is fixed to (2.10) and $\langle\kappa_T\rangle=0.5$ GeV; thus, apart from the gluon shape parameter γ, our results are free of arbitrary parameters.

We see that inclusion of moderate κ_T effects accounts fairly well for the magnitude and the p_T dependence of the ISR data, down to $p_T\simeq2$. The basic model (including QCD effects) accounts well for the recent very large p_T ($\gtrsim$7-GeV) data.[46] This is certainly true for scaling $G_{C/c}$. As one expects, inclusion of scale violation in the quark fragmentation functions $G_{C/c}$ somewhat increases the p_T dependence and lowers the predictions (Fig. 3).

The p_T dependence of the Fermilab data ($\sqrt{s}$

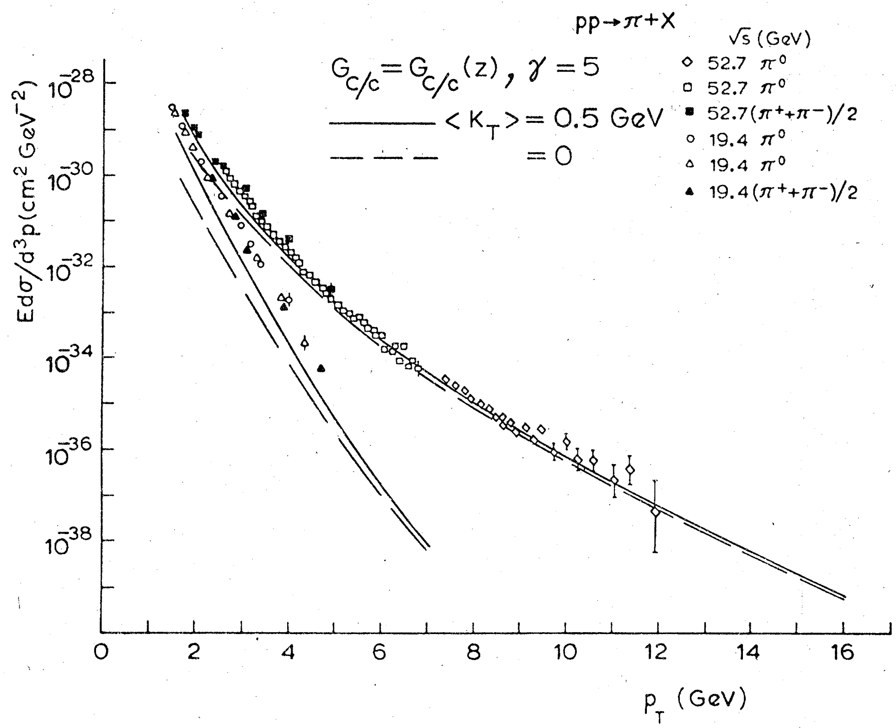

FIG. 2. Inclusive cross sections for $pp \to \pi^0 + X$ and $pp \to \frac{1}{2}(\pi^+ + \pi^-) + X$ at $\theta = 90°$. Data: ◇ Ref. 46, □ 51, ■ 52, ○ 53, △ 54, ▲ 55. Calculations for scaling fragmentation functions $G_{C/c}$ and gluon shape parameter $\lambda = 5$ (Sec. V).

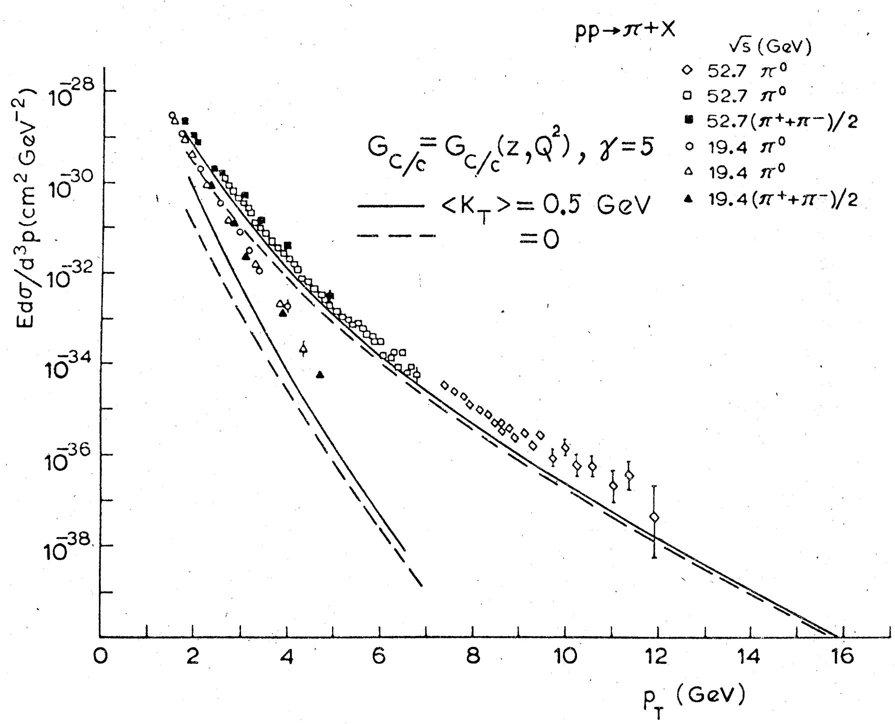

FIG. 3. As in Fig. 2 for nonscaling quark $G_{C/c}$ and $\gamma = 5$.

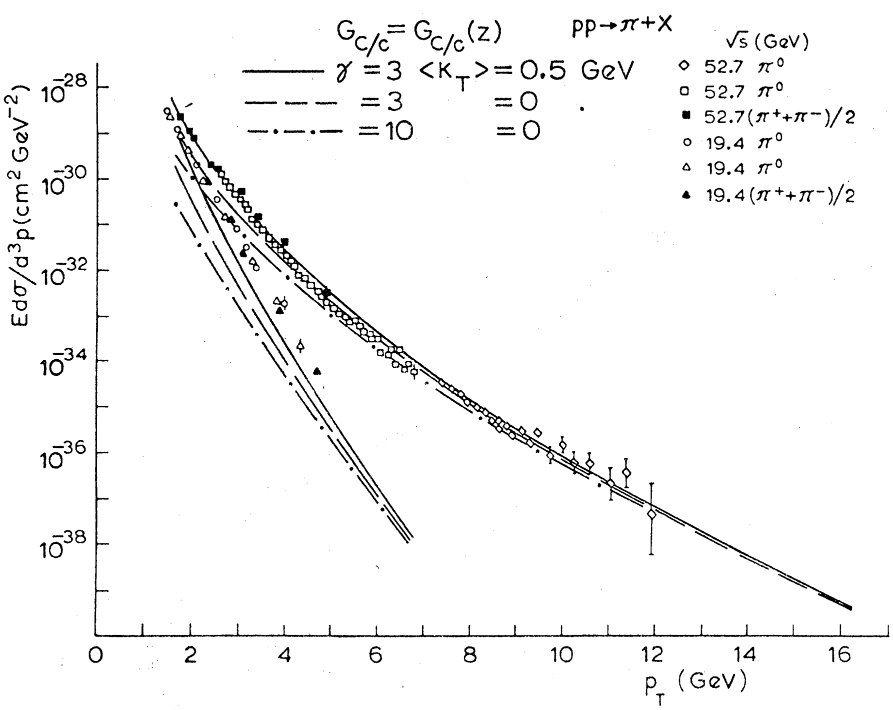

FIG. 4. As in Fig. 2 for scaling $G_{C/c}$, $\gamma=3$ and $\gamma=10$ (without κ_T effects).

$=19.4$) is also predicted reasonably well (in particular, Fig. 2); however, the predicted cross sections lie somewhat below experiment. The model is in difficulty to account for the correct energy dependence at fixed p_T; this has also been observed in other applications of the scale violating approach.[35,8,23]

A very important role in the energy dependence is played by the exact shape of the gluon distribution at $Q=Q_0$[the exponent γ in (5.1)]. Increasing γ strengthens the x_T dependence and thus suppresses the gg and qg distributions particularly at Fermilab energies (larger x_T).

To show the effect we present in Fig. 4 calculations for $\gamma=3$ and 10. For $\gamma=3$ with the inclusion of parton κ_T the predictions are in good agreement with ISR data, and in somewhat better agreement (than for $\gamma=5$, Fig. 3) with Fermilab data, in particular at $p_T \simeq 2$.

Finally, within the framework of QCD we have considered the following possibility: An important (and yet unresolved) question is the choice of the best scaling variable [Bjorken x, x' (Ref. 47), or ξ (Ref. 48)] that properly accounts for mass effects [corrections of $O(1/Q^2)$]. It is possible that such effects are still important at $\sqrt{s}=19.4$ but they practically disappear at $\sqrt{s}=52.7$ GeV. To investigate this we have replaced in the gluon distribution the variable x by

$$\xi = \frac{2x}{1+(1+4x^2M^2/Q^2)^{1/2}} \tag{5.3}$$

and have calculated, according to Appendix B and Ref. 15, the necessary changes in the sea distribution. We find that at $\sqrt{s}=19.4$ this replacement somewhat improves the agreement and at $\sqrt{s}=52.7$, it leaves our results unaffected. However, for M=nucleon mass[48] the change is very small; for larger M it becomes more significant.

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APPENDIX A

In this appendix we present the essentials of the kinematics and of our calculation of the integrals in $Ed\sigma/d^3p$, Eq. (2.3).

The kinematics of the subprocess $a+b\to c+d$ with the quarks a, b having transverse momenta $\vec{\kappa}_{Ta}, \vec{\kappa}_{Tb}$ has already been considered. We follow essentially the notation of Ref. 27, take $m_a=m_b=m$, $m_c=m_d=0$, and set

$$m_{Ti}^2=\kappa_{Ti}^2+m^2\,,\quad i=a,b,$$
$$\epsilon_{ab}=\vec{\kappa}_{Ta}\cdot\vec{\kappa}_{Tb}/m_{Ta}m_{Tb}\,,\quad \epsilon_{ic}=\vec{\kappa}_{Ti}\cdot\vec{p}_{Tc}/m_{Ti}p_{Tc}\,,$$
$$\sinh y_a=\frac{x_a\sqrt{s}}{2m_{Ta}}\,,\quad \sinh y_b=-\frac{x_b\sqrt{s}}{2m_{Tb}}\,,\quad \sinh y_c=\frac{p_{zc}}{p_{Tc}}\,. \tag{A1}$$

In the c.m. of the colliding hardons A,B the invariant variables of the subprocess are

$$\hat{s}=2m_{Ta}m_{Tb}[\cosh(y_a-y_b)-\epsilon_{ab}]+2m^2\,,$$
$$\hat{t}=-2m_{Ta}p_{Tc}[\cosh(y_a-y_c)-\epsilon_{ac}]+m^2\,, \tag{A2}$$
$$\hat{u}=-2m_{Tb}p_{Tc}[\cosh(y_b-y_c)-\epsilon_{bc}]+m^2\,.$$

Let $\vec{P}_C$ be the momentum of the observed hadron C and $\vec{\kappa}_{Tc}$ be its component perpendicular to $\vec{p}_c$; then

$$\vec{p}_c=\frac{1}{z}\,(\vec{P}_C-\vec{\kappa}_{Tc})\,. \tag{A3}$$

As stated in Sec. II, the region of integration in (2.3) is defined from

$$\hat{s}+\hat{t}+\hat{u}=2m^2 \tag{A4}$$

together with the conditions $0\le x_i,\ z\le 1$. The boundary of this region corresponds to $z=1$, when (A4) reduces to

$$Ae^{2y_a}+Be^{y_a}+c=0 \tag{A5}$$

with

$$A=e^{-y_b}-\lambda_2e^{-y_c}\,,\quad B=2\lambda_3-\lambda_1(e^{y_b-y_c}+e^{y_c-y_b})\,,$$
$$c=e^{y_b}-\lambda_2e^{y_c}\,,$$
$$\lambda_1=\frac{p_{Tc}}{m_{Ta}}\,,\quad \lambda_2=\frac{p_{Tc}}{m_{Tb}}\,, \tag{A6}$$
$$\lambda_3=\lambda_1\epsilon_{bc}+\lambda_2\epsilon_{ac}-\epsilon_{ab}+\frac{m^2}{m_{Ta}m_{Tb}}\,,$$

and

$$p_{Tc}=|\vec{p}_{Tc}|=|\vec{P}_{TC}-\vec{\kappa}_{Tc}|\,.$$

To carry the integrations $\int d^2\kappa_T$ in (2.3) we take in the c.m. of the colliding hadrons A,B

$$\vec{P}_A=\left(0,0,\frac{\sqrt{s}}{2}\right),\quad \vec{P}_B=\left(0,0,-\frac{\sqrt{s}}{2}\right),$$
$$\vec{P}_C=P_C(0,\sin\theta,\cos\theta)\,, \tag{A7}$$
$$\vec{\kappa}_{Ti}=\kappa_i(\sin\alpha_i,\cos\alpha_i,0)\,,$$
$$\vec{\kappa}_{Tc}=\kappa_c(\sin\alpha_c\sin\beta_c,\sin\alpha_c\cos\beta_c,\cos\alpha_c)\,.$$

The requirement that $\vec{\kappa}_{Tc}$ be normal to the momentum $\vec{p}_c$ of the quark c fixes

$$\cos\beta_c=\left(\frac{\kappa_c}{p_c}-\cos\alpha_c\cos\theta\right)\Big/\sin\alpha_c\sin\theta$$

and restricts κ_c to

$$P_C\cos(\alpha_c+\theta)\le\kappa_c\le P_C\cos(\alpha_c-\theta)\,. \tag{A8}$$

In terms of the angles α_i,α_c,

$$\epsilon_{ab}=\kappa_a\kappa_b\cos(\alpha_a-\alpha_b)/m_{Ta}m_{Tb}\,,$$
$$\epsilon_{ic}=\frac{1}{zp_{Tc}}\frac{\kappa_i}{m_{Ti}}[P_C\cos\alpha_i\sin\theta-\kappa_c\sin\alpha_c\cos(\alpha_i-\beta_i)]\,.$$

The integrals over $\vec{\kappa}_{Ti}$ become simply

$$\int d^2\kappa_{Ti}\to\int_0^\infty\kappa_i d\kappa_i\int_0^{2\pi}d\alpha_i \tag{A9}$$

and the integral over $\vec{\kappa}_{Tc}$ becomes

$$\int d^2\kappa_{Tc}\to\frac{2}{P_C\sin\theta}\int_0^\pi d\alpha_c\int_0^\infty d\kappa_c\frac{\kappa_c(P_C^2-\kappa_c^2)^{1/2}}{\sin\beta_c} \tag{A10}$$

together with the restriction (A8).

The numerical evaluation of the multifold integral in (2.3) has been carried out with Monte Carlo techniques. We have found that a useful procedure is to change the variables

$$\kappa_i=\kappa\ln\frac{1}{q_i}\,, \tag{A11}$$

so that

$$\int_0^\infty\kappa_i d\kappa_i\to\kappa^2\int_0^1\frac{dq_i}{q_i}\ln\frac{1}{q_i}\,.$$

In terms of the new variable q_i the exponential distribution function (4.1) becomes

$$D(\kappa_i)=\frac{b^2}{2\pi}q_i^{b\kappa}\,.$$

By choosing κ so that $b\kappa$ is slightly greater than 1 we can obtain a relatively smooth integrand. For a Gaussian distribution function a similar procedure can be used with the change

$$\kappa_i=\kappa\left(\ln\frac{1}{q_i}\right)^{1/2}\,. \tag{A11$'$}$$

Finally, for large values of κ_i, as discussed in Sec. II, x_a and/or x_b may become very small causing $f_{a/A}$ of Eq. (2.1) to become very large. To place more emphasis on the corresponding kinematical region (importance sampling) we make the change

$$x_i=w_i^h \tag{A12}$$

with some constant $h>1$.

APPENDIX B

Here we present for quarks and gluons the detailed forms of the distributions and the parameters of the fragmentation functions. We also specify the form of the subprocess cross section $d\sigma/d\hat{t}$.

Throughout the work we use the distributions of Ref. 15. The valence u and d quark inside the proton

$$\nu_u(x,Q^2)=\frac{3}{2}\frac{x^{\eta_1}(1-x)^{\eta_2}}{B(\eta_1,1+\eta_2)}-\tfrac{1}{2}\nu_d(x,Q^2)\,, \tag{B1}$$

$$\nu_d(x,Q^2)=\frac{x^{\eta_3}(1-x)^{\eta_4}}{B(\eta_3,1+\eta_4)} \tag{B2}$$

with

$$\eta_i=\eta_i(\bar{s})=\eta_i(0)+\gamma_i G\bar{s}\quad (i=1,\ldots,4)\,, \tag{B3}$$

where $G=\frac{4}{25}$ and

$$\eta_1(0)=0.70\,,\quad \eta_2(0)=2.60\,,$$

$$\eta_3(0)=0.85\,,\quad \eta_4(0)=3.35\,,$$

$$\gamma_1=-1.1\,,\quad \gamma_2=5.0\,,\quad \gamma_3=-1.5\,,\quad \gamma_4=5.1\,.$$

The sea and the gluon distribution are of the form

$$t(x,Q^2)=\frac{1}{6}\frac{\tau_2}{\tau_3}(\tau_2-\tau_3)(1-x)^{(1/\tau_3)(\tau_2+2\tau_3)}\,, \tag{B4}$$

$$g(x,Q^2)=\frac{G_2}{G_3}(G_2-G_3)(1-x)^{(G_2/G_3)-2} \tag{B5}$$

where $\tau_j=\tau_j(\bar{s})$ and $G_j=G_j(\bar{s})$, $j=2,3$. In Eq. (5.1) we set

$$G_2(0)/G_3(0)=\gamma+2\,. \tag{B6}$$

We carry out calculations for $\gamma=3$, 5, and 10 (Sec. V). Varying γ has little effect on the parameters of $t(x,Q^2)$.[14,15] The calculations of Figs. 1–3 correspond to $\gamma=5$ which leads to the following solution[15]:

$$\begin{aligned}
&\tau_j=\tfrac{1}{4}A_j+\tfrac{3}{4}B_j\,,\quad j=2,3\\
&A_2=0.11\exp(-0.427\bar{s})\,,\\
&A_3=-4.068\,68\times10^{-3}\exp(-0.667\bar{s})\,,\\
&B_2=0.429+0.169\exp(-0.747\bar{s})\\
&\qquad-0.488\exp(-0.427\bar{s})\,,\\
&B_3=2.814\,53\times10^{-3}\exp(-1.386\bar{s})\\
&\qquad+0.170\,235\,68\exp(-0.609\bar{s})\\
&\qquad-0.157\exp(-0.667\bar{s})\,,
\end{aligned} \tag{B7}$$

and

$$\begin{aligned}
&G_2=0.571-0.169\exp(-0.747\bar{s})\,,\\
&G_3=4.433\,221\times10^{-2}\exp(-0.609\bar{s})\\
&\qquad+1.306\,779\times10^{-2}\exp(-1.386\bar{s})\,.
\end{aligned} \tag{B8}$$

The solution corresponding to $\gamma=10$ is presented in detail in Ref. 15 and that corresponding to $\gamma=3$ (Fig. 4) can be easily deduced from the formalism of Ref. 15. Clearly, increasing γ produces qg and particularly gg distributions decreasing faster with x_T (Sec. V).

The parameters $m_{C/c}(0)$ and $g_{C/c}$ of the quark fragmentation functions are specified as follows: At first, in the sum rule (3.8) we introduce only the contribution of pions and kaons, leaving out baryons.[8] When c is a valence quark of $\pi^\pm$ or $K^\pm$ (e.g., u of π^+) we take

$$m_{\pi^\pm/c}(0)=m_{K^\pm/c}(0)=1\,; \tag{B9}$$

this is in accord with hadron leptoproduction analyses[49,24] as well as with counting rules.[50] When c' is a nonvalence quark of $\pi^\pm$ or $K^\pm$ (e.g., u of π^-) we take[49,24]

$$m_{\pi^\pm/c}=m_{K^\pm/c}=1.5\,. \tag{B10}$$

The same leptoproduction analyses require

$$g_{\pi^+/u}\simeq 2g_{\pi^-/u}\,, \tag{B11}$$

and fits to $pp\to K^\pm+X$ require[8]

$$g_{\pi^+/u}\simeq 2g_{K^+/u}\simeq 4g_{K^-/u}\,. \tag{B12}$$

Also, we take as usual

$$G_{\pi^0/c}(z,Q^2)=\tfrac{1}{2}[G_{\pi^+/c}(z,Q^2)+G_{\pi^-/c}(z,Q^2)]\,. \tag{B13}$$

Then the sum rule (3.8) is satisfied by

$$g_{\pi^+/u}\simeq 0.75 \tag{B14}$$

in fair agreement with Refs. 49 and 24.

Concerning the gluon fragmentation function (3.10) we also assume that the sum rule (3.11) is saturated by $C=\pi^\pm$, π^0, and $K^\pm$ (no baryons). For all these mesons we take

$$m_{C/g}=1\,; \tag{B15}$$

this (or a similar) value is also used in other calculations[23-25] and corresponds to applying usual counting rules[50] with $m_{C/g}$ in the form $m_{C/g}=2n_g-1$ and $n_g=1$. Now we take

$$g_{\pi^+/g}=g_{\pi^-/g}=g_{\pi^0/g}\,,\quad g_{K^+/g}=g_{K^-/g}\,, \tag{B16}$$

and

$$g_{\pi^+/g}=2g_{K^+/g}\,, \tag{B17}$$

as suggested by the first of Eq. (B12). Then the sum rule (3.11) implies

$$g_{\pi^+/g}\simeq 0.5\,. \tag{B18}$$

To specify $d\sigma/d\hat{t}$ for the subprocess $a+b\to c+d$ we give the form of the function $\Sigma(ab)$ of Eq. (2.9) for the cases of interest in our calculation[23,24]:

$$q_1q_2\to q_1q_2:\quad \Sigma(q_1q_2)=\tfrac{4}{9}(\hat{s}^2+\hat{u}^2)\times\frac{1}{\hat{t}^2}\,, \tag{B19}$$

$$qg \to qg: \quad \Sigma(qg) = (\hat{s}^2 + \hat{u}^2)\left(\frac{1}{\hat{t}^2} - \frac{4}{9}\frac{1}{\hat{s}\hat{u}}\right), \tag{B20}$$

$$gg \to gg: \quad \Sigma(gg) = \frac{9}{2}\left(3 - \frac{\hat{t}\hat{u}}{\hat{s}^2} - \frac{\hat{u}\hat{s}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2}\right). \tag{B21}$$

In addition, there are also contributions from the subprocesses $q\bar{q} \to gg$ and $gg \to q\bar{q}$; they are, however, very small[24] and have been neglected throughout this work.

A problem we have faced concerns the choice of the variable Q^2 in the functions $F_{a/A}(x, Q^2)$, $G_{C/c}(z, Q^2)$ and in the QCD coupling Eq. (2.10). For $qq \to qq$ Q is the four-momentum of the exchanged gluon and there is no problem: $Q^2 = -\hat{t}$ or $Q^2 = -\hat{u}$, and since we are interested in hadron production at $\theta = 90°$,

$$Q^2 = -\hat{t} = -\hat{u}. \tag{B22}$$

For $qg \to qg$ and $gg \to gg$ the choice of the proper variable is complicated by questions of gauge invariance.[23,24] In the presented calculations (Figs. 1–4) we have always chosen Q to be the four-momentum of the constituent exchanged in $ab \to cd$ irrespective of whether the exchange takes place in the $\hat{t}$, $\hat{u}$, or $\hat{s}$ channel. We carried also calculations taking always $Q^2 = -\hat{t}$ (for hadron production at $\theta = 90°$); the results were not significantly altered. Other similar calculations[23,24] have also shown that a variety of choices of Q^2 does not significantly change the results.

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