Electromagnetic muon-pair contibutions in neutrino trimuon production

V. Barger and T. Gottschalk
Physics Department, University of Wisconsin, Madison, Wisconsin 53706

R. J. N. Phillips
Rutherford Laboratory, Chilton, Didcot, Oxon, England
(Received 28 November 1977)

The electromagnetic production of $\mu^+\mu^-$ pairs in charged-current neutrino scattering is calculated for a scaling parton model. This process gives trimuons at 0.4×10^{-4} of the single-muon rate at 200 GeV. Energy and invariant-mass distributions for radiative trimuons are presented and characteristic features of azimuthal correlations are discussed.

The trimuon events[1-3] reported in deep-inelastic neutrino scattering may signal important new physical effects such as heavy-lepton, heavy-quark, or Higgs-boson production.[4] However, there are mechanisms in "old" physics that can produce trimuons, and these must be evaluated before any new physics can be fully separated. One such mechanism is the associated production of charm plus anticharm in the recoiling hadron jet, with muonic charm decays; quantum-chromodynamic calculations indicate that this gives few trimuons, of order 10^{-6}, of the single-muon rate at 200 GeV.[5] A more direct mechanism is the electromagnetic production of muon pairs via virtual photons radiated from the final muon and the interacting parton; in the present paper we calculate and discuss the contributions from this process.[6]

We find non-negligible rates for the radiative contributions to trimuon production. At $E=200$ GeV, the rate for $\nu N\to\mu^-\mu^+\mu^-X$ is 0.4×10^{-4} of the single-muon rate. Averaged over the Fermilab quadrupole triplet spectrum, for $E>100$ GeV, the predicted rate for νN including the $E_\mu>4$ GeV acceptance cuts is

$$\left(\frac{\sigma(3\mu)}{\sigma(\mu)}\right)_{\text{radiative}}=0.25\times10^{-4}\ . \tag{1}$$

This is a non-negligible background to the reported experimental rate[2]

$$\left(\frac{\sigma(3\mu)}{\sigma(\mu)}\right)_{\text{experimental}}\simeq5\times10^{-4}\ . \tag{2}$$

The quoted rates in Refs. 1 and 3 are lower. In this article we present characteristics of radiative trimuon production that can be used in identifying events of this origin.

Our calculations are based on the scaling parton model with a four-fermion weak-interaction vertex. For gauge invariance, the virtual photon must be attached in all possible ways to the charged particles in the $\nu d\to\mu^-u$ parton process. The contributions of the three graphs in Fig. 1 are

$$\begin{aligned}\mathfrak{M}_1&=-\frac{Ge^2}{\sqrt2}\bar u(p_2)\gamma_\lambda(1-\gamma_5)u(p_1)\\&\quad\times\bar u(k_2)\not\epsilon\frac{[(\not k_2+\not q)+m_\mu]}{(k_2+q)^2-m_\mu{}^2}\gamma_\lambda(1-\gamma_5)u(k_1)\ ,\\\mathfrak{M}_2&=\frac23\frac{Ge^2}{\sqrt2}\bar u(p_2)\not\epsilon\frac{[(\not p_2+\not q)+m_2]}{(p_2+q)^2-m_2{}^2}\gamma_\lambda(1-\gamma_5)u(p_1)\\&\quad\times\bar u(k_2)\gamma_\lambda(1-\gamma_5)u(k_1)\ ,\\\mathfrak{M}_3&=-\frac13\frac{Ge^2}{\sqrt2}\bar u(p_2)\gamma_\lambda(1-\gamma_5)\frac{[(\not p_1-\not q)+m_1]}{(p_1-q)^2-m_1{}^2}\not\epsilon u(p_1)\\&\quad\times\bar u(k_2)\gamma_\lambda(1-\gamma_5)u(k_1)\ ,\end{aligned}\tag{3}$$

where $\epsilon_\alpha=\bar u(k_4)\gamma_\alpha v(k_3)/q^2$ and $q=k_3+k_4$. The complete matrix element for distinguishable trileptons is

$$\mathfrak{M}=\mathfrak{M}_1+\mathfrak{M}_2+\mathfrak{M}_3\ .$$

We note that $\mathfrak{M}=0$ for $\epsilon=q$, as required for gauge invariance. For a real photon with a polarization vector ϵ, Eq. (3) is identical to the bremsstrahlung calculation of Kiskis.[7] For $\mu^-\mu^+\mu^-$ production we must antisymmetrize the matrix element,

$$\mathfrak{M}(\mu^-\mu^+\mu^-)=\mathfrak{M}(k_2,k_4)-\mathfrak{M}(k_4,k_2)\ , \tag{4}$$

and divide the phase space by a factor of 2.

To evaluate $|\mathfrak{M}|^2$ we have used the computer reduction program REDUCE.[8] After calculating the differential cross section $d\sigma(\nu_\mu d\to(3\mu)u)$ for the parton process, we set $p_1=xP_1$, $m_2=m_1=xM_N$, where P_1 is the target-nucleon momentum, and M_N is the nucleon mass. The observed cross section is then obtained by integrating over x, weighting with the parton distribution function $d(x)$,

$$d\sigma(\nu_\mu p\to(3\mu)X)=\int d\sigma(\nu_\mu d\to(3\mu)u)d(x)\,dx\ , \tag{5}$$

with $x=Q^2/(2M\nu)$. Sea-parton contributions to

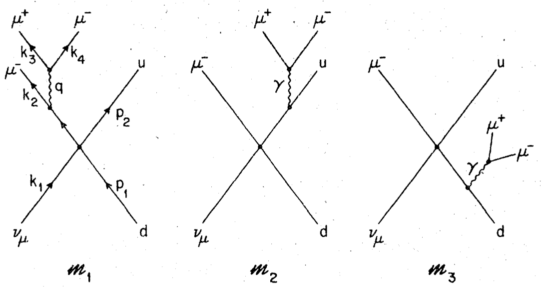

FIG. 1. Parton-model diagrams for radiative trimuon production by neutrinos.

trimuon production are very small and hence ignored here; we use the valence distribution of Ref. 9.

Over the energy range 30–350 GeV, our results for the $\nu N \to 3\mu X$ total cross section on an average nucleon target $N=(p+n)/2$ are well represented by the empirical expression

$$\frac{\sigma(3\mu)}{\sigma(\mu)} = \alpha^2(0.035 \ln^2 E - 0.19) \ , \qquad (6)$$

where α is the fine structure constant, and E is in GeV. Acceptance cuts $E_\mu > 4$ GeV reduce the rate at $E=200$ GeV from $\sigma(3\mu)/\sigma(\mu) = 0.4\times 10^{-4}$ to 0.3×10^{-4}.

Figure 2 shows invariant-mass distributions for radiative trimuon production, with spectrum average and acceptance cuts appropriate to Ref. 2. The $m(3\mu)$ distribution peaks at 1.5 GeV and has an average value $\langle m(\mu)\rangle = 3.3$ GeV. The distribution in the lower of the two $\mu^+\mu^-$ mass combinations in each event is sharply peaked at very low mass: 85% of all radiative events have $m(\mu^-\mu^+)_{\text{lower}} < 1$ GeV, 92% have $m(\mu^-\mu^+)_{\text{lower}} < 1.3$ GeV. By restricting our attention to observed trimuon events which have $m(\mu^-\mu^+)_{\text{lower}} \gtrsim 1.3$ GeV, the radiative background is largely eliminated, independent of rate considerations. Several of the reported events[2] survive such a mass cut.

Figure 3 shows the energy distributions for the radiative trimuon events for the spectrum and cuts of Ref. 2. The $E(3\mu)$ distribution is broad, with 6% of the events occurring above 200 GeV. The μ^+ and slow μ^- distributions are fairly similar, reflecting a usual common radiative-pair origin. The average energies are $\langle E(\mu^+)\rangle = 22$, $\langle E(\mu_S^-)\rangle = 18$, and $\langle E(\mu_F^-)\rangle = 64$ GeV. Events in which all three muons are energetic are highly improbable: For example, the probability that an event occurs with each muon carrying energy in excess of 60 GeV is less than 1%. One of the reported events with three highly energetic muons, $E(\mu_F^-)=92$, $E(\mu^+)=79$, and $E(\mu_S^-)=69$ GeV, is thus highly unlikely to be of radiative origin.

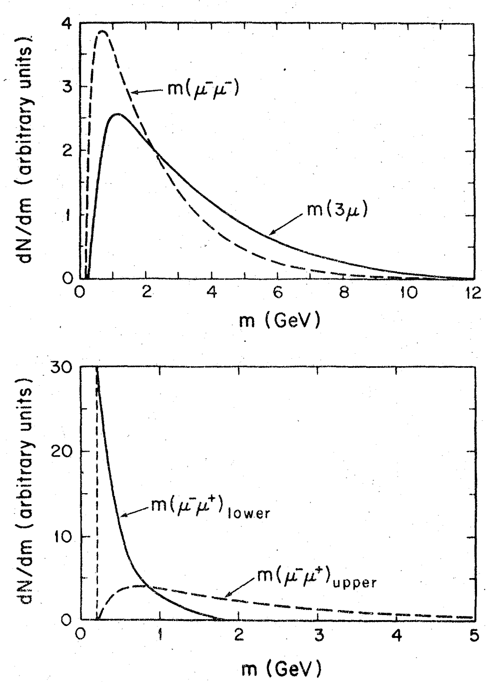

FIG. 2. Invariant-mass distributions for radiative trimuon events, with spectrum average and acceptance cuts appropriate to Ref. 2. The vertical dashed line is the kinematic lower limit $m=2m_\mu$.

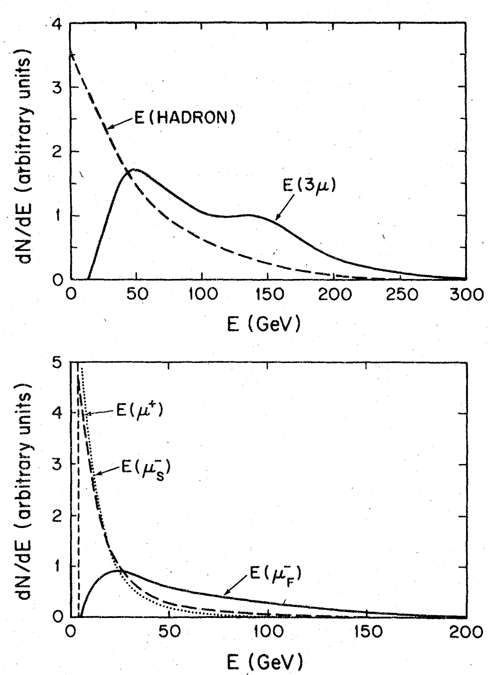

FIG. 3. Energy distributions for radiative trimuon events, with the spectrum average appropriate to Ref. 2 and $E_\mu > 4$ GeV acceptance cuts (denoted by the vertical dashed line in the lower figure).

For radiative trimuons, the azimuthal angular separation $\Delta\phi(\mu^- \mu^+)$ of the μ^- and μ^+ about the beam axis has a distinctive distribution, especially for the $\mu^-\mu^+$ pair that gives $m(\mu^-\mu^+)_{\rm lower}$. This $\Delta\phi(\mu^-\mu^+)_{\rm lower}$ distribution has a sharp peak at 0°, with 80% of the events occurring below 60°. For the spectrum and E_μ cuts of Ref. 2, the calculated radiative $\Delta\phi(\mu^-\mu^+)_{\rm lower}$ distribution can be parameterized as

$$\frac{dN}{d(\Delta\phi)} \propto \exp[-(\Delta\phi/22.3)+(\Delta\phi/81.6)^2] \ , \qquad (7)$$

with $\Delta\phi$ in degrees. This peaking of $\Delta\phi_{\rm lower}$ at 0° is correlated with the peaking of $m(\mu^-\mu^+)_{\rm lower}$ at low mass. In contrast, new physics mechanism for trimuons, such as the heavy-lepton cascade model, predict a relatively flat distribution in $\Delta\phi(\mu^-\mu^+)_{\rm lower}$ and an $m(\mu^-\mu^+)_{\rm lower}$ distribution which tends to zero at low mass.

Another interesting azimuthal separation is that between the momentum of the $\mu^-\mu^+$ pair with the lower $m(\mu^-\mu^+)$ and the momentum of the other μ^-. This distribution is approximately symmetric about $\Delta\phi = 90°$ with peaks at 0° and 180°. The results can be represented by

$$\frac{dN}{d(\Delta\phi)} \propto \exp[(\Delta\phi-90)^2/(83.3)^2] \ , \qquad (8)$$

for the spectrum and E_μ cuts of Ref. 2.

The radiative process does not lead to $\mu^-\mu^-$ events, except when the μ^+ energy is below the acceptance cutoff. This occurs at the level of 17% of the observed 3μ rate for a $E_\mu > 4$ GeV cutoff, which gives $\sigma(\mu^-\mu^-)/\sigma(\mu) \sim 0.4\times 10^{-5}$. Observation of $\mu^-\mu^-$ events at a rate comparable to trimuons[10] likely signals new-physics origins.

Theoretical uncertainty in our calculation arises from the necessary assumption that radiative μ-pair emission from the hadron vertex can be calculated in the parton model. In this connection, the Lorentz-gauge contributions to $\sigma(\nu N \to 3\mu X)/\sigma(\mu)$ at $E = 200$ GeV of the three diagrams in Fig. 1 are (in units 1.2×10^{-4})

$\lvert\mathfrak{M}_1\rvert^2$	$\lvert\mathfrak{M}_2\rvert^2$	$\lvert\mathfrak{M}_3\rvert^2$	$2\,{\rm Re}(\mathfrak{M}_1^*\mathfrak{M}_2)$	$2\,{\rm Re}(\mathfrak{M}_1^*\mathfrak{M}_3)$	$2\,{\rm Re}(\mathfrak{M}_2^*\mathfrak{M}_3)$
1	0.41	0.10	−0.98	−0.54	0.35 .

Each of these terms includes the antisymmetrization in Eq. (4). It is interesting to note that all contributions scale fairly closely to the products of the charges involved, namely,

1 0.44 0.11 −1.33 0.67 −0.44 ,

though the sign of the $\mathfrak{M}_3$ interference is reversed. Intuitively this suggests that all charges involved receive comparable accelerations. The destructive interferences reduce the final rate (compared to the diagram with only muon radiation) by a factor of 3. The extent to which this cancellation takes place may be dependent on our parton-model treatment of the radiation from the hadron lines. Each of the above terms includes the antisymmetrization in Eq. (4).

Based on our comparisons of the radiative trimuon rate and distributions with the experimental results of Ref. 2, we conclude that the observed trimuons and $\mu^-\mu^-$ events are highly likely to be of new physics origin.

We especially wish to thank J. Smith and J. Vermaseren for detailed comparisons with some of their calculations prior to publication. We also thank J. D. Bjorken, K. W. Chen, D. Cline, C. Goebel, G. Grammer, and D. Reeder for discussions. This was supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation, and in part by the Energy Research and Development Administration under Contract No. EY-76-C-02-0881, COO-881-9.

[1]B. C. Barish *et al.*, Phys. Rev. Lett. 38, 577 (1977).
[2]A. Benvenuti *et al.*, Phys. Rev. Lett. 38, 1110 (1977); 38, 1183 (1977); and report submitted to Hamburg Conference, 1977 (unpublished).
[3]M. Holder *et al.*, Phys. Lett. 69B, 377 (1977).
[4]C. Albright *et al.*, Phys. Rev. Lett. 38, 1187 (1977); Phys. Rev. D 16, 3182 (1977); V. Barger *et al.*, Phys. Rev. Lett. 38, 1190 (1977); Phys. Rev. D 16, 214 (1977); 16, 3170 (1977); 16, 3177 (1977); Phys. Lett. 70B, 243 (1977); 70B, 329 (1977); UW-Madison Report No. COO-604, 1977 (unpublished); R. Barnett and L. Chang, Phys. Lett. 72B, 233 (1977); A. Soni, Univ. Calif. Santa Barbara Report No. Th-4-77 (unpublished); Phys. Lett. 71B, 435 (1977); V. K. Cung and C. W. Kim, Phys. Lett. 69B, 359 (1977); A. De Rújula *et al.*, Phys. Rev. D 17, 151 (1978); J. Ellis *et al.*, Nucl. Phys. B131, 285 (1977); D. Horn and G. Ross, Phys. Lett. 69B, 364 (1977); P. Langacker and G. Segrè, Phys. Rev. Lett. 39, 259 (1977); Univ. of Pennsylvania Report Nos. UPR-0072T, 0079T (unpublished); B. Lee

and S. Weinberg, Phys. Rev. Lett. 38, 1237 (1977); M. A. Bêg *et al*., *ibid*. 39, 1054 (1977); Y. Tomozawa, Michigan Report No. UM-HE-77/22, 1977 (unpublished); A. Zee *et al*., Phys. Lett. 68B, 369 (1977).

[5]H. Goldberg, Phys. Rev. Lett. 39, 1598 (1977); T. Walsh *et al*., (unpublished).

[6]J. Smith and J. A. M. Vermaseren Phys. Rev. D 17, 2288 (1978) have independently calculated radiative trimuon production with similar results to those reported here.

[7]J. Kiskis, Phys. Rev. D 8, 2129 (1973).

[8]A. C. Hearn, Univ. of Utah Report No. UCP-19, 1973 (unpublished).

[9]V. Barger and R. J. N. Phillips, Nucl. Phys. B73, 269 (1974).

[10]A. Benvenuti *et al*., Phys. Rev. Lett. 35, 1199 (1975); D. Reeder, invited talk at the Ben Lee Memorial International Conference, Fermilab, 1977 (unpublished).