# Does the heavy charged lepton have its own neutrino?*

John F. Donoghue and Lincoln Wolfenstein
*Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213*
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In $SU \times U(1)$ theories with both left-handed and right-handed lepton doublets, we show that the new charged heavy lepton must have its own neutrino. Experiments are discussed which would force a similar conclusion in theories with only left-handed doublets.

If the particle causing $\mu e$ events in $e^+e^-$ annihilation is a new heavy lepton $\tau^\pm$ one must consider how to accommodate it in gauge models of the weak interaction. In particular, it is of great interest to know whether $\tau^\pm$ has its own neutrino. The apparent absence[1] (roughly an order of magnitude below the expectations of the standard Weinberg-Salam model) of parity-violating effects in atomic bismuth places severe constraints on possible models. We consider these problems within the context of $SU(2)\times U(1)$ gauge theories in which leptons are classified as doublets and singlets.

We first consider models in which the electrons and muons are coupled in right-hand doublets together with a heavy neutral lepton in addition to their usual left-handed coupling to the neutrino. Such models have no axial-vector neutral current for electrons and so provide an explanation for the absence of parity-violating effects in heavy nuclei. We discuss models without right-handed currents briefly at the end.

The simplest lepton model with right-handed currents has been analyzed in detail by Cheng and Li[2]; it involves two left-handed and two right-handed doublets. To extend the model to include the $\tau$ we assume that the $\tau$ also enters only in doublets; the alternative of a singlet $\tau$ (either left or right) leads in general to $\mu \to 3e$ in first-order weak interactions.[3] One possibility then is that in spite of its large mass the $\tau$ has associated it with a third massless neutrino $\nu_\tau$ so that the extended model has six doublets and three singlets[4]:

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L,\begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L,\begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L,\begin{pmatrix}N_e\\ e^-\end{pmatrix}_R,\begin{pmatrix}N_\mu\\ \mu^-\end{pmatrix}_R,\begin{pmatrix}N_\tau\\ \tau^-\end{pmatrix}_R,$$
$$(N_e)_L,\quad (N_\mu)_L,\quad (N_\tau)_L\,. \tag{1}$$

The alternative, which we analyze here, is based on the assumption that there are only two massless neutrinos so that $\nu_\tau$ is replaced by $(N_\tau)_L$ in the left-handed doublet. As a result the normal $\tau$ decay results from the mixing $(N_\tau)_L$ with $\nu_e$ and $\nu_\mu$. Our analysis indicates that this alternative is unacceptable so that we are left with the model of Eq. (1).

Our starting point then is six doublets and two singlets:

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L,\ \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L,\ \begin{pmatrix}N_\tau\\ \tau^-\end{pmatrix}_L,\ \begin{pmatrix}N_e\\ e^-\end{pmatrix}_R,\ \begin{pmatrix}N_\mu\\ \mu^-\end{pmatrix}_R,\ \begin{pmatrix}N_\tau\\ \tau^-\end{pmatrix}_R,$$
$$(N_e)_L,\quad (N_\mu)_L\,. \tag{2}$$

With only Higgs singlets and doublets[5] the mass matrix takes the form

$$\begin{array}{cc} R\backslash L & \begin{matrix}\nu_e & \nu_\mu & N_\tau & N_e & N_\mu\end{matrix}\\ \begin{matrix}N_e\\ N_\mu\\ N_\tau\end{matrix} & \begin{pmatrix} m_e & 0 & 0 & m_{ee} & m_{e\mu}\\ 0 & m_\mu & 0 & m_{\mu e} & m_{\mu\mu}\\ 0 & 0 & m_\tau & m_{\tau e} & m_{\tau\mu}\end{pmatrix}\end{array}. \tag{3}$$

In terms of the mass eigenstates $N_1, N_2, N_3$ the doublets have the form

$$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L,\ \begin{pmatrix}\beta_\mu\nu_\mu+m_\mu\sum_i \dfrac{v_{2i}}{m_i}N_i\\ \mu\end{pmatrix}_L,\ \begin{pmatrix}\beta_\tau\nu_\mu+m_\tau\sum_i \dfrac{v_{3i}N_i}{m_i}\\ \tau\end{pmatrix}_L,$$
$$\begin{pmatrix}\sum_i v_{1i}N_i\\ e\end{pmatrix}_R,\ \begin{pmatrix}\sum_i v_{2i}N_i\\ \mu\end{pmatrix}_R,\ \begin{pmatrix}\sum_i v_{3i}N_i\\ \tau\end{pmatrix}_R, \tag{4}$$

where we have neglected $m_e/m_i$ and $\beta_\mu, \beta_\tau$ are determined by orthonormality relations. In the approximation that $(m_\mu/m_i)^2 \ll 1$ these relations yield

$$\beta_\mu \simeq 1, \quad \beta_\tau \simeq -m_\mu m_\tau \sum_i \frac{v_{2i}v_{3i}}{m_i^2}, \qquad m_\tau^2 \sum_i \frac{v_{3i}^2}{m_i^2} \approx 1. \tag{5}$$

The quantities $v_{ji}$ are the elements of a $3\times 3$ unitary matrix.

There are two important physical consequences of this model that distinguish it from that of Eq. (1). The "normal" $\tau$ decays

$$\tau \to \nu + e + \bar{\nu}_e, \tag{6a}$$

$$\tau \to \nu + \mu + \bar{\nu}_\mu, \tag{6b}$$

$$\tau \to \nu + \text{hadrons} \tag{6c}$$

are suppressed in rate by the factor $\beta_\tau^2$ and thus the lifetime of $\tau$ may become observably long. Furthermore, the lightest of the $N_i$ (labeled $N_1$) has a mass less than $m_\tau$ and thus the competing decays

$$\tau \to N_1 + e + \bar{\nu}_e, \tag{7a}$$

$$\tau \to N_1 + \mu + \bar{\nu}_\mu, \tag{7b}$$

$$\tau \to N_1 + \text{hadrons} \tag{7c}$$

become possible. The result we find and discuss below is that in order to suppress the decays (7) adequately it is necessary to increase the lifetime of $\tau$ to an unacceptable value.

To illustrate the result, we give an explicit solution to the truncated model in which the electron and its associated particles are left out so that there are four doublets and one singlet. The results are extended to the complete model in the Appendix.

The mass matrix in the truncated model has the form

$$\begin{array}{cc} R\backslash L & \nu_\mu \quad N_\tau \quad N_\mu \\ \begin{array}{c} N_\mu \\ N_\tau \end{array} & \begin{pmatrix} m_\mu & 0 & m_0 \\ 0 & m_\tau & b \end{pmatrix} \end{array}. \tag{8}$$

Neglecting terms of order $(m_\mu/m_i)^2$ the $N_\tau$-$N_\mu$ matrix may be diagonalized yielding

$$m_1 = A_+ - A_-, \tag{9a}$$

$$m_2 = A_+ + A_-, \tag{9b}$$

$$A_\pm = \tfrac{1}{2}[b^2 + (m_0 \pm m_\tau)^2]^{1/2}, \tag{9c}$$

and the $2\times 2$ matrix $v_{ji}$ is given by

$$v_{11} = v_{22} = \cos\beta = \frac{m_1}{m_\tau}\left(\frac{m_2^2 - m_\tau^2}{m_2^2 - m_1^2}\right)^{1/2}, \qquad v_{12} = -v_{21} = \sin\beta = \frac{m_2}{m_\tau}\left(\frac{m_\tau^2 - m_1^2}{m_2^2 - m_1^2}\right)^{1/2}. \tag{10}$$

The normal decays of the $\tau$ [Eq. (6)] are proportional to $\beta_\tau^2$, which, from Eq. (5) and algebra, is given by

$$\beta_\tau^2 = \left(\frac{m_\mu}{m_\tau}\right)^2 \lambda, \qquad \lambda = (b/m_0)^2 = \left(\frac{m_\tau^2 - m_1^2}{m_1^2}\right)\left(\frac{m_2^2 - m_\tau^2}{m_2^2}\right). \tag{11}$$

For fixed values of $m_1$ and thus of the phase space for the abnormal decays, the ratio of normal to abnormal decays is proportional to $(\beta_\tau/v_{22})^2$ and thus by Eqs. (10) and (11) to $(m_2^2 - m_1^2)/m_2^2$. It follows then that to maximize this ratio we should let $m_2$ approach infinity. This leads to the inequality for fixed $m_1$

$$\left(\frac{\beta_\tau}{v_{22}}\right)^2 \le m_\mu^2(m_\tau^2 - m_1^2)/m_1^4. \tag{12}$$

This inequality is derived in the Appendix for the complete model. At the same time as the ratio of abnormal decays is minimized, the absolute lifetime of $\tau$ is minimized corresponding to

$$\beta_\tau^2 \le \left(\frac{m_\mu}{m_\tau}\right)^2 \left(\frac{m_\tau^2 - m_1^2}{m_1^2}\right). \tag{13}$$

As $m_1$ is increased toward $m_\tau$ the abnormal decays are suppressed relative to the normal, but the lifetime of the $\tau$ is increased.

The decay probability of the $\tau$ is given by

$$\Gamma(\tau) = \left(\frac{m_\tau}{m_\mu}\right)^3 \lambda C (5\times 10^5)(1+B)\ \text{sec}^{-1}, \tag{14}$$

where $C$ is the ratio of the sum of all $\tau$ decays to the decay (6a) and is approximately equal to 5, and

$$B = \Gamma(\tau \to N + \text{all})/\Gamma(\tau \to \nu_\mu + \text{all}). \tag{15}$$

The main abnormal decay when $m_1$ is not much smaller than $m_\tau$ is that of (7a) which has a decay probability

$$\Gamma(\tau \to N_1 + e^- + \bar{\nu}_e) = \frac{G^2}{15\pi^3}(m_\tau - m_1)^5 \times (1 - \tfrac{1}{2}\lambda), \tag{16}$$

where we have used the nonrelativistic $\beta$-decay formula and set $m_e = 0$. Since this decay has almost equal left and right contributions, it is almost completely vector.

In Fig. 1 we show the minimum value of the branching ratio $\Gamma(\tau \to N_1 + e + \bar{\nu}_e)/\Gamma(\tau)$ as determined from Eqs. (13) and (16) as a function of $\Gamma(\tau)$ determined from Eq. (14). For small values of $\lambda$ the results are approximately

$$\rho \equiv \frac{\Gamma(\tau \to N_1 + e + \bar{\nu}_e)}{\Gamma(\tau)} \ge 4\times 10^2(\tfrac{1}{2}\lambda)^4, \qquad \Gamma(\tau) \approx 3\times 10^{10}\ \text{sec}^{-1}(\tfrac{1}{2}\lambda) \le 7\times 10^9 \rho^{1/4}\ \text{sec}^{-1}. \tag{17}$$

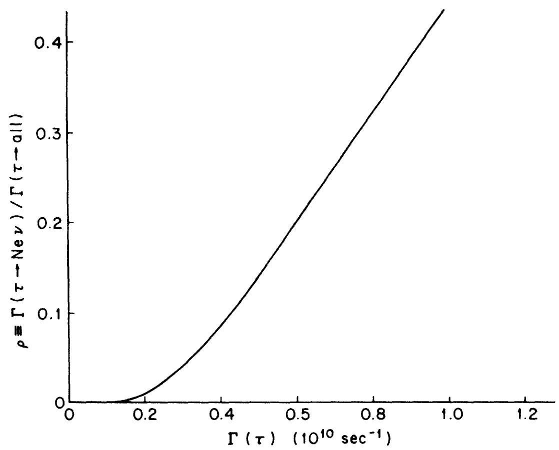


FIG. 1. The minimum value of the branching ratio $\Gamma(\tau\to N_1+e+\bar{\nu}_e)/\Gamma(\tau)$ as a function of $\Gamma(\tau)$.

As can be seen from Fig. 1 or Eq. (17) even a loose upper limit on the branching ratio of $\tau\to N_1 e\nu$ such as 25% leads to an unacceptably long lifetime of the order $2\times10^{-10}$ sec. The decays $\tau\to N_1 e\nu$ are observable because of the subsequent decay of $N_1$. It follows from the model that the principal decays of the $N_1$ are due its right-handed couplings to $e$ and $\mu$; the left-handed couplings including the neutral-current coupling to $\nu_\mu$ are down by a factor $m_\mu/m_1$ or $m_\mu/m_\tau$. The lifetime of the $N_1$ is much shorter than that of the $\tau$ in this model so that $N_1$ would indeed be expected to decay in experiments at SPEAR. Thus these abnormal $\tau$ decays show up mainly as

$$\tau^\pm\to N_1+e_a^\pm+\nu,\qquad N_1\to e_b^\pm+\begin{cases}\mu^\pm\nu\\ e_c^\pm+\nu\\ \text{hadrons}\end{cases}\tag{18}$$

In $e^+e^-$ annihilation experiments if one $\tau$ decays by the normal mode (6b) and one decays by the abnormal mode, the result is $\mu e$ events with more than two charged particles, of which about half involve charged hadrons. Note that for the case of interest, $(m_\tau-m_1)\ll m_\tau$, the decay electron $e_b$ in (18) has a spectrum similar to that from the normal $\tau$ decay and so would contribute to $\mu e$ events with the usual cuts. Thus published results[6] that no more than about 10% of the $\mu e$ events have extra charged hadrons provide a limit of about 20% on the abnormal decays. Clearly the abnormal decays would also produce a variety of events with four leptons.

So far we have considered the case that $N_1$ has a mass close to $m_\tau$; an alternative possibility is that $N_1$ is so light that the abnormal decays cannot be distinguished experimentally from the decays (6). For masses below about 400 MeV the $N_1$ would live too long to be detected in $e^+e^-$ experiments and such a low mass cannot be distinguished from zero mass in present experiments. Masses between $m_\mu$ and 400 MeV, however, would lead to anomalous $K$ decays such as

$$\begin{aligned}&K^+\to e^+ +N\,,\\&K^+\to\pi^0+e^+ +N\,,\\&K^+\to\mu^+ +N\,.\end{aligned}\tag{19}$$

From Eq. (4) these decays are proportional to $v_{11}^2$ and $v_{12}^2$; these cannot both be small since $v_{11}^2+v_{12}^2=1-O(m_1/m)^2$. It is probable then that $K$ decay experiments can rule out this possibility except conceivably for masses just around 400 MeV. The mass matrix does not allow masses below $m_\mu$.

We turn now briefly to models involving only left-handed currents. These require either ignoring the results on atomic bismuth or explaining them in some other way. The possibility that the $\tau$ can be added as a left-handed singlet to the two usual doublets, decaying then via its mixing with $\mu$ and $e$, has been ruled out by the failure to see the decay mode $\tau^-\to 3$ charged leptons.[7] Turning to models with three left-handed doublets, there are again the two possibilities (a) that as in Eq. (1) the neutral particle associated with $\tau_L$ is a third massless neutrino; (b) that as in Eq. (2) there are only two massless neutrinos, that the neutral particle associated with $\tau^-$ is a massive particle $N_\tau$, and that the normal decays of $\tau$ occur via mixing of $N_\tau$ with the massless neutrinos. In this latter scheme, which has been discussed by many authors,[8] the mixing must be fairly small leading to the structure[9]

$$\begin{pmatrix}(1-\frac{1}{2}\epsilon_e^2)\nu_e'-\frac{1}{2}\epsilon_e\epsilon_\mu\nu_\mu'-\epsilon_e N_\tau\\ e^-\end{pmatrix}_L,\quad\begin{pmatrix}(1-\frac{1}{2}\epsilon_\mu^2)\nu_\mu'-\frac{1}{2}\epsilon_e\epsilon_\mu\nu_e'-\epsilon_\mu N_\tau\\ \mu^-\end{pmatrix}_L,\quad\begin{pmatrix}N_\tau+\epsilon_e\nu_e'+\epsilon_\mu\nu_\mu'+0(\epsilon^2)\\ \tau^-\end{pmatrix}_L,$$
$$e_R\mu_R\tau_R(N_\tau)_R,\tag{20}$$

where terms $O(\epsilon^3)$ or larger are omitted. $N_\tau$ can now be heavier than $\tau$ and the mixing parameters $\epsilon_e,\epsilon_\mu$ are not constrained as $\beta_\tau$ was in the right-handed model. Thus this model is not subject to the arguments that we have used to rule out the model of Eq. (2).

Without making assumptions about the quark sector there are two constraints. The $\nu$ produced in pion decay has a probability $|\epsilon_e\epsilon_\mu|^2$ of converting into an electron. Bubble-chamber experiments give a limit

$$|\epsilon_e\epsilon_\mu|<0.1. \qquad (21a)$$

The ratio of $\pi\to e\nu$ to $\pi\to\mu\nu$ is consistent with $\mu e$ universality, but if $\mu e$ universality is violated it should be in the direction[10]

$$\epsilon_\mu \geq \epsilon_e. \qquad (21b)$$

This yields a limit on the lifetime

$$\tau(\tau^-)\gtrsim 10^{-12}\ \text{sec}.$$

However, such a short lifetime is associated with a $\epsilon_\mu{}^2=0.1$ in which case the reaction

$$\nu_\mu + N \to \tau^- + x \qquad (22)$$

has a cross section 10% as large as normal $\nu_\mu$ reactions well above the threshold and should be detectable via the electrons from $\tau^-$ decay. As the rate for reaction (22) is reduced by lowering the value of $|\epsilon_\mu|^2$ the lifetime $\tau(\tau^-)$ increases, and it becomes possible to detect reaction (22) in bubble chambers by the finite track length of $\tau^-$. Thus, a search for reaction (22) coupled with a lifetime limit from SPEAR experiments might well be able to rule out this model.

Our conclusion is that in models with right-handed currents one must associate with the heavy lepton $\tau^\pm$ its own neutrino. The alternative model in which the $\tau$ decays by sharing $\nu_e$ and $\nu_\mu$ can be ruled out by present experimental information. In the case of models with left-handed currents it should be possible by future experiments to test the model with neutrino sharing. Of course, we do not know that neutinos are exactly massless; if, as we argue, the $\tau$ has its own neutrino the mass limits on it are not nearly so sharp as for $\nu_e$ and $\nu_\mu$.

## APPENDIX

In the text we presented exact results for a truncated model in which we neglect the electron and its associated neutral particles. For the full theory we have not found an explicit solution, but the physics involved and the bounds obtained are the same. Here we describe the methods used in obtaining the bounds.

The ordering of the masses will be taken as $m_1 \leq m_2 \leq m_3$, and the current-coupling matrix $v$,

$$v=\begin{bmatrix} S_1S_2 & C_1S_2C_3+C_2S_3 & C_1S_2S_3-C_2C_3 \\ S_1C_2 & C_1C_2C_3-S_2S_3 & C_1C_2S_3+S_2C_3 \\ C_1 & -S_1C_3 & -S_1S_3 \end{bmatrix}. \qquad (A1)$$

The important constraint on the masses and angles, due to the unit normalization of the particles coupled to $\tau_L$, is, neglecting $m_\mu{}^2/m_\tau{}^2$, $m_e{}^2/m_\tau{}^2$,

$$m_\tau{}^2\left(\frac{C_1{}^2}{m_1{}^2}+\frac{S_1{}^2C_3{}^2}{m_2{}^2}+\frac{S_1{}^2S_3{}^2}{m_3{}^2}\right)=1. \qquad (A2)$$

From this it can be seen that at least one of the neutrals must be lighter than the $\tau$. We will analyze the case that only one is lighter. The ratio of normal decays to the unusual decays is governed by the coupling constants $\beta_\tau/C_1$, which are given by

$$\frac{\beta_\tau}{C_1}=\frac{-m_\mu m_\tau}{m_1{}^2}S_1\left\{C_2\left[1-m_1{}^2\left(\frac{C_3{}^2}{m_2{}^2}+\frac{S_3{}^2}{m_3{}^2}\right)\right]+\frac{C_3S_3S_2}{C_{11}}\left(\frac{m_1{}^2}{m_2{}^2}-\frac{m_1{}^2}{m_3{}^2}\right)\right\}. \qquad (A3)$$

Using Eq. (A1) this can be rewritten

$$\frac{\beta_1}{C_1}=\frac{-m_\mu(m_\tau{}^2-m_1{}^2)^{1/2}}{m_1{}^2}A \qquad (A4)$$

with

$$A=C_2\left(1-\frac{m_1{}^2C_3{}^2}{m_2{}^2}-\frac{m_1{}^2S_3{}^2}{m_3{}^2}\right)^{1/2}+C_3S_2\frac{(m_3{}^2-m_2{}^2)^{1/2}m_1}{m_2m_3}\frac{S_3}{\left[S_3{}^2+\dfrac{(m_2{}^2-m_\tau{}^2)}{m_3{}^2-m_2{}^2}\dfrac{m_3{}^2}{m^2}\right]^{1/2}}. \qquad (A5)$$

We will obtain a bound $A\leq 1$ which corresponds to Eq. (12) in the truncated version and allows us to obtain all the results shown in the text.

We have not analyzed in detail the situations when two or three heavy neutral particles are lighter than the $\tau$, due to the complexities of dealing with the many phase spaces and coupling constants at once. However, we do not expect that such a situation could be more favorable in suppressing the abnormal decays than the case where only one abnormal decay channel is allowed.

The task is to find the maximum value of $A$. The angles are assumed to always add to produce the largest value, and we may vary any parameter subject to Eq. (A2). It can be seen by inspection that, for fixed $m_1$, $m_2$ and angles, increasing $m_3$ always increases $A$. This is similar to what happened in the truncated model. Setting $m_3=\infty$, we have

$$A = C_2\left(1 - \frac{m_1^2}{m_2^2}C_3^2\right)^{1/2} + C_3S_2\frac{m_1}{m_2}\frac{S_1}{\left(S_3^2 + \frac{m_2^2 - m_\tau^2}{m_\tau^2}\right)^{1/2}}. \quad \text{(A6)}$$

This expression, as a function of $m_2$, has only one extremum, a minimum, which may either occur for $0 \le m_2 \le m_\tau$ or $m_2 > m_\tau$. Thus the maximum value of $A$ will occur at one of the end points of the range of $m_2$, i.e., $m_2 = m_\tau$ or $m_2 = \infty$. In these cases

$$\max A = \max \begin{cases} C_2\left(S_3^2 - \frac{m_\tau^2 - m_1^2}{m_\nu^2}\right)^{1/2} + C_3S_2\frac{m_1}{m_\tau} \\ C_2\,. \end{cases} \quad \text{(A7)}$$

For arbitrary values of the angles this is always less than unity.

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