Comments and Addenda

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Pontryagin density in Yang's R gauge

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A simple and computationally useful expression for the Pontryagin density in Yang's R gauge for self-dual SU(2) gauge fields is exhibited.

Yang[1] has shown how, with a suitable choice of gauge (the R gauge), the self-dual SU(2) gauge field equations reduce to Laplace-type equations for one real variable (φ) and one complex variable (ρ). The familiar Corrigan-Fairlie-Wilczek-'t Hooft ansatz turns out to be an especially simple solution to these equations. The purpose of this note is to exhibit a simple and computationally useful expression for the Pontryagin density in terms of the variables φ and ρ. As in Ref. 1 all considerations are local in character, and do not refer to global properties.

For self-dual gauge fields, the action density $S=\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}$ and the Pontryagin density $*S=\frac{1}{4}{*F}^a_{\mu\nu}F^a_{\mu\nu}=\frac{1}{8}\epsilon_{\mu\nu\lambda\rho}F^a_{\mu\nu}F^a_{\lambda\rho}$ are equal to each other, with (using the notation of Ref. 1)

$$S=F^a_{y\bar z}F^a_{\bar y z}+\tfrac{1}{2}(F^a_{y\bar y}F^a_{\bar y y}+F^a_{z\bar z}F^a_{\bar z z})\,. \tag{1}$$

A direct substitution of the R-gauge potentials into (1) gives

$$S=2\varphi^2\left[\left(\frac{\rho_y}{\varphi^2}\right)_{\bar y}\left(\frac{\bar\rho_{\bar y}}{\varphi^2}\right)_y+\left(\frac{\rho_z}{\varphi^2}\right)_{\bar z}\left(\frac{\bar\rho_{\bar z}}{\varphi^2}\right)_z+\left(\frac{\rho_z}{\varphi^2}\right)_{\bar y}\left(\frac{\bar\rho_{\bar z}}{\varphi^2}\right)_y+\left(\frac{\rho_y}{\varphi^2}\right)_{\bar z}\left(\frac{\bar\rho_{\bar y}}{\varphi^2}\right)_z\right]$$
$$+2\left\{\left[\left(\frac{\varphi_y}{\varphi}\right)_{\bar y}+\frac{\rho_y\bar\rho_{\bar y}}{\varphi^2}\right]^2+\left[\left(\frac{\varphi_z}{\varphi}\right)_{\bar z}+\frac{\rho_z\bar\rho_{\bar z}}{\varphi^2}\right]^2+2\left[\left(\frac{\varphi_y}{\varphi}\right)_{\bar z}+\frac{\rho_y\bar\rho_{\bar z}}{\varphi^2}\right]\left[\left(\frac{\varphi_{\bar y}}{\varphi}\right)_z+\frac{\rho_z\bar\rho_{\bar y}}{\varphi^2}\right]\right\}. \tag{2}$$

Equation (2), as it stands, does not make the topology of the gauge fields transparent. We attempt therefore to simplify (2) by repeated use of the self-duality equations [i.e., Eq. (27) of Ref. 1]. A slightly tedious amount of algebra leads one to the following result:

$$S={*S}=-\tfrac{1}{2}\Box\Box\ln\varphi+2\left[\partial_y\partial_{\bar y}\left(\frac{\varphi_z\varphi_{\bar z}-\rho_y\bar\rho_{\bar y}}{\varphi^2}\right)-\partial_y\partial_{\bar z}\left(\frac{\varphi_z\varphi_{\bar y}+\rho_z\bar\rho_{\bar y}}{\varphi^2}\right)\right.$$
$$\left.+\partial_z\partial_{\bar z}\left(\frac{\varphi_y\varphi_{\bar y}-\rho_z\bar\rho_{\bar z}}{\varphi^2}\right)-\partial_z\partial_{\bar y}\left(\frac{\varphi_y\varphi_{\bar z}+\rho_y\bar\rho_{\bar z}}{\varphi^2}\right)\right]. \tag{3}$$

We were unable to reduce the expression (3) any further except to note that for all published[2] self-dual gauge fields the quantity in the square brackets vanishes identically so that $S={*S}=-\frac{1}{2}\Box\Box\ln\varphi$. It is tempting to conjecture, though we have no proof, that $S={*S}=-\frac{1}{2}\Box\Box\ln\varphi$ for all self-dual gauge fields. The expression ${*S}=-\frac{1}{2}\Box\Box\ln\varphi$ can be trivially integrated over four-dimensional Euclidean space to yield a topological characteristic of the gauge field, i.e., the Pontryagin index.

Note added. Recently E. Corrigan *et al.* [Phys. Lett. 72B, 354 (1978)] have constructed the sequence of Atiyah-Ward (AW) *Ansätze* A_l ($l=1,2,3,\ldots$) for self-dual SU(2) gauge fields. The Pontryagin density ${*S}_l$ for A_l satisfies the following striking "superposition" principle:

$$^*S_l=\sum_{k=1}^{l}\left(-\tfrac{1}{2}\Box\Box\ln{}^{(k)}\varphi\right),$$

where $^{(k)}\varphi$ is Yang's R-gauge φ function for the AW *Ansatz* A_k. This result follows from the very simple transformation properties of Eq. (3) under the Backlund transformation used in generating A_l.

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[1]C. N. Yang, Phys. Rev. Lett. 38, 1377 (1977).
[2]R. Jackiw, C. Nohl, and C. Rebbi, Phys. Rev. D 15, 1642 (1977).