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Influence of form factors on the energy bounds for dileptons

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Dileptons originating from the three-particle decay of scalar, spin-1/2, and vector particles are considered. The influence of form factors on the energy bounds of Pais and Treiman is investigated. The analysis shows that no large deviations from the case without form factors are to be expected as long as the form factors are symmetric functions of the dilepton momenta.

I. INTRODUCTION

In recent years, dileptons have been observed in several processes. They have been produced in e^+e^- annihilation,[1] inelastic $\nu(\bar{\nu})N$,[2] μN,[3] and pp scattering.[4] In any of these processes the most likely explanation seems to be the production of some new particles. There is no difficulty in identifying these particles with particles in already existing theories since gauge theories of unified weak, electromagnetic, and possibly strong interactions provide us with a very rich particle spectrum, even if one excludes the scalar Higgs particles. For example the observed $e\mu$ events in e^+e^- annihilation can be explained by the pair production of heavy charged leptons,[1] the e and μ originating in the leptonic decays of the two charged heavy leptons, respectively. These heavy leptons then could be identified with the charged heavy leptons introduced in Weinberg-Salam–type $SU(2)\times U(1)$ models with three left-handed quark doublets. There an additional charged lepton is needed if one insists on cancellation of triangle anomalies, which is required for the renormalizability of the theory. But the same $e\mu$ events can be explained by the pair production of charged red quarks of the Pati-Salam model,[5] each of which decays sequentially into a charged lepton and two neutrinos. In this case it is the existence of charged exotic gauge bosons, coupling to quarks and leptons and mixing via a symmetry-breaking mass term with the charged weak gauge bosons which makes the above decays possible. Since the decays are of sequential type one has also to assume the existence of colored vector mesons (gluons) with mass below the mass of the charged red quarks.

In inelastic μN scattering, dimuons can be explained by the production of a charmed particle with semileptonic decay.[6]

Most of the data for high-mass dilepton pairs originating in proton-nucleus collisions seem to be explainable by the production of J/ψ and ψ' particles and a color-added parton model.[4] Here the occurrence of μ-e events could be a sign for pair production of charmed mesons.

High-energy neutrino and antineutrino interactions are also a source for dilepton production. It looks as if the observed events could be explained by the production of a new (charmed) particle at the hadronic vertex, which decays semileptonically.[2] Of course, as in the other cases of dilepton production, there exist also other possible explanations for their origin. Among these, the production of neutral heavy leptons decaying into two leptons and a neutrino has been discussed as a possible mechanism responsible for dilepton events.[7] Subsequently, this mechanism has been ruled out by Pais and Treiman[8] on the basis of the experimental value for the ratio of the expectation values for the two lepton energies. Assuming the most general four-fermion pointlike interaction for a spin-$\frac{1}{2}$ heavy lepton they derived lower and upper bounds for this ratio incompatible with the experimental results. In the same way Daumens and Noirot[9] obtained similar results for spin-$\frac{3}{2}$ neutral heavy leptons. Bounds on ratios for higher moments of energy expectation values of the two leptons have been calculated by Nilles[10] who also obtained conditional bounds by relating the bounded quantities. For a spin-$\frac{1}{2}$ neutral heavy lepton, the influence of a mass for the associated neutrino on the result of Pais and Treiman has been discussed by Baulieu[11] and found to be at most 20%, which is still outside the experimental result. Assuming a sequential type of decay for the neutral heavy lepton of the form $L^0 \to L^-\mu^+\nu_\mu$, where L^- is another charged heavy lepton which in turn decays via $L^- \to \mu^-\bar{\nu}_\mu\nu_L$, he obtains for the energy ratio a value within the experimental limits for appropriate mass ratios of the two heavy leptons.

This whole discussion shows that the observation of dileptons and their energy ratio may give quite an amount of information on the basic (and composite) particle structure of the underlying theory. It seems therefore worthwhile to investigate the results of Pais and Treiman in even more detail. The general picture that emerges, if one looks for dilepton decays of neutral particles in unified gauge theories, is that such transitions may occur, even for certain spin-$\frac{1}{2}$ particles, with internal structure. In the spin-$\frac{1}{2}$ case one may, for example, consider neutral "heavy leptons" participating in strong interactions. The strong interactions in this case may be specific to the "heavy leptons"[12] or just a consequence of the unification of weak, electromagnetic, and strong interactions in an appropriate energy range.[13] Therefore the effective phenomenological Lagrangian for these decays will contain form factors. To see in a not-too-specialized way what the influence of such form factors may be on the bounds for the energy ratio, one may just multiply the pointlike matrix elements with an overall momentum-dependent form factor of suitable structure. In Secs. II, III, and IV the consequences of such a procedure are calculated for neutral spin-$\frac{1}{2}$, spin-0, and spin-1 particles, respectively. In all three cases the masses of the decay products are neglected, which allows an exact calculation of phase-space integrals with arbitrary positive powers of the external momenta. In addition, not the most general effective four-particle interaction is considered, but only a typical part giving different energy spectra for the charged lepton and antilepton, since inclusion of all possible interactions is not likely to change the result significantly. The general emerging picture is that only in the case of an overall form factor, depending asymmetrically on the two charged-lepton momenta, can considerable deviations from a pure pointlike structure be expected. This puts obvious general restrictions on the decay mechanism of the neutral particles if the experimental value of the energy ratio for the two leptons is outside the bounds for an effective pointlike structure, as is the case in high-energy neutrino interactions.

For completeness, the relevant phase-space integrals needed in the calculations are given in the Appendix.

II. DILEPTONS FROM SPIN-$\frac{1}{2}$ PARTICLES

The first case considered, and probably the most interesting one, is the decay $L^0 \to \mu^+\mu^-\nu$, where L^0 is a heavy spin-$\frac{1}{2}$ neutral particle ("heavy lepton"). Since, in the following, only decays of the above structure, that is, $A \to \mu^+\mu^- B$, with A and B some neutral particles, will be investigated, the momentum variables and masses are fixed once and for all to denote

$$p_A = p, \quad p_{\mu^-} = p_1, \quad p_{\mu^+} = p_2, \quad p_B = k\ , \tag{2.1}$$

$$m_A = m, \quad m_{\mu^\pm} = m_B = 0\ . \tag{2.2}$$

For simplicity, obvious from (2.2), mass influences of the decay products are neglected. In general, the notation follows that of Bjorken and Drell.[14]

Assuming left-handed neutrinos, the most general invariant decay amplitude originating from a vector- and axial-vector interaction is (in the charge-retention order) of the form

$$f = NF\bar{u}(k)\gamma_\mu(1-\gamma_5)u(p)\bar{u}(p_1)\gamma^\mu(1-\lambda\gamma_5)v(p_2), \tag{2.3}$$

where a nonpointlike structure has been approximated by an overall form factor F. N denotes some normalization factor which does not influence any results since it cancels in the calculation of energy ratios. In general, for simplicity, a factor N will always denote some overall normalization constant (absorbing all uninteresting factors) which drops out in the energy ratios and hence does not influence any results. Squaring (2.3), summing over final-state polarizations but keeping the spin s_μ of the initial lepton, gives the well-known result

$$\sum{}' |f|^2 = N|F|^2 k^\mu p_2^\nu\{(1-\lambda)^2 g_{\mu\nu}[p\cdot p_1 - m s\cdot p_1] + (1+\lambda)^2(p_\nu p_{1\mu} - m s_\nu p_{1\mu})\}\ . \tag{2.4}$$

At this point one has to make an assumption about the form factor F in order to be able to calculate the relevant phase-space integrals. The two most simple choices are a symmetric and an asymmetric dependence on the charged-lepton momenta, which means that one has to calculate the energy distribution for the charged lepton with form factors of the kind

$$|F_s|^2 \propto (p_1\cdot p_2)^n \tag{2.5a}$$

and

$$|F_a|^2 \propto (p_1\cdot k)^n\ . \tag{2.5b}$$

Results for more general functions can be obtained in principle from the above functions by taking power series, this will be done for some special cases.

(a) First, the case (2.5a) is considered. Taking the phase-space integral of (2.4) over the second lepton and neutrino momenta using Eq. (A4), the momentum distribution of lepton 1 in the rest system of the decaying lepton is given by

$$\rho_{s,n}^{(1)}(x,\theta) = \int \frac{d^3k}{2E_k}\frac{d^3p_2}{2E_2}\,\delta(P-p_2-k)\sum{}'|f|^2$$
$$= N\frac{m^{2(n+2)}}{2^n(n+2)(n+3)}\,x^{n+1}(1+\lambda^2)\left[g(x,\theta,\lambda) - \frac{n}{n+1}\,\frac{(3-n)(1+\lambda^2)+2\lambda(n+1)}{4(1+\lambda^2)}\,(1-x)(1+\epsilon\cos\theta)\right], \tag{2.6}$$

with

$$g(x,\theta,\lambda)=3(1-x)+\frac{(1+\lambda)^2}{4(1+\lambda^2)}(4x-3)+\epsilon\cos\theta\,\frac{1-4\lambda+\lambda^2}{1+\lambda^2}\left[(1-x)-\tfrac{3}{4}\frac{(1+\lambda)^2}{1-4\lambda+\lambda^2}(\tfrac{4}{3}x-1)\right],\quad 0\le x=2E_1/m\le 1, \tag{2.7}$$

where ϵ denotes the degree of polarization of the decaying lepton, and θ denotes the angle between polarization direction and $\vec{p}_1$. The energy expectation value for lepton 1 in the system of the decaying lepton moving with velocity v is obtained from (2.6) by making use of the Lorentz-transformation properties of the involved expressions, that is,

$$\begin{aligned}\langle E_1\rangle_{s,n}&=\pi\int_0^1 dx\int_0^\pi \sin\theta\, d\theta\, x^2(1-v\cos\theta)\rho^{(1)}_{s,n}(x,\theta)\\&=N\frac{m^{2(n+2)}}{2^{n+2}}\frac{n!}{(n+5)!}\{(\lambda^2+1)[3(13+11n+2n^2)-(\vec{\epsilon}\cdot\vec{v})(3-n)]+2\lambda(n+1)[3+(\vec{\epsilon}\cdot\vec{v})(2n+9)]\},\end{aligned} \tag{2.8}$$

where $\vec{\epsilon}$ is the polarization vector of the decaying lepton and the result has been rewritten in a rotationally invariant form. The corresponding expectation value for lepton 2 is obtained by just changing the sign of λ in (2.8). This allows one to calculate bounds for the energy ratio R of the two charged decay leptons by taking the extrema of the resulting expression over all occurring variables. In Table I the result of such a procedure for three different types of the form factor F is given. The restriction to $|b|<1$ in the third row of Table I is necessary since one has to use a power-series expansion in $p_1\cdot p_2$ in order to obtain the result. Obviously there are only small deviations from the case without form factor, where one has $\frac{1}{2}\le R\le 2$.[8]

(b) Next, the case of an asymmetric form factor, that is, Eq. (2.5b), has to be considered. The corresponding equation to (2.6) is now given by

$$\rho^{(1)}_{a,n}(x,\theta)=N\frac{m^{2(n+2)}}{2^n(n+2)(n+3)}x^{n+1}(1+\lambda^2)\left[g(x,\theta,\lambda)+\frac{n}{n+1}\,\frac{n(1+\lambda^2)+2\lambda}{2(1+\lambda^2)}(1-x)(1+\epsilon\cos\theta)\right]. \tag{2.9}$$

Owing to the asymmetry of the form factor, the momentum distribution of lepton 2 is now

$$\rho^{(2)}_{a,n}(x,\theta)=N\frac{m^{2(n+2)}}{2^{n+1}\times 3}x(1-x)^n(1+\lambda^2)g(x,\theta,-\lambda). \tag{2.10}$$

From this the energy expectation values for the two leptons, again in the system where the initial lepton moves with velocity v, are obtained to be

$$\begin{aligned}\langle E_1\rangle_{a,n}&=N\frac{m^{2(n+2)}}{2^{n+2}}\frac{n!}{(n+5)!}\{(\lambda^2+1)[3(13+14n+3n^2)-(\vec{\epsilon}\cdot\vec{v})(3+2n+n^2)]+2\lambda[3(1+4n+n^2)+(\vec{\epsilon}\cdot\vec{v})(9+8n+n^2)]\},\\\langle E_2\rangle_{a,n}&=N\frac{m^{2(n+2)}}{2^{n+2}}\frac{n!}{(n+5)!}\{(\lambda^2+1)[3(9n+13)-(\vec{\epsilon}\cdot\vec{v})(7n+3)]+2\lambda[3(3n-1)-(\vec{\epsilon}\cdot\vec{v})(5n+9)]\}.\end{aligned} \tag{2.11}$$

Calculation of the extrema of the energy ratio R gives the results collected in Table II. This result tells one that using an asymmetric overall form factor, essentially any value of R can be obtained. Of course, the above increase of R with n is just a consequence of choosing $F\propto(p_1\cdot k)^n$, if one uses $F\propto(p_2\cdot k)^n$ instead, one would obviously obtain a corresponding decreasing R.

TABLE I. Bounds for dileptons from spin-$\frac{1}{2}$ particles with a symmetric form factor.

$F(p_1\cdot p_2)$	Remarks	Bounds for $R=\frac{\langle E_1\rangle}{\langle E_2\rangle}$
$(p_1\cdot p_2)^{n/2}$	$n\ge 0$ Extrema for $n=2$ or 3	$\frac{1}{2.2}\le R\le 2.2$
$1+a(p_1\cdot p_2)+b(p_1\cdot p_2)^2$	a,b real	$0.45\le R\le 2.21$
$\left[1-\frac{2b}{m^2}(p_1\cdot p_2)\right]^{-1}$	b real, $\|b\|<1$	$0.47\le R\le 2.13$

TABLE II. Bounds for dileptons from spin-$\frac{1}{2}$ particles with an asymmetric form factor.

$F(p_1 \cdot k)$	Bounds for $R = \frac{\langle E_1 \rangle}{\langle E_2 \rangle}$
$(p_1 \cdot k)^{n/2}$	$\frac{n+2}{4} \le R \le \frac{n+4}{2}$
$1 + a(p_1 \cdot k)$	$0.34 \le R \le 3.56$

III. DILEPTONS FROM SPIN-0 PARTICLES

The decay of a scalar particle $a^0 \to b^0 \mu^+ \mu^-$, with $a^0 \ne \bar{a}^0$, $b^0 \ne \bar{b}^0$, is considered with respect to the energy ratio R of the produced dileptons. To get a nontrivial result, that is $R \ne 1$, one has to add to a scalar and pseudoscalar interaction term at least a term with tensor interaction. Of course, this means derivative couplings, but the whole amplitude is anyhow only to be considered as a phenomenological approximation to the exact amplitude resulting from some underlying theory. Therefore the invariant amplitude f is assumed to be

$$f = NF\bar{u}(p_1)[g_S(1-\lambda\gamma_5) - i g_T p_\mu k_\nu \sigma^{\mu\nu}] v(p_2). \tag{3.1}$$

Squaring this and summing over spins results in

$$\sum{}' |f|^2 = N|F|^2 \{ g_S{}^2(1+\lambda^2) p_1 \cdot p_2 + 2 g_S g_T (p \cdot p_1 k \cdot p_2 - p \cdot p_2 k \cdot p_1) + g_T{}^2 [2p \cdot k(p \cdot p_1 k \cdot p_2 + p \cdot p_2 k \cdot p_1) - (p \cdot k)^2 p_1 \cdot p_2 - 2p^2 k \cdot p_1 k \cdot p_2] \}. \tag{3.2}$$

Integrating this over phase space using the relevant integrals of the Appendix gives, for the momentum distribution and the energy expectation values in the case of a symmetric form factor (2.5a),

$$\rho^{(1)}_{s,n}(x, g_S) = N \frac{m^{2(n+1)}}{2^{n+1}} \frac{x^{n+1}}{n+2} \left\{ 4g_S{}^2(1+\lambda^2) + 4g_S g_T m^2 \left[1 - x\frac{n+4}{n+3}\right] + g_T{}^2 m^4 \left[1 - 2x\frac{n+4}{n+3} + x^2 \frac{n^2+9n+22}{(n+3)(n+4)}\right] \right\}, \tag{3.3}$$

$$\langle E_1 \rangle_{s,n} = N \frac{m^{2(n+1)}}{2^n} \frac{(n+1)!}{(n+6)!} [2g_S{}^2(1+\lambda^2)(n+3)(n+5)(n+6) - 2g_S g_T m^2(n+6) + g_T{}^2 m^4 (n+4)]. \tag{3.4}$$

There is no dependence on the velocity v of the initial scalar particle since there exists no fixed direction for a scalar particle. Therefore an additional term to $\langle E_1 \rangle$, due to a Lorentz transformation from the rest system, vanishes upon integration over the solid angle of lepton 1. The corresponding equations for lepton 2 are obtained by just changing the sign of g_S ($g_S \to -g_S$) in Eqs. (3.3) and (3.4). In the same way one gets the following results for an asymmetric form factor (2.5b):

$$\rho^{(1)}_{a,n}(x, g_S) = N \frac{m^{2(n+1)}}{2^{n+1}} \frac{x^{n+1}}{(n+1)(n+2)} \left\{ 4g_S{}^2(1+\lambda^2) + 4g_S g_T m^2 \left[1 - 2x\frac{n+2}{n+3}\right] + g_T{}^2 m^4 \left[1 - 4x\frac{n+2}{n+3} + 2x^2 \frac{2n^2+10n+11}{(n+3)(n+4)}\right] \right\},$$

$$\rho^{(2)}_{a,n}(x, g_S) = \left(\frac{m^2}{2}\right)^n (1-x)^n \rho^{(1)}_{a,n=0}(x, -g_S), \tag{3.5}$$

$$\langle E_1 \rangle_{a,n} = N \frac{m^{2(n+1)}}{2^{n+1}} \frac{n!}{(n+6)!} [4g_S{}^2(1+\lambda^2)(n+3)(n+5)(n+6) - 4m^2 g_S g_T (n^2+4n+1)(n+6) + m^4 g_T{}^2 (n^3+6n^2+9n+8)],$$

$$\langle E_2 \rangle_{a,n} = N \frac{m^{2(n+1)}}{2^{n+1}} \frac{n!}{(n+6)!} [12 g_S{}^2(1+\lambda^2)(n+5)(n+6) - 4m^2 g_S g_T (3n-1)(n+6) + m^4 g_T{}^2 (3n^2+n+8)]. \tag{3.6}$$

The bounds for the energy ratio R after taking the extrema are collected in Table III. Only the most simple form factors have been considered for, in analogy to the spin-$\frac{1}{2}$ case, the essential features already appear in this way.

TABLE III. Bounds for dileptons from spin-0 particles.

F	Bounds for $R = \frac{\langle E_1\rangle}{\langle E_2\rangle}$	Remarks; definition of parameters
$(p_1 \cdot p_2)^{n/2}$	$\frac{\alpha-\beta}{\alpha+\beta} \leq R \leq \frac{\alpha+\beta}{\alpha-\beta}$	$\alpha = [2(n+3)(n+4)(n+5)]^{1/2}$ $\beta = (n+6)^{1/2}$
1	$0.64 \leq R \leq 1.58$	Extremum in the case of a symmetric form factor
$(p_1 \cdot k)^{n/2}$	$\frac{a(b-c)-d}{f(b-c)-g} \leq R \leq \frac{a(b+c)-d}{f(b+c)-g}$	$a = n^3 + 6n^2 + 9n + 8$ $b = 2n(n+2)(n+5)$ $c = [3(n+3)(n+5)(11n^2 + 26n + 16)]^{1/2}$ $d = (n^2 + 4n + 1)k$ $f = 3n^2 + n + 8$ $g = (3n-1)k$ $k = (n+6)(2n^3 + 3n^2 - 9n - 8)$
$(p_1 \cdot k)^3$	$1.66 \leq R \leq 4.60$	In asymmetric case, bounds increase with n

IV. DILEPTONS FROM SPIN-1 PARTICLES

Finally, the decay of a neutral spin-1 particle, $A_\mu^0 \to b^0 \mu^+ \mu^-$, where again $A_\mu^0 = \bar{A}_\mu^0$, $b^0 \neq \bar{b}^0$, is investigated with respect to consequences for the energy ratio of the two leptons. The calculation proceeds in analogy to the spin-0 case. Again, one has to introduce a phenomenological derivative coupling in order to get a nontrivial result for the bounds on R. A suitable choice for the invariant decay amplitude is given by

$$f = NF\epsilon_\mu k_\nu \bar{u}(p_1)[g_S g^{\mu\nu}(1-\lambda\gamma_5) + i g_T \sigma^{\mu\nu}] v(p_2) \,. \tag{4.1}$$

Keeping the polarization vector ϵ_μ of the initial particle A_μ^0 fixed, the square of (4.1) is

$$\sum{}' |f|^2 = N|F|^2 \{ g_S^2(1+\lambda^2)(\epsilon\cdot k)^2 p_1\cdot p_2 + 2g_S g_T \epsilon\cdot k(\epsilon\cdot p_2\, k\cdot p_1 - \epsilon\cdot p_1\, k\cdot p_2) + g_T^2[2\epsilon\cdot k(\epsilon\cdot p_2\, k\cdot p_1 + \epsilon\cdot p_1\, k\cdot p_2) - (\epsilon\cdot k)^2 p_1\cdot p_2 + 2k\cdot p_1 k\cdot p_2]\}. \tag{4.2}$$

Actually there is no need to keep the polarization vector ϵ_μ since, after the first phase-space integration, terms containing ϵ_μ in the momentum-distribution function $\rho(x)$ are proportional to $(\epsilon\cdot p_1)^2$. The effect of the integration over the solid angle of lepton 1 in the calculation of $\langle E_1\rangle$ is then just the same as if one had started with the corresponding distribution averaged over all polarizations of the initial particle, owing to the assumption of vanishing lepton mass. Again the dependence on the Lorentz frame of A_μ^0 drops out in the calculation of $\langle E\rangle$. Denoting by $\bar{\rho}(x)$ the averaged momentum-distribution function, the following results are obtained for the symmetric and the asymmetric cases [(2.5a) and (2.5b), respectively]:

$$\bar{\rho}_{s,n}^{(1)}(x, g_S) = N \frac{m^{2(n+2)}}{2^{n+1}} \frac{x^{n+1}}{n+2} \left\{ g_S^2(1+\lambda^2)\left(1 - 2x\frac{n+2}{n+3} + x^2\frac{n+2}{n+4}\right) - 2g_S g_T\left[1 - 2x + x^2\frac{(n+2)(n+5)}{(n+3)(n+4)}\right] + g_T^2\left[\frac{n+9}{n+1} - 2x\frac{n^2+9n+16}{(n+1)(n+3)} + x^2\frac{n^2+9n+22}{(n+3)(n+4)}\right]\right\}. \tag{4.3}$$

$$\bar{\rho}_{s,n}^{(2)}(x, g_S) = \bar{\rho}_{s,n}^{(1)}(x, -g_S)\,,$$

$$\langle E_{1,2}\rangle_{s,n} = N\frac{m^{2(n+2)}}{2^n}\frac{n!}{(n+6)!}[3g_S^2(1+\lambda^2)(n+1)(n+4) \pm 4g_S g_T(n+1) + g_T^2(5n^2+41n+76)]\,, \tag{4.4}$$

TABLE IV. Bounds for dileptons from spin-1 particles.

F	Bounds for $R=\frac{\langle E_1\rangle}{\langle E_2\rangle}$	Remarks; definition of parameters
$(p_1\cdot p_2)^{n/2}$	$\frac{\alpha-\beta}{\alpha+\beta}\le R\le\frac{\alpha+\beta}{\alpha-\beta}$	$\alpha=[3(n+4)(5n^2+41n+76)]^{1/2}$ $\beta=2(n+1)^{1/2}$
1	$0.88\le R\le 1.14$	Extremum in the case of a symmetric form factor
$(p_1\cdot k)^{n/2}$	$\frac{a+bz_-}{c+dz_-}\le R\le\frac{a+bz_+}{c+dz_+}$ $z_\pm=\frac{1}{f}(-g\pm h)$	$a=n^3+8n^2+15n+4$ $b=9n^3+86n^2+225n+152$ $c=3n^2+9n-4$ $d=27n^2+169n+152$ $f=23n^3+225n^2+636n+608$ $g=n(n+3)(7n+40)$ $h=\{8(n+3)[9(n+3)^5+16n^4+109n^3+100n^2-189n+245]\}^{1/2}$
$(p_1\cdot k)^3$	$2.96\le R\le 3.34$	Increase of bounds with n in asymmetric case

$$\bar\rho^{(1)}_{a,n}(x,g_S)=N\frac{m^{2(n+2)}}{2^{n+1}}\frac{x^{n+1}}{(n+1)(n+2)}\left\{g_S^2(1+\lambda^2)\left[1-4x\frac{1}{n+3}+6x^2\frac{1}{(n+3)(n+4)}\right]-2g_Sg_T\left[1-2x+2x^2\frac{2n+5}{(n+3)(n+4)}\right]+g_T^2\left[8n+9-4x\frac{2n^2+9n+8}{n+3}+2x^2\frac{2n^2+10n+11}{(n+3)(n+4)}\right]\right\}. \tag{4.5}$$

$$\bar\rho^{(2)}_{a,n}(x,g_S)=\left(\frac{m^2}{2}\right)^n(1-x)^n\bar\rho^{(1)}_{a,n=0}(x,-g_S),$$

$$\langle E_1\rangle_{a,n}=N\frac{m^{2(n+2)}}{2^{n+1}}\frac{n!}{(n+6)!}[g_S^2(1+\lambda^2)(n+3)(n^2+7n+8)+2g_Sg_T(n^3+8n^2+15n+4)+g_T^2(9n^3+86n^2+225n+152)],$$
$$\langle E_2\rangle_{a,n}=N\frac{m^{2(n+2)}}{2^{n+1}}\frac{n!}{(n+6)!}[g_S^2(1+\lambda^2)(n+3)(3n+8)+2g_Sg_T(3n^2+9n-4)+g_T^2(27n^2+169n+152)]. \tag{4.6}$$

Taking the extrema in the usual manner gives the results collected in Table IV. Again only the most simple cases have been taken into account.

V. CONCLUSION

The whole calculation shows that only in the case of an asymmetric overall form factor can appreciable deviations from a value close to 1 for the ratio R be obtained. This means that, in order to obtain such a deviation, one has to build the basic interactions in such a way that structure effects emerge in a different way for the two leptons. In neutrino interactions this puts severe restrictions on models which try to explain the experimental value of R by a three-particle decay.

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APPENDIX

For completeness, the relevant phase-space integrals are given here. Owing to the approximation of vanishing masses, there is no difficulty in calculating them, which would be rather tedious without this approximation. The general phase-space integral which is needed to calculate all occurring ones is given by

$$I_n = \int \frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\delta(P-p_1-p_2)p_2^{\mu_1}\times\cdots\times p_2^{\mu_n}$$

$$=A_n^0P^{\mu_1}\times\cdots\times P^{\mu_n}+A_n^1P^2\sum_Q{}' g^{\mu_{Q_1}\mu_{Q_2}}P^{\mu_{Q_3}}\times\cdots\times P^{\mu_{Q_n}}$$

$$+\cdots+\begin{cases}(P^2)^{n/2}A_n^{n/2}\sum_Q{}' g^{\mu_{Q_1}\mu_{Q_2}}\times\cdots\times g^{\mu_{Q(n-1)}\mu_{Q_n}}, & \text{if } n \text{ even}\\ (P^2)^{(n-1)/2}A_n^{(n-1)/2}\sum_Q{}' g^{\mu_{Q_1}\mu_{Q_2}}\times\cdots\times g^{\mu_{Q(n-2)}\mu_{Q(n-1)}}P^{\mu_{Q_n}}, & \text{if } n \text{ odd}\end{cases} \tag{A1}$$

$$A_n^k=(-1)^k\frac{\pi}{2^{k+1}(n+1)\cdots(n-k+1)},\quad 0\le k\le\left[\frac{n}{2}\right],$$

where $\sum_Q'$ means sum over all inequivalent permutations Q and $[n/2]$ means the largest integer contained in $n/2$. Of course, Eq. (A1) is valid only for vanishing masses, that is, $p_1^2=p_2^2=0$. From the general formula (A1) it is easy to obtain the two phase-space integrals necessary for the spin-1 case, that is,

$$\int\frac{d^3k}{2E_k}\frac{d^3p_2}{2E_2}\delta(P-p_2-k)p_2^{\mu}k^{\nu}k^{\lambda}(p_1\cdot p_2)^n$$

$$=\frac{\pi}{2^3}\frac{(p\cdot p_1)^{n-2}}{(n+1)(n+2)(n+3)(n+4)}$$

$$\times\{8(n+1)(p\cdot p_1)^2P^{\mu}P^{\nu}P^{\lambda}+2(n+1)P^2(p\cdot p_1)^2[2(P^{\lambda}g^{\mu\nu}+P^{\nu}g^{\mu\lambda})-(n+2)P^{\mu}g^{\nu\lambda}]$$

$$+4nP^2p\cdot p_1[(n+1)(p_1^{\nu}P^{\mu}P^{\lambda}+p_1^{\lambda}P^{\mu}P^{\nu})-3p_1^{\mu}P^{\nu}P^{\lambda}]+n(n+1)P^4p\cdot p_1(p_1^{\lambda}g^{\mu\nu}+p_1^{\nu}g^{\mu\lambda}+p_1^{\mu}g^{\nu\lambda})$$

$$+n(n-1)P^4[(n+1)p_1^{\lambda}p_1^{\nu}P^{\mu}-3(p_1^{\mu}p_1^{\nu}P^{\lambda}+p_1^{\mu}p_1^{\lambda}P^{\nu})]+Gp_1^{\nu}p_1^{\mu}p_1^{\lambda}\} \tag{A2}$$

and

$$\int\frac{d^3k}{2E_k}\frac{d^3p_2}{2E_2}\delta(P-p_2-k)p_2^{\mu}k^{\nu}k^{\lambda}(p_1\cdot k)^n$$

$$=\frac{\pi}{2^3}\frac{(p\cdot p_1)^{n-2}}{(n+1)(n+2)(n+3)(n+4)}\{4(n+1)(n+2)(p\cdot p_1)^2P^{\mu}P^{\nu}P^{\lambda}-4(n+1)P^{\mu}P^2p\cdot p_1[g^{\lambda\nu}p\cdot p_1+n(p_1^{\lambda}P^{\nu}+p_1^{\nu}P^{\lambda})]$$

$$+2(n+1)(n+2)P^2p\cdot p_1[p\cdot p_1(g^{\mu\nu}P^{\lambda}+g^{\mu\lambda}P^{\nu})+np_1^{\mu}P^{\nu}P^{\lambda}]$$

$$-n(n+1)P^4[p\cdot p_1(p_1^{\lambda}g^{\mu\nu}+p_1^{\nu}g^{\mu\lambda}+p_1^{\mu}g^{\nu\lambda})+(n-1)(p_1^{\nu}p_1^{\mu}P^{\lambda}+p_1^{\mu}p_1^{\lambda}P^{\nu})]$$

$$+3n(n-1)P^4P^{\mu}p_1^{\lambda}p_1^{\nu}+G'p_1^{\mu}p_1^{\lambda}p_1^{\nu}\}. \tag{A3}$$

In (A2) and (A3) the relation $P=p-p_1$ has been used. Terms proportional to $p_1^{\mu}p_1^{\nu}p_1^{\lambda}$ are not needed, since they do not contribute in the calculation owing to the approximation of vanishing masses.

The phase-space integrals for the spin-$\frac{1}{2}$ case are obtained from (A3) by contracting with p_1^{λ} and replacing n by $n-1$. This gives

$$\int\frac{d^3k}{2E_k}\frac{d^3p_2}{2E_2}\delta(P-p_2-k)k^{\mu}p_2^{\nu}(p_1\cdot p_2)^n=\int\frac{d^3k}{2E_k}\frac{d^3p_2}{2E_2}\delta(P-p_2-k)p_2^{\mu}k^{\nu}(p_1\cdot k)^n$$

$$=\frac{\pi}{2}\frac{(p\cdot p_1)^{n-1}}{(n+2)(n+3)}\left[p\cdot p_1\left(P^{\mu}P^{\nu}+\frac{P^2}{2}g^{\mu\nu}\right)+\frac{n}{2}P^2\left(p_1^{\mu}P^{\nu}-\frac{2}{n+1}p_1^{\nu}P^{\mu}\right)+Ap_1^{\mu}p_1^{\nu}\right], \tag{A4}$$

where again the term proportional to $p_1^{\mu}p_1^{\nu}$ does not contribute in the calculation.

Finally the integrals for the spin-0 case are obtained from (A4) by multiplication with $(2/P^2)g^{\mu\nu}$, that is

$$\int\frac{d^3k}{2E_k}\frac{d^3p_2}{2E_2}\delta(P-p_2-k)(p_1\cdot p_2)^n=\int\frac{d^3k}{2E_k}\frac{d^3p_2}{2E_2}\delta(P-p_2-k)(p_1\cdot k)^n=\frac{\pi}{2(n+1)}(p\cdot p_1)^n. \tag{A5}$$

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