PHYSICAL REVIEW D VOLUME 17, NUMBER 8 15 APRIL 1978

Local and covariant gauge quantum field theories. Cluster property, superselection rules, and the infrared problem*

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(Received 30 March 1977)

General structure properties of local and covariant gauge quantum field theories are investigated and a generalized cluster property is proved. Necessary and sufficient conditions are given for the failure of the cluster property which is strictly related to "quark" confinement. The deep relation between superselection rules and the infrared problems is discussed.

I. INTRODUCTION

In recent years there has been increasing evidence (both experimental and theoretical) that gauge quantum field theories may be the only quantum field theories relevant to elementary particle physics, so that it is of some physical interest to analyze some structure properties of those theories, without relying on perturbation theory. Gauge quantum field theories (GQFT's) have a very peculiar property with respect to the other standard QFT's; whereas standard QFT can be entirely formulated in terms of fields satisfying all the standard axioms (positivity included), gauge QFT's cannot (see Sec. II). This property seems to be strongly related to the possibility of explaining some basic features of elementary particle interactions such as (i) scaling in deep-inelastic scattering and asymptotic freedom,[1] (ii) good high-energy behavior of electromagnetic and weak interactions in unified theories of these interactions,[2] (iii) nonobservability of quarks and the mechanism of quark confinement.[3,4] Also from a more fundamental or theoretical point of view GQFT's share some very nice properties not common to standard QFT's:

(a) GQFT's with asymptotic freedom exhibit a very good ultraviolet behavior, the small-distance singularities being essentially those of free fields.[1]

(b) Non-Abelian GQFT's provide a nontrivial combination of internal-symmetry and space-time groups, the internal symmetry being connected with gauge groups of the second kind.

(c) GQFT's allow a mechanism of spontaneous symmetry breaking which does not imply the existence of a massless particle for each broken generator.[2,5] On the contrary, it predicts a massless particle associated with the unbroken generator, a situation exactly realized in elementary particle physics where there does not seem to be any candidate for Goldstone bosons corresponding to internal-symmetry groups and the only existing massless boson (the photon) is associated with an unbroken internal-symmetry group.

(d) Non-Abelian GQFT's exhibit a nontrivial relation with general relativity, and they seem to incorporate as a first-order approximation some of the basic properties of general relativity, i.e., a nontrivial interplay between dynamics and space-time geometry.

It has been stressed by several authors that the above properties *cannot* be shared by standard QFT's, and one may therefore ask whether the experimental (and theoretical) evidence is in fact in the direction of a strong departure from the standard QFT scheme for elementary-particle interactions. More specifically one can ask:

(i) Are GQFT's really theories of quantized fields or do they require abandoning the concept of quantized fields in favor of Green's functions? How strong are the modifications required for the standard Wightman axioms[6] in order to include GQFT?

(ii) What remains of the analytic properties of standard QFT in GQFT?

(iii) Does the cluster property *necessarily* hold in GQFT as in standard QFT?

(iv) Can one provide a general treatment of spontaneous symmetry breaking in GQFT's at the same level of rigor as in standard QFT?

(v) How are the infrared problem and the definition of physical states in GQFT's related to the presence of local gauge groups and to the superselection rules they generate?

The present paper is an attempt to answer some of the above questions and more generally to explore the structure properties of GQFT's in the hope that one eventually gets some insight into the more fundamental question: Why does nature choose GQFT's among all the possible QFT's to describe elementary-particle interactions?

II. LOCAL AND COVARIANT GAUGE QUANTUM FIELD THEORY

In order to avoid any question of semantics and to allow an unambiguous discussion, it is necessary to spell out what we mean by a local and covariant GQFT. The following properties can be regarded as basic properties or axioms for GQFT's. Essentially, one has to give a precise meaning to the key words: *local field, covariance, gauge transformations*.

By a *local and covariant gauge quantum field theory* we mean a quantum theory satisfying the following properties.

(A) (Local fields). It is defined in terms of a set of fields ϕ_α, $\alpha = 1, \ldots, n$, i.e., operator-valued (tempered) distributions in a Hilbert space $\mathcal{H}$, with scalar product denoted by (,), having a common dense domain D. The fields ϕ_α are local, i.e., they satisfy local commutativity, and the polynomial $*$-algebra generated by the smeared fields $\phi_\alpha(f)$ will be denoted by $\mathfrak{F}$.

Physically interesting quantities such as transition amplitudes, vacuum expectation values, or Green's functions, $*$ operation, etc., are computed in terms of a bounded, Hermitian, nondegenerate[7] sesquilinear form $\langle\,,\,\rangle = (\,,\eta\,)$, and η is called the metric operator.

(B) (Covariance). There is a weakly continuous representation $U(a,\Lambda)$ of the Poincaré group, defined on the dense domain D, such that the operators $U(a,\Lambda)$ are "unitary" with respect to the product $\langle\,,\,\rangle$, i.e., $\langle U\Psi, U\Phi\rangle = \langle\Psi,\Phi\rangle\ \forall\Psi,\Phi\in D$, and the fields ϕ_α transform covariantly under $U(a,\Lambda)$.

(C) (Physical states). There is a distinguished (nontrivial and maximal) subspace $\mathcal{H}'\subset\mathcal{H}$, such that (c1) $\langle\Psi,\Psi\rangle\geq 0$, $\forall\Psi\in\mathcal{H}'$. (c2) There is a common dense domain $D'\in\mathcal{H}'$ such that $U(a,\Lambda)D'\subset D'$. (c3) The unique translationally invariant state Ψ_0 in $\mathcal{H}$ [i.e., such that $U(a,1)\Psi_0=\Psi_0$] called the vacuum state is a cyclic vector with respect to the local field algebra $\mathfrak{F}$, and it belongs to D'.

(D) (Spectral condition). The Fourier transform $\tilde{W}(q_1,\ldots,q_{n-1})$ of the n-point vacuum expectation values $\mathcal{W}(x_1,\ldots,x_n) = W(x_1-x_2,\ldots,x_{n-1}-x_n)$ of the fields ϕ_α satisfy

$$\tilde{W}(q_1,\ldots,q_{n-1}) = 0 \text{ if } q_j \notin \bar{V}_+ . \tag{2.1}$$

(E) (Gauge transformations). There exists a (nontrivial) group of local automorphisms α_Λ of $\mathfrak{F}$, depending on real C^∞ vector-valued functions Λ with components $\Lambda_a \in O_M$, $a=1,\ldots,n$ (i.e., of at most slow increase), such that the infinitesimal action of α_Λ on the fields ϕ_α

$$\phi_\alpha(f) \to \phi_\alpha(f) + \delta^\Lambda\phi_\alpha(f)$$

is generated by local operators $J^\Lambda_\mu(x)$ in the following sense:

$$-i\,\delta^\Lambda\phi_\alpha(f) = \lim_{R\to\infty}[Q^\Lambda_R, \phi_\alpha(f)] \equiv \lim_{R\to\infty}\int f_R(|\vec{x}|)f_d(x_0)[J^\Lambda_0(x),\phi_\alpha(f)]\,d^4x . \tag{2.2}$$

[$f_R\in\mathfrak{D}(\mathrm{R}^3)$, $f_R(|\vec{x}|)=1$ for $|\vec{x}|<R$, $f_R(|\vec{x}|)=0$ for $|\vec{x}|>R+\epsilon$, $f_d\in\mathfrak{D}(\mathrm{R})$, $\int f_d(x_0)dx_0=1$.] $\delta^\Lambda\phi_\alpha$ depends linearly and continuously (in the O_M topology) on Λ, and $\partial^\mu J^\Lambda_\mu(x)=0$. The automorphisms α_Λ are called *local gauge transformations*. The elements α_Λ corresponding to nonconstant Λ are called *gauge transformations of the second kind*.

The elements α_Λ corresponding to $\Lambda\equiv(\Lambda_a=\text{const}, \Lambda_b=0,\ b\neq a)$ and such that there is at least one gauge transformation of the second kind with $\Lambda\equiv(\Lambda_a\neq\text{const}, \Lambda_b=0,\ b\neq a)$ define the *subgroup G of local gauge transformations of the first kind.* The corresponding local generators are denoted by $J^a_\mu(x)$. They are assumed to be local translationally covariant conserved currents, with the property that there is a skew-symmetric local field $G^{\mu\nu}_a = -G^{\nu\mu}_a$ such that

$$\langle\Psi, J^\mu_a\Phi\rangle = \langle\Psi, \partial_\nu G^{\nu\mu}_a\Phi\rangle, \quad \forall\Psi,\Phi\in D', \tag{2.3}$$

$$\mathfrak{a}^a_\mu(f)D' \equiv [J^a_\mu(f) - \partial^\nu G^a_{\nu\mu}(f)]D'\subset D'. \tag{2.4}$$

The above properties are a slight adaptation of the similar well-known properties which characterize a local and covariant formulation (or gauge) of quantum electrodynamics (QED).[8] For a familiar and simple interpretation of the above concepts we refer to the Gupta-Bleuler formulation of free QED.[8,9] The following remarks are meant to provide general motivations and discussion.

Property A is just the standard definition of local fields. The requirement that the algebra $\mathfrak{F}$ is large enough to allow approximation of every vector in $\mathcal{H}$ by vectors $\in D_0\equiv\{\mathfrak{F}\Psi_0\}$ [cyclicity of the vacuum, assumption (c3)] can be regarded as a characteristic feature of what one should mean by a local "gauge." The only nonstandard element in property A is the allowed possibility that the physically interesting matrix elements are computed by using a product $\langle\,,\,\rangle$ which is not the natural scalar product in $\mathcal{H}$ ($\eta=1$ would correspond to the standard case).

Property B is essentially the standard definition of covariance in QFT. The main difference is that no commitment is made about $\eta=1$ or $\eta\neq1$. Since the covariance of the Wightman functions only requires the "unitarity" of $U(a,\Lambda)$ with respect to the product $\langle\,,\,\rangle$, in general the operators $U(a,\Lambda)$ need not be unitary and/or bounded operators.

In order to have a physically acceptable interpretation of the theory in the general case $\eta \neq 1$, one has to specify which vectors of $\mathcal{H}$ describe physical states (the correspondence need not be one-to-one). This is essentially the reason for *condition C*.

Since the matrix elements between two "physical" states $\Phi, \Psi \in \mathcal{H}'$ do not change if one adds to Φ and/or to Ψ elements $\chi \in \mathcal{H}'$ with vanishing "η norm", i.e., $\langle \chi, \chi \rangle = 0$, it is more convenient to characterize the physical state corresponding to Φ by the equivalence class $[\Phi]$. The quotient space $\mathcal{H}_{\text{phys}} = \overline{\mathcal{H}'/\mathcal{H}''}$, $\mathcal{H}'' \equiv \{\chi \in \mathcal{H}' |\ \langle \chi, \chi \rangle = 0\}$ will be called the space of physical states (without quotation marks). $\mathcal{H}_{\text{phys}}$ is equipped with the positive-definite scalar product $([\Phi], [\Psi])_{\mathcal{H}_{\text{phys}}} \equiv \langle \Phi, \Psi \rangle$ induced by $\mathcal{H}'$.

The assumption of the uniqueness of the vacuum is essentially the statement that a physical theory corresponds to a "pure phase." The physical motivations for this have been discussed in the literature[10] and we do not insist on this point, the essential argument being that the presence of more than one vacuum state would give rise to a decomposition of the Hilbert space into phases, which describe, in general, different physical theories.

The spectral *condition D* is equivalent to

$$\int d^4a\, e^{ipa} \langle \Psi, U(a)\Phi \rangle = 0, \text{ if } p \notin \overline{V}_+ , \qquad (2.5)$$

for any $\Psi, \Phi \in D_0 \equiv \{\mathfrak{F}\Psi_0\}$. The physical interpretation of the theory requires only a weak form of the spectral condition, namely,

$$\int e^{ipa} d^4a \langle \Psi, U(a)\Phi \rangle = 0, \text{ if } p \notin \overline{V}_+ \qquad (2.5')$$

for any $\Psi, \Phi \in D' \subset \mathcal{H}'$. The stronger form (2.5) is suggested by perturbation theory, and it can be justified by arguments based on the asymptotic condition, the "support of the spectrum" of $U(a)$ then being dictated by the free in/out fields for which in all the free-field gauges known to us Eq. (2.1) is satisfied.

Furthermore, the above spectral condition is strictly connected to the possibility of formulating the theory in terms of the Schwinger function (Euclidean formulation) by analytic continuation.

Property E provides a precise definition of local gauge transformations. The term local here is meant to indicate a symmetry at the level of local fields, i.e., a local automorphism of the algebra of local fields (in the Lagrangian-field-theory language such are the local transformations of the fields which leave the Lagrangian or the equations of motion invariant). Local symmetry has to be contrasted to global symmetry (or global automorphism) which is realized when the local symmetry generates a symmetry of the states (one briefly characterizes the latter case by saying that the local symmetry is not spontaneously broken). Gauge transformations depending on the space-time points ($\Lambda \neq$ const) and the corresponding subgroup of constant-phase gauge transformations are distinguished by calling them of the second and of the first kind, respectively. (Some authors use the terms local and global for this purpose.)

The group G is by definition a local internal-symmetry group of the theory; it has, however, a very special property with respect to other local internal-symmetry groups that the theory may have, namely, that of being associated with nontrivial gauge transformations of the second kind. In the Lagrangian-quantum-field-theory language one would say that the equations of motion are not only invariant under the following transformation of the "charged" fields

$$\psi(f) \to \psi(f) + i\Lambda_a Q^a \psi(f). \qquad (2.6)$$

$\Lambda_a =$ const, Q^a being a finite-dimensional representation of G, but also under the corresponding transformations of the second kind, e.g.,

$$\begin{aligned} &\psi(f) \to \psi(f) + iQ^a \psi(\Lambda_a f), \\ &A_\mu(f) \to A_\mu(f) + (\partial_\mu \Lambda)(f), \end{aligned} \qquad (2.7)$$

for suitable nonconstant functions $\Lambda \equiv [\Lambda_a = \Lambda_a(x)]$. The property of being associated in the above sense to local gauge transformations of the second kind, gives a very special character to G. It is interesting to note that the presently available experimental and theoretical evidence indicates that most of the (local) internal-symmetry groups which are relevant to elementary-particle physics are of this type. This suggests that perhaps the most interesting feature of GQFT is that they offer the possibility of such a *nontrivial combination of internal-symmetry and space-time groups*. Furthermore, the relation between the internal-symmetry group G and the space-time-dependent gauge transformations of the second kind is of the same type as the relation between the Lorentz transformations and the space-time-dependent coordinate transformations of general relativity.

Conditions (2.3), (2.4) require some comment. In describing local automorphisms of the field algebra in axiomatic quantum field theory one has to abstract their basic properties, e.g., from Lagrangian QFT, so that one may give a characterization which does not rely on the equations of motions and/or the Lagrangian function. In the case of a continuous Lie group of local automorphisms the basic feature is provided by Noether's theorem, according to which each generator Q^a of the group is not only a constant of motion but also the space integral of the fourth component of a local conserved current $J_\mu^a(x)$:

$$Q^a = \int d^3x\, J_0^a(x), \quad \partial^\mu J_\mu(x) = 0 . \tag{2.8}$$

The above equations are the basic properties which can be easily translated into QFT as a characterization of a continuous local group of automorphisms of the algebra of local fields, through its infinitesimal action

$$-i\delta^a \phi_\alpha(f) = \lim_{R\to\infty} [Q_R^a, \phi_\alpha(f)] = \lim_{R\to\infty} [J_0^a(f_R f_d), \phi_\alpha(f)], \tag{2.8'}$$

$$\partial^\mu J_\mu^a(x) = 0$$

Equations (2.8′) play a crucial role in understanding local symmetries in QFT and their spontaneous breaking. It is important to remark that the current conservation provides a local version of "charge conservation" which still retains a meaning even if a global conservation law fails to exist (symmetry breaking).

In passing from continuous (finite-dimensional) Lie groups to continuous infinite-dimensional (or gauge) Lie groups, the invariance of the Lagrangian or of the equations of motion yield an additional characterization. Clearly the Λ = const subgroup G of the gauge group is a finite-dimensional continuous group, so that again Noether's theorem applies, but now because of the connection with an infinite-dimensional (or gauge) Lie group, the currents $J_\mu^a(x)$ associated with its local generators Q_R^a have a very special property: They can be written as the divergence of antisymmetric tensors $G_{\mu\nu}^a$, $J_\mu^a = \partial^\nu G_{\nu\mu}^a$.

Condition (3) is the QFT translation of this basic feature, and we take it as *a characterization of those local internal symmetries which are associated with local gauge groups of the second kind*.[11] The deep physical consequences of condition (3) will be discussed in Sec. IV.

Conditions (3) and (4) can also be regarded as a kind of gauge invariance of the matrix elements between "physical" states. In fact, putting $\mathfrak{A}_\mu^a \equiv J_\mu^a - \partial^\nu G_{\nu\mu}^a$, by Eq. (2.3) one has $\mathfrak{A}_\mu^a \mathcal{K}' \subset \mathcal{K}''$ and therefore condition (3) can be read as the statement that the physical states are equivalence classes with respect to the gauge-type transformations $\Psi \to \Psi + \mathfrak{A}_\mu^a \mathcal{K}'$, $\Psi \in \mathcal{K}'$. It is worthwhile to stress that the existence of local gauge automorphisms (of the second kind) is strictly related to the existence of the unphysical field $\mathfrak{A}_\mu$, which has vanishing expectation values between "physical" states, but it has a nontrivial action on local fields (gauge transformations).

In condition A the possibility was left open that in some cases one might choose $\eta = 1$ or at least positive. This would imply that all the standard Wightman axioms are satisfied, and one would have a standard QFT. As we will see, the existence of a nontrivial internal-symmetry group G associated with local gauge transformations of the second kind in the sense of property E (i.e., a nontrivial combination of internal and space-time groups) implies that η must be indefinite: Thus a characteristic feature of local gauge quantum field theories is the *lack of positivity*.

Theorem 1. In a local gauge quantum field theory, if the local (internal) symmetry group G associated with local gauge transformations of the second kind (property E) has a nontrivial representation on $\mathfrak{F}$, i.e., the charges Q_R^a do not induce the trivial automorphism on $\mathfrak{F}$, then the form $\langle\ ,\ \rangle$ must be indefinite.

Proof. Consider the state $\Phi_f \equiv [J_\mu^a(f) - \partial^\nu G_{\nu\mu}^a(f)]\Psi_0$. Since $\Psi_0 \in D'$ by (c3), so does Φ_f as a consequence of condition (4). Hence, by condition (3)

$$\langle \Phi_f, \Phi_f \rangle = 0 \quad \forall f \in \mathfrak{D}(\mathrm{R}^4) . \tag{2.9}$$

If η would be semidefinite (≥ 0), the above Eq. (2.9) would imply

$$\langle \Psi, \Phi_f \rangle = 0 \quad \forall \Psi \in \mathcal{K}$$

as a consequence of a generalized Schwarz inequality, and then

$$\Phi_f = 0 \quad \forall f \in \mathfrak{D}(\mathrm{R}^4) , \tag{2.10}$$

since η is not degenerate.

One can show that in a local (and covariant) gauge quantum field theory a generalized version of the Reeh-Schlieder theorem holds (Sec. III) so that Eq. (2.10) yields

$$J_\mu^a = \partial^\nu G_{\nu\mu}^a , \tag{2.11}$$

and therefore

$$\lim_{R\to\infty} [Q_R^a, \phi_\alpha(f)] = 0 ,$$

for any local field $\phi_\alpha(f)$, contrary to the assumption that G has a nontrivial representation on $\mathfrak{F}$. Thus, η cannot be semidefinite.

In many of the formulations of GQFT discussed in the literature, the emphasis is on the property of positivity at the price of losing locality and (manifest) covariance of the basic fields (nonlocal and noncovariant gauges). The above theorem says that this is a general fact, i.e., gauge quantum field theories are nonstandard QFT's.

The simplest candidate of a local gauge quantum field theory is provided by a local formulation of quantum electrodynamics. In this case the gauge transformations of the second kind, corresponding to the c-number functions Λ satisfying $\Box\Lambda = 0$, are generated by the local currents

$$J_\mu^\Lambda(x) = -\Lambda(x)\,\overleftrightarrow{\partial}_\mu \partial^\rho A_\rho(x) + \alpha\partial^\nu[\Lambda(x)F_{\mu\nu}(x)]\,,$$

α being an arbitrary constant, and A_ρ denoting the vector potential.

In order to discuss the energy-momentum spectrum it is convenient to introduce the truncated vacuum expectation values $\mathcal{W}^T$ through the recursive relations

$$\mathcal{W}_1(x_1) = \mathcal{W}_1^T(x_1)\,, \qquad (2.12)$$
$$\mathcal{W}_2(x_1, x_2) = \mathcal{W}_1^T(x_1)\mathcal{W}_1^T(x_2) + \mathcal{W}_2^T(x_1, x_2), \ldots .$$

Clearly, with the above definition, the truncated expectation values are translationally invariant since so are the $\mathcal{W}$'s and, therefore, one may introduce the truncated Wightman functions

$$W_{n-1}^T(x_1 - x_2, x_2 - x_3, \ldots, x_{n-1} - x_n) \equiv W_n^T(x_1, \ldots, x_n)\,. \qquad (2.13)$$

Definition 1. The Wightman functions $W(q_1 \cdots q_n)$ are said to have a "*mass gap*" or, equivalently, to satisfy a *strong spectral condition* if there is a positive μ such that

$$W^T(q_1 \cdots q_n) = 0$$

if $q_j^2 < \mu^2$ for some j. The Wightman functions are said to have no mass gap if there exists at least one Wightman function $W(q_1 \cdots q_n)$ for which there is no positive μ such that $W^T(q_1 \cdots q_n) = 0$, if $q_j^2 < \mu^2$.

Remark. Since we are working in an indefinite-metric theory the absence of a mass gap for the Wightman functions does not imply the absence of a mass gap in the physical spectrum, i.e., in the spectrum of $U(a)$ in $\mathcal{H}'$.

Definition 2. The physical spectrum of $U(a)$ is defined as the union of the supports of the $\mathcal{S}'$ distributions

$$\int e^{ipa}\langle\Psi, U(a)\,\Phi\rangle d^4a \equiv U_{\Psi\Phi}(p)$$
$$\Psi \in \mathcal{H}', \quad \Phi \in D'.$$

Clearly, by the spectral condition $U_{\Psi\Phi}(p) = 0$ if $p \not\in \overline{V}^+$. We will say that the physical spectrum of $U(a)$ has a mass gap $(0, \mu)$ if $\exists$ a positive μ such that $\forall\,\Psi \in \mathcal{H}'$, $\Phi \in D'$

$$U_{\Psi\Phi}(p) = 0 \quad \forall\, p \neq 0, \quad p^2 < \mu^2\,.$$

Otherwise the physical spectrum of $U(a)$ is said to have no mass gap.

Example. Two-dimensional QED where in the local Gupta-Bleuler gauge the Wightman functions have no mass gap, whereas the physical spectrum has a mass gap.

III. GENERAL (MATHEMATICAL) PROPERTIES OF LOCAL AND COVARIANT GAUGE QUANTUM FIELD THEORIES

Since, as shown in the previous section, gauge quantum field theories cannot satisfy all the standard Wightman axioms, it is of some interest to analyze how much of the results of standard axiomatic QFT can be generalized to GQFT. For the reasons explained in the previous section we will discuss this problem for local and covariant GQFT.

The first question of principle is whether a QFT with indefinite metric, and in particular a GQFT, is completely defined by its vacuum expectation values. More precisely, since the Green's or the Wightman functions are ultimately the objects of direct physical use, it is natural to ask whether one can define a QFT as a set of Wightman functions, satisfying some basic properties, without necessarily implying the existence of quantized fields, i.e., operator-valued distributions, in a Hilbert space $\mathcal{H}$. In the standard positive-metric case, the positivity property guarantees that quantized fields can always be constructed whose expectation values are the given set of Wightman functions (Wightman reconstruction theorem).[12,13] In the indefinite-metric case, such reconstruction is not possible in general, and one may take the attitude that indefinite-metric quantum field theories represent a substantial departure from the standard scheme of QFT's. One may, however, find a substitute of the positivity condition, which allows the introduction of a Hilbert-space structure and the reconstruction of the quantized fields *(Hilbert-space structure condition)*.[14] This condition can be formulated in terms of regularity properties of the Wightman functions, and most of the mathematical properties of the theory, such as the unitarity of space-time translations, are direct consequences of how strongly the Hilbert-space structure condition is fulfilled.[15]

The main technical difficulties in extending the standard results of axiomatic QFT to GQFT are (a) the lack of positivity and (b) the possible non-unitarity of the representation of the translation group $U(a)$. In fact, the translation invariance of the Wightman functions only requires that the space-time translation operators $U(a)$ are "unitary" with respect to the indefinite product $\langle\ ,\ \rangle$, i.e.,

$$U(a)^\dagger\,\eta U(a) = \eta\,, \qquad (3.1)$$

where the $U(a)^\dagger$ denotes the Hilbert-space Hermitian conjugate of $U(a)$, and the above equation is required to hold on the domain D. The lack of unitarity of $U(a)$ has sometimes been regarded as a violation of the spectral condition since for nonunitary operators $U(a)$ the standard spectral theorem does not apply. However, this only means that the theory may have worse singularities in momentum space than in the standard case. [For example, the Fourier transform of the two-point function need not be a measure and, e.g., $\delta'(p^2)$ singularities may appear.] This does not imply a violation of the spectral condition given in condition D, which concerns only the support properties of the Wightman functions in momentum space.

One of the main advantages of using a local and covariant formulation of GQFT is that the analyticity properties of the standard Wightman QFT carry through easily.

Theorem 2. In a QFT satisfying A–D the vacuum expectation values $W(x_1, \ldots, x_n)$ are boundary values of holomorphic functions $W(z_1, \ldots, z_n)$, $z_j = x_j - i\eta_j$, $j = 1, \ldots, n$, the domain of holomorphy being the tube $\mathcal{T}_n = \{z_1, \ldots, z_n |\ \eta_j \in V_+\}$.

As a consequence of the covariance property B, the holomorphic functions W have a single-valued continuation into the extended tube $\mathcal{T}_n' \equiv \{L_+(\mathbb{C})\mathcal{T}_n$, $L_+(\mathbb{C})$ being the set of all proper complex Lorentz transformations$\}$ and, when so extended, W transforms covariantly under $L_+(\mathbb{C})$.

Furthermore, by locality (condition A), W has an analytic continuation to the permuted extended tube.

A direct consequence of theorem 2 is that local and covariant GQFT's have PCT symmetry.

Theorem 3 (PCT symmetry). A QFT satisfying A–D has the PCT symmetry:

$$\langle\Psi_0, \varphi_\alpha(x_1)\cdots\psi_\beta(x_n)\Psi_0\rangle = i^F(-1)^J\langle\Psi_0, \psi_\beta(-x_n)\cdots\varphi_\alpha(-x_1)\Psi_0\rangle, \quad (3.2)$$

where J is the total number of undotted indices in the spinor fields $\varphi_\alpha, \ldots, \psi_\beta$ appearing on the right-hand side, and F is the number of half-odd integer spin fields.

Conversely, if the PCT condition (3.2) holds in a QFT satisfying B–D, the weak local commutativity holds in a real neighborhood of the Jost points

$$\langle\Psi_0, \varphi_\alpha(x_1)\cdots\psi_\beta(x_n)\Psi_0\rangle = i^F\langle\Psi_0, \psi_\beta(x_n)\cdots\varphi_\alpha(x_1)\Psi_0\rangle. \quad (3.3)$$

The above theorem 3 is particularly relevant since it provides an explanation of the observed PCT symmetry in elementary-particle physics, where there is evidence that some of the elementary-particle interactions are described by GQFT. This result shows how suitable is a local and covariant formulation (as defined in Sec. II) for discussing general structure properties of GQFT's, in contrast to nonlocal and noncovariant formulations.

Another deep result of standard (positive-metric) QFT, which extends to GQFT is the Reeh-Schlieder property.

Theorem 4 (Reeh-Schlieder property). Let $\mathcal{O}$ denote an open set of space-time and $\mathfrak{F}(\mathcal{O})$ the set of polynomials in the fields $\varphi_\alpha(f)$, smeared with test functions f having support contained in $\mathcal{O}$. Then in a QFT satisfying A–D Ψ_0 is a cyclic vector for $\mathfrak{F}(\mathcal{O})$, i.e., any state $\Psi \in \mathcal{H}$ can be approximated by local states $\{\mathfrak{F}(\mathcal{O})\Psi_0\}$ as closely as one likes.

Proof. One first notices that the nondegeneracy of the metric η implies that $\eta\mathcal{H}$ is dense in $\mathcal{H}$, since

$$(\Psi, \chi) = 0, \quad \forall\, \Psi \in \eta\mathcal{H}, \text{ i.e., } \Psi = \eta\Psi', \quad \Psi' \in \mathcal{H}$$

implies

$$\langle\Psi', \chi\rangle = 0, \quad \forall\, \Psi' \in \mathcal{H},$$

and therefore $\chi = 0$.

Thus, if $\mathfrak{F}(\mathcal{O})\Psi_0$ is not dense in $\mathcal{H}$, there must be at least one vector $\Psi \in \eta\mathcal{H}$, say $\Psi = \eta\Psi'$, which is orthogonal to any vector χ of the form

$$\sum_{j=0}^{N} \varphi_{\alpha_1}(f_1)\cdots\varphi_{\alpha_j}(f_j)\Psi_0, \quad f_k \in \mathfrak{D}(\mathrm{R}^4), \quad \mathrm{supp} f_k \subset \mathcal{O}:$$

$(\Psi, \chi) = 0$

or, equivalently,

$$\langle\Psi', \chi\rangle = 0.$$

By using an analyticity argument as in the standard case, one deduces

$$\langle\Psi', \Phi\rangle = 0 \quad (12)$$

for any Φ of the form

$$\sum_{j=0}^{N} \varphi_{\alpha_1}(f_1)\cdots\varphi_{\alpha_j}(f_j)\Psi_0, \quad f_k \in \mathfrak{D}(\mathrm{R}^4).$$

Since Ψ_0 is cyclic with respect to $\mathfrak{F}$, Eq. (12) implies

$$\langle\Psi', \Phi\rangle = 0, \quad \forall\, \Phi \in \mathcal{H}$$

and therefore, by the nondegeneracy of η, $\Psi' = 0$ and $\Psi = \eta\Psi' = 0$.

Theorem 5. If $\mathcal{O}$ is an open set such that $\mathcal{O}' = \{$set of points which are spacelike with respect to every point of $\mathcal{O}\}$ is not empty and $T \in \mathfrak{F}(\mathcal{O})$, then

$$T\Psi_0 = 0$$

implies $T = 0$.

Proof. If $\Psi = A\Psi_0$, $A \in \mathfrak{F}(\mathcal{O}')$, then for any $T \in \mathfrak{F}(\mathcal{O})$

$$\langle T\Psi, \Phi\rangle = \langle AT\Psi_0, \Phi\rangle = 0, \quad \forall\, \Phi \in \{\mathfrak{F}\Psi_0\},$$

since the vectors Φ run over a dense set and the metric is nondegenerate, the above equation implies $T\Psi = 0$. This in turn yields $T = 0$, because

$\{\mathfrak{F}(\mathcal{O}')\Psi_0\}$ is dense.

By fully exploiting the nondegeneracy of the metric, one similarly proves the analog of the theorem on the irreducibility of the field operators (theorem 4-4 of Ref. 6).

IV. SUPERSELECTION RULES AND INFRARED PROBLEM

In this section we will discuss the case in which there is a nontrivial internal-symmetry group G associated with local gauge transformations of the second kind, in the sense discussed in property E, and the group G is not spontaneously broken. A theory of this type has been suggested to describe strong interactions (quantum chromodynamics or QCD).[16] For the following discussion it is useful to recall briefly the basic expected features of such a theory and their physical motivations.

The problem of the classification of low-lying baryon states in the quark model and the $\pi^0 \to 2\gamma$ decay strongly suggest that if the hadrons are made out of "quarks," the quarks obey a parastatistics of rank three.[17,18] This is equivalent[19] to the existence of an unbroken SU(3) group which commutes with all the observables, called *SU(3) color group*.

On the other hand, the results of the SLAC experiments on deep-inelastic scattering and, in particular, the observed scaling property cannot be explained in a standard QFT and the basic feature of asymptotic freedom requires a non-Abelian gauge QFT. It has then been suggested[16] that the SU(3) color group, motivated by the low-energy properties of strong interactions, has to be associated with gauge transformations of the second kind in order to explain the (high-energy) small-distance behavior of strong interactions. The result is a non-Abelian gauge theory of the type discussed in Sec. II with a nontrivial *unbroken* group $G = \mathrm{SU}(3)$, associated with gauge transformations of the second kind in the sense of property E. For the sake of concreteness we will discuss this particular case in the following, the generalization to an arbitrary compact Lie group G being straightforward.

In the definition of the QFT model of strong interactions it was explicitly assumed[16] that such a theory should exhibit three basic properties: (i) the SU(3) color group is unbroken, (ii) all observables are color neutral, (iii) all physical states are color singlets. We will discuss point iii in Sec. V. Here we will discuss i and ii and some of their consequences.

First, it is important to stress that property ii can be proved to be a consequence of i in a local gauge QFT.

Theorem 6. In a local GQFT, if the local internal-symmetry group G associated with gauge transformations of the second kind in the sense of property E is not broken, then its generators Q^i commute with all the observables, i.e., they define superselection rules.

Proof. A necessary condition for an operator A to describe an observable is that (a) it satisfies locality or miscroscopic causality, (b) its matrix elements between physical states are well defined. The locality property and its motivations have been discussed at length in the literature.[20,6] In a local GQFT the second property b means that, for any $\Psi, \Phi \in \mathcal{H}'$, the matrix elements $\langle\Psi, A\,\Phi\rangle$ depend only on the equivalence classes $[\Psi]$, $[\Phi]$, which uniquely define the corresponding physical states (see Sec. II, remark on condition C) as elements of $\mathcal{H}_{\rm phys} \equiv \overline{\mathcal{H}'/\mathcal{H}''}$. Thus, property b implies

$$\langle(\Psi+\chi_1), A(\Phi+\chi_2)\rangle = \langle\Psi, A\,\Phi\rangle \tag{4.1}$$

for any $\chi_1, \chi_2 \in \mathcal{H}''$, $\Psi, \Phi \in \mathcal{H}'$. Hence if A describes an observable one has $\forall\, \Psi, \Phi \in \mathcal{H}'$ and for any generator Q^i of G [see Eq. (2.8′)]

$$\begin{aligned}\langle\Psi, [Q^i, A]\Phi\rangle &\equiv \lim_{R\to\infty}\langle\Psi, [Q^i_R, A]\Phi\rangle\\ &= \lim_{R\to\infty}\langle\Psi, [Q^i_R - (\partial G)^i_R, A]\Phi\rangle\\ &= \lim_{R\to\infty}[\langle\mathfrak{A}^i_R\Psi, A\,\Phi\rangle - \langle\Psi, A\,\mathfrak{A}^i_R\Phi\rangle]\\ &= 0\end{aligned} \tag{4.2}$$

The first equality follows from the locality of A and Gauss's theorem,[21,8] the last equality follows from property E, Eqs. (2.3), (2.4).

Theorem 6 has a very interesting consequence peculiar to the non-Abelian case. As it is clear from the discussion in Sec. II, the physical meaning of the local charge Q^i_R is roughly that of the charge contained in the space region O_R of radius R. In fact for local charged states Ψ_{qi} localized in a region O contained in the causal domain of O_R

$$\langle\Psi_{qi}, Q^i_R\Psi_{qi}\rangle \simeq q^i\langle\Psi_{qi}, \Psi_{qi}\rangle .$$

In the Abelian (QED) case Q_R describes the electric charge confined in the region O_R, and it corresponds to a well-defined physical observable. In the non-Abelian case we have the following.

Corollary. In QCD, if the SU(3) color group is not broken, the local color charges Q^i_R cannot be observable.

Proof. The proof follows from the structure of the algebra of charges $[Q^a, Q^b_R] = if^{abc}Q^c_R$ and from theorem 6.

The above corollary implies the existence of non-Abelian superselection rules in QCD, and it has strong consequences for the labeling of physical states. Clearly, since the charges Q^i are not observable, only the Casimir operators of

$SU(3)_{color}$ can be used to label the physical states. If we decompose $\mathcal{H}'$ into a direct sum of irreducible subspaces $\mathcal{H}'_I$, with respect to the color SU(3) group we have

$$\mathcal{H}' = \oplus \mathcal{H}'_I ,$$

where the index I identifies an irreducible representation of SU(3). Thus, if $\Psi_1, \Psi_2 \in \mathcal{H}'_I$ differ *only* on some color quantum number (e.g., they are eigenstates of color charges with different eigenvalues) they describe *the same physical states*, since all the observables commute with $SU(3)_{color}$. Similarly, any coherent superposition of Ψ_1 and Ψ_2 is not observable and two different superpositions of Ψ_1, Ψ_2 define the same physical state. This means that *physical states are described by mixtures with respect to the color quantum numbers*, within a given irreducible representation of SU(3), and they are labeled only by the index I.

It is important to stress that the above argument does not involve what is usually called the mechanism of confinement; it is only a consequence of the (local) underlying (non-Abelian) SU(3) color symmetry. Furthermore, the above argument implies that if such a theory has particlelike states with nonvanishing I (e.g., quark and gluon states), they will exhibit rather peculiar properties and the physical interpretation of the theory will then require a careful analysis.[22] For example, the result of a scattering experiment will strongly depend on *how* the incoming states have been prepared.[23]

The above characterization of physical states leads to an important clarification of the infrared problem. It requires in fact that a scattering amplitude for $a+b \to c+d$ is labeled in the color space by the indices I_a, I_b, I_c, I_d which label the irreducible color SU(3) representations to which a, b, c, d belong, respectively. Thus, in a QFT formulation based on local fields one has to average over the color number of each incoming particle and sum over the color number of each outcoming particle. The important result is that with the above "averaging" procedure *required by a correct definition of physical states infrared singularities cancel.*[24]

Moreover, one can easily define a renormalized gauge-invariant coupling constant for nonneutral ($I \neq 0$) channels, a problem which has attracted much attention recently.[25] The amplitude $\langle I_a I_b | S | I_a I_b \rangle$, S being the scattering matrix, is in fact gauge invariant if S is, and this allows one to define a renormalized coupling constant by following the standard procedures.

A much deeper question is whether a non-Abelian GQF has "asymptotic" particlelike states with nonvanishing color, i.e., $I \neq 0$, and in particular whether there are particlelike states with quark and/or gluon quantum numbers. Clearly, these questions are of nonperturbative character, and one cannot rely on perturbation theory (typically infinite sums of diagrams are involved). A mechanism which prevents the existence of particlelike states with nonvanishing color (*quark confinement*) will be discussed in Sec. V.

V. CLUSTER PROPERTY IN LOCAL GAUGE QUANTUM FIELD THEORY AND QUARK CONFINEMENT

The success of the quark model and the failure of detecting quarks have given rise to speculations about possible mechanisms which would prevent the existence of particle states with the quark quantum numbers (*quark confinement*). The main (heuristic) line of thought has been that of regarding the observed hadrons as bound states of quarks with a binding potential which does not decrease at infinity. Typically for $q\bar{q}$ states speculations have been made about a $q\bar{q}$ potential, $V(r)$ behaving like r^N as $r \to \infty$, $N > 0$. Actually, there is strong experimental evidence from the P-wave charmonium states that a nonrelativistic approximation of the $q\bar{q}$ interaction involves a linearly rising potential.[26] Thus, in order to explain the phenomenon of quark confinement one should not only account for the nonobservability of quarks, but also for the experimental evidence that the heuristic picture of a linearly rising potential is valid.

When translated into the QFT language, the nonobservability of quarks means that "quarks" are associated with a basic set of local fields $\psi_i(x)$, but no particlelike asymptotic states exist with the quark quantum numbers. In this picture the quark model should be interpreted as a sort of one-particle approximation to the quantum-field-theory Green's functions. Moreover, the behavior of the potential for large spacelike separations is connected to the cluster property,[27] and then the linearly rising $q\bar{q}$ potential can be regarded as the nonrelativistic trace of the failure of the cluster property. Since the validity of the cluster property plays a crucial role in the existence of the asymptotic limit of a field operator, the failure of the cluster property for the quark fields $\psi_i(x)$ is strictly related to the fact that the states $\psi_i(f)\Psi_0$ do not have an asymptotic limit belonging to $\mathcal{H}'$. This agrees with the picture of two-dimensional QED where the cluster property fails, and one can view the dipole states as a sort of bound states of "electrons" interacting through a potential increasing at infinity.

It has been pointed out[28] that it is difficult to understand such a mechanism in a local quantum field theory since in a QFT satisfying all the standard Wightman axioms the validity of the cluster property is a theorem,[6,29] and it is strictly related to the property of locality. In the two-dimensional QED case, the failure of the cluster property has a rather accidental origin since it is crucially connected to the pathologies of the massless scalar field in two dimensions; more precisely, it is related to the fact that the Fourier transform of the two-point function of a massless scalar field is not a measure.[30,31] It is, therefore, natural to ask whether this phenomenon can survive in four dimensions without requiring a departure from local QFT, as has been suggested in the literature.

The above arguments should make it clear that the question at issue for a mechanism of quark confinement is the cluster property in GQFT.[32] Since GQFT cannot satisfy all the standard Wightman axioms, the cluster property has to be investigated anew. The advantage of using a local formulation of GQFT (as defined in Sec. II) is that in this case the above problem can be clearly posed and discussed.

Quite independently of the above physical motivations the discussion of the cluster property in indefinite-metric QFT has its own justification as a necessary step in the extension of the results of the standard (positive-metric) Wightman field theory to QFT's with indefinite metric.

We first discuss the case of an indefinite-metric local QFT in which the Wightman functions have a mass gap (Sec. II, definition 1). This implies that the theory is free from infrared singularities and in particular that the physical spectrum has a mass gap (see Sec. II, definition 2). In the standard (positive-metric) case with mass gap $(0,m)$, the cluster property holds and the clustering for large r is approached exponentially fast ($\sim e^{-mr}$) as in the large-distance behavior of the Yukawa potential.[29] In the indefinite-metric case one has a rather close analogy.

Theorem 7. In an indefinite-metric local quantum field theory satisfying properties A–D of Sec. II, if the Wightman functions have a mass gap $(0,\mu)$ (or satisfy a strong spectral condition) (definition 1, Sec. II), then the cluster property holds, i.e., for any spacelike vector a

$$\lim_{\lambda\to+\infty} W(x_1,\ldots,x_j,x_{j+1}+\lambda a,\ldots,x_n+\lambda a) - W(x_1,\ldots,x_j)W(x_{j+1},\ldots,x_n)\to 0\,,$$

the convergence being in $\mathcal{S}'$. Moreover, if $B_1(x_1)$, $B_2(x_2)$ denote two clusters

$$B_i(x_i)=\int dx_1'\cdots dx_{r(i)}' f_i(x_1',\ldots,x_{r(i)}')\times\phi(x_1'+x_i)\cdots\phi(x_{r(i)}'+x_i)\,,$$

$$f_i\in\mathcal{D}(\mathrm{R}^{4r(i)}),\quad i=1,2,\quad \xi\equiv x_1-x_2\,,$$

$D_{\varphi_1\varphi_2}\equiv$ the set of points ξ for which $\langle[B_1(x_1),B_2(x_2)]\rangle$ vanishes by locality, $D_1\equiv$ the convex closure of the complement in the plane $\{\xi,\ \xi_0=0\}$ of the intersection $D_{\varphi_1\varphi_2}\cap\{\xi,\ \xi_0=0\}$, then if ξ is spacelike to every point in D_1 and $[\xi]\equiv$ the shortest distance between ξ and D_1 is $\geq\delta>0$, one has

$$|\langle\Psi_0,B_1(x_1)B_2(x_2)\Psi_0\rangle-\langle\Psi_0,B_1(x_1)\Psi_0\rangle\langle\Psi_0,B_2(x_2)\Psi_0\rangle|\leq C[\xi]^{-3/2}\exp(-\mu[\xi])[\xi]^{2N}(1+|\xi_0|/[\xi])\,,\tag{5.1}$$

where C is a constant independent of ξ and N is a non-negative integer. The above equation (5.1) implies that if the fields $B_1(x_1),B_2(x_2)$ are separated by a large spacelike distance R, $\langle B_1(x_1)B_2(x_2)\rangle^T$ tends to zero at least as fast as $R^{-3/2}e^{-\mu R}R^{2N}$.

As already mentioned in Sec. III, the difficulties in extending the results of standard QFT to gauge field theories are the indefinite metric and the possible nonunitarity of space-time translations.[33-35] This implies that in the Jost-Lehmann-Dyson representation the spectral functions $\rho_i(m^2,\vec{y})$ are not measures and one has to modify[4] the Araki-Hepp-Ruelle proof.

The above theorem makes clear that the picture of a $q\bar{q}$ potential increasing at infinity and, in general, a failure of the cluster property is incompatible with locality if the Wightman functions have a mass gap. An infrared mechanism is therefore crucial for realizing a quark confinement in local gauge QFT[36] (*infrared slavery*).

Theorem 8.[4] In an indefinite-metric local quantum field theory satisfying properties A–D of Sec. II, if the Wightman functions do not satisfy the mass-gap condition (definition 1, Sec. II), then Eq. (5.1) of theorem 7 is replaced by

$$|\langle\Psi_0,B_1(x_1)B_2(x_2)\Psi_0\rangle^T|\leq C[\xi]^{-2}[\xi]^{2N}(1+|\xi_0|/[\xi]^2)\,,\tag{5.2}$$

and therefore the cluster property may fail if $N>0$.

Proof.[37] One considers the truncated vacuum expectation values

$$h_{12}(\xi)\equiv\langle B_1(x_1)B_2(x_2)\rangle^T,\ h_{21}(-\xi)\equiv\langle B_2(x_2)B_1(x_1)\rangle^T\,.\tag{5.3}$$

By the spectral condition, h_{12} is the boundary value of an analytic function $h_{12}(z)$, analytic in the forward tube $\mathcal{T}_+$. Similarly, $h_{21}(-\xi)$ is the boundary value of an analytic function $h_{21}(z)$, analytic in $\mathcal{T}_-$. For ξ sufficiently spacelike $h_{12}(\xi)=h_{12}(-\xi)$ by locality and, therefore, there is an analytic function $h(z)$ in $\mathcal{T}_+\cup\mathcal{T}_-\cup\{$neighborhood of sufficiently large Jost points$\}$ such that $h_{12}(z)=h(z)$ for $z\in\mathcal{T}_+$, $h_{21}(z)=h(z)$ for $z\in\mathcal{T}_-$. As a consequence of the Bros-

Epstein-Glaser theorem[38] one can write

$$h(z) = z^{2N}H(z),$$

where N is a nonpositive integer and $H(z)$ is such that $H_{12}(z) \equiv H(z)$ for $z \in \mathcal{T}_+$ is the Laplace transform of a continuous function $\tilde{H}_{12}(p)$ of at most polynomial increase and with support in $\overline{V}_+$ [similarly for $H_{21}(z) \equiv H(z)$ for $z \in \mathcal{T}_-$]. For the function

$$G(\xi) \equiv H_{12}(\xi) - H_{21}(-\xi)$$

one can therefore repeat the Araki-Hepp-Ruelle argument, and one gets Eq. (5.2).

One might think that the upper bound (5.2) on the generic behavior of the truncated expectation value $\langle B_1(x_1)B_2(x_2)\rangle^T$ for large spacelike separations is too weak and that there could be room for a stronger bound implying the validity of the cluster property, in general. This is not so, since one may find soluble four-dimensional local GQFT models in which the bound (5.2) is saturated and the cluster property fails. Thus, the above theorem cannot be improved if one wants to cover the general case, and it is clear that the cluster property is allowed to fail in a local GQFT compatible with locality and without requiring a nonunique vacuum.[39,40]

The next problem is to characterize the case in which the large spacelike behavior of $\langle B_1(x_1)B_2(x_2)\rangle^T$ is given by the right-hand side of Eq. (5.2) with $N \neq 0$.

Proposition. If $[U(a), \eta] = 0$, then $N = 0$.

Proof. The proposition follows essentially from the argument of Ref. 29.

The above proposition clarifies the role of the indefinite metric in the failure of the cluster property in a local QFT and it shows that *standard local QFT's cannot exhibit the mechanism of quark confinement* discussed at the beginning of this section. Since local GQFT require an indefinite metric, they appear as natural candidates for a mechanism of quark confinement. This statement points in the same direction as the exclusion of standard QFT's for accounting for the scaling behavior in deep-inelastic scattering.[1]

For a refined characterization of the failure of the cluster property it is useful to introduce the following.

Definition. The space-time translations $U(a)$ are said to be *unitary* (or to commute with η) *on the light cone* if there is a suitable neighborhood of the light cone $\{p^2 = 0\}$ on which the distributions

$$U_{k,j}(p) \equiv \int d^4a\, e^{ipa} \langle \Psi_0, \phi_{\alpha_1}(f_1)\cdots\phi_{\alpha_k}(f_k)U(a) \times \phi_{\beta_1}(f_1)\cdots\phi_{\beta_j}(f_j)\Psi_0\rangle$$

are measures $\forall k, j$.[41]

Theorem 9. In a local QFT with indefinite metric the cluster property fails if and only if the space-time translations are not unitary on the light cone.

Proof. From the proof of theorem 8 it follows that the cluster property fails in those channels in which the translations fail to be measures on the light cone, since then the large spacelike behavior is given by the right-hand side of Eq. (5.2) with $N \neq 0$.

A simple example of the above theorem is provided by dipole fields, for which the two-point function in momentum space is proportional to $\delta'(p^2)\theta(p_0)$ [the corresponding propagator is $\sim 1/p^4$ (Refs. 42 and 43)]. Clearly, the above theorem covers a much more general case, and it reduces the failure of the cluster property to the occurrence of (infrared) singularities in the Fourier transform of the Wightman functions. (Clearly the quark confinement requires that such singularities occur in colored channels.)

In two-dimensional QED this phenomenon looks rather accidental since the space-time translations fail to commute with the metric on the light cone because of the pathologies of the massless scalar field in two dimensions.

VI. CHARGE SCREENING AND THE MECHANISM OF CONFINEMENT

In Sec. V we discussed a possible mechanism by which charged local fields do not give rise to "asymptotic" particlelike states with the same quantum numbers. At this point it is worthwhile to distinguish two quite different situations in which this may happen.

The first case is the *charge screening* induced by the Higgs mechanism. This is crucially connected to a phenomenon of spontaneous symmetry breaking. As a consequence, the Wightman functions are not invariant under the local symmetry associated with the local charge which becomes screened: The long-range force disappears (i.e., the vector boson acquires a mass) and the physical states are all neutral. This is what happens in the Abelian Higgs-Kibble model.[44,45]

The second possibility is realized when the symmetry associated with the local charge is not broken and the physical states are neutral (*confinement mechanism*) as a consequence of an infrared mechanism. As in the Higgs phenomenon the $\delta(p^2)$ singularities become unphysical and disappear from the physical spectrum, but without inducing a symmetry breaking. This is the case of two-dimensional QED (local gauge) where no Higgs phenomenon occurs,[46] and the charge disappears because of the bad infrared behavior of the massless scalar field.

It is the second case which should be realized in QCD since one wants the color SU(3) to be unbroken. The sum over the color charges in the $\pi^0 \to 2\gamma$ decay and in other physical processes (see Sec. IV) requires the invariance of the Green's functions under the color group. No genuine Higgs mechanism is therefore expected in this case.

We will distinguish the two phenomena discussed above by calling them charge screening and confinement mechanism respectively.

Since the two mechanisms have the basic feature in common that the infrared singularities are not physical, one might speculate about the possibility of using the Higgs mechanism as an infrared regularization of non-Abelian gauge theories. The mass M of the Higgs model would play the role of an infrared regulator, which is removed by letting $M \to \infty$ at the end. It is expected that such a limiting procedure will define a non-Abelian gauge theory in which the confinement mechanism occurs. As a matter of fact, there are strong indications that the Higgs mechanism necessarily requires the failure of the cluster property [i.e., nonunitary $U(a)$ on the light cone]; thus, in the above limiting procedure the cluster property fails at each step, and it looks very likely that it will also fail in the limit $M \to \infty$ when the symmetry is restored.[47] This appears as a possible mechanism for resolving the ambiguities (infrared problem) in the definition of a non-Abelian GQFT, in such a way that the so-defined theory exhibits a quark confinement.

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*Based on an invited talk at the Workshop on Quantum Electrodynamics, Aspen, Colorado, June 7–27, 1976.

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[32]The cluster property can be regarded as a statement

about what may be measurable in a given field theory, since clusters of fields which have nonvanishing correlations at infinite spacelike separations cannot give rise to well-defined measurements, the result of the measurement being strongly dependent on what happens at infinite spacelike distance.

[33]In the case of unitary $U(a, \Lambda)$ the cluster property for the Wightman functions follows independently of locality and/or temperedness of the fields (Ref. 34). A refined form of cluster property, i.e., the vanishing of the truncated vacuum expectation values

$$\langle \Psi_0, B(x_1) \cdots B(x_n)\Psi_0\rangle^T$$

faster than any negative power of the diameter of the point sets $\{x_i\}$ when the x_i are in a spacelike plane and the testing functions are in $\mathcal{S}$, has been proved by Ruelle (Ref. 35) under the assumption of temperedness of the fields, translation invariance, mass-gap condition, and locality, but without using Lorentz covariance or positivity of the metric. In Ref. 29 Ruelle's result has been generalized to the case in which the temperedness of the field and the mass-gap condition are not assumed, but the space-time translation operators $U(a)$ are required to be unitary.

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[37]Theorem 7 is proved in a similar way.

[38]J. Bros, H. Epstein, and V. Glaser, Commun. Math. Phys. 6, 77 (1967).

[39]According to the general wisdom (Ref. 10) a local QFT satisfying the Wightman axioms describes a *pure phase* and, therefore, even in the case of spontaneous symmetry breaking the vacuum is unique (Ref. 40). The general belief that spontaneous symmetry breaking is related to vacuum degeneracy must be regarded as a statement about the possible theories (or phases) corresponding to a given Lagrangian. This situation is exactly the same as in statistical mechanics, where the infinite-volume limit of the correlation functions (which are the analog of the functional integral of QFT) is in general not unique, i.e., the given Hamiltonian describes more than one phase. When the infinite-volume limit describes a mixture (i.e., a case of a nonunique "ground" state) the correlation functions decompose into extremal invariant ones which in turn define a pure phase with unique "ground" state. For these reasons a *local* GQFT as defined in Sec. II was required to have a unique vacuum. A formulation of local GQFT in terms of a nonunique vacuum looks rather artificial to us and, in fact, we do not know of any case in which the introduction of many vacuums is unavoidable and/or preferable. Actually, in the local formulation of two-dimensional QED, which is usually regarded as a sort of prototypic model of quark confinement, the vacuum is unique.

[40]D. W. Robinson, in *Symmetry Principles and Fundamental Particles* (Freeman, San Francisco, 1967).

[41]A tempered distribution is a measure if its singularities are smooth enough so that a smearing with a continuous function is enough to yield a finite result. For a more precise definition see L. Schwartz, *Théories des Distributions* (Hermann, Paris, 1967).

[42]QFT models with $1/p^4$ propagators have been discussed in the literature (Ref. 43), but even in these simple cases the relation with the indefinite metric does not seem to have been realized.

[43]See, e.g., S. Blaha, Phys. Lett. 56B, 427 (1975).

[44]P. W. Higgs, Phys. Rev. 145, 1156 (1966); T. W. B. Kibble, *ibid.* 155, 1554 (1967).

[45]J. A. Swieca, Phys. Rev. D 13, 312 (1976).

[46]We do not agree with the statement appearing in the literature according to which the symmetry is broken in the local formulations of two-dimensional (QED). One can easily check that the Wightman functions are invariant under the symmetry $\psi \to e^{i\alpha}\psi$, $A_\mu \to A_\mu$. These transformation, induced by the electric charge, are therefore implementable in $\mathcal{H}$, but have only the trivial representation in $\mathcal{H}'$.

[47]Since the mechanism has a nonperturbative character the Kinoshita-Lee-Nauenberg theorem is not applicable.