Dynamical method for generating the gravitational interaction

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We present a model for self-coupled scalar fields which, when solved subject to a self-consistent constraint, exhibits the same diagrammatic structure as the covariantly quantized Einstein theory of gravity. The graviton appears naturally as a dynamical Goldstone boson. Formal expressions for the graviton propagator, as well as scalar-graviton and graviton-graviton vertices, are derived.

I. INTRODUCTION

A model recently proposed by Adler[1] suggests photon pairing as an origin for the gravitational interaction at the quantum level. The gravitational field is described by composite pairing amplitudes, which arise as expectation values of local quantum fields, but are not themselves local quantum fields.

We present here a heuristic "pairing" model for gravitation which, similarly, does not introduce a fundamental quantum field for the graviton. However, the graviton *is* described by a quantum field, a composite operator constructed from the interacting matter fields. The Lagrangian contains only matter terms; for simplicity we treat here the case of self-interacting massless scalar fields.

The theory is of the type originally considered by Nambu and Jona-Lasinio,[2] which breaks vacuum chiral invariance in order to generate the "pion" field; this theory has been shown to reproduce the linear σ model.[3,4] Bjorken,[5] and later Guralnik,[6] have shown that a theory of this type, containing only a four-fermion current $\times$ current interaction and solved subject to a self-consistent constraint, is completely equivalent to spinor quantum electrodynamics. The Bjorken model assumes a non-Lorentz-invariant vacuum; the photon appears as a dynamical Goldstone boson.

The same method is applied here to generate the graviton dynamically. We assume the existence of an infinitely degenerate set of non-Lorentz-invariant vacuums. Unless otherwise noted, all vacuum expectation values (VEV's) are taken with respect to one such vacuum state, denoted as $|b\rangle$. Solving the theory subject to certain constraints imposed upon the VEV of the scalar stress tensor results in a theory possessing the same diagrammatic structure as the covariantly quantized Einstein theory of gravitation (QGR).

The organization of this paper is as follows: In Sec. II the model is presented, and the equivalence to lowest-order QGR is demonstrated. In Sec. III the zeroth-order solution for the scalar propagator is found, and the lowest-order perturbative corrections are calculated, giving rise to expressions for the graviton propagator, scalar-graviton, and graviton-graviton vertices. Finally, Sec. IV presents a summary and discussion.

II. THE MODEL

We consider the case of a massless, Hermitian scalar field, with derivative self-coupling, described by the Lagrangian

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{2}\partial_\mu\varphi(x)\partial^\mu\varphi(x) \\ &\quad -g_0\,\overline{T}_{\mu\nu}(x)T^{\mu\nu}(x) - J_{\mu\nu}(x)T^{\mu\nu}(x) \\ &\equiv \mathcal{L}_F + \mathcal{L}_I\,. \end{aligned} \tag{2.1}$$

Here, $T^{\mu\nu}(x)$, given by

$$T^{\mu\nu}(x) \equiv \tfrac{1}{2}\left[\partial^\mu\varphi(x)\partial^\nu\varphi(x) - \tfrac{1}{2}\eta^{\mu\nu}\partial_\alpha\varphi(x)\partial^\alpha\varphi(x)\right], \tag{2.2}$$

is the scalar stress tensor, computed in the usual manner from the free part of the Lagrangian $\mathcal{L}_F$. In our notation

$$\overline{T}_{\mu\nu}(x) \equiv T_{\mu\nu}(x) - \tfrac{1}{2}\eta_{\mu\nu}T(x)\,, \tag{2.3a}$$

$$T(x) \equiv \eta_{\mu\nu}T^{\mu\nu}(x)\,, \tag{2.3b}$$

and the analogous form of Eq. (2.3a) will define

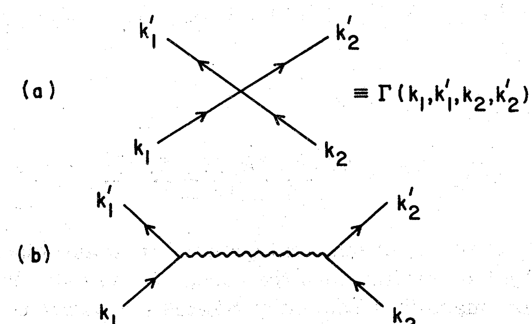

FIG. 1. (a) Four-scalar vertex computed from $\mathcal{L}_I$. (b) Lowest-order Feynman graph describing scalar-scalar scattering. The wavy line represents the graviton.

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any "barred" second-rank tensor. The bare coupling constant g_0 is assumed to be positive, and has dimension $(\text{mass})^{-4}$. The external source for the stress tensor $J_{\mu\nu}(x)$ has been introduced for convenience; after all calculations the limit $J_{\mu\nu}\to 0$ will be taken.

Unlike the Nambu-type theories mentioned previously,[2,5,6] the interaction here may be iterated. A new stress tensor, ${}^{(1)}T^{\mu\nu}(x)$, may be computed from the full Lagrangian (2.1). This tensor is then used to define a new Lagrangian,

$$ {}^{(1)}\mathcal{L} \equiv \mathcal{L}_F + {}^{(1)}\mathcal{L}_I , \tag{2.4a}$$

$$ {}^{(1)}\mathcal{L}_I \equiv \mathcal{L}_I[{}^{(1)}T^{\mu\nu}(x)] , \tag{2.4b}$$

and the process is repeated; the stress-tensor self-coupling reflects the essential nonlinearity of the gravitational interaction. In general ${}^{(n)}T^{\mu\nu}(x)$ goes as $(\partial\varphi)^{2n}$. Since, as will be shown, the gravitational field operator goes as $(\partial\varphi)^2$, this nonlinearity gives rise to 2-scalar-n-graviton vertices.

A stress tensor $\times$ stress tensor self-coupling is the obvious generalization of Bjorken's current $\times$ current interaction. However, we have chosen the coupling to be $\overline{T}_{\mu\nu}T^{\mu\nu}$ rather than $T_{\mu\nu}T^{\mu\nu}$ in order that the four-scalar vertex derived from $\mathcal{L}_I$, $\Gamma(k_1, k_1', k_2, k_2')$, shown in Fig. 1(a), and written as

$$\Gamma(k_1,k_1',k_2,k_2') = (2\pi)^4\delta^4(k_1+k_2-k_1'-k_2')[(k_1\cdot k_2)(k_1'\cdot k_2') + (k_1\cdot k_2')(k_2\cdot k_1') - (k_1\cdot k_1')(k_2\cdot k_2')] , \tag{2.5}$$

has the same tensor structure as the lowest-order Feynman graph, shown in Fig. 1(b), describing scalar-scalar scattering via one-graviton exchange in the QGR coupled (scalar) matter-gravity system.[7]

The field equation satisfied by $\varphi(x)$,

$$\{[\eta_{\mu\nu} - (g_0 T_{\mu\nu}(x) + \overline{J}_{\mu\nu}(x))]\partial^\mu\partial^\nu - \partial^\mu(g_0 T_{\mu\nu}(x) + \overline{J}_{\mu\nu}(x))\partial^\nu\}\varphi(x) = 0 , \tag{2.6}$$

is used to calculate the functional Schwinger-Dyson (SD) equation for

$$G(x,y) \equiv \frac{1}{i}\,\frac{\langle b,\text{out}\,|\,T(\varphi(x)\varphi(y))\,|\,b,\text{in}\rangle^J}{\langle b,\text{out}\,|\,b,\text{in}\rangle^J} , \tag{2.7}$$

the scalar two-point function. The superscript J indicates the VEV's are taken in the presence of the external source. The two-point function satisfies

$$\left\{[\eta_{\mu\nu} - (ig_0 t_{\mu\nu}(x) + \overline{J}_{\mu\nu}(x))]\partial^\mu\partial^\nu - \partial^\mu(ig_0 t_{\mu\nu}(x) + \overline{J}_{\mu\nu}(x))\partial^\nu - ig_0\partial^\mu\frac{\delta}{\delta J^{\mu\nu}(x)}\partial^\nu - ig_0\frac{\delta}{\delta J^{\mu\nu}(x)}\partial^\mu\partial^\nu\right\}G(x,y) = -\delta^4(x-y) , \tag{2.8}$$

where we have defined

$$t_{\mu\nu}(x) \equiv \frac{1}{i}\,\frac{\langle b,\text{out}\,|\,T(T_{\mu\nu}(x))\,|\,b,\text{in}\rangle^J}{\langle b,\text{out}\,|\,b,\text{in}\rangle^J} , \tag{2.9}$$

and the T symbol denotes the usual time ordering.

The corresponding SD equation in QGR is easily computed. One finds

$$\left\{[\eta_{\mu\nu} - \langle\kappa\overline{h}^g_{\mu\nu}(x)\rangle^J]\partial^\mu\partial^\nu - \partial^\mu\langle\kappa\overline{h}^g_{\mu\nu}(x)\rangle^J\partial^\nu - i\partial^\mu\frac{\delta}{\delta J^{\mu\nu}(x)}\partial^\nu - i\frac{\delta}{\delta J^{\mu\nu}(x)}\partial^\mu\partial^\nu\right\}G(x,y) = -\delta^4(x-y) . \tag{2.10}$$

Here,

$$\langle\kappa\overline{h}^g_{\mu\nu}(x)\rangle^J \equiv \kappa\,\frac{\langle 0,\text{out}\,|\,T(\overline{h}^g_{\mu\nu}(x))\,|\,0,\text{in}\rangle^J}{\langle 0,\text{out}\,|\,0,\text{in}\rangle^J} , \tag{2.11}$$

$h^g_{\mu\nu}(x)$ is the fundamental quantum field associated with the graviton, and the constant κ with dimension $(\text{mass})^{-1}$ is related to Newton's constant G through $\kappa^2 = 32\pi G$. The vacuum states $|0,\text{in}\rangle$, and $\langle 0,\text{out}|$ are the usually defined Lorentz-invariant states.

To lowest order in κ the two systems are formally identical if we make the identification

$$ig_0 t_{\mu\nu}(x) + \overline{J}_{\mu\nu}(x) = \langle\kappa\overline{h}_{\mu\nu}(x)\rangle^J , \tag{2.12a}$$

or

$$ig_0\overline{t}_{\mu\nu}(x) + J_{\mu\nu}(x) = \langle\kappa h_{\mu\nu}(x)\rangle^J ; \tag{2.12b}$$

that is, we identify the VEV of the scalar stress tensor taken between broken vacuum states with the "normal" VEV of the graviton field.

The theory is to be solved subject to the symmetry-breaking condition

$$g_0\left.\frac{\langle b,\text{out}\,|\,T(\overline{T}_{\mu\nu}(x))\,|\,b,\text{in}\rangle^J}{\langle b,\text{out}\,|\,b,\text{in}\rangle^J}\right|_{J_{\mu\nu}=0} \equiv \epsilon_{\mu\nu} \neq 0 , \tag{2.13}$$

with $\epsilon_{\mu\nu}$ a constant tensor, thus ensuring that vacuum translation invariance is maintained. Self-consistency is ensured by requiring

$$\frac{\langle b,\text{out}\,|\,T(T_{\mu\nu}(x))\,|\,b,\text{in}\rangle^J}{\langle b,\text{out}\,|\,b,\text{in}\rangle^J}=\lim_{\delta\to 0} T_{\mu\nu}[G(x+\delta,x-\delta)]\,; \tag{2.14}$$

that is, the VEV of $T_{\mu\nu}(x)$ is written as a function of the scalar two-point function, *as determined from the Schwinger-Dyson equation.*

In the evaluation of the VEV of the scalar stress tensor a form of "partial regularization" is required to ensure that only the contribution due to the breaking of vacuum Lorentz invariance is included as a gravitational effect. The VEV of $T_{\mu\nu}(x)$, taken with respect to the Lorentz-invariant vacuum $|0\rangle$, need not vanish. Lorentz invariance will be maintained if

$$\left.\frac{\langle 0,\text{out}\,|\,T(T_{\mu\nu}(x))\,|\,0,\text{in}\rangle^J}{\langle 0,\text{out}\,|\,0,\text{in}\rangle^J}\right|_{J_{\mu\nu}=0}=C\eta_{\mu\nu} \tag{2.15}$$

with C a (possibly divergent) constant. In the following, $T_{\mu\nu}(x)$ will everywhere be replaced by: $:T_{\mu\nu}(x):$, defined by

$$:T_{\mu\nu}(x):\equiv T_{\mu\nu}(x)-\left.\frac{\langle 0,\text{out}\,|\,T(T_{\mu\nu}(x))\,|\,0,\text{in}\rangle^J}{\langle 0,\text{out}\,|\,0,\text{in}\rangle^J}\right|_{J_{\mu\nu}=0}. \tag{2.16}$$

It is seen that the VEV of $:T_{\mu\nu}(x):$ taken with respect to the broken vacuum vanishes as $|b\rangle\to|0\rangle$. Since the expectation of the gravitational field is associated with the VEV of $:T_{\mu\nu}(x):$, gravitational effects vanish in the absence of symmetry breaking.

III. GRAVITON PROPAGATOR AND VERTEX FUNCTIONS

We seek a lowest-order solution to (2.8) which respects the symmetry-breaking condition (2.13). Higher-order corrections will be calculated perturbatively about this solution.

To this end, we write $G(x,y)$ as a power series in a parameter ϵ:

$$G(x,y)=G^{(0)}(x,y)+\epsilon G^{(1)}(x,y)+\epsilon^2 G^{(2)}(x,y)+\cdots; \tag{3.1}$$

all functions of the scalar propagator are expanded similarly. Also, we assume that the functional derivatives appearing in the SD equation are of first order in ϵ.[8] At the end of the calculation ϵ may be set equal to unity. Collecting terms, we find to zeroth order

$$\{[\eta_{\mu\nu}-\langle\kappa\bar{h}^{(0)}_{\mu\nu}(x)\rangle^J]\partial^\mu\partial^\nu-\partial^\mu\langle\kappa\bar{h}^{(0)}_{\mu\nu}(x)\rangle^J\partial^\nu\}G^{(0)}(x,y)=-\delta^4(x-y)\,, \tag{3.2}$$

while to first order in ϵ,

$$\{[\eta_{\mu\nu}-\langle\kappa\bar{h}^{(0)}_{\mu\nu}(x)\rangle^J]\partial^\mu\partial^\nu-\partial^\mu\langle\kappa\bar{h}^{(0)}_{\mu\nu}(x)\rangle^J\partial^\nu\}G^{(1)}(x,y)=ig_0\partial^\mu\left[\left(\frac{\delta}{\delta J^{\mu\nu}(x)}+t^{(1)}_{\mu\nu}(x)\right)\partial^\nu G^{(0)}(x,y)\right]. \tag{3.3}$$

The first approximation to the scalar propagator is obtained by evaluating (3.2) in the $J_{\mu\nu}\to 0$ limit. In momentum space the propagator is given by

$$G^{(0)}(k)\,\big|_{J_{\mu\nu}=0}=\frac{1}{k^2-\overline{\epsilon}_{\alpha\beta}k^\alpha k^\beta}\,, \tag{3.4}$$

and describes a massless scalar propagating in a constant gravitational potential. A constant potential produces no physical effects. In obtaining (3.4) use has been made of the symmetry-breaking condition (2.13). Combining (2.14) and (3.4) allows the constraint condition to be cast in the momentum-space form

$$\epsilon_{\mu\nu}=\tfrac{1}{2}ig_0\int\frac{d^4k}{(2\pi)^4}k_\mu k_\nu\left(\frac{1}{k^2-\overline{\epsilon}_{\alpha\beta}k^\alpha k^\beta}-\frac{1}{k^2}\right). \tag{3.5}$$

As in all theories of the Nambu-type, the constraint or "gap equation" involves a divergent integral, introducing the question of a cutoff.[9] We leave as an open question whether an actual cutoff Λ shall be imposed. One possible heuristic interpretation is that as $\Lambda\to\infty$ the coupling constant $g_0\to 0$ in such a way that a weak-coupling singularity exists, and (3.5) is satisfied.

We now turn to the calculation of the perturbative corrections to the scalar propagator. Noting that

$$\int d^4z\,G^{-1}(x,z)G(z,y)=\delta^4(x-y)\,,$$

we may express the functional derivatives of $G^{(0)}(x,y)$ appearing in (3.3) in terms of the variations of $G^{(0)-1}(x,y)$; these may be calculated from the zeroth-order Schwinger-Dyson equation (3.2). The result is

$$G^{(1)}(k)\,\big|_{J_{\mu\nu}=0}=\langle\kappa\bar{h}^{(1)}_{\mu\nu}\rangle^J\,\big|_{J_{\mu\nu}=0}G^{(0)}(k)k^\mu k^\nu G^{(0)}(k)+G^{(0)}(k)\Sigma^{(1)}(k)G^{(0)}(k)\,, \tag{3.6a}$$

$$\Sigma^{(1)}(k)\equiv ig_0\int\frac{d^4q}{(2\pi)^4}\left[\tfrac{1}{2}\eta^{\alpha\beta}k\cdot(k-q)-k^\alpha(k-q)^\beta\right]\times D^{(0)}_{F\alpha\beta\mu\nu}(q)\left[\tfrac{1}{2}\eta^{\mu\nu}k\cdot(k-q)-k^\mu(k-q)^\nu\right]\times G^{(0)}(k-q)\,, \tag{3.6b}$$

and we have defined the lowest-order "graviton propagator" $D^{(0)}_{F\alpha\beta\mu\nu}(x,y)$ by

$$D^{(0)}_{F\alpha\beta\mu\nu}(x,y)\equiv\frac{\delta}{\delta\overline{J}^{\mu\nu}(y)}[ig_0\overline{t}^{(0)}_{\alpha\beta}(x)+J_{\alpha\beta}(x)]\,. \tag{3.7}$$

Up to constants, $\Sigma^{(1)}(k)$ is identical in structure to the (lowest order in κ) gravitational radiative corrections to the scalar propagator; the scalar-graviton vertices, which will be defined formally in the following, are identical to those obtained in QGR. The momentum-space version of $\Sigma^{(1)}(k)$ is

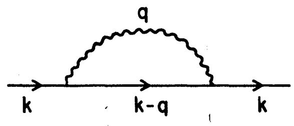

FIG. 2. Lowest-order perturbative correction to the scalar propagator.

shown graphically in Fig. 2.

A formal solution for the graviton propagator may now be obtained. As before, we use the SD equation to evaluate the functional derivative appearing in (3.7), and find

$$D^{(0)}_{F\alpha\beta\mu\nu}(q) = \tfrac{1}{2}(\eta_{\alpha\mu}\eta_{\beta\nu} + \eta_{\alpha\nu}\eta_{\beta\mu} - \eta_{\alpha\beta}\eta_{\mu\nu}) + \Pi^{(1)\lambda\gamma}_{\alpha\beta}(q)D^{(0)}_{F\lambda\gamma\mu\nu}(q)\,, \tag{3.8a}$$

$$\Pi^{(1)\lambda\gamma}_{\alpha\beta}(q) \equiv \tfrac{1}{2}ig_0\int \frac{d^4k}{(2\pi)^4}G^{(0)}(k)G^{(0)}(k-q) \times k_\alpha(k-q)_\beta[k^\lambda(k-q)^\gamma - \tfrac{1}{2}\eta^{\lambda\gamma}k\cdot(k-q)]. \tag{3.8b}$$

$\Pi^{(1)\lambda\gamma}_{\alpha\beta}(q)$ is, to this order, the graviton self-energy. It has the QGR structure of a scalar loop with a graviton vertex at one end and a vertex for a "barred" graviton at the other.

In an earlier work discussing the possibility of describing the graviton as a Goldstone boson, Phillips[10] assumes (3.8a) as a starting point[11]; we have shown how it may arise as a consequence of a particular self-interaction, as well as deriving a specific form for the graviton self-energy.

Differentiating the constraint condition (3.5) with respect to $\epsilon^{\lambda\gamma}$ results in

$$\Pi^{(1)\lambda\gamma}_{\alpha\beta}(0) = \tfrac{1}{2}(\delta^\lambda_\alpha\delta^\gamma_\beta + \delta^\gamma_\alpha\delta^\lambda_\beta)\,; \tag{3.9}$$

thus

$$D^{(0)}_{F\alpha\beta\mu\nu}(0) = \tfrac{1}{2}(\eta_{\alpha\mu}\eta_{\beta\nu} + \eta_{\alpha\nu}\eta_{\beta\mu} - \eta_{\alpha\beta}\eta_{\mu\nu}) + D^{(0)}_{F\alpha\beta\mu\nu}(0)\,, \tag{3.10}$$

and $D^{(0)}_{F\alpha\beta\mu\nu}(q)$ possesses a pole at $q=0$.

In Ref. 10 the conditions under which an expression of the form (3.8) may be inverted are discussed. All those conditions are not met by the graviton self-energy derived here. However, there is no reason to believe that the "gauge condition" required by Phillips, i.e.,

$$q^\mu[\Pi^{(1)\alpha\beta}_{\mu\nu}(q) - \Pi^{(1)\alpha\beta}_{\mu\nu}(0)] = 0\,, \tag{3.11}$$

must be obeyed. This was chosen to correspond to the condition derived by Bjorken,[5] $q^\nu[\Pi^{(1)}_{\mu\nu}(q) - \Pi^{(1)}_{\mu\nu}(0)] = 0$, which in his model is analogous to the QED condition $q^\nu\Pi_{\mu\nu}(q) = 0$, where $\Pi_{\mu\nu}(q)$ is the photon vacuum polarization tensor. No such condition $q^\mu\Pi^{\alpha\beta}_{\mu\nu}(q) = 0$ holds in QGR; rather, gravitational gauge invariance merely provides a relation between the three lowest-order graviton self-energy graphs, the scalar loop, seagull, and tadpole,[12] shown in Fig. 3.

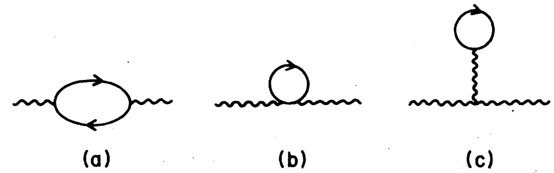

FIG. 3. Lowest-order graviton self-energy diagrams in QGR: (a) scalar loop, (b) seagull, (c) tadpole.

The vertices of the theory are defined in the usual manner. The 2-scalar–n-graviton vertex is given formally by

$$\Gamma^{(\varphi)^2(h)^n}_{\mu\nu,\ldots,\alpha\beta}(x,y,z_1,z_2,\ldots,z_n) \equiv \frac{\delta^n G^{-1}(x,y)}{\delta\langle h_{\mu\nu}(z_1)\rangle^J \cdots \delta\langle h_{\alpha\beta}(z_n)\rangle^J}\bigg|_{J_{\mu\nu}=0}. \tag{3.12}$$

For $n=1$ the expression is trivially evaluated. To lowest order, the momentum-space version, shown in Fig. 4(a), is given by

$$\Gamma^{(0)}_{\mu\nu}(k,p,q) = \kappa(2\pi)^4\delta^4(k+p+q) \times (\tfrac{1}{2}\eta_{\mu\nu}k\cdot p - k_\mu p_\nu)\,, \tag{3.13}$$

which is identical to the corresponding vertex found in QGR. The $n>1$ vertices arise only when

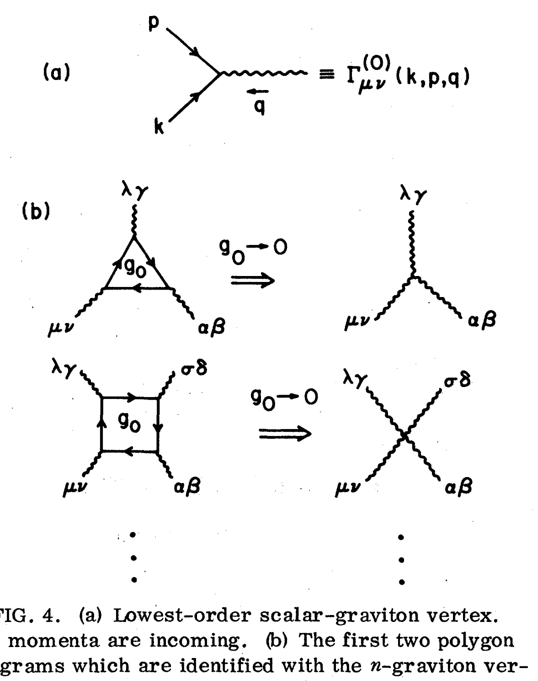

FIG. 4. (a) Lowest-order scalar-graviton vertex. All momenta are incoming. (b) The first two polygon diagrams which are identified with the n-graviton vertices.

the interaction term in the Lagrangian is iterated, as discussed in Sec. II.

The n-graviton vertices are defined similarly. We have

$$\Gamma^{(h)^n}_{\alpha\beta,\mu\nu,\lambda\gamma,\ldots,\sigma\delta}(x,y,z_1,z_2,\ldots,z_{n-2})$$

$$\equiv \frac{\delta^{n-2}D^{-1}_{F\alpha\beta\mu\nu}(x,y)}{\delta\langle h_{\lambda\gamma}(z_1)\rangle^J\cdots\delta\langle h_{\sigma\delta}(z_{n-2})\rangle^J}\bigg|_{J_{\mu\nu}=0}. \qquad (3.14)$$

The three-graviton vertex is easily calculated; we find a three-scalar triangle diagram. All $n \geqslant 3$ vertices are given by "polygon" diagrams, the first two of which are shown in Fig. 4(b). All are quartically divergent, and contain one factor of g_0. As discussed previously, the coupling constant can be thought of as "absorbing" the quartic divergence as $g_0 \to 0$, resulting in finite n-graviton vertices.

IV. DISCUSSION AND SUMMARY

We have shown that a self-coupled scalar theory, when solved subject to a self-consistent constraint, generates the structure present in the coupled gravity-matter system. Tensor bound states emerge, which couple to the scalars in the usual gravitational manner, and to themselves. These bound states produce radiative corrections to the scalar propagator having the same form as gravitational radiative corrections, while the formal expression obtained for the bound-state graviton propagator exhibits a $q=0$ pole, and possesses the same tensor structure as the QGR propagator written in the harmonic gauge.

The self-consistency condition quarantees that the leading quartic divergence in the graviton self-energy is absorbed, thus leaving at worst only quadratic divergences, as assumed in Ref. 10. However, the form of the self-energy generated here differs from that which was assumed by Phillips.

Owing to the presence of two methods of expansion in the pairing model, the iteration of the interaction, and the "ϵ expansion" employed in solving the theory, the expression for the lowest-order graviton self-energy obtained here differs from that in QGR. Only the scalar loop is present; the seagull and tadpole terms do not arise to lowest order in each expansion. Whether this formal simplification results in any headway being made on the problem of renormalization is left for a later work.

As in the Bjorken model, we have assumed here that vacuum Lorentz invariance is broken. However, as in that model, the symmetry-breaking condition may be considered as a gauge condition on the bound-state "gauge" field; the lack of Lorentz invariance is only a matter of interpretation, and is not observable in any physical process.

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