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Polarization of the microwave background radiation. I. Anisotropic cosmological expansion and evolution of the polarization states

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We investigate the rotation of the polarization planes for radiation freely traveling in homogeneous anisotropic spaces (Brans effect). Solving the parallel-transport equations in Bianchi type-I and type-V spaces, we find that the rotation angle is at most about 1 rad. The scrambling of the polarization states due to the Brans effect and the finite thickness of the last-scattering surface is negligible in any realistic model, so that any primeval polarization can survive up to the present epoch.

I. INTRODUCTION

During the last twelve years much attention has been devoted to the features of the microwave background, because of their bearing on cosmological problems. In particular, deviations from the blackbody frequency spectrum are related to such phenomena as the secondary ionization of the cosmological plasma,[1] any kind of energy injections at temperatures lower than $\sim 10^8$ °K (Refs. 2 and 3), and possibly the large-scale anisotropy of the cosmological expansion.[4] Angular anisotropies of the background are connected to the same phenomena,[1-3,5-7] and also to local perturbations of cosmological matter, i.e., density fluctuations and turbulent motions.[8-12]

Perhaps less attention has been devoted to the polarization properties of the background radiation: To the best of our knowledge, only Nanos[13] has so far measured the large-scale polarization degree of the background at a 3.2-cm wavelength, finding a null result to within 0.1%.

Rees[4] showed that early anisotropies of the cosmological expansion should induce a partial polarization into the radiation through Thomson scattering, and pointed out that the resulting polarization degree of the background may be detectable if the secondary ionization occurred early enough to allow photons to be scattered a few times by the reheated plasma. Under favorable circumstances such a polarization degree may be even larger than the angular anisotropy of the background.[4]

The above mechanism can produce a large-scale (quadrupole) polarization. Also, a small-scale polarization can arise from local perturbations of the cosmological fluid. The following processes are possible: (a) If lumps of matter have peculiar velocities (associated with turbulent motions or density fluctuations) the scattered radiation would be partially polarized.[4] (b) Density perturbations and gravitational waves produce temperature anisotropies,[8-10] which in turn give rise to a partial polarization owing to the same mechanism of Thomson scattering. The influence of gravitational fields on the temperature anisotropies is large when the characteristic scale is at least $L \sim 10^3$ Mpc at the present epoch. Since at these scales L becomes comparable to the Hubble length, the conversion of temperature anisotropies into partial polarization is effective if reheating occurred at a redshift $z \gtrsim 10$. Therefore we expect to find some background polarization also at "intermediate" angular scales, namely, $1° < \alpha < 180°$, as discussed in the following paper.[14]

Now the problem arises as to whether a nonzero polarization of the cosmic background can survive up to the present epoch, if it exists at some point of the cosmological evolution.

The interesting work of Brans[15] shows that, if the expansion of the universe is anisotropic, the polarization plane of an electromagnetic wave rotates along the path from the last scattering to us. As the radiation arriving at the detector is contributed by photons that traveled different path lengths since their last scatterings, any primeval polarization, produced by the anisotropic expansion itself or by local perturbations before the last-scattering red-shift, might be smeared out through the random rotations of the polarization planes.

In the present paper we explicitly calculate the rotation of the polarization planes for two sets of world models: More precisely, we integrate the parallel-transport equations for the polarization four-vector, during the free propagation in Bianchi type-I and type-V spaces. Our choice has been motivated by the following reason: Type-I spaces are the simplest homogeneous world models admitting a quasi-isotropic stage, and where the background radiation shows a quadrupole anisotropy. Type-V spaces, on the other hand, are the simplest models where the so-called Milne epoch eventually takes place, and the background begins to develop a spotlike pattern.[6,7,16]

In the former case we find that the influence of the cosmological expansion on the polarization vector of any wave is very small during the free propagation, if the space is quasi-isotropic, as required by the available experimental data. Thus a nonzero polarization of the background, existing at the last scattering, cannot be appreciably affected. Since the last-scattering surface has a finite thickness, different polarization states should be averaged even in the absence of the Brans rotation. However, such a smearing effect, which is not connected to the phenomenon that we investigate in the present paper, is perfectly analogous to the averaging of angular anisotropies.[17]

In the latter case (type-V spaces) the rotation of the polarization plane is quite small, even if the cosmic scale factors associated with different sky directions show a large anisotropy, provided that the anisotropy energy has been small since the last-scattering red-shift z_1 (see Sec. IV). The rotation angle approaches ~1 rad when the anisotropy energy density[18] becomes equal to the critical (i.e., flat-space) energy density at $z=z_1$.

The depolarizing effect is not determined by the magnitude of the rotation angle, but, more precisely, by the dispersion of the rotation angles associated with different z_1. If the radiation was scattered by the reheated plasma near $z_1 \sim 10$, then the thickness of the last-scattering surface is at most $\Delta z_1 \sim 10$. On the other hand, if radiation definitively decoupled from matter at the recombination of hydrogen, then $\Delta z_1 \sim 10^2$.[10] In both cases the dispersion of the rotation angles about the mean values is found to be less than 10^{-1} rad in any realistic type-V model. Therefore in these spaces, too, any initial polarization can survive without any appreciable decrease during the free propagation.

In the present paper we do not take into account the possible existence of intergalactic magnetic fields, which may affect the polarization of the background by means of the Faraday rotation. Rees[4] estimates that the background would be depolarized by a magnetic field stronger than ~3×10^{-8} G, but at present there is no evidence for the existence of such a field.[19]

In all the models considered here the curvature tensor of space is isotropic. At present we cannot exclude that, in more general (anisotropic-curvature) spaces, a stronger depolarization may occur due to the Brans effect. However, our results show that the background radiation may indeed be partially polarized. Further measurements are then desirable in any angular range. In the companion paper,[14] we give experimental upper limits on the background polarization in the angular range $0.5° < \alpha < 40°$ and discuss their significance.

II. PARALLEL TRANSPORT OF THE POLARIZATION FOUR-VECTOR

Since we are dealing with type-I and type-V homogeneous spaces, it is extremely convenient to take an orthonormal tetrad of vectors as a local reference frame at each point of spacetime. Using geometrical units ($8\pi G = c = 1$), the metric takes the form

$$ds^2 = -dt^2 + \sum_{i=1}^{3} (\Omega^i)^2 , \tag{1}$$

where Ω^i is the differential form[20] associated with the frame unit vector e_i. Such a unit vector can be written as

$$e_i = [R_i(t)]^{-1} u_i , \tag{2}$$

where the vector u_i is directed along the ith principal axis of the Bianchi space and satisfies the canonical commutation relations

$$[u_i, u_k] = C_{ik}{}^n u_n . \tag{3}$$

In anisotropic flat spaces (i.e., Bianchi type-I) all the commutation coefficients vanish. In type-V spaces, on the other hand, the only nonzero coefficients are

$$C_{31}{}^1 = C_{32}{}^2 = 1 , \quad C_{13}{}^1 = C_{23}{}^2 = -1 . \tag{4}$$

In Eq. (2) three scale factors $R_i(t)$ appear, describing the anisotropic expansion of the universe. They are related to the directional Hubble constants H_i by

$$H_i = \frac{R_i'}{R_i} , \tag{5a}$$

where the prime denotes differentiation with respect to time. The average Hubble constant H is given by

$$H = \tfrac{1}{3}(H_1 + H_2 + H_3) = R'/R , \tag{5b}$$

where R is the average scale factor.

Any radiation field can be described, in the high-frequency (geometrical-optics) limit,[20,15] by the wave four-vector k and the polarization four-vector m. Aside from a phase factor the electric field $\vec{E}$ is given by

$$E^i = k^0 m^i - k^i m^0 .$$

Both k and m are parallel-transported along geodesic null paths. The parallel-transport equations, once they are written down in the orthonormal tetrad patch, take the form

$$\begin{aligned} k^0 v^{0\prime} &= H_i v^i k^i , \\ k^0 v^{1\prime} &= -H_1 v^0 k^1 + (a/R_3) v^3 k^1 , \\ k^0 v^{2\prime} &= -H_2 v^0 k^2 + (a/R_3) v^3 k^2 , \\ k^0 v^{3\prime} &= -H_3 v^0 k^3 - (a/R_3)(v^1 k^1 + v^2 k^2) , \end{aligned} \tag{6}$$

where $a=0$ for type-I spaces and $a=1$ for type-V spaces, according to Eq. (4). Also, the four-vector v in Eq. (6) denotes either k or m. The functions $R_3(t)$ and $H_i(t)$ should be derived from the dynamics of the considered cosmological model.

In order to avoid spurious effects due to the rotation of the three-vector $\vec{k}$, it is useful to introduce also a couple of unit vectors e_ϑ and e_φ always orthogonal to $\vec{k}$, and to project the transverse polarization in such a frame. Taking a local polar system whose $\vartheta=0$ axis is set along the direction of e_3, the angles ϑ and φ specify the point of the celestial sphere from which a light ray impinges on a local observer. Thus

$$\begin{aligned} k^1 &= \alpha_1 k^0 = -\sin\vartheta \sin\varphi k^0 , \\ k^2 &= \alpha_2 k^0 = -\sin\vartheta \cos\varphi k^0 , \\ k^3 &= \alpha_3 k^0 = -\cos\vartheta\, k^0 , \end{aligned} \tag{7}$$

where the minus sign denotes ingoing radiation. The transverse polarization is then described by

$$\begin{aligned} m^\vartheta &= \cos\vartheta(\sin\varphi\, m^1 + \cos\varphi\, m^2) - \sin\vartheta\, m^3 , \\ m^\varphi &= \cos\varphi\, m^1 - \sin\varphi\, m^2 . \end{aligned} \tag{8}$$

In the following we shall deal with a polarization angle β defined by

$$\tan\beta = m^\vartheta / m^\varphi . \tag{9}$$

The difference $\Delta\beta = \beta(z) - \beta(0)$ will express the Brans effect quantitatively.

III. POLARIZATION EFFECTS IN TYPE-I SPACES

A Bianchi type-I space might give a good description of the real world only in the quasi-isotropic stage, namely, if $|R_k - R|/R \lesssim 10^{-3}$ when $z = z_1$.[18] This fact allows us to treat Eq. (6) by a perturbative method, taking the solution for an isotropic space as a zero-order approximation. (Such a solution gives no polarization effect.)

For the wave vector k the exact solution is known[15]:

$$\begin{aligned} k^i &= \gamma_i / R_i , \\ k^0 &= \left[\sum_{i=1}^{3} \left(\frac{\gamma_i}{R_i} \right)^2 \right]^{1/2} , \end{aligned} \tag{10}$$

where γ_i are constants. For the polarization vector we shall consider the perturbative expansion

$$m_i = \sum_{k=0}^{\infty} m_i^{(k)} .$$

The ratio $m_i^{(k+1)}/m_i^{(k)}$ is of order $\Delta R_i/R$, where

$$|\Delta R_i| = |R_i - R| \ll R .$$

The zero-order solution is given by

$$\begin{aligned} m_1^{(0)} &= M\gamma_1 e^\lambda + N(1-\gamma_1^2) - S\gamma_1\gamma_2 , \\ m_2^{(0)} &= M\gamma_2 e^\lambda - N\gamma_2\gamma_1 + S(1-\gamma_2^2) , \\ m_3^{(0)} &= M\gamma_3 e^\lambda - N\gamma_3\gamma_1 - S\gamma_3\gamma_2 , \end{aligned} \tag{11}$$

where M, N, and S are arbitrary constants and

$$\lambda = -\int H dt .$$

In Eq. (11) the λ-dependent terms describe the component of $\vec{m}$ parallel to the wave vector. Since they do not affect the transverse polarization, we can take $M=0$ without any loss of generality.

The first-order equation is

$$\frac{dm_i^{(1)}}{d\lambda} = \gamma_i\gamma_k \left[m_k^{(1)} + \left(\frac{\Delta R_i'}{R'} - 2\frac{\Delta R_i}{R} - \frac{\Delta R_k}{R} + 2\gamma_s^2 \frac{\Delta R_s}{R} \right) m_k^{(0)} \right] . \tag{12}$$

From Eqs. (11) and (12) we get

$$m_i^{(1)} = \gamma_i e^\lambda \int^\lambda d\lambda' e^{-\lambda'} f(\lambda') , \tag{13}$$

where

$$f(\lambda') = \rho\gamma_1 \left(\gamma_s^2 \frac{\Delta R_s}{R} - \frac{\Delta R_1}{R} \right) + \left(\gamma_s^2 \frac{\Delta R_s}{R} - \frac{\Delta R_2}{R} \right) \sigma\gamma_2 ,$$

and ρ and σ are constants. Equation (13) shows that the three-vector $\vec{m}^{(1)}$ is parallel to $\vec{k}$. Therefore we conclude that the transverse polarization vector is constant to the first order. However, since the three-vector $\vec{k}$ does rotate, then, even at the first order, the polarization state does change: By means of Eqs. (8) we find

$$\begin{aligned} \delta m^\vartheta &\equiv m_\vartheta^{(1)} = \cos\vartheta^{(0)} m_\varphi^{(0)} \delta\varphi , \\ \delta m^\varphi &\equiv m_\varphi^{(1)} = -\cos\vartheta^{(0)} m_\vartheta^{(0)} \delta\varphi , \end{aligned} \tag{14}$$

where $\cos\vartheta^{(0)}$, $m_\vartheta^{(0)}$, and $m_\varphi^{(0)}$ constitute the zero-order solution, and $\delta\varphi$ is given by

$$\delta\varphi = \sin\varphi^{(0)} \cos\varphi^{(0)} \left(\frac{\Delta R_2 - \Delta R_1}{R} \right) . \tag{15}$$

It may be interesting to note that the covariant differentials Dm_ϑ and Dm_φ over the celestial sphere (treated as a two-dimensional space of constant curvature) do vanish. Thus the transverse polarization is parallel-transported on such a sphere.

It may also be asked if the above first-order effect has an appreciable influence on the polarization degree of the radiation. To this purpose let us write the radiation intensity at the detector $I(\vartheta, \varphi, \beta)$ as a sum of contributions coming from different red-shifts z:

$$I(\vartheta, \varphi, \beta) = \int dz\, P(z)[1 + A(\vartheta', \varphi', \beta', z)], \qquad (16)$$

where $P(z)$ is the averaged contribution of the red-shift z and ϑ', φ', and β' are functions of z. As far as large-scale anisotropies are considered, we find $A \sim \Delta R_i/R$ and

$$\frac{\partial A}{\partial \vartheta'} \sim \frac{\partial A}{\partial \varphi'} \sim \frac{\partial A}{\partial \beta'} \sim \frac{A}{2\pi}.$$

If we write

$$\begin{aligned} I(\vartheta, \varphi, \beta) = \int dz\, P(z)[1 + A(\vartheta, \varphi, \beta, z)] \\ + \int dz\, P(z)\, \frac{\partial A}{\partial \vartheta'}(\vartheta' - \vartheta) + \frac{\partial A}{\partial \varphi'}(\varphi' - \varphi) \\ + \frac{\partial A}{\partial \beta'}(\beta' - \beta) \quad , \end{aligned} \qquad (17)$$

then the rotation of the polarization planes is taken into account in the second integral of Eq. (17), and gives an effect of order $(\Delta R_i/R)^2$. Thus there is no appreciable smearing out of the polarization states during the free propagation of photons. More generally, if the angular scale of the anisotropies is α, then effects of order A^2/α are expected to occur.

IV. POLARIZATION EFFECTS IN TYPE-V SPACES

The results of the previous section are related to the fact that the space was assumed to be quasi-isotropic. A rather different situation comes out in type-V spaces where a Milne epoch finally occurs. In such an epoch the ratios R_1/R_3 and R_2/R_3 tend to arbitrary constants, whereas the directional Hubble constants H_i tend to their average value H.

In these spaces the parallel-transport equations take the form

$$\begin{aligned} v_1' &= -H\left[1 - \left(\frac{\nu}{R_3}\right)^2 (HR)^{-1}\right]\frac{v^0 k_1}{k^0} + \frac{1}{R_3}\,\frac{v_3 k_1}{k^0}, \\ v_2' &= -H\left[1 + \left(\frac{\nu}{R_3}\right)^2 (HR)^{-1}\right]\frac{v^0 k_2}{k^0} + \frac{1}{R_3}\,\frac{v_3 k_2}{k^0}, \\ v_3' &= -H\frac{v^0 k_3}{k^0} - \frac{1}{R_3}\,\frac{v_1 k_1 + v_2 k_2}{k^0}, \end{aligned} \qquad (18)$$

where

$$k^0 v^0 = k_i v_i. \qquad (19)$$

The evolution of the world model is described by

$$H = \frac{1}{R_3}\left[1 + \frac{1}{3}\,\frac{\mu}{R_3} + \frac{1}{3}\left(\frac{\nu}{R_3}\right)^4\right]^{1/2}, \qquad (20)$$

where μ and ν are arbitrary constants describing the matter content and the anisotropy energy, respectively.

The exact analytic solution of Eqs. (18) can be found in the late Milne epoch, namely, when we can set $\mu = \nu = 0$ and[21,22]

$$R_i = P_i t, \quad H_i = 1/t. \qquad (21)$$

The constants P_1 and P_2 are arbitrary, whereas $P_3 = 1$. Using (18) and (21) we get the direction cosines α_i of the wave vector, introduced by Eq. (7):

$$\begin{aligned} \alpha_1(t) &= \frac{2\alpha_1(t_1)\, t_1 t}{[1 - \alpha_3(t_1)]\, t^2 + [1 + \alpha_3(t_1)] t_1^2}, \\ \alpha_2(t) &= \frac{\alpha_2(t_1)}{\alpha_1(t_1)}\alpha_1(t), \\ \alpha_3(t) &= \frac{[\alpha_3(t_1) - 1] t^2 + [\alpha_3(t_1) + 1] t_1^2}{[1 - \alpha_3(t_1)] t^2 + [\alpha_3(t_1) + 1] t_1^2}, \end{aligned} \qquad (22)$$

where t_1 is the last-scattering time. The polarization vector obeys the equations

$$\begin{aligned} m_1' &= \frac{\alpha_1}{t} m_3, \\ m_2' &= \frac{\alpha_2}{t} m_3, \\ m_3' &= -\frac{\alpha_2}{t}\left(m_2 + \frac{\alpha_1}{\alpha_2} m_1\right). \end{aligned} \qquad (23)$$

Integrating (23) we find

$$\begin{aligned} m_1 &= \frac{\alpha_1 \alpha_3}{\alpha_2} + A_0, \\ m_2 &= \alpha_3 - A_0 \frac{\alpha_1}{\alpha_2}, \\ m_3 &= -\frac{\alpha_1^2 + \alpha_2^2}{\alpha_2}, \end{aligned} \qquad (24)$$

where A_0 is an integration constant. The transverse polarization turns out to be

$$\begin{aligned} m_\vartheta &= \frac{(\alpha_1^2 + \alpha_2^2)^{1/2}}{\alpha_2}, \\ m_\varphi &= -A_0 m_\vartheta. \end{aligned} \qquad (25)$$

The inspection of Eqs. (22) and (25) shows that in the late Milne epoch both m_ϑ and m_φ are constants. We conclude that the polarization angle $\beta = \tan^{-1}(m_\vartheta/m_\varphi)$ is also constant.

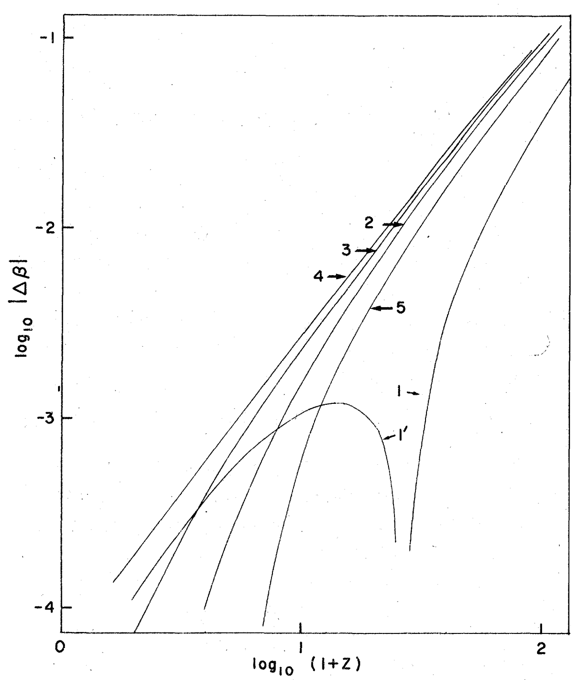

FIG. 1. The absolute value of the rotation angle $\Delta\beta$ as a function of red-shift z, for $z \lesssim 10^2$, in the cosmological model where $z_a = 99$, $z_m = 0$. The curves refer to $\varphi = 225°$ and to different values of θ, where φ and θ are defined by Eq. (7). The curves are labeled according to the following scheme: 1 and 1′: $\theta = 20°$; 5: $\theta = 40°$; 2: $\theta = 60°$; 3: $\theta = 100°$; 4: $\theta = 160°$. $\Delta\beta$ is positive everywhere, except in the 1′ branch.

In spite of the constance of β, some smearing effect may appear because of the z dependence of ϑ', φ', and $A(\vartheta', \varphi', \beta, z)$ in Eq. (16). However, as remarked in the Introduction, this effect reduces the temperature anisotropies as well as the polarization degree.[17]

At earlier epochs, namely, when the matter term and the anisotropy term are not negligible in Eq. (20), exact solutions of Eqs. (18) are not available, and numerical methods are needed. By integration of (18) we calculated the polarization angle $\beta(z)$ for several world models. In order to classify such models, we introduce the red-shifts z_m and z_a at which the matter density and anisotropy energy density, respectively, become equal to $\frac{1}{3}R_3^{-2}$ in geometrical units. The quasi-isotropic stage begins at $z \sim z_a$ and the Milne epoch at $z \sim z_m$. In Figs. 1–3 we report the rotation angle $\Delta\beta = \beta(z) - \beta(0)$ as a function of the red-shift z for some models, where $0.1 \leq 1 + z_m \leq 3$ and $10 \leq 1 + z_a \leq 100$. In such a model the ratio of the present-time matter density to the critical density ($\sim 10^{-29}$ g cm^{-3}) is $\lesssim 0.8$.

In all the cases that we investigated the rotation angle $\Delta\beta$ does not depend on the polarization state at the present epoch, namely, on $\beta(0)$, within the accuracy of our computation (always better than 10^{-4} rad). Thus the rotation due to the Brans mechanism does not induce any polarization on unpolarized radiation. On the other hand, $\Delta\beta$ apparently depends on the direction of the incoming radiation. As shown by the figures, $\Delta\beta$ is a monotonically increasing function of ϑ at a fixed red-shift.

When $1 + z_m \gtrsim 1$, at small red-shifts $\Delta\beta$ is slightly negative, but becomes positive very soon: Thus the region where $\Delta\beta < 0$ can be put in evidence in Figs. 1 and 3 is only for $k_3 < 0$. When $1 + z_m < 1$, some values of k_3 exist, for which $\Delta\beta$ is negative in the whole range $z > 0$.

In all cases we found the inequality

$$|\Delta\beta| \lesssim \left[\sum_i \left(\frac{H_i - H}{H}\right)^2\right]^{1/2}, \tag{26}$$

where all the quantities are to be evaluated at a given red-shift. In particular, the approximate equality holds when $\cos\vartheta \approx -1$.

For $z \lesssim 10$ we found $\Delta\beta \lesssim 10^{-1}$ rad, except in those models where the universe has not entered the late Milne stage yet and the anisotropy energy

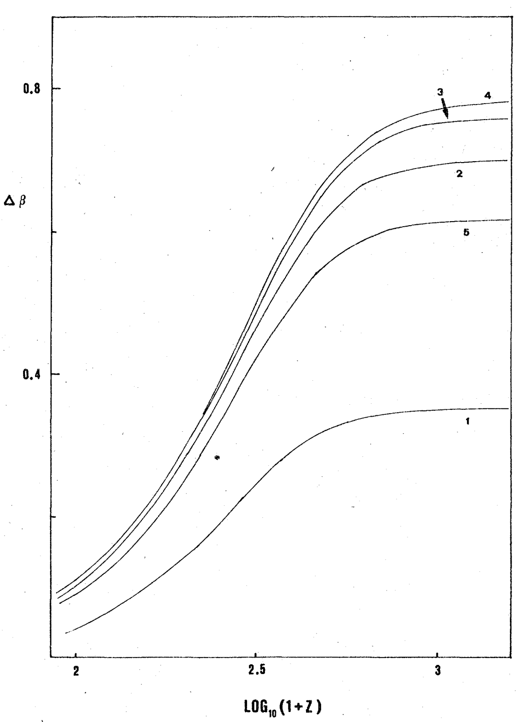

FIG. 2. The rotation angle $\Delta\beta$ as a function of red-shift z, for high values of z, in the cosmological model where $z_a = 99$ and $z_m = 0$. The labeling scheme is the same as in Fig. 1.

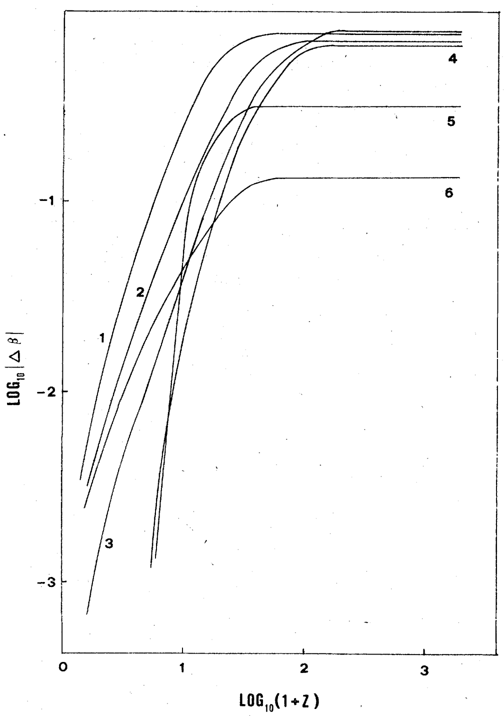

FIG. 3. The absolute value of the rotation angle $\Delta\beta$ as a function of red-shift for some cosmological models. All the curves refer to $\varphi=225°$, and are labeled according to the following scheme: 1: $z_a=9$, $z_m=0$, $\theta=140°$; 2: $z_a=9$, $z_m=-0.9$, $\theta=140°$; 3: $z_a=29$, $z_m=2$, $\theta=140°$; 4: $z_a=29$, $z_m=2$, $\theta=40°$; 5: $z_a=9$, $z_m=0$, $\theta=40°$; 6: $z_a=9$, $z_m=-0.9$, $\theta=40°$. $\Delta\beta$ is positive everywhere, except in the 6 curve.

was still large at $z \gtrsim 10$ (as an extreme example, see the case $z_a=10$, $z_m=1$ in Fig. 3). Such exceptional models are not consistent with the high isotropy of the microwave background.

In the range $900 \lesssim z \lesssim 1500$ the rotation angle is large, but depends weakly on the red-shift. Since the last-scattering surface had a thickness $\Delta z_1 \sim 10^2$ at the recombination of hydrogen,[10] the dispersion of $\Delta\beta$ is only $\sim 10^{-2}$ rad.

We conclude that in realistic type-V models the rotations of the polarization planes do not affect the polarization properties of the background, since the dispersion of $\Delta\beta$ is much less than π rad.

V. CONCLUSION

As shown in the previous sections, the polarization degree of the microwave background cannot be appreciably decreased during the free propagation of photons, as far as the universe can be represented by a Bianchi space with an isotropic curvature tensor. Should the inequality (26) still hold in anisotropic-curvature spaces, our result might turn out to be quite general. Whether the secondary ionization of plasma greatly affected the propagation of photons or not, our conclusion about type-I and type-V spaces remains valid. However, the large-scale polarization of the background may be easily detectable only if the radiation has been scattered by the reheated plasma. If a polarization degree as large as $\sim 10^{-4}$ will be found, this fact will give us information both about the anisotropy of the cosmological expansion and about the secondary ionization of plasma.

Note added. A. M. Anile and R. A. Breuer [Astrophys. J. 217, 353 (1977)] have recently calculated the correction to geometrical optics, due to the finite frequency ω of the radiation, for type-I spaces. These authors neglect the finite thickness of the last-scattering surface, and find that during the free propagation the polarization state varies by an amount $\lesssim 10^2 H_0/\omega$, where $H_0 \simeq 10^{-10}$ yr^{-1} is the Hubble constant. Clearly, this effect is exceedingly small in the case of the microwave background. In the present paper we show that, owing to the thickness of the last-scattering surface, the smearing of the polarization states remains finite in the high-frequency limit, but is rather small. Incidentally, the work of Anile and Breuer proves that the geometrical-optics approach, used in the present paper, is completely adequate.

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