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η mesons with inert components

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It is assumed that the $\eta(549)$ and $\eta'(958)$ mesons are each composed of a nonstrange-quark part, a strange-quark part, and an "inert" part that interacts weakly with mesons. This type of structure was suggested previously by the author on the basis of self-consistency conditions postulated by Odorico. Various experimental results on hadronic processes and electromagnetic decay rates are used to analyze the η and η' wave functions. The results are compared with the predictions that follow from the Odorico postulate. It is suggested that the inert component is a gluon bound state. It is predicted that a third pseudoscalar meson with mass less than 2 GeV/c^2 exists; this meson should be produced copiously in π^-p and K^-p collisions at sufficiently high energy.

I. INTRODUCTION AND PROCEDURE

Several years ago the author proposed that the light P (pseudoscalar) meson nonet is part of a decimet, composed of an octet and two singlets of SU(3).[1] The three physical, isotopic-scalar P mesons were postulated to be specific mixtures of the isoscalar member of the quark-model $q\overline{q}$ octet, the quark-model singlet, and a singlet that is "inert" in the sense that it does not interact strongly with mesons. Two of the physical mesons, denoted here by η_α and η_β, are identified with the 549 and 958 MeV η's, respectively.

This proposal resulted from a combination of duality with a postulate of Odorico, the postulate that the zeros in PP scattering amplitudes are linear in the Mandelstam plane.[2] When combined with SU(3) symmetry, the Odorico postulate led to a specific η_α-η_β mixing angle, and to predicted interaction ratios of tensor mesons with PP states that are quite different from those of the standard quark model.[2,3] [The interaction ratios of the standard quark model are defined as those specified by the Okubo-Zweig-Iizuka (OZI) rule.[4]] The inert meson state is required if one assumes both Odorico's solution to his conditions and the interaction ratios of the standard quark model.[1]

Alexander, Lipkin, and Scheck have used the quark model and the OZI rule to derive two sum rules for differential cross sections near the forward direction.[5] These rules are

$$\sigma(\pi^-p\to\eta_\alpha n)+\sigma(\pi^-p\to\eta_\beta n)+\sigma(\pi^-p\to\pi^0 n) = \sigma(K^+n\to K^0 p)+\sigma(K^-p\to\overline{K}^0 n), \quad (1)$$

$$\sigma(K^-p\to\eta_\alpha\Lambda)+\sigma(K^-p\to\eta_\beta\Lambda) \doteq \sigma(K^-p\to\pi^0\Lambda) + \sigma(\pi^-p\to K^0\Lambda). \quad (2)$$

Experimentally, the sums of $\eta_\alpha(549)$ and $\eta_\beta(958)$ production are consistently smaller than predicted by these equations.[6,7] Lipkin has proposed that these results are caused by a large inert part in the η_β wave function.[8]

The purpose of this paper is to discuss the quark-model wave functions of the η_α and η_β, under the assumption that they may contain large components of an inert state. It is suggested in Sec. II that the inert state is a gluon bound state; the implications of this suggestion are discussed. In Sec. III the specific predictions that follow from the Odorico hypothesis are listed and discussed.

Various experimental data are used to analyze the η_α and η_β wave functions in Sec. IV. This analysis does not depend on the nature of the inert components. Most of the experiments considered are not new. However, most earlier analyses involved the assumption that the η_α and η_β do not contain inert parts.[9]

The possible identity of the third, isoscalar P meson is discussed in Sec. V. Some of the properties of this meson are predicted.

Throughout this paper the three physical η mesons are denoted by η_α, η_β, and η_γ. Their wave functions are written

$$\eta_i = a_{iu}\Psi_u + a_{is}\Psi_s + a_{i0}\Psi_0\,, \quad (3)$$

where $\Psi_u=(u\overline{u}+d\overline{d})/\sqrt{2}$, $\Psi_s=s\overline{s}$, Ψ_0 is an inert state, u, d, and s denote up, down, and strange quarks, and a is an orthogonal matrix. The SU(3) singlet and octet $q\overline{q}$ states are given by

$$\eta_1 = (\tfrac{2}{3})^{1/2}\Psi_u + (\tfrac{1}{3})^{1/2}\Psi_s\,, \quad (4a)$$

$$\eta_8 = (\tfrac{1}{3})^{1/2}\Psi_u - (\tfrac{2}{3})^{1/2}\Psi_s\,. \quad (4b)$$

II. THE NATURE OF THE INERT STATE

In this section we discuss the possible physical structure of the inert state Ψ_0. One possibility is that Ψ_0 is a $q\overline{q}$ pair made of charmed quarks or other heavy quarks. However, there are two arguments against this possibility. First, if one uses the observed $J/\psi \to \eta_\alpha \gamma$ and $J/\psi \to \eta_\beta \gamma$ decay rates to estimate the amplitudes of the $c\bar{c}$ pair in the η_α and η_β, the results are very small.[10] Second, the mass of a heavy $q\bar{q}$ pair must be much larger than those of the Ψ_u and Ψ_s states; it is unlikely that a large mixing occurs among states with such different rest masses.

A second possibility, emphasized by Lipkin, is that the Ψ_0 is a radial excitation of some combination of $s\bar{s}$ and $(u\bar{u}+d\bar{d})/\sqrt{2}$ states.[8] This is a plausible suggestion; I know of no strong arguments against it.

Another possibility is that Ψ_0 is a gluon bound state. It is usually assumed that the dominant mechanism for the mixing of Ψ_u and Ψ_s in the η states is virtual annihilation into gluon bound states, i.e., the mechanism $\Psi_u \to G \to \Psi_s$.[11] The intermediate gluon bound states G do not contain quarks or antiquarks; the simplest component of the G consists of two gluons. A gluon is assumed to be a V (vector) particle that corresponds to the octet of color-SU(3) symmetry. One of the attractive features of this mechanism is that color-symmetry and charge-conjugation invariance require that the simplest term in the corresponding virtual-annihilation mechanism for V mesons involves three gluons. Therefore, the intermediate-gluon-bound-state assumption is consistent with the fact that Ψ_u-Ψ_s mixing is smaller for V mesons than for P mesons. Although this assumption does not lead to a prediction of the η_α and η_β masses, I assume the mechanism is important for mixing the η states.

It may be that all gluon bound states are too heavy to be mixed appreciably with Ψ_u and Ψ_s. However, in this section I postulate that this is not the case, and that Ψ_0 is a gluon bound state. The mechanism discussed above contributes to Ψ_u-Ψ_0 and Ψ_s-Ψ_0 coupling to lower order than it does to Ψ_u-Ψ_s coupling. Therefore, if Ψ_0 is a gluon bound state, one expects the inequalities

$$a_{\alpha s}{}^2 < a_{\alpha 0}{}^2 < a_{\alpha u}{}^2, \quad a_{\beta u}{}^2 < a_{\beta 0}{}^2 < a_{\beta s}{}^2, \tag{5}$$

so that the η_α is primarily a Ψ_u-Ψ_0 mixture and the η_β is primarily a Ψ_s-Ψ_0 mixture. Equation (5) is called here the "partially pure P" hypothesis.

This hypothesis is not used in making the data analysis of Sec. IV. The assumption made there is simply that the wave functions of the η's are of the type of Eq. (3).

III. SOLUTIONS OF THE ODORICO CONDITIONS

I next consider the conditions that follow from the Odorico hypothesis, since these conditions lead to specific predictions of the wave functions. The application of these conditions to $PP \to PP$ amplitudes is discussed thoroughly in Refs. 1 and 2. Therefore, I give only the results here. In every solution to the conditions the ratio of the a_s and a_u of Eq. (3) is fixed for each η meson. Any solution also corresponds to fixed ratios of the constants of interaction of various vector and tensor Regge trajectories to the PP states. I consider only solutions for which the tensor coupling ratios are those of the standard quark model, as discussed in Sec. I.

A solution of this type is given in Eqs. (9) and (13) of Ref. 1, with $\theta = \tan^{-1}(\frac{1}{2})^{1/2}$. In terms of the basis of Eq. (3), this solution is

$$\begin{aligned} \eta_\alpha &= (\tfrac{1}{2})^{1/2}\Psi_u - \tfrac{1}{2}\Psi_s - \tfrac{1}{2}\Psi_0 , \\ \eta_\beta &= -(\tfrac{1}{2})^{1/2}\Psi_s + (\tfrac{1}{2})^{1/2}\Psi_0 , \\ -\eta_\gamma &= (\tfrac{1}{2})^{1/2}\Psi_u + \tfrac{1}{2}\Psi_s + \tfrac{1}{2}\Psi_0 . \end{aligned} \tag{6}$$

This solution is on the borderline for satisfying the partially pure P conditions of Eq. (5).

I next consider whether or not there are any other soltuions, in addition to those obtained by permuting the α, β, and γ in Eq. (6). If there is only one inert state, it can be shown that in all other solutions that have the interaction ratios of the standard quark model, one of the a_{iu} is of magnitude one, or one of the a_{is} is of magnitude one. These solutions are in strong disagreement with experimental data, and so need not be considered. The solution of Eq. (6) is compared with experiment in Sec. VI.

If there are two or more inert P states, the conditions are not so restrictive. However, one can use the methods of Ref. 1 to show that in any solution to the Odorico conditions with the standard-quark-model interaction ratios, the $q\bar{q}$ components for every η meson must satisfy one of the following four conditions:

$$a_{iu} = 0, \quad a_{is} = 0, \text{ or } a_{is} = \pm(\tfrac{1}{2})^{1/2} a_{iu} . \tag{7}$$

It is interesting that while $a_s = (\frac{1}{2})^{1/2} a_u$ represents a pure singlet, there is no solution corresponding to a pure octet η.

The Odorico consistency conditions were derived in Refs. 1 and 2 without considering the effects of mass differences among the P mesons. However, one can show that the limitations on the η wave functions arise only from considering the ηK-ηK amplitude and the crossed amplitudes of the other channels. Since the η_α and K masses are not greatly different, one would expect the wave-function condition of Eq. (7) to be applicable particularly to the η_α.

IV. EXPERIMENTAL ANALYSIS OF η WAVE FUNCTIONS

In this section I discuss various experimental evidence concerning the quark-model wave functions of the $\eta_\alpha(549)$ and $\eta_\beta(958)$. I continue to assume that the Ψ_0 state is not coupled to mesons.

In many theoretical treatments of the η_α and η_β the Gell-Mann–Okubo (GMO) mass formula is assumed, either for the masses or for the squares of the masses. In terms of the squares this formula may be written

$$\langle \eta_8 | m^2 | \eta_8 \rangle = \tfrac{1}{3}(4m_K^2 - m_\pi^2),$$

where η_8 is the wave function of Eq. (4b). The right side of this equation is not far from the square of the mass of the η_α, so use of the rule forces the η_α to have a large octet component.

Since the masses of the η_α and η_β are not understood theoretically, I do not assume the GMO formula and do not require the η_α to have a large octet component. No attempt is made here to calculate the η_α and η_β masses.

A. The tensor-meson decays

We first turn to the PP decays of members of the tensor-meson nonet. Since the $\pi\pi$ mode of the $f'(1516 \text{ MeV})$ is unobserved, I make the usual assumptions that the OZI rule applies and that the structures of the $f(1271)$ and f' are $(u\bar{u}+d\bar{d})/\sqrt{2}$ and $s\bar{s}$, respectively. Those decays that do not involve an η fit the predictions of SU(3) symmetry and the OZI rule remarkably well, if the partial widths are related to coupling constants g by the simple formula

$$\Gamma_i = g_i^2 p_i^5 / m_i^2, \tag{8}$$

where p is the decay momentum and m is the mass of the tensor meson. This is shown in Table I. The experimental data are taken from the compilation of the Particle Data Group,[12] and the method of determining the errors in $g^2(\text{exp})$ is explained in a previous analysis.[13] The overall normalization of the $g^2(\text{theory})$ was determined by the condition that the quantity

$$\Sigma_i [g_i^2(\text{exp}) - g_i^2(\text{theory})]/E_i^2$$

be a minimum, where the sum is over the five non η modes of Table I and E_i is the error in $g_i^2(\text{exp})$.

The value of $a_{\alpha u}^2$ corresponding to the $A_2 \to \pi\eta_\alpha$ decay is

$$a_{\alpha u}^2 = 0.43 \pm 0.04, \tag{9a}$$

where the error does not include any error in $g^2(\text{theory})$. The prediction and experiment disagree somewhat for the $A_2 \to K\bar{K}$ decay; consequently, if one considers only the $\pi\eta_\lambda/K\bar{K}$ branching ratio of the A_2 the prediction is different, i.e.,

$$a_{\alpha u}^2 = 0.55 \pm 0.07. \tag{9b}$$

Comparison of these two values gives a rough idea of the accuracy of this determination.

TABLE I. Experimental and theoretical values of g^2 (in GeV^{-2}) for PP decays of the tensor-meson nonet.

Decay	$g^2(\text{exp})$	$g^2(\text{theory})$
$A_2(1310) \to K\bar{K}$	0.57 ± 0.07	0.74
$f(1271) \to \pi\pi$	2.57 ± 0.29	2.23
$f \to K\bar{K}$	0.82 ± 0.21	0.74
$f'(1516) \to K\bar{K}$	1.50 ± 0.53	1.48
$K^*(1421) \to \pi K$	1.38 ± 0.14	1.12
$A_2 \to \pi\eta_\alpha$	0.63 ± 0.06	$1.48a_{\alpha u}^2$
$A_2 \to \pi\eta_\beta$	<1.04	$1.48a_{\beta u}^2$
$f' \to \eta_\alpha\eta_\alpha$	<0.8	$1.48a_{\alpha s}^4$
$K^* \to K\eta_\alpha$	<0.35	$0.37(a_{\alpha u} + \sqrt{2}\,a_{\alpha s})^2$

It is seen from Table I that if the experimental upper limits on the decays $A_2 \to \pi\eta_\beta$, $f' \to \eta_\alpha\eta_\alpha$, and $K^* \to K\eta_\alpha$ were decreased, significant information about the η_α and η_β wave functions would result. A tighter limit on the $K^* \to K\eta_\alpha$ decay would be particularly desirable, since the Odorico solution of Eq. (6) predicts this decay to be absent.

B. Other hadronic processes involving the Ψ_u terms

We consider next the charge-exchange sum rule, Eq. (1). A derivation is given in Ref. 5. I will write the amplitudes of Eq. (1) in a form convenient for analyzing the η wave functions. It is assumed in the derivation that something is exchanged between the meson and the baryon. If one sets the amplitude for the $K^+n \to K^0p$ process equal to that of the inverse process, then the baryon vertex for all the processes involves the conversion of a p to an n; the interaction at this vertex may be represented by a common factor B. In the simple quark model of Ref. 5 (which includes the OZI rule), there are two amplitudes at the meson vertex, $M(du)$ and $M(\bar{u}\bar{d})$, where $M(ij)$ is the amplitude for converting an i quark of the initial meson to a j quark of the final meson. It is convenient to use the amplitudes $M_\pm$ defined by

$$M_\pm = [M(du) \pm M(\bar{u}\bar{d})]/\sqrt{2}. \tag{10}$$

The amplitudes A for the processes of the sum rule are

$$A(K^- \bar{K}^0) = B(M_+ - M_-)/\sqrt{2}\,,$$
$$A(K^+ K^0) = B(M_+ + M_-)/\sqrt{2}\,,$$
$$A(\pi^- \pi^0) = BM_-\,,$$
$$A(\pi^- \eta_i) = Ba_{iu} M_+\,, \qquad (11)$$

where $A(jk)$ is the amplitude for which the initial and final mesons are j and k. Each amplitude may be a sum of contributions of several different exchanges; for example, there may be several contributing Regge trajectories. Nevertheless, the amplitudes may be written in the form of Eq. (11). The cross sections near the forward direction include contributions from both helicity flip and nonflip, in general. The sum rule of Eq. (1) follows if $a_{\alpha u}{}^2 + a_{\beta u}{}^2 = 1$.

Since the ratio $a_{\beta u}{}^2/a_{\alpha u}{}^2$ may be measured in many ways, I use a modification of Eq. (1) that does not involve the η_β. It is seen from Eq. (11) that $a_{\alpha u}{}^2$ is given by

$$a_{\alpha u}{}^2 = \frac{\sigma(K^+K^0) + \sigma(K^-\bar{K}^0) - \sigma(\pi^-\pi^0)}{\sigma(\pi^-\eta_\alpha)}\,. \qquad (12)$$

It has been shown that Eq. (12) is consistent with 6-GeV data over the range of four-momentum transfer squared 0–1 GeV^2, if $a_{\alpha u}{}^2$ has the value[14]

$$a_{\alpha u}{}^2 \sim \tfrac{1}{3}\,. \qquad (13)$$

This result is usually presented differently.[14] If the η_α is a pure octet particle containing no inert piece, then the η_β term may be dropped from Eq. (1) if a factor of 3 is inserted in front of the η_α term. This is called the octet sum rule and has the advantage that its derivation does not require the OZI rule. However, satisfaction of the octet sum rule implies only Eq. (13) and provides no evidence concerning the relative sizes of $a_{\alpha s}$ and $a_{\alpha 0}$.

The results of Eqs. (9a), (9b), and (13) are summarized in Table II, along with other experimental results discussed later in this section.

We next consider the up-down quark probability for the $\eta_\beta(958)$. Okubo and Jagannathan have considered several hadronic processes that involve an η_α or η_β and other hadrons that are made exclusively of up and down quarks and antiquarks in the quark model.[15] (The nonstrange baryons and isotriplet mesons have this property.) If these processes are described by simple quark diagrams that satisfy the OZI rule, the predicted ratios of η_β and η_α rates are proportional to the ratio $a_{\beta u}{}^2/a_{\alpha u}{}^2$. The processes analyzed include the $\pi^- p \to \eta n$ cross sections of the charge-exchange sum rule. The statistics are not good for many of the experiments; however, Okubo and Jagannathan show that most of the data are consistent with the approximate relation, $K = a_{\beta u}{}^2/a_{\alpha u}{}^2 \sim 0.50$.[15]

On the other hand, it is pointed out in Ref. 15 that determination of the η_β/η_α ratio for the processes $\pi^+ p \to \eta_i \Delta^{++}$ at 5.45 GeV/c leads to the value $K = 0.24 \pm 0.11$,[16] and 7.1-GeV/c $\pi^- p \to \eta_i \Delta^0$ data imply $K = 0.25 \pm 0.025$.[17] Furthermore, the η_β/η_α ratio from the $\pi^- p \to \eta_i n$ reactions at 8.4 GeV/c depends on $t' = |t - t(0°)|$, where t is the square of the four-momentum transfer.[18] The η_β/η_α ratio is close to unity for very small t', is about $\frac{1}{2}$ for 0.15 $(\mathrm{GeV}/c)^2 < t' < 0.3$ $(\mathrm{GeV}/c)^2$, and is smaller in the t' range 0.4–0.9 $(\mathrm{GeV}/c)^2$. Fuchs has pointed out that the spin-flip cross section is more suitable than the non-spin-flip cross section for measuring K, because absorptive corrections and Regge cuts are more important for the non-spin-flip process.[19] He estimates the spin-flip contributions to the 8.4-GeV/c data, and concludes that K is approximately 0.27. Our conclusion from these analyses is that K is probably in the range

$$0.25 \lesssim a_{\beta u}{}^2/a_{\alpha u}{}^2 \lesssim 0.5\,. \qquad (14)$$

TABLE II. Summary of evidence concerning η_α and η_β wave functions.

Parameter	Value	Source
$a_{\alpha u}{}^2$	0.33 to 0.55	$\Gamma(A_2 \to \pi\eta_\alpha)$, $\sigma(\pi^- p \to \eta_\alpha n)$, $\Gamma(\rho \to \eta_\alpha \gamma)$
$a_{\alpha s}{}^2$	~0.2	$\Gamma(\phi \to \eta_\alpha\gamma)$, consistent with $\sigma(K^- p \to \eta_\alpha \Lambda)$
Sign $(a_{\alpha s}/a_{\alpha u})$	negative	$\Gamma(\eta_\alpha \to \gamma\gamma)$, $\sigma(K^- p \to \eta_\alpha\Lambda)$ dip
$a_{\beta u}{}^2/a_{\alpha u}{}^2$	0.25 to 0.5	Refs. 15 and 19
$a_{\beta s}{}^2$	not small	$\sigma(K^- p \to \eta_\beta \Lambda)$
Sign $(a_{\beta s}/a_{\beta u})$	positive unless $a_{\beta u}$ very small	$\Gamma(\eta_\beta \to \gamma\gamma)/\Gamma(\eta_\beta \to \rho\gamma)$

Lipkin has pointed out that another measure of $a_{\beta u}^2/a_{\alpha u}^2$ in this model is the cross-section ratio $R_{\text{back}} = \sigma(K^-p \to n_\beta\Lambda)/\sigma(K^-p \to n_\alpha\Lambda)$ in the backward direction, since there are no strange quarks in the proton or exchanged baryon.[20] The experimental ratio at 4.2 GeV/c is[7]

$$R_{\text{back}} = (2.0 \pm 0.68)/(1.27 \pm 0.39),$$

suggesting an $a_{\beta u}^2/a_{\alpha u}^2$ ratio much larger than 0.5. If this result persists experimentally, a possible interpretation is that the Ψ_0 part of the η_α and η_β wave functions interacts weakly with mesons but strongly with baryons.

Further evidence concerning the interactions of η_α mesons with baryons may be obtained from decays of baryon resonances that correspond to the first excited level in the quark model. We consider the $\eta_\alpha N$ partial widths of the $J^P = \frac{1}{2}^-$, N^* resonances. These widths cannot be predicted from non-η widths and SU(3) symmetry alone, because the $\frac{1}{2}^-$ resonances probably are mixtures of states of different SU(3) multiplets. However, it was shown in previous papers that SU(6)$_W$ symmetry is valid approximately for most P-meson decays of states of this quark-model level, and this symmetry leads to a prediction of the sum of the squares of the coupling constants to $\eta_\alpha N$ states of the $N_{1/2^-}(1516)$ and $N_{1/2^-}(1668)$.[13,21] The experimental value of this sum is (106 ± 40) GeV^2, while the theoretical prediction is 45 $a_{\alpha u}^2$ GeV^2 in the notation of this paper.[13] [The experimental value results entirely from the $N^*(1516)$.] Although there is a large error in the experimental value, it appears that the prediction is very low. A possible resolution of this conflict is a strong coupling of the inert state Ψ_0 to baryons. This requires that the η_α has an appreciable Ψ_0 term, of course.

C. Electromagnetic decays involving η mesons

In order to analyze electromagnetic processes involving η mesons, one must make additional assumptions. I make the usual vector-dominance assumption in a simple form. It is assumed that the γ interactions are a linear combination of those of the ρ^0, ω, and ϕ, and that the VVP interaction ratios are in accordance with SU(3) symmetry and the OZI rule. It is convenient to express the γ interactions in terms of the $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$ V states rather than the observed V states. The requirement that the γ interacts as a U-spin scalar then allows one to write the γ interactions in terms of two parameters f and S, i.e.,

$$\gamma \sim f[(\tfrac{1}{2}S - 2)(u\bar{u}) + (\tfrac{1}{2}S + 1)(d\bar{d} + s\bar{s})]. \tag{15}$$

The parameter S measures the relative contribution of an SU(3) singlet component of the photon; $S = 0$ if the photon interacts as an octet particle.[22]

I consider first the $P\gamma$ decays of V mesons and certain $V\gamma$ decays of the η_β meson. The partial width for a $V \to P\gamma$ decay is taken to be

$$\Gamma_i = C g_i^2 p_i^3, \tag{16}$$

where C is a constant that is the same for all $V \to P\gamma$ decays, and p is the decay momentum, i.e., $p = \frac{1}{2}(m_V^2 - m_P^2)/m_V$. The predicted coupling constants g_i are listed for the relevant decays in Table III, along with the experimental values of the widths. Andrews *et al.* list two solutions to their data analysis for the $\rho \to \eta_\alpha\gamma$ and $\omega \to \eta_\alpha\gamma$ widths.[25] The solution not listed in Table III is $\Gamma(\rho \to \eta_\alpha\gamma) = 76 \pm 15$ keV; $\Gamma(\omega \to \eta_\alpha\gamma) = 29 \pm 7$ keV. This solution is neglected here because it disagrees with vector dominance.

Even if the appropriate phase-space factor is more complicated than the p^3 factor of Eq. (16), the three possible ω/ρ ratios are nearly independent of these factors. These ratios are predicted to be

$$\frac{\Gamma(\omega\pi)}{\Gamma(\rho\pi)} \approx \frac{\Gamma(\rho\eta_\alpha)}{\Gamma(\omega\eta_\alpha)} \approx \frac{\Gamma(\eta_\beta\rho)}{\Gamma(\eta_\beta\omega)} = \frac{9}{(1-S)^2}\,\Omega, \tag{17}$$

where $\Gamma(ij)$ denotes the partial width for the decay $i \to j\gamma$, and Ω represents the phase-space ratio.

If $S = 0$, the predicted $\omega \to \pi\gamma/\rho \to \pi\gamma$ ratio is too large by a factor of more than 2. The experimental $\Gamma(\omega\pi)$ and $\Gamma(\rho^-\pi^-)$ of Table III lead to the relation $|1 - S| = 0.615$. The choice $S = 1.61$ leads to large contradictions for other processes, so I take $S = 0.385$.

Recently, Zanfino *et al.* have determined the $\rho\gamma/\omega\gamma$ branching ratio of the η_β to be approximately 9.9 ± 2.0.[27] It is seen from Eq. (17) that this result is consistent with $S = 0$ but not $S = 0.385$. The measured $\Gamma(\rho\eta_\alpha)/\Gamma(\omega\eta_\alpha)$ ratio of Table III is large, as predicted, but the errors are so large

TABLE III. Experimental partial widths and calculated coupling constants for some $V \to P\gamma$ decays.

Decay	Γ (keV)	g
$\omega \to \pi\gamma$	870 ± 61 [a]	-3
$\rho \to \pi\gamma$	$35 \pm 10(\rho^-\pi^-)$ [b]	$-1+S$
$\omega \to \eta_\alpha\gamma$	3.0 ± 2.5 [c]	$(-1+S)a_{\alpha u}$
$\rho^0 \to \eta_\alpha\gamma$	50 ± 13 [c]	$-3a_{\alpha u}$
$K^{0*} \to K^0\gamma$	75 ± 35 [d]	$2+S$
$\phi \to \eta_\alpha\gamma$	74 ± 15 [a]	$(2+S)a_{\alpha s}$

[a] Reference 23.
[b] Reference 24.
[c] Reference 25.
[d] Reference 26.

that this ratio is consistent with either $S=0$ or $S=0.385$. I shall compute other $V\to P\gamma$ results for the two values $S=0$ and 0.385. The $\omega\to\pi\gamma$ decay rate will be used as a standard, i.e., to determine the constant C in Eq. (16).

This model does not fit the $K^{0*}\to K^0\gamma$ decay. The predicted $\Gamma(K^{0*}K^0)$ is 290 ± 140 keV if $S=0.385$ or 210 ± 100 keV if $S=0$. This does not agree with the experimental number in Table III. Bohm and Teese have assumed a complicated dependence of the phase-space factor on the V and P masses in order to fit this rate and other $V\to P\gamma$ rates.[23] Unfortunately, there are so few measured rates that the model loses most of its predictive power if many symmetry-breaking effects are present. Consequently, I will continue using Eq. (16), with the g_i equal to those of Table III.

It is seen from Table III that the ratio $\Gamma(\rho\eta_\alpha)/\Gamma(\omega\pi)$ is a measure of $a_{\alpha u}{}^2$. The experimental numbers lead to the value

$$a_{\alpha u}{}^2=0.45\pm0.12\,. \tag{18}$$

This is consistent with the value obtained from purely hadronic processes, and so lends support to our simple model of $V\to P\gamma$ decays.

The ratio $\Gamma(\varphi\eta_\alpha)/\Gamma(\omega\pi)$ measures the strange-quark probability of the η_α. The numbers in Table III and the formula of Eq. (16) lead to the result

$$a_{\alpha s}{}^2=0.22\pm0.05 \quad \text{if } S=0\,,$$

or (19)

$$a_{\alpha s}{}^2=0.16\pm0.035 \quad \text{if } S=0.385\,.$$

This is the most direct measure of $a_{\alpha s}{}^2$ that is possible at present, and is included in Table II.

We turn next to the $\gamma\gamma$ decays of the self-conjugate P mesons. The vector-dominance assumption is extended in the usual way, i.e., the amplitude $A(P\to\gamma\gamma)$ is assumed proportional to $\Sigma_i A(P\to V_i\gamma)A(V_i\to\gamma)$, where $A(V_i\to\gamma)$ is equal to the appropriate coefficient of Eq. (15). I take the singlet term S to be zero. This leads to the result

$$\frac{\Gamma(\eta_\alpha\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}=\frac{25}{9}\frac{m_\eta{}^3}{m_\pi{}^3}\left(a_{\alpha u}+\frac{\sqrt{2}}{5}a_{\alpha s}\right)^2. \tag{20}$$

If we take $\Gamma(\eta_\alpha\to\gamma\gamma)$ to be 324 eV (Ref. 28) and $\Gamma(\pi^0\to\gamma\gamma)$ to be 7.86 eV,[12] the result is

$$[a_{\alpha u}+(\sqrt{2}/5)a_{\alpha s}]^2=0.22\pm0.035\,. \tag{21}$$

For comparison, the η_α amplitude of the Odorico solution of Eq. (6) leads to 0.32 for the quantity of Eq. (21). Because of the number of assumptions involved, I do not use this result to estimate $a_{\alpha u}$ or $a_{\alpha s}$. However, Eq. (21) is consistent with the approximate $a_{\alpha u}{}^2$ and $a_{\alpha s}{}^2$ values shown in Table II only if the relative $a_{\alpha u}-a_{\alpha s}$ sign is negative. Thus, this result is evidence for the negative sign. This is the sign that is usually assumed, since it corresponds to a large $q\bar{q}$ octet component for the η_α.

A formula analogous to Eq. (20) applies to the $\eta_\beta\to\gamma\gamma$ decay. Unfortunately, the total η_β decay width is not known, although the $\gamma\gamma/\rho^0\gamma$ branching ratio of the η_β is known approximately. Consequently, we consider the ratio R defined as follows:

$$R=\frac{\Gamma(\eta_\beta\to\gamma\gamma)}{\Gamma(\pi^0\to\gamma\gamma)}\frac{\Gamma(\omega\to\pi\gamma)}{\Gamma(\eta_\beta\to\rho\gamma)}\,. \tag{22}$$

Our model predicts the ratio $\Gamma(\omega\to\pi\gamma)/\Gamma(\eta_\beta\to\rho\gamma)$ to be $\frac{1}{3}a_{\beta u}{}^{-2}(p_{\omega\pi}/p_{\eta\rho})^3$, where the p are the decay momenta. The factor $\frac{1}{3}$ results from the sum over spins in the $\rho\gamma$ final state. Consequently, the predicted value of R is

$$R=\frac{25}{27}\left[\frac{m_\eta{}^2(m_\omega{}^2-m_\pi{}^2)}{m_\pi m_\omega(m_\eta{}^2-m_\rho{}^2)}\right]^3\left(1+\frac{\sqrt{2}}{5}\frac{a_{\beta s}}{a_{\beta u}}\right)^2, \tag{23}$$

where η denotes the η_β. If the $\omega\to\pi\gamma$ width is taken from Table III and the $\pi^0\to\gamma\gamma$ width and η_β branching ratio are taken from Ref. 12, the result is

$$[1+(\sqrt{2}/5)(a_{\beta s}/a_{\beta u})]^2=1.87\pm0.35. \tag{24}$$

The two solutions to this equation are

$$(a_{\beta s}/a_{\beta u})=1.3\pm0.5 \text{ and } -8.4\pm0.5\,. \tag{25}$$

The positive sign is probably the correct one, since $a_{\beta u}$ does not appear to be nearly zero. If the ratio is positive, one cannot consider this to be a reliable determination of the magnitude of $a_{\beta s}/a_{\beta u}$.

D. Strangeness-exchange reactions

In order to understand the relevance of the strangeness-exchange sum rule of Eq.(2), we follow the procedure used for the charge-exchange rule in part B of this section. The baryon vertices of the reactions of Eq.(2) all involve the transition $p\to\Lambda$, and will be represented by the factor B'. The meson vertices involve the quark transitions $s\to u$ and $\bar{u}\to\bar{s}$, and are denoted by $M(su)$ and $M(\bar{u}\bar{s})$. The amplitudes involved in the sum rule may be written

$$\begin{aligned}
&A(K^-\pi^0)=B'M(su)/\sqrt{2}\,,\\
&A(\pi^-K^0)=B'M(\bar{u}\bar{s})\,,\\
&A(K^-\eta_i)=B'\{[a_{iu}M(su)/\sqrt{2}]+a_{is}M(\bar{u}\bar{s})\}\,.
\end{aligned} \tag{26}$$

The cross sections are the sums of the squares of the helicity-flip and nonflip amplitudes. If the

TABLE IV. Average experimental values of $d\sigma/dt'$ [in μb/GeV^2] in different t' regions for strangeness-exchange processes at 4.2 GeV/c.

t' region	$K^-p\to\eta_\alpha\Lambda$	$K^-p\to\eta_\beta\Lambda$	$K^-p\to\pi^0\Lambda$	$\pi^-p\to K^0\Lambda$
0–0.05	138 $\pm$ 8.8	211 $\pm$ 14	234 $\pm$ 9.6	280 $\pm$ 22
0.05–0.15	62.5 $\pm$ 3.9	150 $\pm$ 8.1	162 $\pm$ 5.6	165 $\pm$ 12
0.15–0.3	12 $\pm$ 2	75.3 $\pm$ 4.6	88.7 $\pm$ 3.3	66.6 $\pm$ 6.0
0.3–0.6	3.2 $\pm$ 0.6	20.3 $\pm$ 1.9	42.8 $\pm$ 1.7	19.6 $\pm$ 2.3
0.6–1.0	6.7 $\pm$ 0.6	2.6 $\pm$ 0.5	18.5 $\pm$ 0.9	10.7 $\pm$ 1.5
Average				
0–1.0	18.6 $\pm$ 0.7	44.0 $\pm$ 1.4	61.5 $\pm$ 1.1	50.7 $\pm$ 2.0

inert components of the η_α and η_β vanish and if the a matrix is an orthogonal two-by-two matrix, the sum rule follows.

The strangeness-exchange sum rule has been checked at 4.2 GeV/c. The two sides of Eq. (2) are plotted as a function of momentun transfer in Ref. 7. I list the 4.2-GeV/c differential cross sections for all four processes in different t' ranges in Table IV, where $t' = |t - t(0°)|$. The experimental cross sections for the three K^-p processes are taken from Ref. 7. The $\pi^-p\to K^0\Lambda$ cross sections are interpolations between the 3.9-GeV/c measurements of Abramovich *et al.* and the 4.5-GeV/c measurements of Crennell *et al.*[29,30] The errors in Table IV are essentially root mean squares of the errors given for the smaller t' bins in the experimental papers. More precisely, the error given here for any of the K^-p processes is $(\Sigma_i E_i^2 F_i f_i)^{1/2}$, where E_i is the error in $d\sigma/dt'$ for the experimental bin i, F_i is the ratio of the bin size to the t' range of this paper, and $f_i = F_i 0_i$, where 0_i is the fraction of the experimental bin that overlaps the t' range used here. In most cases this overlap fraction is one. In the $\pi^-p\to K^0\Lambda$ process, the errors for the 3.9- and 4.5-GeV data were each determined by this procedure; the results given here both for the cross sections and errors are simple averages of the results at these two energies.

No kinematic correction factors have been introduced, although the meson mass difference may be important for a lab momentum as small as 4.2 GeV/c.

A few preliminary formulas are useful for the interpretation of the data. First, we note from Eq. (26) that if either of the $q\bar{q}$ components vanishes for either of the η's, a simple prediction results, i.e.,

$$\sigma(K^-\eta_i) = a_{iu}^2\sigma(K^-\pi^0) \text{ if } a_{is}=0\,, \tag{27a}$$

$$\sigma(K^-\eta_i) = a_{is}^2\sigma(\pi^-K^0) \text{ if } a_{iu}=0\,, \tag{27b}$$

Another simple way to analyze the data is in terms of two Regge trajectories corresponding to the vector and tensor mesons $K^*(892)$ and $K^{**}(1421)$. For these trajectories individually, the amplitudes of Eq.(26) are related as follows:

$$M(su) = -M(\bar{u}\bar{s}) \text{ for the } K^*\,, \tag{28a}$$

$$M(su) = M(\bar{u}\bar{s}) \text{ for the } K^{**}\,. \tag{28b}$$

We now consider the $K^-p\to\eta_\alpha\Lambda$ cross section. If $a_{\alpha s}$ were zero, it would follow from Eq.(27a) and Table II that $\sigma(K^-\eta_\alpha)/\sigma(K^-\pi^0)$ is about $\frac{1}{3}$ or $\frac{1}{2}$. This is consistent with the data for $t'<0.15$ (GeV/c)^2 and $t'>0.6$ (GeV/c)^2, and so supports the conclusion that $a_{\alpha s}$ is not large. However, the most striking feature of the $K^-p\to\eta_\alpha n$ cross section is the dip at about $t'=0.4$ (GeV/c)^2.[7] It is usually assumed that this dip is caused by a wrong-signature zero in the $K^*(892)$ exchange amplitude, in which case the contribution of the $K^{**}(1421)$ trajectory must be small. In Ref. 2 Odorico uses this argument to support his solution for the TPP couplings; it is seen from Eqs. (26) and (28b) that the $K^{**}\to Kn_\alpha$ coupling vanishes if the η_α has the structure of Eq.(6).

On the other hand, it is difficult to interpret all the data in a simple Regge model. If $K^*(892)$ exchange is unimportant near $t'=0.4$ (GeV/c)^2, it follows from Eqs. (26) and (28b) that the $\pi^-p\to K^0\Lambda$ cross section should be significantly larger than the $K^-p\to\pi^0\Lambda$ cross section. This contradicts the data.

Finally, we consider the $K^-p\to\eta_\beta\Lambda$ cross section. It is striking that this cross section is significantly larger than the $\eta_\alpha\Lambda$ cross section for all $t'<0.6$ (GeV/c)^2, despite the fact that the η_β mass is large. Since much evidence exists that $a_{\beta u}^2$ is small (reviewed in Sec. IV B), this is strong evidence that $a_{\beta s}^2$ is not small.

V. POSSIBLE MANIFESTATIONS OF A THIRD η MESON

If the η_α or η_β contains an appreciable component of an inert P state, then at least one other physical η meson should exist that contains an appreciable component of at least one of the quark states Ψ_u or Ψ_s. Therefore, the observation of a third η meson is crucial to the hypothesis that the η_α or η_β contains a significant inert piece.

The third η (called here the η_γ) must be even under charge conjugation, and so should not decay into a ρ^0 and π^0. However, if the η_γ is sufficiently massive, it might decay into a $K\bar{K}$ or possibly even a $\rho\rho$.

The 1420-MeV E meson is a possible candidate for the η_γ. Ths spin and parity of the E are not known, but the two possible assignments that are favored experimentally are 0^- and 1^+.[12] An appreciable $K\bar{K}^*(892)$ decay mode is observed. If the E were an SU(3) singlet, this would imply a strong $\pi^0\rho^0$ decay, a decay forbidden by charge-conjugation invariance. On the other hand, if the E is a pure octet particle, it is a mystery that other members of the octet have not been discovered in the (1100-1600)-MeV energy region. Therefore, the E is probably a singlet-octet mixture, and may be the η_γ.

The possibility that the E is one of the η mesons of Eq.(6) has been discussed previously.[31] It was shown in Ref. 31 that if the phase-space factor for $V \to PP$ decays is p^3/m_V^2, the corresponding phase-space factor for $P \to VP$ decays is p^3/E_V^2. If the E is the η_γ, SU(3) symmetry is used for the VPP interactions, and a K^*-$K\pi$ partial width of 49 MeV is used to fix the overall coupling constant, then the predicted $(K\bar{K}^* + \bar{K}K^*)$ partial width Γ_{KK^*} of the E is

$$\Gamma_{KK^*} = (54\ \text{MeV})\, a_8^2\,,$$

where a_8 is the component of the octet $q\bar{q}$ state in the E. This is given by $a_8 = (\frac{1}{3})^{1/2} a_{\gamma u} - (\frac{2}{3})^{1/2} a_{\gamma s}$. The experimental partial width is about 12 MeV, implying $a_8^2 \sim 0.22$. In the Odorico solution of Eq.(6), the η_γ has no octet component.[32]

The most crucial experiments to test the concept of the inert P state involve measuring the forward cross section of the sum rules of Eqs. (1) and (2) at sufficiently high energies that production of the η_γ is not inhibited very much because of its mass. If the a matrix of Eq. (3) is orthogonal, and if the η_γ is included in the sum, then the sum rules should be satisfied if the mesonic vertices of the Ψ_0 are zero. If these vertices are not zero, then the sum of the η_α, η_β, and η_γ cross sections should be greater than the predictions of the sum rules. Because of the large predicted η_γ production cross sections, the $\pi^- p \to \eta_\gamma n$ and $K^- p \to \eta_\gamma \Lambda$ processes may be useful in identifying the η_γ in the first place. After such a meson is identified, the sum rules test the extent to which the Ψ_0 is inert.

VI. CONCLUDING REMARKS

The sums of η_α and η_β forward cross sections in both $\pi^- p \to \eta n$ and $K^- p \to \eta\Lambda$ processes are smaller than expected. This is evidence that one or both of these η mesons contains a part that is different from the $q\bar{q}$ parts of the quark model. This part interacts relatively weakly at the mesonic vertices of these processes. I have used experimental data to analyze the η_α and η_β wave functions under the assumption that each is a sum of a nonstrange-quark part, a strange-quark part, and an inert part that does not interact with mesons. The OZI rule is assumed for the interactions of the quark parts.

The results of the analysis are summarized in Table II. A simple vector-dominance assumption was made for photon processes; the resulting prediction of the $K^{0*} \to K^0\gamma$ decay width is not in agreement with experiment. Consequently, results that depend only on observation of electromagnetic decays cannot be considered solid.

The results of Table II are in approximate agreement with the predictions of Eq.(6), obtained by combining Odorico's consistency conditions with the interaction ratios of the standard quark model. The one significant difference between the experimental results and the predictions is that the experimental $a_{\beta u}^2$ is not close to zero, although it may be as small as 0.1.

It is suggested that the inert P state is a gluon bound state. The interaction between such a gluon bound state and a $q\bar{q}$ state is of lower order than the interaction between two $q\bar{q}$ states that proceeds through virtual gluon bound states. This suggests the "partially pure P" hypothesis of Eq. (5). The results of Table II are consistent with this hypothesis.

The search for gluon bound states is an important problem of particle physics. However, in the case of P states, it may be that one will see mixtures of gluon bound states and quark states, rather than nearly pure gluon bound states.

It is pointed out in Sec. IV B that experimental results on the $\eta_\alpha N$ decays of N^* resonances, and on backward production of the η_α and η_β, lead to conflicts with the rest of the analysis. This may indicate that the inert components interact fairly strongly with baryons.

Since we lack a theory of gluon-bound-state interactions, the experimental data do not provide evidence concerning whether the inert state (or states) is a gluon bound state or something else.

Another possibility is that this state is a baryon-antibaryon bound state of the $q\bar{q}q\bar{q}$ type.[33] It has been argued in the literature that these states should interact much more strongly with baryons than with mesons.[34]

A third physical η meson, the η_γ, is predicted in this model. The 1420-MeV E meson is a possible candidate for the η_γ. At sufficiently high energies the η_γ should be produced copiously in $\pi^- p \to \eta_\gamma n$ and $K^- p \to \eta_\gamma \Lambda$ reactions.

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