# Calculating the electron mass in terms of measured quantities

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We construct a class of models in which the electron mass can be calculated in terms of the fine-structure constant, the muon mass, and the Weinberg angle. We obtain $m_e/m_\mu = N(\alpha/\pi\sin^2\theta_W)$ where $N$ is a pure number of order 1 which depends on the specific model. (There are corrections to this formula which depend on parameters of the model, but which can be made arbitrarily small by arranging certain exotic gauge bosons to be heavy.) We show that within this class of models it is possible to obtain agreement with neutral-current data and the experimental value of $m_e/m_\mu$.

## I. INTRODUCTION

The electron mass is certainly one of the most basic parameters in physics. The advent of renormalizable gauge theories of the weak and electromagnetic interactions has raised hopes of understanding the origin of the electron's mass and has provided a natural framework for discussing the age-old electron-muon problem.

The smallness of $m_e$ compared to $m_\mu$ (and, indeed, compared to all other known fermion masses) has led many to suspect that the electron might, in fact, be massless in the tree order of perturbation theory, becoming massive as a result of radiative corrections. The numerical fact $m_e/m_\mu \sim O(\alpha)$ lends further credence to this general idea.

Georgi and Glashow, in a pioneering work,[1] and other authors have constructed models in which the electron mass is calculable. The "trick" for constructing such models is to impose on the Lagrangian symmetries (discrete or continuous or a combination of both) which rule out the appearance of an electron bare-mass term, and which further prevent the electron from acquiring mass through the Higgs mechanism in tree order. This latter condition can be accomplished in two ways. In the first possibility, symmetry forbids any Higgs field from coupling the left-handed components of the electron to the right-handed components. The second possibility is that the Higgs field coupling $e_L^-$ to $e_R^-$ has vanishing vacuum expectation value in tree order. It is apparent that in such models the electron mass will be finite and calculable. This follows from the fact that, in a renormalizable theory, ultraviolet divergences can always be canceled by counterterms which respect the symmetries of the Lagrangian (if the theory can be regulated in an invariant way). Thus no divergence can appear in $m_e$ to any order of perturbation theory.

One of the most disappointing features of all models hitherto published in which $m_e$ is calculable is that the calculated value of the electron mass ends up depending sensitively on several parameters which, while measurable in principle, are not accessible to experiments in the immediate future. (These are the masses and mixing angles of superheavy gauge bosons, or heavy leptons, or couplings in interactions which do not contribute measurably to known phenomenology.)

In this paper we present an example of a class of models in which the leading contribution to the calculated value of $m_e$ can be expressed in terms of quantities already measured. Specifically, the electron mass in these models has the simple form

$$m_e = N\frac{\alpha m_\mu}{\pi \sin^2\theta_W} + (\text{corrections})\,. \qquad (1)$$

Here $\sin\theta_W \equiv e/g$ (where $g$ is the gauge coupling of the weak interactions) and can be measured in neutral-current interactions. $N$ is a pure number which varies from model to model depending upon the dimensions of the lepton multiplets and the choice of gauge group. The corrections referred to in Eq. (1) are dependent upon unknown parameters but are down in magnitude by one power of the logarithm of a large quantity (the ratio of the mass of a heavy gauge boson to the mass of a heavy lepton).

In this paper we adopt the reasonable assumption that gauge bosons are much more massive than all relevant leptons.

There is, of course, no fundamental reason why, in the true theory, the electron mass should be expressible in terms of the other physical constants which are currently measurable by physicists. On the other hand, it is obviously desirable to give first consideration to models which make some contact with experiment.

In Sec. II we discuss briefly previously published models, show where the parameter dependence arises, and outline the basic idea for a class of models which lead to a calculated value of $m_e$ in the form given by Eq. (1). In Sec. III we give a simple example of such a model, based on the group SU(2)×SU(2)×U(1). In Sec. IV we discuss how the value of the numerical factor $N$ of Eq. (1) depends upon the details of the model, and for which models good agreement with experiment is possible. Section V is a summary.

## II. GENERAL SCHEME

We propose to consider models in which the electron mass arises principally from diagrams of the type shown in Fig. 1. Simple power counting indicates that such a diagram is logarithmically divergent. Therefore, in any model of this type where the electron mass is calculable, there will be in general more than one diagram which contributes to the electron mass, the divergent parts of which cancel. This cancellation may occur between two (or more) diagrams involving bosons of different mass, or the cancellation may occur between two (or more) diagrams with fermions of different mass on the internal line. We will consider models of the latter variety.[2]

As an example, suppose that the electron and muon are in the same representation multiplet of some gauge group. Call the off-diagonal gauge boson which couples the electron to the muon $W_h$ and the gauge coupling constant $g_h$. Further, suppose the muon mixes with some heavy lepton $X^-$. (This is the type of model considered by Georgi and Glashow.) The undiagonalized mass matrix for $\mu^-$ and $X^-$ will have the form

$$(\mu_L^- \; X_L^-)^{(0)} \begin{bmatrix} 0 & a \\ b & c \end{bmatrix} \begin{bmatrix} \mu_R^- \\ X_R^- \end{bmatrix}^{(0)} . \tag{2}$$

There is to be allowed no direct coupling between the multiplets containing $e_L^-$ and $e_R^-$, and therefore none between $\mu_L^-$ and $\mu_R^-$ (remember $e$ and $\mu$ are in the same multiplet). This explains the zero entry in the mass matrix exhibited above. Diagonalizing this asymmetric mass matrix we find

$$\begin{aligned} \begin{bmatrix} \mu_L^- \\ X_L^- \end{bmatrix}^{(0)} &= \begin{bmatrix} \cos\lambda & \sin\lambda \\ -\sin\lambda & \cos\lambda \end{bmatrix} \begin{bmatrix} \mu_L^- \\ X_L^- \end{bmatrix}, \\ \begin{bmatrix} \mu_R^- \\ X_R^- \end{bmatrix}^{(0)} &= \begin{bmatrix} \cos\rho & \sin\rho \\ -\sin\rho & \cos\rho \end{bmatrix} \begin{bmatrix} \mu_R^- \\ X_R^- \end{bmatrix}, \\ m_\mu \cos\lambda \cos\rho &= -m_X \sin\lambda \sin\rho . \end{aligned} \tag{3}$$

Thus the two diagrams shown in Fig. 2 will contribute to the electron mass. An evaluation of these diagrams gives (in the Landau gauge)

$$m_e = m_\mu (g_h \cos\lambda)(g_h \cos\rho) \int \frac{d^4k}{(2\pi)^4} \frac{3}{(k^2+M_{W_h}{}^2)(k^2+m_\mu{}^2)} + m_X (g_h \sin\lambda)(g_h \sin\rho) \int \frac{d^4k}{(2\pi)^4} \frac{3}{(k^2+M_{W_h}{}^2)(k^2+m_X{}^2)} . \tag{4}$$

The mass relation given in Eq. (3) indicates that the two terms in Eq. (4) combine to give a finite result:

$$m_e = m_\mu \cos\lambda \cos\rho \, \frac{3g_h^2}{16\pi^2} \times \left[ \frac{m_X{}^2}{M_{W_h}{}^2 - m_X{}^2} \ln\left(\frac{M_{W_h}}{m_X}\right)^2 + \frac{m_\mu{}^2}{M_{W_h}{}^2 - m_\mu{}^2} \ln\left(\frac{M_{W_h}}{m_\mu}\right)^2 \right]. \tag{5}$$

In a particular model $g_h$ may be related in a known way to $e$. However, in general the parameters

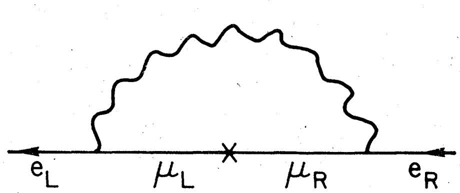


FIG. 1. Prototypical diagram contributing to the electron mass. The wavy line is a gauge boson.

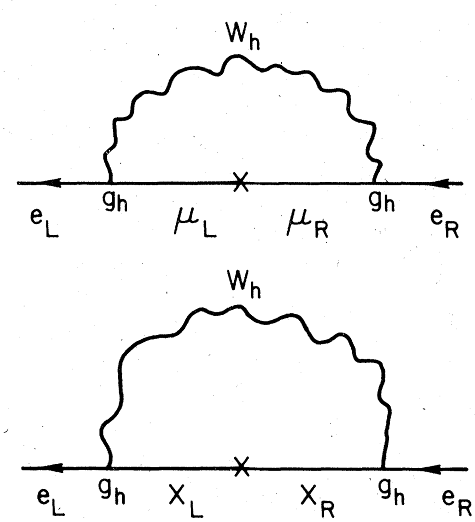


FIG. 2. Two diagrams contributing to the electron mass in a model in which the muon mixes with a heavy lepton $X^-$. $W_h$ is a heavy "exotic" gauge boson.

$\lambda$, $\rho$, $m_X$, and $M_{W_h}$ are *not* known. Thus a formula for $m_e$ such as Eq. (5), which typifies the results obtained so far in the literature,[1] is not particularly exciting. As explained in the Introduction we have assigned ourselves the task of doing better.

We can see from the foregoing discussion that most of the parameter dependence of the electron mass is a result of the fact that the muon mixes with another (heavy) lepton. Such mixing appears unavoidable if the electron mass arises from one-loop diagrams with the divergences canceling among diagrams with different leptons on the internal lines. We propose to eliminate much of this parameter dependence by not mixing the muon.

Suppose that in some models both the electron *and* the muon are massless at the tree level, that the muon derives its mass at the one-loop level from diagrams of the sort shown in Fig. 3(a), and that the electron derives its mass at the two-loop level from diagrams of the sort shown in Fig. 3(b). (In Fig. 3, $W_h$ and $W_h'$ are heavy gauge bosons and $X^-$ and $Y^-$ are heavy leptons.) The point is that much of the unknown parameter dependence may be expected to be common to the expressions for $m_e$ and $m_\mu$, and thus to cancel out in the ratio $m_e/m_\mu$. That this indeed can be arranged shall be demonstrated presently.

The expression for the muon mass which results from the diagrams of Fig. 3(a) has the form (ignoring numerical constants for now)

$$m_\mu \sim \frac{g_h'^2}{(2\pi)^2} m_X \cos\lambda \cos\rho \left[\left(\frac{m_Y}{M_{W_h'}}\right)^2 \ln\left(\frac{M_{W_h'}}{m_Y}\right)^2 + \left(\frac{m_X}{M_{W_h'}}\right)^2 \ln\left(\frac{M_{W_h'}}{m_X}\right)^2\right] + O\left(\left(\frac{m_Y}{M_{W_h'}}\right)^4, \left(\frac{m_X}{M_{W_h'}}\right)^4\right). \tag{6}$$

Here $\lambda, \rho$ are the mixing angles of the left- and right-handed heavy leptons ($X^-$ and $Y^-$). [Compare to Eq. (4).] We have made the reasonable assumption that $m_X, m_Y \ll M_{W_h}, M_{W_h'}$. The electron mass will have the form

$$m_e \sim g_h^2 g_h'^2 (m_X \cos\lambda \cos\rho) \times I_{(2\text{ loop})}\left(\frac{m_Y}{M_{W_h'}}, \frac{m_X}{M_{W_h'}}, \frac{M_{W_h'}}{M_{W_h}}\right), \tag{7}$$

where $I_{(2\text{ loop})}$ is the result of evaluating the two-loop momentum integrals in Fig. 3(b). The expression $g_h'^2 m_X \cos\lambda \cos\rho$ is indeed common to Eqs. (6) and (7). Moreover, it is no problem to construct a model wherein $g_h$ is related in a known way to $e$ (as we shall see). It remains, however, to be seen whether the parameter dependence of

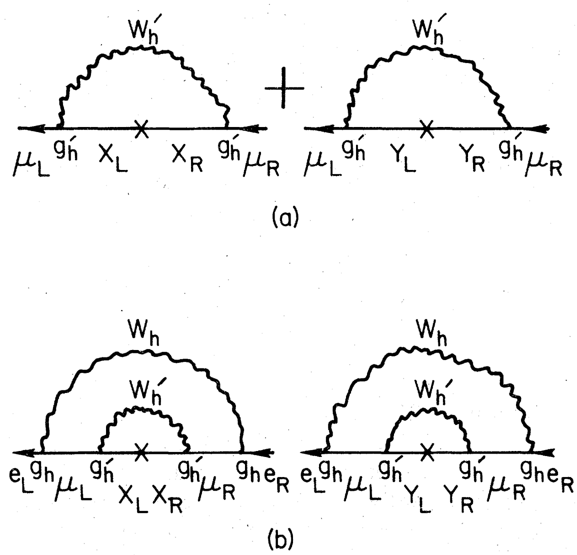


FIG. 3. Diagrams contributing to the muon and electron masses in the class of models proposed here. $X^-$ and $Y^-$ are heavy leptons. $W_h$ and $W_h'$ are heavy "exotic" gauge bosons.

$I_{(2\text{ loop})}$ will cancel out in the ratio $m_e/m_\mu$. If we call $(M_{W_h}/M_{W_h'})^2 = R$, for the sake of brevity, then an evaluation of the two-loop momentum integral[3] can be found to give an expression of the form (again, assuming $m_X, m_Y \ll M_{W_h}, M_{W_h'}$)

$$I_{(2\text{ loop})} \sim \frac{\text{const}}{(2\pi)^4} \times \frac{\ln R}{R-1}\left[\left(\frac{m_Y}{M_{W_h'}}\right)^2 \ln\left(\frac{M_{W_h'}}{m_Y}\right)^2 + \left(\frac{m_X}{M_{W_h'}}\right)^2 \ln\left(\frac{M_{W_h'}}{m_X}\right)^2 + O\left(\left(\frac{m_Y}{M_{W_h'}}\right)^2, \left(\frac{m_X}{M_{W_h}}\right)^2\right)\right] \tag{8}$$

so that, up to a numerical constant,

$$\frac{m_e}{m_\mu} \sim (\text{const}) \times \frac{\ln R}{R-1}\left(\frac{g_h}{2\pi}\right)^2 \times\left[1 + O\left(\frac{1}{\ln(M_{W_h'}/m_Y)^2}\right)\right]. \tag{9}$$

Introducing an angle $\theta$ defined by $e = g_h \sin\theta$, this may be rewritten as

$$\frac{m_e}{m_\mu} \sim (\text{const}) \times \frac{\ln R}{R-1} \frac{\alpha}{\pi \sin^2\theta} \times [1 + (\text{logarithmic corrections})]. \tag{10}$$

If $g_h = g$ (the coupling of ordinary weak interactions) then $\theta = \theta_W$ (the usual Weinberg angle).

We now proceed to implement this general approach in specific models. To summarize, we seek models in which the diagrams shown in Fig. 3 can arise, $R$ is fixed, $\sin\theta \equiv e/g_h$ is measurable, and $m_X, m_Y \ll M_{W_h}, M_{W'_h}$. Actually, in the simplest type of models we consider, $W_h$ and $W'_h$ turn out to be one and the same gauge boson so that the factor $\ln R/(R-1)$ can be replaced by unity. In this case Eq. (10) becomes

$$\frac{m_e}{m_\mu} \sim (\text{const}) \times \frac{\alpha}{\pi \sin^2\theta} \times [1 + (\text{logarithmic corrections})]. \quad (11)$$

## III. AN EXAMPLE

By way of illustration we present here a simple example based on the gauge group $SU(2)_h \times [SU(2) \times U(1)]_{wk+em}$. The coupling constants are $g_h$, $g$, and $g'$, respectively. The charge is given by $Q = I_3 + Y/2$, where $I_3$ is the third component of weak isospin and $Y$ is the generator of the U(1) hypercharge group. Denoting the multiplets by $(I_h, I; Y)$, the left-handed leptons belong to a $(1, \frac{1}{2}; -1)$, a $(1, 0; 0)$, and a $(0, \frac{1}{2}; -1)$ as follows:

$$L_1 \equiv \begin{bmatrix} \nu_e & \nu_\mu & \nu_X \\ e^- & \mu^- & X^- \end{bmatrix}_L, \quad L_2 \equiv (N_e \; N_\mu \; N_X)_L,$$

$$L_3 \equiv \begin{bmatrix} N_Y \\ Y^- \end{bmatrix}_L.$$

The horizontal direction is the $I_{3h}$ direction and the vertical is $I_3$. The right-handed components are in a $(1, \frac{1}{2}; -1)$ and a $(0, \frac{1}{2}; -1)$:

$$R_1 \equiv \begin{bmatrix} N_e & N_\mu & N_X \\ e^- & \mu^- & X^- \end{bmatrix}_R, \quad R_3 \equiv \begin{bmatrix} N_Y \\ Y^- \end{bmatrix}_R.$$

[As is evident, this example is a generalization of the Cheng-Li[4] vectorlike models. It is possible to generalize the Weinberg-Salam model in the same way by eliminating the heavy neutrinos $N_e, N_\mu, N_X$ and putting the right-handed leptons in $(1, 0; -2)$ and $(0, \frac{1}{2}; -1)$ multiplets.] The Higgs multiplets are $\phi_1$: $(1, 0; 0)$, $\phi_2$: $(1, 0; 0)$, and $\phi_3 = (0, \frac{1}{2}, -1)$, with vacuum expectation values[5] (VEVs)

$$\langle\phi_1\rangle = (0\;0\;a),$$
$$\langle\phi_2\rangle = (0\;0\;b),$$
$$\langle\phi_3\rangle = \begin{bmatrix} c \\ 0 \end{bmatrix},$$
$$a, b \gg c.$$

The Yukawa couplings are schematically summarized in Fig. 4. No bare mass or Higgs coupling of $L_1$ to $R_1$ is permitted by symmetry in the Lagrangian. This ensures that at tree level $m_e = m_\mu = 0$ "naturally." Diagonalizing the lepton mass matrix one finds

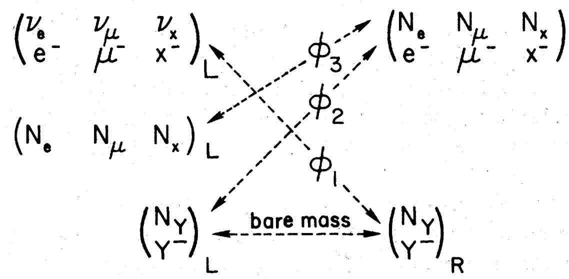


FIG. 4. Schematic representation of the Yukawa couplings in the Lagrangian in illustrative model. $\phi_1$, $\phi_2$ and $\phi_3$ are Higgs multiplets which couple left-handed leptons to right-handed leptons.

$$\begin{bmatrix} X_L^- \\ Y_L^- \end{bmatrix}^{(0)} = \begin{bmatrix} \cos\lambda & \sin\lambda \\ -\sin\lambda & \cos\lambda \end{bmatrix} \begin{bmatrix} X_L^- \\ Y_L^- \end{bmatrix}^{(m)},$$

$$\begin{bmatrix} X_R^- \\ Y_R^- \end{bmatrix}^{(0)} = \begin{bmatrix} \cos\rho & \sin\rho \\ -\sin\rho & \cos\rho \end{bmatrix} \begin{bmatrix} X_R^- \\ Y_R^- \end{bmatrix}^{(m)}, \quad (12)$$

where the superscript (0) means "undiagonalized" and $(m)$ means "mass eigenstate." From the pattern of the Higgs vacuum expectation values it is clear that the $SU(2)_h$ gauge bosons are not mixed with the $SU(2)\times U(1)$ gauge bosons. The gauge boson masses are given by

$SU(2)\times U(1)$: $(M_{W^\pm})^2 = \frac{1}{2} g^2 c^2$ (ordinary weak intermediate vector boson)

$$(M_Z)^2 = \frac{1}{\cos^2\theta_W} (M_{W^\pm})^2$$
$$(M_A)^2 = 0 \text{ (photon)},$$

$SU(2)_h$: $(M_{W_h})^2 = g_h^2(a^2 + b^2)$

$$(M_{Z_h})^2 = 2g_h^2(a^2 + b^2),$$

with $\tan\theta_W \equiv g'/g$. The $SU(2)\times U(1)$ subgroup we identify with the gauge group of the observed weak and electromagnetic interactions. With $a$ and $b$ very large, $SU(2)_h$ becomes irrelevant for low-energy phenomenology.

There are three diagrams that contribute at the one-loop level to the muon mass in this model. These are shown in Fig. 5. The unphysical Higgs boson (the would-be Goldstone boson associated with $I_{3h}$) is represented by $S^0$ and the physical Higgs boson (with mass $m_H$) by $g^0$. It is straightforward to compute that

$$m_\mu = -\frac{3g_h^2}{16\pi^2}(m_X\cos\lambda\cos\rho) \times\left[\left(\frac{m_Y}{M_{W_h}}\right)^2 \ln\left(\frac{M_{W_h}}{m_Y}\right)^2 + \left(\frac{m_X}{M_{W_h}}\right)^2\ln\left(\frac{M_{W_h}}{m_X}\right)^2\right] \times\left[1+O\left(\frac{m_H^2}{m_Y^2}\right)\right] \qquad (13)$$

It is perhaps surprising that the physical Higgs-boson contribution *increases* as $m_H$ increases. The origin of this effect[6] is simply that the Higgs-boson–lepton couplings depend on the lepton masses. This means that in the sum of the two diagrams shown in Fig. 5(b) the divergences do not cancel. Rather the divergence from the diagrams with unphysical Higgs bosons must be canceled by the divergence from the diagrams with physical Higgs bosons [Fig. 5(c)]. This means that instead of assuming $m_H$ very large, in order to render the effect of the Higgs boson negligible one must assume that $m_H$ is sufficiently small.

This does not cause any phenomenological problems, however, since the required bound on $m_H$ is not very restrictive ($m_H$ may still be ~100 GeV without spoiling the expression for $m_e/m_\mu$). Furthermore, the physical Higgs field $g^0$ always couples to a heavy lepton $X^-$ or $Y^-$, is neutral, and thus is difficult to detect.

Some thought shows that making $m_H$ small is equivalent to making those terms in the Higgs potential, $V_{\text{Higgs}}$, which couple $\phi_1$ to $\phi_2$ small.

There are also diagrams involving physical Higgs fields which contribute to the electron mass in both one- and two-loop order. Again, by making the terms in $V_{\text{Higgs}}$ which couple $\phi_1$ to $\phi_2$ small we can render such contributions negligible. This is obvious since $\phi_1$ couples to $e_L^-$ and not $e_R^-$, whereas $\phi_2$ couples to $e_R^-$ and not $e_L^-$. Thus to compute $m_e$ one need only consider the diagrams of Fig. 6. Notice that the graph in Fig. 5(c) is not of the form indicated in Fig. 1. Thus Fig. 3 is actually incomplete.

Evaluating these diagrams in the generalized $R_\xi$ gauge one finds that

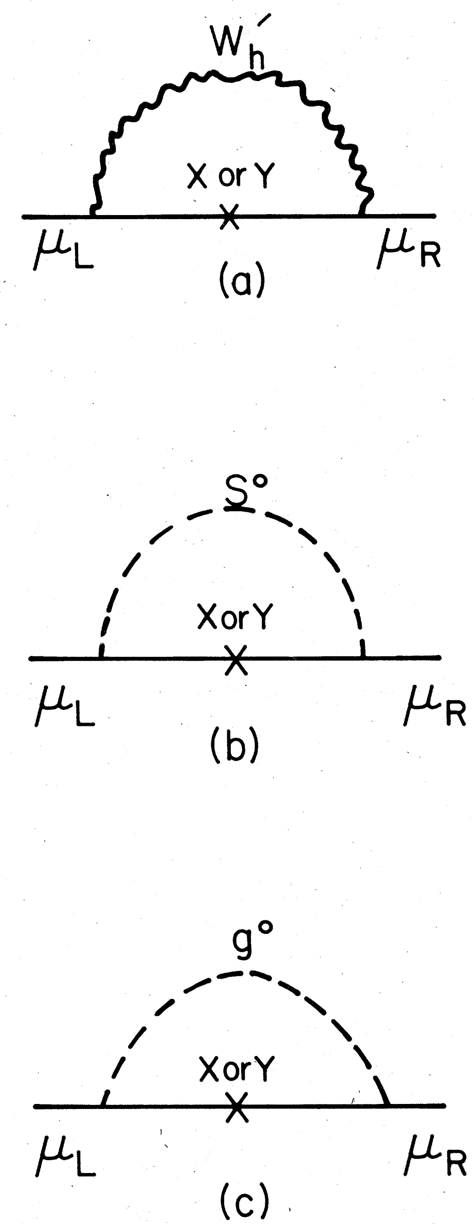


FIG. 5. Diagrams contributing to $m_\mu$ in illustrative model.

$$m_e = -\frac{g_h^4}{(2\pi)^8}\frac{(m_Y^2-m_X^2)}{M_{W_h}^2}\int d^4k\int d^4l\,\frac{\{12l^2+(l^2-k\cdot l)\times(\xi\text{-dependent expression})\}}{[(k-l)^2+X][(k-l)^2+Y][k^2(k^2+1)l^2(l^2+1)]}, \qquad (14)$$

where $X\equiv m_X^2/M_{W_h}^2 \ll 1$, $Y\equiv M_Y^2/M_{W_h}^2 \ll 1$. The dominant contribution comes only from the first term in the curly brackets.[7] (This is apparent as the second term becomes relatively small when $l_\lambda \approx k_\lambda$, which is precisely the region of momentum space where the denominator becomes small.) Therefore one finds that

$$m_e = -\frac{g_h^4}{2^8\pi^4}(m_X\cos\lambda\cos\rho)(12)\left[\left(\frac{m_Y}{M_{W_h}}\right)^2\ln\left(\frac{M_{W_h}}{m_Y}\right)^2+\left(\frac{m_X}{M_{W_h}}\right)^2\ln\left(\frac{M_{W_h}}{m_X}\right)^2+O\left(\left(\frac{m_Y}{M_{W_h}}\right)^2,\left(\frac{m_X}{M_{W_h}}\right)^2\right)\right],$$

and hence

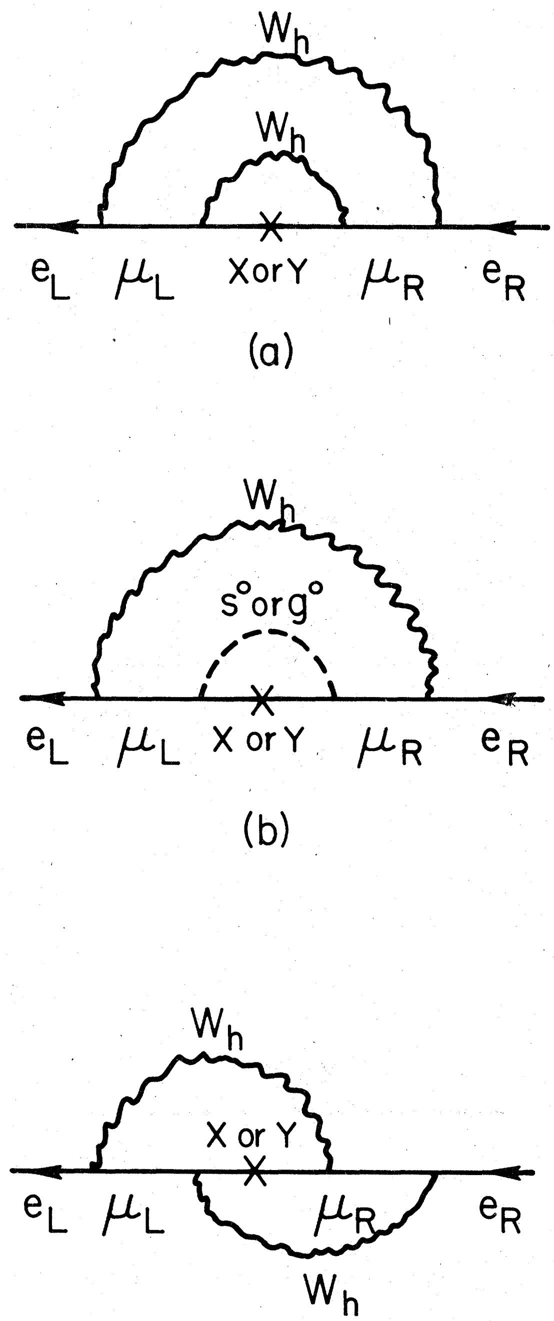


FIG. 6. Diagrams contributing to $m_e$ in illustrative model.

$$\frac{m_e}{m_\mu} = \frac{{g_h}^2}{4\pi^2}\left[1 + O\left(\frac{1}{\ln(M_{W_h}/m_Y)^2}\right)\right]. \tag{15}$$

It remains to relate $g_h$ to $e$. This can be done by imposing upon the model the discrete symmetry of interchanging the two SU(2) groups thus forcing $g = g_h$.[8] This can be done at the cost of introducing additional lepton and Higgs multiplets. For example, corresponding to $L_1$: $(1, \frac{1}{2}, -1)$ there must be a $(\frac{1}{2}, 1, -1)$ sufficiently massive so as not to be relevant for phenomenology. The discrete symmetry is to be broken by the vacuum. This makes the model perhaps a bit ungainly, but let us choose to overlook this sort of consideration at present.

Using this additional constraint we obtain

$$\frac{m_e}{m_\mu} = \frac{\alpha}{\pi \sin^2\theta_W} + (\text{corrections}). \tag{16}$$

This has the expected form. To agree with the experimental value of $m_e/m_\mu$ one needs $\sin^2\theta_W \cong 0.48$. In this vectorlike model the usual[9] neutral-current couplings, $g_V$ and $g_A$, that parametrize $\nu_\mu e^-$ elastic scattering have the values $g_V = -1 + 2\sin^2\theta_W \cong -0.04$ and $g_A = 0$. These do not give a very satisfactory fit to the neutral-current data.

## IV. FURTHER CONSIDERATIONS

The question naturally arises whether a model of the type described can give a value of $m_e/m_\mu$ in accord with experiment with a value of $\sin\theta_W$ consistent with neutral-current phenomenology. This leads us to consider what the overall constant factor $N$ of Eq. (1) will be in a general model.

Let us consider models where the overall gauge group of leptonic interactions is $G = \mathrm{SU}(N)_I \times \mathrm{U}(1)_Y \times G'$. We assume there is a superstrong breaking of $G$ down to $\mathrm{SU}(2)_I \times \mathrm{U}(1)_Y$ which is the gauge group of the observed weak and electromagnetic interactions, and that this symmetry is further broken down to the exact U(1) of electromagnetism by a single Higgs-field vacuum expectation value which is responsible for the masses of the $W^\pm$ and $Z$ bosons, the mediators of charged- and neutral-current weak interactions. Here $\mathrm{SU}(2)_I \subset \mathrm{SU}(N)_I$. $\mathrm{U}(1)_Y$ is the hypercharge group. We assume that $G$ contains a subgroup $\mathrm{SU}(2)_h$ to which there corresponds an off-diagonal gauge boson $W_h$ coupling the electron to the muon, and further that $G$ contains a subgroup $\mathrm{SU}(2)_h'$ with a corresponding off-diagonal gauge boson $W_h'$ coupling the muon to the heavy leptons $X^-$ and $Y^-$. It may be that $\mathrm{SU}(2)_h$ and $\mathrm{SU}(2)_h'$ are one and the same group, in which case $W_h$ and $W_h'$ are the same boson. This is true of the illustrative model described in Sec. III. It is equally possible, however, to construct models where $\mathrm{SU}(2)_h$ and $\mathrm{SU}(2)_h'$ are separate groups. For example, one may construct a model with $G = \mathrm{SU}(2)_h \times \mathrm{SU}(2)_h' \times \mathrm{SU}(2)_I \times \mathrm{U}(1)_Y$ in which the largest lepton multiplets are a left-handed $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}; Y = -1)$:

$$\left[\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \atop \begin{pmatrix} \nu_f \\ f^- \end{pmatrix} \begin{pmatrix} \nu_X \\ X^- \end{pmatrix}\right]_L$$

and a right-handed $(\frac{1}{2}, \frac{1}{2}, 0; Y=-2)$:

$$\begin{bmatrix} e^- & \mu^- \\ f'^- & X^- \end{bmatrix}_R .$$

As in the models previously discussed, one arranges through the Yukawa couplings that $X^-$ mixes with another heavy lepton $Y^-$.

In the general case, the groups $SU(2)_h$ and $SU(2)'_h$ may be subgroups of $SU(N)_I$ or of $G'$. In the latter case one must impose discrete symmetries on the Lagrangian to require $g_h = g'_h = g$ [the coupling constants of $SU(2)_h$, $SU(2)'_h$, and $SU(2)_I$, respectively].

Because we have assumed that one Higgs vacuum expectation value does the breaking of the weak gauge group $SU(2)_I \times U(1)_Y$, the observed weak interactions may be parametrized by an angle $\theta$ reflecting the relative magnitude of $g$ and $g'$ [$g'$ is the coupling constant of $U(1)_Y$]. This parameter $\theta$ will appear in the expression for $m_e/m_\mu$. In particular the covariant derivative is

$$D_\mu = \partial_\mu + i(g/2)A_\mu^a \lambda^a + ig'(Y/2)B_\mu + (\text{terms involving the generators of } G') .$$

Here the $SU(N)_I$ generators $\lambda^a$ are normalized by $\text{Tr}\,\lambda^a\lambda^b = 2\delta^{ab}$. The charge operator of the model if of the form $Q = Y/2 + I$ where $(\frac{1}{2}\text{Tr}I^2)^{-1/2}\, I$ is a properly normalized generator of $SU(N)_I$. We can then define the parameter $\theta$ by $\tan\theta = (g'/g)(2\,\text{Tr}I^2)^{1/2}$. In this case $e = g'\cos\theta = g\sin\theta/(2\,\text{Tr}I^2)^{1/2}$ is the electromagnetic coupling constant. (For example, in the Weinberg-Salam model $N=2$ and $\text{Tr}\,I^2 = \frac{1}{2}$, so that $\tan\theta = g'/g$ and $e = g\sin\theta$.)

Finally, suppose the electron to have $I_h = t$ and $I_{3h} = m$ and the muon to have $I_h = t$ and $I_{3h} = m-1$. $I_h$ represents the "horizontal" isospin of the group $SU(2)_h$ and $I_{3h}$ its third component.

If the model is of the first type, where $W_h = W'_h$ are the same boson (as in the example of Sec. III), then we write

$$\frac{m_e}{m_\mu} = \frac{(\text{Tr}I^2)}{2(\text{Tr}I_{3h}{}^2)}\,(t+m)(t-m+1)\,\frac{\alpha}{\pi\sin^2\theta} . \qquad (17)$$

If, on the other hand, the model is of the second type, where $W_h$ and $W'_h$ are different bosons [with, possibly, different masses, so that $R \equiv (M_{W_h}/M'_{W_h})^2 \neq 1$], then

$$\frac{m_e}{m_\mu} = 2\,\frac{\ln R}{R-1}\,\frac{(\text{Tr}I^2)}{2(\text{Tr}I_{3h}{}^2)}\,(t+m)(t-m+1)\,\frac{\alpha}{\pi\sin^2\theta} . \qquad (18)$$

[The factor of 2 that appears in Eq. (18) can be traced to the fact that for every two-loop diagram contributing to the electron mass there is another obtained by interchanging $W_h$ and $W'_h$ that gives the same contribution[10]: Thus there are twice as many diagrams in the second type of model as in the first where $W_h = W'_h$.]

The range of $\sin^2\theta$ that will fit the neutral-current data depends upon the details of the particular model. We have not found, so far, a simple model where $\sin^2\theta$ gives at the same time a good agreement with the neutral-current data and the mass ratio $m_e/m_\mu$. In the various models we have examined, the computed value of $m_e/m_\mu$ turns out to be somewhat too large. Notice that the factors $\text{Tr}I^2$ and $(t+m)(t-m+1)$ are bounded below: $\text{Tr}I^2 \geq \frac{1}{2}$ and $(t+m)(t-m+1) \geq 1$.

Now, there is a way in which in models of the second type one may arrange to get the computed value of $m_e/m_\mu$ to agree with experiment—though it is admittedly quite *ad hoc* and artificial. It is possible by a judicious choice of Higgs multiplets and vacuum expectation values to control the factor $\ln R/R-1$ which appears in Eq. (18). Specifically, one may introduce a Higgs multiplet, call it $\chi$, which does not couple to the leptons, is a singlet under $SU(2)_I$, and is a nonsinglet under $SU(2)_h$ and $SU(2)'_h$. Supposing that $g_h = g'_h$ and that the vacuum expectation value of one of the components of $\chi$ is large compared to the other Higgs-field vacuum expectation values, then $\langle\chi\rangle$ will give the largest contributions to $M_{W_h}$ and $M_{W'_h}$ without interfering with the ordinary weak phenomenology of the model. To take a specific case, if $\chi$ has $I_h = \frac{3}{2}$ and $I'_h = \frac{1}{2}$ and the $(I_{3h} = +\frac{3}{2}, I'_{3h} = +\frac{1}{2})$ component of $\langle\chi\rangle$ is large[11] then $R \cong 3$.

The model we mentioned above with $G = SU(2)_h \times SU(2)'_h \times SU(2)_I \times U(1)_y$ and where the largest multiplets are $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}; -1)$ on the left and $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}; -2)$ on the right may be constructed so as to reduce to the Weinberg-Salam model for low-energy phenomenology. As is well known, in the Weinberg-Salam model an excellent fit to the data is possible with $\sin^2\theta = (e/g)^2 \cong 0.27$. So if, in this model, one arranges that $R \cong 3$ then [referring to Eq. (18)]

$$\frac{m_e}{m_\mu} \cong 2 \times \frac{\ln 3}{2} \times \frac{1}{2}\,\frac{\alpha}{\pi(0.27)} \cong 0.0047 .$$

The experimental value is 0.004 836.

The point of the foregoing example is merely to demonstrate that it is *possible* in principle to obtain satisfactory numerical results within our approach. Of course, the expedient we have suggested by which $R$ may be adjusted to bring this about is extremely contrived and is not put forward as a realistic possibility. It is to be hoped that satisfactory numerical results may be achieved in a more natural manner. Whether this is possible is an open question.

An undesirable feature of the model given in Sec. III was the necessity of imposing a discrete symmetry to force $g_h = g$. If $SU(2)_h \times SU(2)'_h \subset SU(N)_I$ there is no need for such a discrete symmetry. [For example, our approach can be implemented in an SU(4)×U(1) model with the leptons in 1 and 15 representations of the SU(4) group.]

At first glance it appears that the muon $(g-2)$ will pose a problem for all such models, as there will be contributions to it proportional to $m_Y$ and $m_X$ which are very large masses. However, such contributions are actually suppressed by a factor $(M_{W^\pm}/M_{W'_h})^2$ and are completely negligible.

We remark, finally, that the incorporation of hadrons presents no special difficulty in this class of models if the known quarks are allowed to transform as singlets under the "exotic" gauge interactions. For example, in the model of Sec. III one may put the quarks into $(0, \frac{1}{2}, \frac{1}{3})$, $(0, 0, -\frac{2}{3})$, and $(0, 0, \frac{4}{3})$ representations.

## V. CONCLUSION

We have presented here a general class of models in which the electron mass is finite, calculable, and expressible (up to corrections which may be made arbitrarily small) in terms of currently measured quantities. It is perhaps not unreasonable to hope that, given the possibility of such models, the electron mass may afford another clue to the underlying structure of the weak interactions.

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[1]G. 't Hooft, Nucl. Phys. B35, 167 (1971); S. Weinberg, Phys. Rev. D 5, 1962 (1972); H. Georgi and S. L. Glashow, *ibid.* 7, 2457 (1973); R. Mohapatra, *ibid.* 9, 3461 (1974); A. A. Ansel'm, and D. I. D'yakanov, Zh. Eksp. Teor. Fiz. 71, 1268 (1976) [Sov. Phys.—JETP 44, 4 (1976)]; S. M. Barr and A. Zee, Phys. Rev. D 15, 2652 (1977).

[2]A detailed example is presented by Georgi and Glashow (Ref. 1) based on the group SU(3) ×U(1).

[3]M. J. Levine and R. Roskies, Phys. Rev. D 9, 421 (1974). Note the absence in Eq. (8) of terms of order $(m_Y/M_{W'_h})^2 \ln^2(m_Y/M_{W'_h})^2$ which might naively be expected to occur. This fact is the key to the success of our approach.

[4]S. M. Belenky, S. T. Petcov, and B. Pontecorvo, Phys. Lett. 67B, 309 (1977); T. P. Cheng and L.-F. Li, Phys. Rev. Lett. 38, 381 (1977); S. B. Treiman, F. Wilczek, and A. Zee, Phys. Rev. D 16, 152 (1977); T. P. Cheng and L.-F. Li, *ibid.* 16, 1425 (1977); B. W. Lee and R. E. Shrock, *ibid.* 16, 1444 (1977); J. Bjorken, K. Lane, and S. Weinberg, *ibid.* 16, 1474 (1977).

[5]An SU(2) triplet of real fields must have a vacuum expectation value of the form $(X+iY, Z, X-iY)$. Thus to get the proper VEV's one may introduce another U(1) symmetry group so that $\phi_1$ and $\phi_2$ may be complex triplets. More simply, however, one may have $\phi_1 = \alpha_1 + i\beta_1$ and $\langle\alpha_1\rangle = (a/2, 0, a/2)$, $\langle\beta_1\rangle = (ia/2, 0, -ia/2)$, and similarly for $\phi_2$. This can be accomplished by means of a discrete symmetry.

[6]D. Yevick, doctoral thesis, Princeton University, 1977 (unpublished); S. M. Barr and S. Wandzura, Phys. Rev. D 16, 707 (1977). The same phenomenon occurs at the one-loop level in many models in which unphysical Higgs fields are linear combinations of members of more than one isospin multiplet.

[7]For this term, gauge invariance is manifest. To show the gauge invariance of the second term in the braces one must evaluate the momentum integrals. This is beside our purpose here, however.

[8]Then the gauge group is isomorphic to O(4) ×U(1).

[9]See for example, J. J. Sakurai, International School of Subnuclear Physics, Erice, Sicily, 1976 (unpublished); UCLA Report No. UCLA/76/TEP/21 (unpublished).

[10]Note that Eq. (8) is invariant [to leading order in $\ln(m_Y/M_{W'_h})$] under the interchange $W_h \longleftrightarrow W'_h$ and $R \longleftrightarrow 1/R$.

[11]We assume that one can achieve this condition through minimizing some Higgs effective potential.