Decays of a heavy lepton involving the hadronic vector current

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The decays of a heavy lepton involving the hadronic vector current are calculated from electron-positron-annihilation data. The result, $\Gamma(\tau^- \to \nu_\tau + (2n \text{ pions})^-) = 1.69\Gamma(\tau^- \to \nu_\tau + e^- \bar{\nu}_e)$ for $M_\tau = 1.9$ GeV and $m_{\nu_\tau} = 0$, and its implications are discussed.

I. INTRODUCTION

Over the past few years evidence has been accumulating from electron-positron annihilation experiments for a class of events with low multiplicity and charged leptons among the final particles. The properties of these events, as measured in several independent experiments, are such that only a small fraction could originate from the production and weak semileptonic decay of charmed hadrons. The only surviving single explanation for these events is that they are due to the pair production and subsequent weak decay of a new charged heavy lepton.[1-9]

Such a lepton, called τ,[9] might be expected to couple to a neutrino ν_τ via the charged weak current. If this is the same charged weak current as that responsible for the leptonic and semileptonic decays of the "ordinary" particles, we must expect the decays $\tau^- \to \nu_\tau e^- \bar{\nu}_e$, $\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu$, and $\tau^- \to \nu_\tau + (\text{hadrons})^-$. This last decay, if pictured as occurring by the production of a light-quark pair which then dress themselves as hadrons, is naively expected (because of three colors) to occur at three times the rate of $\tau^- \to \nu_\tau e^- \bar{\nu}_e$ or $\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu$.

These decays, $\tau^- \to \nu_\tau + (\text{hadrons})^-$, are of considerable interest; for, not only does one want to know for theoretical reasons if the naively calculated rate agrees with the observed sum over the physical hadronic channels, but also experimentally these modes and their detailed properties serve to clarify the existence and nature of the τ and of its couplings.

A number of individual modes (such as $\tau^- \to \nu_\tau \pi^-$) can be calculated from other known quantities (the pion decay constant). The Cabibbo-allowed decays through the hadronic vector current may be related to the total cross section for e^+e^- annihilation into hadrons through the isovector electromagnetic current. In the past, several calculations of $\tau^- \to \nu_\tau + (\text{hadrons})^-$ have been made by combining known couplings to a few channels with estimates of others.[10-12]

In this paper we recalculate the decays through the hadronic vector current. We do this because previous partial calculations plus estimates can now be replaced by a direct integration of colliding-beam data over the entire energy range relevant to τ decay. In the next section we recall the relevant formulas for $\Gamma(\tau^- \to \nu_\tau + (\text{hadrons})^-)$ through the hadronic vector current and show how the ratio of three-charged-prong to one-charged-prong decays can be calculated. Then in Sec. III we present the detailed input and output of the calculation assuming various masses for τ and ν_τ. Section IV is a discussion of our results and their comparison with present experimental information.

II. HEAVY-LEPTON DECAY RATES VIA THE HADRONIC VECTOR CURRENT

The formula for the decay rate for $\tau^- \to \nu_\tau e^- \bar{\nu}_e$ or $\nu_\tau \mu^- \bar{\nu}_\mu$, assuming that the charged current has a $V \pm A$ form and is of universal strength at the τ-ν_τ vertex, is

$$\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e) = \frac{G^2 M_\tau^{\,5}}{192\pi^3} \,. \tag{1}$$

Here $G = 1.02 \times 10^{-5}/M_N^{\,2}$ is the weak-coupling constant, and M_τ, the mass of the τ, is experimentally 1.9 ± 0.1 GeV.[3,8,9] We have assumed that all the final leptons may be taken as massless. With a massive neutrino the decay rate becomes

$$\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e) = \frac{G^2 M_\tau^{\,5}}{192\pi^3} F(\Delta), \tag{2a}$$

with

$$F(\Delta) = 1 - 8\Delta^2 + 8\Delta^6 - \Delta^8 - 12\Delta^4 \ln \Delta^2, \tag{2b}$$

and $\Delta = m_{\nu_\tau}/M_\tau$. The experimental upper bound on the neutrino mass m_{ν_τ} is 0.6 GeV.[3,8,9]

The corresponding decay rate for $\tau^- \to \nu_\tau + (\text{hadrons})^-$, proceeding through the action of the strangeness-nonchanging hadronic vector current, is straightforward to calculate[11]:

$$\Gamma(\tau^- \to \nu_\tau + (\text{hadrons})^-) = \frac{G^2 \cos^2\theta_C}{96\pi^3 M_\tau^3} \int_0^{M_\tau^2} dQ^2 (M_\tau^2 - Q^2)^2 (M_\tau^2 + 2Q^2) \frac{\sigma^{(1)}_{e^+e^-}(Q^2)}{\sigma_{pt}(Q^2)} , \quad (3)$$

where $\cos\theta_C$ is the cosine of the Cabibbo angle, $\sigma^{(1)}_{e^+e^-}(Q^2)$ is the electron-positron cross section to annihilate into hadrons with total isospin 1 at $E_{cm}^2 = Q^2$, and $\sigma_{pt}(Q^2) = 4\pi\alpha^2/3Q^2$ is the cross section for $e^+e^- \to \mu^+\mu^-$. The extension to the case of massive neutrinos is

$$\Gamma(\tau^- \to \nu_\tau + (\text{hadrons})^-) = \frac{G^2 \cos^2\theta_C}{96\pi^3 M_\tau^3} \int_0^{M_\tau^2} dQ^2 [M_\tau^4 + m_{\nu_\tau}^4 + Q^4 - 2m_{\nu_\tau}^2 M_\tau^2 - 2m_{\nu_\tau}^2 Q^2 - 2M_\tau^2 Q^2]^{1/2}$$
$$\times [M_\tau^4 + m_{\nu_\tau}^4 - 2Q^4 - 2m_{\nu_\tau}^4 M_\tau^2 + m_{\nu_\tau}^2 Q^2 + M_\tau^2 Q^2] \frac{\sigma^{(1)}_{e^+e^-}(Q^2)}{\sigma_{pt}(Q^2)} , \quad (4)$$

which reduces to Eq. (3) when $m_{\nu_\tau} = 0$.

The term involving the strangeness-changing vector current which we have neglected is expected to be of order $\tan^2\theta_C \simeq 0.05$ relative to that which we are calculating. Furthermore, its main contribution, through $\tau^- \to \nu_\tau + K^*(890)^-$, may be calculated separately, as we will do in Sec. IV. For the range of integration in Eqs. (3) or (4) of interest to us, purely multipion states very much dominate the final-state hadron channels in electron-positron annihilation. The annihilation cross section into final states with total isospin 1 involves only those channels with even numbers of pions.

The $\pi\pi$ channel must be $\pi^+\pi^-$ in electron-positron annihilation and $\nu_\tau + \pi^0\pi^-$ in τ^- decay, and so it results in a single charged prong for the final τ^- decay products. The four-pion channel must be either $2\pi^+2\pi^-$ or $\pi^+\pi^-2\pi^0$ in colliding beams[13] and $\nu_\tau + \pi^+2\pi^-\pi^0$ or $\nu_\tau + \pi^-3\pi^0$ in τ^- decay. The four-pion states in colliding beams and τ^- decay are total $I_z = 0$ and -1 states, respectively, of the same total $I = 1$ state. This fact allows us to derive[14] a relation between the populations of the two charge states of four pions in colliding beams and the two charge states of four pions in τ^- decay. For any invariant mass Q of the four-pion system, it is

$$\frac{d\Gamma(\tau^- \to \nu_\tau + \pi^+2\pi^-\pi^0)}{d\Gamma(\tau^- \to \nu_\tau + \pi^-3\pi^0)} = 1 + 2\,\frac{\sigma(e^+e^- \to \pi^+\pi^-2\pi^0)}{\sigma(e^+e^- \to 2\pi^+2\pi^-)} . \quad (5)$$

Thus the proportion of four charged pions out of all four-pion final states in colliding beams tells us the proportion of three-charged-prong decays for $\tau^- \to \nu_\tau + (4\pi)^-$. The relative number of three-charged-prong to one-charged-prong decays arising from $\tau \to \nu_\tau + 2\pi$ and $\tau \to \nu_\tau + 4\pi$, which is of some interest experimentally, then can be settled completely from electron-positron annihilation data.

III. EXPERIMENTAL INPUT AND RESULTS

As input to Eq. (4) we need data on electron-positron annihilation into $\pi^+\pi^-$, $2\pi^+2\pi^-$, $\pi^+\pi^-2\pi^0, \ldots$ in the center-of-mass energy range from threshold to M_τ. For this purpose we have taken cross-section data from experiments done at Orsay,[15,16,17] Novosibirsk,[18] and Frascati.[19,20] Our method has been to use what we considered to be the best data on a particular process in a given energy range. We have *not* made a statistical average of all available data. On occasion we have interpolated experimental data points to get a cross section at a desired energy. Our specific choice of data is as follows.

A. $e^+e^- \to \pi^+\pi^-$

From $Q = 0.28$ to 0.90 GeV we use the Orsay[15] fit (taking ρ-ω interference into account) to their data on $|F_\pi(q^2)|^2$:

$$|F_\pi(Q^2)|^2 = \frac{F_0^2 M_\rho^2 \Gamma_\rho^2}{(M_\rho^2 - Q^2)^2 + M_\rho^2 \Gamma_\rho^2 (p/p_0)^6 (M_\rho/Q)^2} , \quad (6)$$

where Q is the total center-of-mass energy, and p is the pion momentum. For this fit the ρ mass $M_\rho = 0.7754$ GeV, $\Gamma_\rho = 0.1496$ GeV, $F_0 = 5.83$, and p_0, the pion momentum at the ρ mass, is 0.3615 GeV. The cross section for $e^+e^- \to \pi^+\pi^-$ is related to $|F_\pi(Q^2)|^2$ by

$$\sigma(e^+e^- \to \pi^+\pi^-) = \frac{\pi\alpha^2}{3Q^2}\left(\frac{2p}{Q}\right)^3 |F_\pi(Q^2)|^2 \quad (7)$$

Between $Q = 0.90$ and 1.34 GeV we use the recent Novosibirsk data[18] on $e^+e^- \to \pi^+\pi^-$, which are significantly above the ρ-meson tail calculated from Eq. (6). Above 1.34 GeV the measurements[19] of $|F_\pi|^2$ are consistent, within rather large error bars, with Eq. (6) once again. We use this formula as input in this region, but in any case this domain makes a very small contribution to $\tau^- \to \nu_\tau + \pi^-\pi^0$.

B. $e^+e^- \to 2\pi^+2\pi^-$

Between $Q = 0.90$ and 1.34 GeV we use the recent data from Novosibirsk[18] along with the Orsay data[16] at 0.91, 0.99, and 1.076 GeV to guide us at the lower end. Above 1.34 GeV our input is based on the recent data[17] from DCI at Orsay, as smoothed by a fit involving both interfering resonance and background contributions.

C. $e^+e^- \to \pi^+\pi^-2\pi^0$

Again we use the Novosibirsk data[18] for this channel up to 1.34 GeV, with earlier Orsay results[16] used to pin down the threshold behavior (0.9 to ~1.0 GeV). Above 1.34 GeV, we turn to the DCI data[17] on the sum of $\pi^+\pi^-2\pi^0$, $2\pi^+2\pi^-2\pi^0$, and $\pi^+\pi^-4\pi^0$. These join on well to the $\pi^+\pi^-2\pi^0$ Novosibirsk data[18] at the lower end.

D. $e^+e \to 6\pi$

As just noted, the six-pion channels involving π^0's are taken into account along with $\pi^+\pi^-2\pi^0$ from using the DCI data above 1.34 GeV. Direct measurements[20] of $e^+e^- \to 3\pi^+3\pi^-$, as well as diffractive photoproduction,[21] shown an effective threshold near 2 GeV.

The input cross sections are summarized in Tables I and II.

We estimate the total error in our calculation due to statistical and systematic errors in the input data to be about ±12%. The largest part of this comes from the $\pi^+\pi^-$ channel below 900 MeV, and is calculated from the statistical errors stated by the Orsay group on the parameters in Eq. (6) combined with their estimates of the systematic errors.[15] That the errors due to the $\pi^+\pi^-$ data dominate the total error is not because the intrinsic statistical or systematic errors in that experiment are particularly large—just that the bulk of the answer comes from that source. Although we have assigned large systematic errors to the multipion data at higher energies, they do not make an important contribution to the overall errors because the magnitude of the multipion contributions is not large and because we have added the errors from different channels and energy regions in quadrature.

It is convenient to state our results for $\Gamma(\tau \to \nu_\tau + (2n \text{ pions}))$ in terms of its magnitude relative to that for $\Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e)$. For a τ mass of 1.9 GeV and massless ν_τ, we find a value for this ratio of 1.69. We expect a value of $1.5\cos^2\theta_C = 1.43$ on the basis of the naive model where $\tau^- \to \nu_\tau + (2n \text{ pions})^-$ is due to $\tau^- \to \nu_\tau + \bar{u}d$, with light $\bar{u}$ and d quarks coming in three colors.[22] Our calculated value is within 20% of this naive result and is even closer to the result obtained with the logarithmic correction due to asymptotic freedom.[23] The contributions to the total result of 1.69 come from individual channels as follows: 1.12 from $\pi^+\pi^-$, 0.22 from $2\pi^+2\pi^-$, and 0.35 from $\pi^+\pi^-2\pi^0$ (plus the six-pion channels involving π^0's).

The variation in $\Gamma(\tau^- \to \nu_\tau + (2n \text{ pions})^-)/\Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e)$ with M_τ is shown in Fig. 1 ($m_{\nu_\tau} = 0$). There is relatively little variation with M_τ as long as it is in the 1.5 to 2 GeV range.

Similarly, the decay width for nonzero values of m_{ν_τ} (with M_τ fixed at 1.9 GeV) is shown in Fig. 2. Here $\Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e)$ is computed from Eq. (2) with $m_{\nu_\tau} \neq 0$. Only when the neutrino mass exceeds about 600 MeV does one see a fairly siz-

TABLE I. Input cross sections for center-of-mass energies between 0.90 and 1.34 GeV.

Q (GeV)	$\sigma(e^+e^- \to \pi^+\pi^-)$ (nb) (Ref. 18)	$\sigma(e^+e^- \to 2\pi^+2\pi^-)$ (nb) (Refs. 16 and 18)	$\sigma(e^+e^- \to \pi^+\pi^-2\pi^0)$ (nb) (Refs. 16 and 18)
0.91	133	0.0	0.0
0.93	115	0.0	0.0
0.95	90.3	0.0	2.0
0.97	68.9	0.0	4.0
0.99	58.7	1.0	6.0
1.01	62.3	1.5	8.0
1.03	49.3	2.0	10.0
1.05	40.6	3.9	7.6
1.07	40.9	3.0	25.5
1.09	41.9	7.0	25.7
1.11	20.0	5.0	18.7
1.13	32.5	5.2	35.1
1.15	37.5	9.6	20.9
1.17	21.1	10.2	29.8
1.19	19.0	10.4	22.5
1.21	21.9	11.7	35.1
1.23	17.2	12.7	30.1
1.25	14.2	15.8	37.1
1.27	10.1	13.6	41.6
1.29	5.7	17.1	19.4
1.31	7.6	19.0	22.6
1.33	4.8	20.0	36.0

TABLE II. Input cross sections for center-of-mass energies between 1.35 and 1.95 GeV.

Q (GeV)	$\sigma(e^+e^- \to 2\pi^+2\pi^-)$ (nb) (Ref. 17)	$\sigma(e^+e^- \to \pi^+\pi^-2\pi^0, 2\pi^+2\pi^-2\pi^0, \pi^+\pi^-4\pi^0)$ (nb) (Ref. 17)
1.35	23.9	33.7
1.40	28.8	38.7
1.45	36.5	44.2
1.50	45.6	57.3
1.55	54.0	62.3
1.60	40.0	64.8
1.65	34.0	54.7
1.70	30.9	79.2
1.75	26.0	62.3
1.80	19.6	44.2
1.85	16.8	35.8
1.90	15.4	33.7
1.95	14.7	27.8

able variation in the ratio $\Gamma(\tau^- \to \nu_\tau + (2n \text{ pions})^-)/\Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e)$.

Employing an integrated version of Eq. (5), we can calculate the ratio $\Gamma(\tau^- \to \nu_\tau + \pi^+2\pi^-\pi^0)/\Gamma(\tau^- \to \nu_\tau + \pi^-3\pi^0)$. For a nominal τ mass of 1.9 GeV and a massless τ neutrino this ratio is 4.18, if we assume that in our input data the six-pion contribution is negligible compared to that from $\pi^+\pi^-2\pi^0$. In other words, under the same assumption ~81% of τ decays involving four pions have three charged prongs. Since

$$\Gamma(\tau^- \to \nu_\tau + (4\pi)^-)/\Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e) \approx 0.57,$$

we conclude that

$$\Gamma(\tau^- \to \nu_\tau + \pi^+2\pi^-\pi^0)/\Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e) \approx 0.46.$$

IV. DISCUSSION

Using data from electron-positron annihilation, we have calculated the decay rate for $\tau^- \to \nu_\tau$ + $(2n$ pions$)^-$ which proceeds through the hadronic vector weak current. There is, in addition to what we have calculated, a small contribution to τ decays coming from the strangeness-changing vector current. This contribution is proportional to $\sin^2\theta_C$ and is likely dominated by the $K^*(890)$ in the same way that the ρ dominates the strangeness-nonchanging contribution. Assuming

$$\Gamma(\tau^- \to \nu_\tau + K^{*-}) = \tan^2\theta_C \Gamma(\tau^- \to \nu_\tau + \rho^-),$$

we estimate that

$$\Gamma(\tau^- \to \nu_\tau + K^{*-})/\Gamma(\tau \to \nu_\tau + e^-\bar{\nu}_e) = 0.05.$$

So, the sum of the τ decay widths to $\nu_\tau + e^-\bar{\nu}_e$, $\nu_\tau + \mu^-\bar{\nu}_\mu$, $\nu_\tau + (2n \text{ pions})^-$, and $\nu_\tau + K^*(890)^-$ is

$$(1 + 0.98 + 1.69 + 0.05)\Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e) = 3.72\Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e).$$

This is a lower bound on the total width; and hence we have an upper bound on the branching ratio into $\nu_\tau + e^-\bar{\nu}_e$:

$$B(\tau^- \to \nu_\tau + e^-\bar{\nu}_e) \leq \frac{1}{3.72} = 0.27.$$

While this bound applies for $M_\tau = 1.9$ GeV and $m_{\nu_\tau} = 0$, the results of Sec. III show that it is not sensitive to variations in these masses by several hundred MeV.

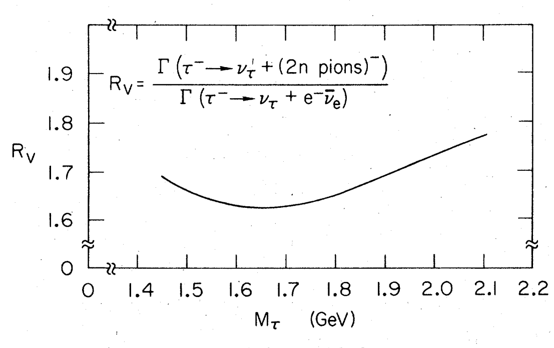

FIG. 1. The ratio R_V of the width for $\tau^- \to \nu_\tau + (2n \text{ pions})^-$, proceeding through the hadronic vector weak current, to that for $\tau^- \to \nu_\tau + e^-\bar{\nu}_e$ as a function of M_τ. $m_{\nu_\tau} = 0$.

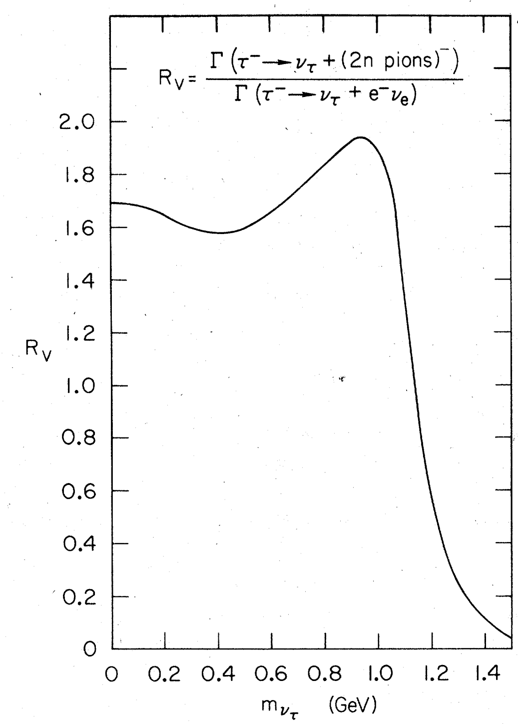

FIG. 2. The dependence of the ratio R_V, as in Fig. 1, on m_{ν_τ} for $M_\tau = 1.9$ GeV. $\Gamma(\tau^- \to \nu_\tau + e^-\bar{\nu}_e)$ is computed from Eq. (2) with the appropriate value of m_{ν_τ}.

The experimental measurements[9] of $B(\tau^- \to \nu_\tau + e^- \bar{\nu}_e)$ are all smaller than our bound and typically less than about 0.2. Most measurements lie in the range 0.15 to 0.20. Since the bound would be saturated if the only τ decays were into $\nu_\tau + e^- \bar{\nu}_e$, $\nu_\tau + \mu^- \bar{\nu}_\mu$, and ν_τ+hadrons through the hadronic vector weak current, we conclude that there must exist other decays. Of course, one does expect decays into ν_τ+hadrons through the hadronic axial-vector weak current. Using our calculation for the vector-current contribution we compute that the width for $\tau^- \to \nu_\tau + (\text{hadrons})^-$ arising from the axial-vector current is 2.95, 2.16 and 1.28 times $\Gamma(\tau^- \to \nu_\tau + e^- \bar{\nu}_e)$ for values of the $\tau^- \to \nu_\tau + e^- \bar{\nu}_e$ branching ratio of 0.15, 0.17, and 0.20, respectively.

Part of these decays through the axial-vector current can be calculated from known quantities: $\Gamma(\tau^- \to \nu_\tau \pi^-)$ and $\Gamma(\tau^- \to \nu_\tau K^-)$ just involve the additional knowledge of the pion and kaon decay constants. For $M_\tau = 1.9$ GeV and $m_{\nu_\tau} = 0$, one finds

$$\frac{\Gamma(\tau^- \to \nu_\tau \pi^-)}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 0.54$$

and

$$\frac{\Gamma(\tau^- \to \nu_\tau K^-)}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 0.03 .$$

However, attempts to find the decay $\tau^- \to \nu_\tau \pi^-$ have so far been unsuccessful, with some indication[4] that the predicted rate is too large to be compatible with the experimental lack of observation.

In any case, even if the $\nu_\tau \pi^-$ and $\nu_\tau K^-$ decays occur at the predicted rate, we have seen that the total width for $\tau^- \to \nu_\tau + (\text{hadrons})^-$ proceeding through the axial-vector current is much larger. There must be decays through the axial-vector current other than $\nu_\tau \pi^-$ and $\nu_\tau K^-$. Specifically, taking our calculation of the vector-current decays and those through the axial-vector current involving only a π^- or K^-, we still have 2.38, 1.59, and 0.71 times $\Gamma(\tau^- \to \nu_\tau + e^- \bar{\nu}_e)$ for the decay widths $\tau^- \to \nu_\tau + (\text{hadrons} \neq \pi^-, K^-)$ through the axial-vector weak current when $B(\tau^- \to \nu_\tau + e^- \bar{\nu}_e)$ is 0.15, 0.17, and 0.20, respectively.

Recently there have been preliminary reports[8,9,24] of the decay $\tau^- \to \nu_\tau A_1^- \to \nu_\tau (3\pi)^-$ at roughly the level we are deducing here.[25] Establishing this and the other semihadronic modes of the τ are important; for, if $\tau^- \to \nu_\tau (3\pi)^-$, $\tau^- \to \nu_\tau \pi^-$, and the decays through the vector current do not all occur at the rates discussed above, then the weak current involved in τ decays is not the one responsible for all other weak decays observed until now.

ACKNOWLEDGMENTS

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