Lepton pair production in hadron collisions

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We construct a model for lepton pair production in the low-mass region ($m \leq 3.1$ GeV), determined by the production characteristics of inclusive hadron production at small p_T. This description agrees well with lepton-pair data up to 3.1 GeV. We then apply our model at higher masses and compare it with predictions from pointlike-parton annihilation.

I. INTRODUCTION

The aim of this work is to examine a hadronlike model for lepton pair production in hadron-hadron collisions. By "hadronlike" we mean that the functional behavior of the transverse- and longitudinal-momentum spectra, their dependence on the incident energy, and all relevant parameters of these distributions, are taken from low-p_T hadron physics.

Such a picture is clearly of interest for the lepton-pair mass region below $m_{J/\psi} = 3.1$ GeV, where pointlike (Drell-Yan[1]) descriptions deviate from the data[2] by a factor of 5 to 10. For masses above 4 GeV, the model can be used to investigate which distributions really call for a pointlike-parton annihilation picture, and how the transition between low and high lepton-pair masses takes place.

In Sec. II, we formulate our model for the lepton-pair mass region $m \lesssim 4$ GeV and discuss the resulting description of the data. In Sec. III, we then extend this description, as fixed in the low-mass region, to high masses, in order to investigate limits of a hadronlike description.

II. THE LOW-MASS REGION

Our basis will be the general behavior of the inclusive production of hadronic secondaries from conventional hadron-hadron collisions, i.e., for $|\vec{p}_T| \lesssim 1-2$ GeV/c. One of the most universal features here is the transverse-momentum behavior of the secondaries: For various single[3] and even multiple[4] hadron systems as secondaries, we have

$$(d\sigma/dp_T^2)/(d\sigma/dp_T^2)_{p_T=0} = f((p_T^2 + m_h^2)^{1/2} - m_h)\,, \tag{1}$$

where m_h denotes the mass of the observed hadronic secondary. In Eq. (1), $f(E_T)$ is a universal function of the transverse kinetic energy

$$E_T \equiv (p_T^2 + m_h^2)^{1/2} - m_h\,; \tag{2}$$

as an illustration, we show in Fig. 1 data[3] for π^+, K^+, and p as secondaries. For comparison, we also show the specific form

$$f(E_T) = e^{-\lambda E_T}\,, \tag{3}$$

with $\lambda = 6$ GeV^{-1}; it is seen to reproduce the data quite well.

For the p_T distribution of lepton pairs in hadronlike production, the above implies, with the function $f(E_T)$ from Eq. (1),

$$(d\sigma/dp_T^2)/(d\sigma/dp_T^2)_{p_T=0} = f((p_T^2 + m^2)^{1/2} - m)\,, \tag{4}$$

where m now denotes the mass of the lepton pair. In Figs. 2(a) and 2(b) we show the data on lepton pair production from $m < 0.45$ GeV to $m = m_{J/\psi}$

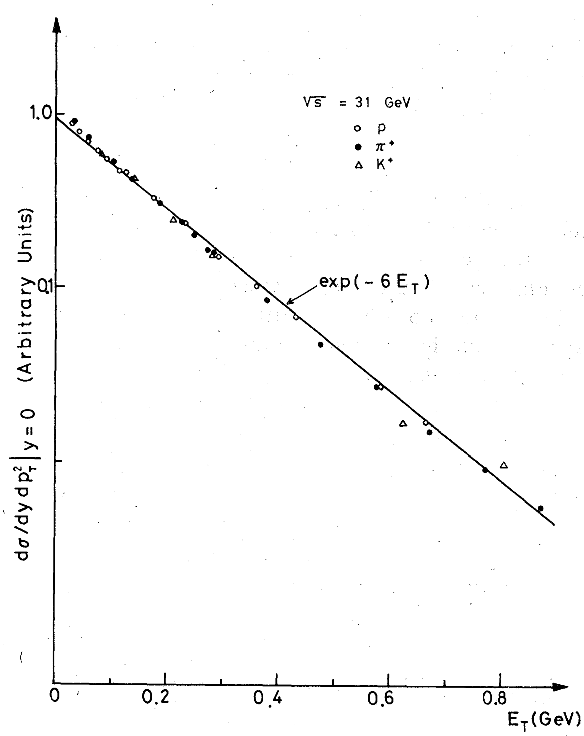

FIG. 1. Data (Ref. 3) on $pp \to (p, \pi^+, K^+) + X$ at 90° compared with $\exp(-6E_T)$.

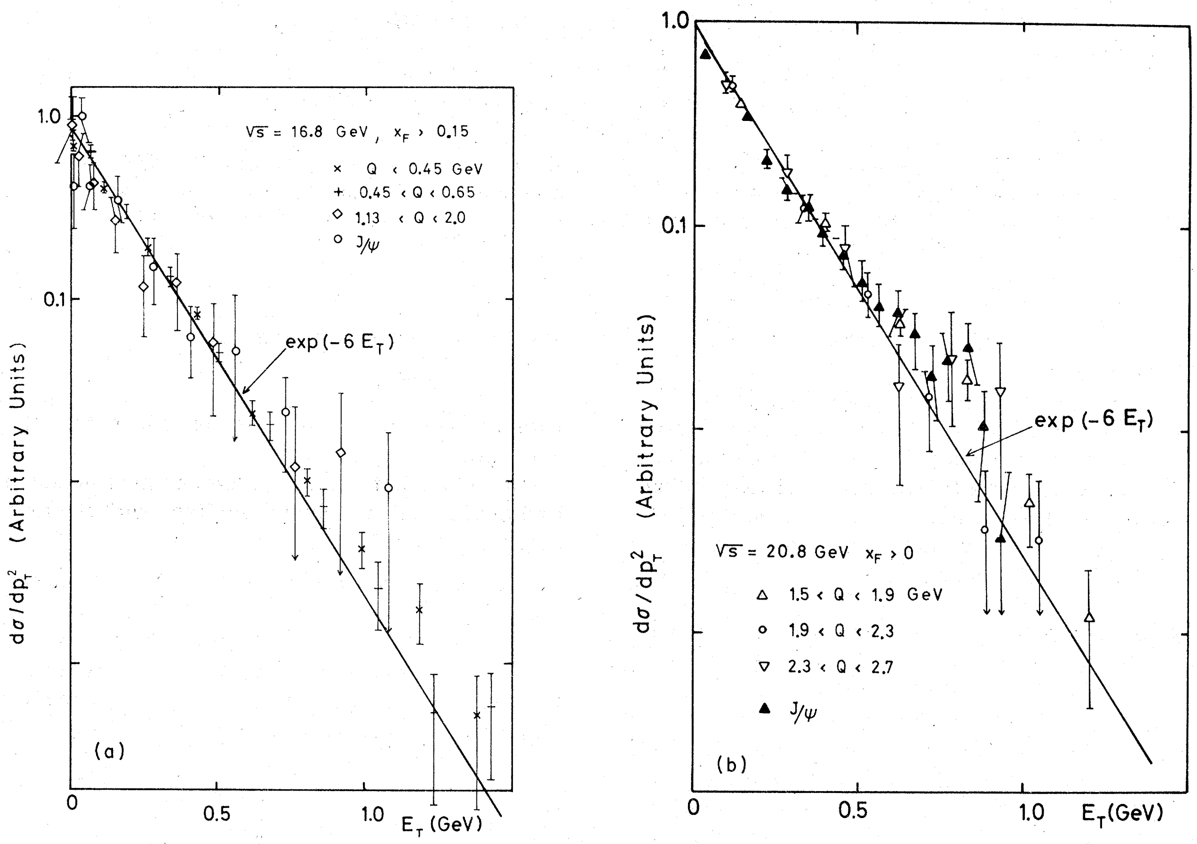

FIG. 2. (a) Data (Ref. 2) on $pp\to\mu^+\mu^-X$ for various mass bins Q compared with $\exp(-6E_T)$. [The original data have been multiplied by a mass- (Q) dependent normalization factor.] (b) Same as Fig. 2(a), but for different initial energy and lepton-pair masses. Data have been taken from Ref. 2.

= 3.1 GeV. All data fall nicely on one curve, and that agrees well with the hadronic parametrization (3) with the universal constant $\lambda = 6\ \text{GeV}^{-1}$. (This conclusion was also reached in Ref. 5.)

Let us now consider longitudinal-momentum spectra and the dependence on incident energy. A suitable variable for these questions is the ratio M_X^2/s, where $M_X^2 \equiv (P-p)^2$ is the missing mass associated with the observed secondary of mass $p^2 = m^2$; $P^2 = s$ is the squared c.m. energy. For m fixed and $s\to\infty$,

$$M_X^2/s = 1 - 2(p_T^2 + p_L^2 + m^2)^{1/2}/s^{1/2} + m^2/s \qquad (5)$$

reduces to

$$M_X^2/s \simeq 1 - x_0 \simeq 1 - |x_F| \,, \qquad (6)$$

with $x_0 \equiv 2p_0/\sqrt{s}$, $x_F \equiv 2p_L/\sqrt{s}$ denoting radial and Feynman scaling variables, respectively. For $p_L = 0$ and $p_T/\sqrt{s} \simeq 0$, but large m, Eq. (5) gives with

$$M_X^2/s = (1 - m/\sqrt{s})^2 \,, \qquad (7)$$

a quantity useful for investigating mass scaling.

Most models for small-p_T hadron production (uncorrelated-jet model, multi-Regge model, dual model, and quark-parton models) agree on the form

$$\frac{x_0}{\sigma_{\text{tot}}}\frac{d\sigma}{dx_F dp_T^2} = c_h(1-x_0)^{\alpha} f(E_T) \,, \qquad (8)$$

for the x dependence of the observed secondary; here c_h and α are constants depending on the nature of the secondary. The origin of this form is perhaps most clearly evident in the uncorrelated jet model,[6] where the single-particle distribution is simply the ratio of the available longitudinal phase-space volumes

$$\frac{x_0}{\sigma_{\text{tot}}}\frac{d\sigma}{dx_F dp_T^2} \sim f(E_T)\Omega_{LPS}[(P-p)^2]/\Omega_{LPS}(P^2) \,. \qquad (9)$$

Since $\Omega_{LPS}(s)$ grows as a power of s,

$$\Omega_{LPS}(s) \sim s^{\alpha} \,, \qquad (10)$$

we obtain

$$\frac{x_0}{\sigma_{\text{tot}}}\frac{d\sigma}{dx_F\,dp_T^{\,2}} \sim f(E_T)(M_X^{\,2}/s)^\alpha\,, \tag{11}$$

$$\simeq \begin{cases} f(E_T)(1-x_0)^\alpha\,, & \text{for } m^2/s \to 0 \quad (12\text{a}) \\ f(E_T)\left(1-\dfrac{m}{\sqrt{s}}\right)^{2\alpha}, & \text{for } \vec{p}^{\,2}/s \to 0 \quad (12\text{b}) \end{cases}$$

for the corresponding distributions in $2p_0/\sqrt{s}$ and $m/\sqrt{s}$. The form (8) has been used to fit a variety of hadron spectra from high-energy collisions; most recent fits[7] for $pp \to \pi +$ anything give $\alpha \simeq 3.5$.

We now assume that lepton pair production of given mass m ($m \lesssim 4$ GeV) is also governed by the hadronic form (8). This immediately provides three types of predictions: At fixed pair-mass m, the form $(M_X^{\,2}/s)^\alpha$ should describe the p_L dependence at fixed s as well as the s dependence at fixed p_L; moreover, the power $\alpha \simeq 3.5$ should hold independently of the mass of the lepton pair.

In Fig. 3, we show the suitably normalized p_L distribution at fixed energies, $\sqrt{s} = 16.8$ and 20.8 GeV, for various bins of mass $m \equiv Q$, both for the resonances and the continuum (the normalization depends on the mass m). The data lie on one universal curve, which agrees with the form $(1-x_0)^{3.5}$ from (8) and (12a). In Fig. 4, the s dependence of lepton pair production at fixed $p_L = 0$ is shown for pairs of fixed mass (equal to the mass of the J/ψ,

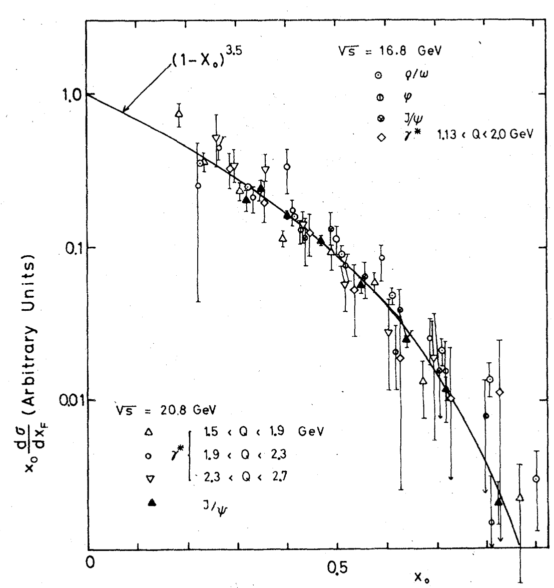

FIG. 3. Data (Ref. 2) on the distribution in $x_0 \equiv 2p_0/\sqrt{s}$ for $pp \to \mu^+\mu^- + X$ at $s = 16.8$ and 20.8 GeV for various lepton-pair masses compared with the hadronlike prediction $(1-x_0)^{3.5}$.

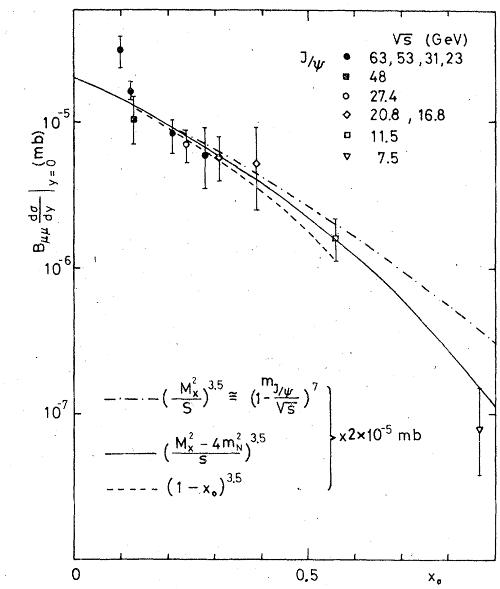

FIG. 4. J/ψ production data (Ref. 8) at various energies compared with the theoretical prediction from (12b).

$m = m_{J/\psi}$)[8] by again plotting the cross section versus x_0. The s dependence agrees with the form of (12b); agreement near the phase-space boundary is improved if $M_X^{\,2}$ in (11) is replaced by $M_X^{\,2} - 4m_N^{\,2}$ to take care of the fact that the final state has to contain at least two nucleons besides the lepton pair. For energies above $\sqrt{s} \sim 15$ GeV both $(1-x_0)^{3.5}$ and $(1-m/\sqrt{s})^7$ from (12a) and (12b) give approximately the same result.

We thus conclude that our hadronlike picture,

$$\frac{x_0\,d\sigma}{dx_F\,dp_T^{\,2}\,dm} = g(m)\left(\frac{M_X^{\,2}}{s}\right)^\alpha \frac{e^{-\lambda E_T}\lambda^2}{2(\lambda m+1)}\,, \tag{13}$$

agrees well with the p_T, p_L, and overall energy dependence of lepton pair production up to and including $m = m_{J/\psi} = 3.1$ GeV. In (13), both the analytical form and the numerical values of the parameters $\lambda = 6$ GeV^{-1} and $\alpha = 3.5$ have been taken from inclusive production in the low-p_T regime. The absolute normalization of the lepton-pair cross section, as well as the relative normalization of the different pair masses [$g(m)$ in (13)], are as yet undetermined. The factor $\lambda^2/2(\lambda m+1)$ in (13) has been extracted from the mass dependence in order to provide a simple normalization of the cross section (13) upon integration over p_T:

$$\frac{\lambda^2}{2(\lambda m+1)}\int e^{-\lambda E_T}\,dp_T^{\,2} = 1\,. \tag{14}$$

Let us note that the hadronic form (13) [with (7)] implies a scaling law for the mass and energy dependence

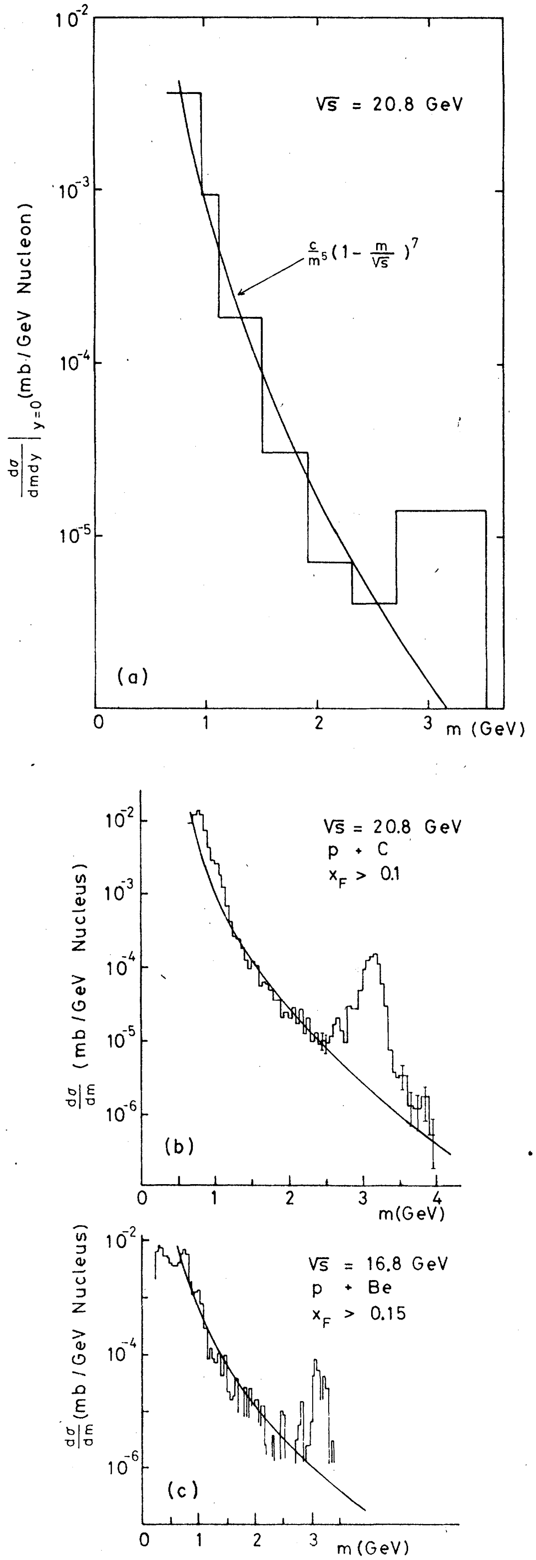

FIG. 5. Data (Ref. 2) on the mass dependence of lepton pair production in the region of $m(e^+e^-) \lesssim 4$ GeV, compared with (17) in (a) and with (18) in (b) and (c).

$$\frac{1}{g(m)}\left(\frac{d\sigma}{dmdy}\right)_{y=0} = (1 - m/\sqrt{s})^{2\alpha}, \qquad (15)$$

in the general sense of a factorizing mass and $m/\sqrt{s}$ dependence. (A scaling law of more general form than the specific one from Drell and Yan[1] has first been conjectured by Sakurai[9].)

To specify the function $g(m)$ in (13), we introduce a power ansatz, $g(m) \sim m^{-\beta}$, containing the free parameter β.

In the spirit of duality,[9] it would be appropriate to determine $g(m)$ from vector-meson production, but for present purposes it seems sufficient to simply adjust the absolute normalization and mass dependence by choosing

$$g(m) = \frac{c}{m^5}, \quad c = 1 \times 10^{-3}\ \text{mb GeV}^4. \qquad (16)$$

This choice provides acceptable fits to the mass distribution, as seen in Figs. 5(a)–5(c), where (15) with (16) is compared with the experimental data from the Chicago-Princeton II collaboration.[2]

Let us briefly comment on the procedure used to obtain Figs. 5(a)–5(c). Data at $y = 0$ are not directly available. We have therefore used in Fig. 5(a) the carbon data from Ref. 2 taken at $x_F \geq 0.1$ and extrapolated to $y = 0$, using the parametrization of Ref. 2. The result is compared with the theoretical ansatz [(15) and (16)]

$$m^5 \frac{d\sigma}{dmdy}\bigg|_{y=0} = c\left(1 - \frac{m}{\sqrt{s}}\right)^7 \qquad (17)$$

in Fig. 5(a). In Figs. 5(b) and 5(c) we then show data for $d\sigma/dm$ on carbon and beryllium at $\sqrt{s} = 20.8$ and 16.8 GeV integrated over $x_F > 0.1$ and $x_F > 0.15$, respectively. In order to compare with (13) and (16), we approximate the x_F integral of (13) by

$$m^5 \frac{d\sigma}{dm}(pA) \cong c'\left(1 - \frac{m}{\sqrt{s}}\right)^7, \qquad (18)$$

with

$$c' = Ac\int_{\min x_F}^{1} (1 - x)^{3.5} dx, \qquad (19)$$

where A is the nucleon number of the target nucleus. Let us note that the mass dependence of the data shown in Fig. 5, due to $m/\sqrt{s} < 0.2$, is dominated by the m^{-5} behavior of (16).

III. THE HIGH-MASS REGIME, $m(e^+e^-) \gtrsim 4$ GeV

It is widely believed that the production of lepton pairs of sufficiently high mass is governed by a dynamical mechanism quite distinct from the one responsible at low lepton-pair masses. Nevertheless, a careful analysis of exactly what features of the data necessitate a departure from concepts

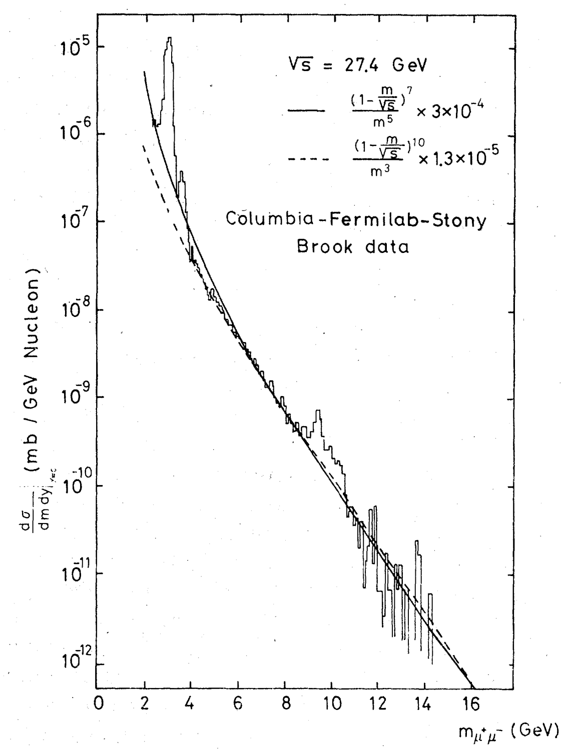

FIG. 6. Data (Ref. 10) on lepton pair production for $m(e^+e^-) \gtrsim 3$ GeV compared with the theoretical predictions from both (20) and (21) at fixed c.m. energy $\sqrt{s}$.

successfully employed (Sec. II) in the low-mass region has, so far, not been provided. Therefore we want to confront the model of Sec. II with the data on high-mass lepton pair production.

In Fig. 6, we compare the most recent data[10] from the Columbia–Fermilab–Stony Brook collaboration on the mass dependence between 3 and 14 GeV with the p_T-integrated form (17)

$$m^5 \frac{d\sigma}{dmdy}\bigg|_{y=0} = c\left(1 - \frac{m}{\sqrt{s}}\right)^7 . \tag{20}$$

As noted in Ref. 2, there is a systematic normalization difference in $d\sigma/dm$ at $m \sim 4$ GeV between the Columbia–Fermilab–Stony Brook data[11] and the Chicago-Princeton II data,[2] which have been used in Figs. 5(a)–5(c). Accordingly, the normalization constant $c = 3 \times 10^{-4}$ used in Fig. 6 is lower by a factor of 3 compared with the one used in Fig. 5. Taking this into account, we conclude from Fig. 6 (and Fig. 5) that the functional form (17) of Sec. II is in satisfactory agreement with the data from the ρ^0 mass to $m(e^+e^-) \sim 14$ GeV.

For comparison, in Fig. 6, we also show a prediction based on the Drell-Yan mechanism of pointlike quark-antiquark annihilation, which is essentially given by

$$m^3 \frac{d\sigma}{dmdy}\bigg|_{y=0} = c_{DY}\left(1 - \frac{m}{\sqrt{s}}\right)^{10} , \tag{21}$$

with $c_{DY} = 1.3 \times 10^{-5}$ mb GeV^{-2}. The power 10 in (21) is motivated by dimensional arguments for parton distributions,[12] and the normalization is compatible with deep-inelastic lepton-hadron scattering data. While agreement with experiment above 4 GeV is very good, there is a well-known strong discrepancy in the low-mass region, as seen in Fig. 6.

The mass dependence at $\sqrt{s} = 27.4$ GeV in the high-mass region thus does not discriminate between (20) and (21). We thus consider the energy dependence, which is predicted to be quite different by (20) and (21). In Fig. 7, we compare with data from the Chicago-Princeton I collaboration at different energies, also including the Columbia–Fermilab–Stony Brook data from Fig. 6. Taking into account the fact that the structure at 9.5 GeV is probably due to the excitation of a new hadronic degree of freedom, we conclude that both (20) and (21) are also in reasonable agreement with the en-

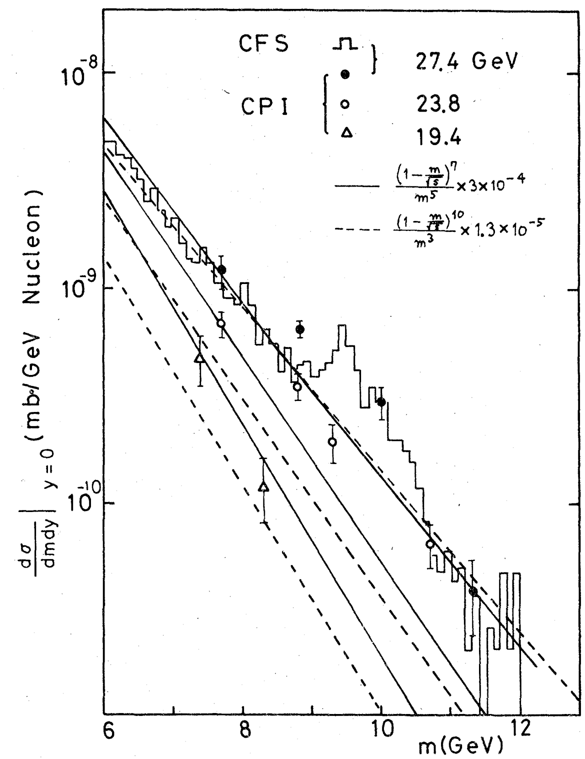

FIG. 7. Data (Ref. 13) on lepton pair production compared with the theoretical predictions (20) and (21) at three different c.m. energies $\sqrt{s}$. The solid curve corresponds to $(1 - m/\sqrt{s})^7/m^5$, while the dashed one is due to $(1 - m/\sqrt{s})^{10}/m^3$.

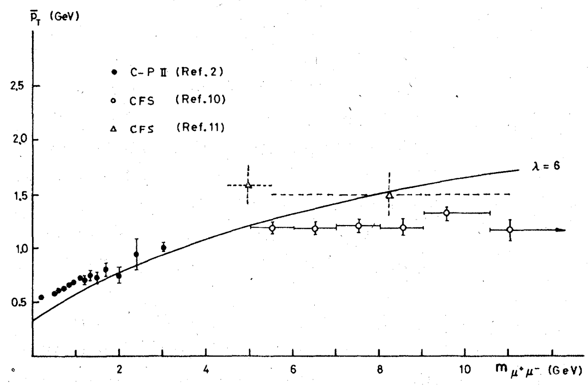

FIG. 8. The average transverse momentum $\bar{p}_T$ as a function of the lepton-pair mass $M(\mu^+\mu^-)$. Data from Refs. 2 and 10.

ergy dependence of the currently available data. More refined data are therefore needed to clearly discriminate between (20) and (21). Let us add that according to (20) or (21), either $m^3 d\sigma/dmdy$ or $m^5 d\sigma/dmdy$ should scale; when performing scaling tests, care should be taken, however, to not compare data from resonant and nonresonant mass bins.

Finally, we turn to transverse-momentum distributions. Here the situation is less well defined both in a hadronlike picture and in pointlike-parton annihilation; no p_T distributions of high-mass single hadrons exist, and at least three different extensions of the (initially longitudinal) parton model to mass-dependent p_T have been proposed.[14-16] In Fig. 8, we compare lepton-pair data with the prediction obtained by integration of Eq. (3):

$$\bar{p}_T = \frac{\lambda m^2}{\lambda m + 1} e^{\lambda m} K_2(\lambda m)\,, \quad \lambda = 6\ \text{GeV}^{-1} \tag{22}$$

where $K_2(x)$ is the modified Bessel function of the second kind. It is seen that the preliminary data of Ref. 10 fall below this prediction. This is also borne out by the corresponding p_T distributions themselves. One should note, however, that multihadron systems yield a slower $\bar{p}_T$ growth with m,[4] more in accord with an independent-emission picture[17] than with the extrapolation of (22) to large m.

Among pointlike-parton descriptions, both covariant parton models[14] and asymptotic-freedom arguments[15] are likely to encounter difficulties with the new data. However, a mass-dependent $\bar{p}_T$ can be understood in the parton picture, if one invokes a new mass scale.[16]

It is difficult at present to decide on the hadronic versus partonic nature of p_T distributions. Nevertheless, here the preliminary data on lepton pairs perhaps indicate a difference between the low- and high-mass regions. It would certainly be of interest to see whether such a difference also exists for massive hadronic systems.

VI. CONCLUSIONS

Let us summarize the main points which have been made by the following concluding remarks:

(1) The p_T and p_L distributions, as well as the energy dependence, of lepton pairs of fixed mass $m(e^+e^-)$, with $m \lesssim 4$ GeV, are essentially the same as the ones found in inclusive meson production. Upon adjusting one free parameter for the mass dependence (and normalizing at ρ^0 mass), we arrive at a complete description of low-mass ($\lesssim 4$ GeV) lepton pair production given by (13) and (16).

(2) The mass and energy dependences, as fixed for m below $m \sim 3$ GeV, are completely consistent with present data up to $m \simeq 14$ GeV. Namely, high-mass data are compatible with mass and energy dependences of the data in the low-mass region, where all production characteristics are hadronlike. High-mass data, however, are also consistent with the mass and energy dependence characteristic for pointlike quark-antiquark annihilation (Drell-Yan mechanism). Using further data one may be able to discriminate between (20) and (21).

(3) Concerning the p_T dependence, the universality in the transverse kinetic energy E_T, relevant at low masses including the J/ψ particle, becomes inconsistent with data at very high masses. This at present seems to be the only definite feature in the data which indicates different physics.

Note added in proof. Concerning the p_T dependence of Fig. 1, it has meanwhile been shown that finite-energy (essentially momentum-conservation) corrections to the universal exponential E_T behavior yield rather good agreement with the flattening of the experimental data as a function of $m_{\mu^+\mu^-}$ shown in Fig. 8 [E. H. de Groot, H. Satz, and D. Schildknecht, Bielefeld Report No. BI-TP 78/01 (in preparation)].

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